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# DEVELOPMENT OF A VARIABLE PARAMETER SIMPLIFIED HYDRAULIC FLOOD ROUTING MODEL FOR RECTANGULAR CHANNELS

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## List of Symbols

| | |
|---|---|
| $A$ | flow area of the channel at a section |
| $A_m$ | flow area at mid-section of the routing reach of length $\Delta x$ |
| $B$ | channel width |
| $B_m$ | channel width at mid section of the routing reach of length $\Delta x$ |
| $C$ | Chezy's friction coefficient |
| $c$ | travel speed corresponding to discharge Q at a section |
| $C_1, C_2, C_3$ | coefficients of the conventional Muskingum difference equation. |
| EVOL | relative error in flow volume |
| $F$ | Froude number |
| $F_o$ | Froude number corresponding to reference discharge $Q_o$ |
| $g$ | gravity due to acceleration |
| $I_i$ | the $i^{th}$ inflow discharge |
| $I_p$ | the inflow hydrograph peak |
| $I_1, I_2$ | respectively inflow at the beginning and end of the routing time interval |
| $j$ | notation identifying the location of cross section in the reach |
| $K$ | the Muskingum travel time |
| $K_o$ | the Muskingum travel time corresponding to initial steady flow |
| $l$ | distance between mid section and section (3) |
| $n$ | Manning's coefficient of roughness; also notation identifying the routing time level |
| $Q$ | discharge at any section of the channel reach during unsteady flow; also discharge at section (2), specifically . |
| $Q_b$ | base flow |

$Q_p$ Peak outflow

$Q_m$ discharge at mid section of the routing reach of length $\Delta x$

$Q_n$ normal discharge

$Q_o$ reference discharge

$Q_3$ discharge at section (3)

$Q_{ci}$ the $i^{th}$ computed discharge

$Q_{oi}$ the $i^{th}$ observed discharge

$Q_{PE}$ relative error in peak discharge (%)

$\bar{Q}_{oi}$ mean of the observed discharges

$S_f$ friction slope

$S_o$ bed slope

$t$ notation for time

$T_{PQE}$ error in time of peak discharge

$T_{PYE}$ error in time of peak stage

$t(y_{pc})$ time corresponding to computed peak stage at the outflow section

$t(y_{po})$ time corresponding to observed peak stage at the outflow section

$t(Q_{pc})$ time corresponding to computed peak discharge

$t(Q_{po})$ time corresponding to observed peak discharge

$v$ velocity of flow at any section

$v_m$ velocity of flow at mid section of the routing reach of length $\Delta x$

$v_o$ velocity corresponding to reference discharge $Q_o$

$v_3$ velocity of flow at section (3)

$x$ notation for distance

$y$ depth of flow

$y_3$ depth of flow at section (3)

| | |
|---|---|
| $y_{PE}$ | error in peak stage |
| $y_m$ | depth of flow at mid section of the routing reach of length $\Delta x$ |
| $y_{pc}$ | computed peak stage at the outflow section |
| $y_{po}$ | observed peak stage at the outflow section |
| $\Delta t$ | routing time interval |
| $\Delta x$ | length of the routing reach |
| $\Theta$ | Muskingum weighting parameter |
| $\Theta_o$ | $\Theta$ corresponding to initial steady flow |
| $\tau$ | dummy time variable |

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ABSTRACT

A variable parameter simplified hydraulic method based on the approximation of the St. Venant's equations which describe the one dimensional flow in a channel or river has been developed for routing floods in channels having uniform rectangular cross section and constant bed slope. The governing equations of this method are same as that of Muskingum flood routing method and it has been demonstrated that these equations can directly account for flood wave attenuation without attributing to it the numerical property of the method as stated by some researchers . The parameters $\Theta$ and K viz., the weighting parameter and the travel time respectively, have been related to the channel and flow characteristics. Using this method the nonlinear behaviour of flood wave movement may be modelled by varying the parameters $\Theta$ and K at every routing time level, but still adopting the linear form of solution equation. The situation for which the routing solution can not be obtained using this method has been brought out, and an alternative solution procedure is suggested for the same. It has been found from this study in general, that the method in which both $\Theta$ and K varying along with multiple routing reaches consideration is able to produce the true solution much closer than the method in which both $\Theta$ and K varying, but with the consideration of single routing reach, or the method in which only K varying and $\Theta$ remaining constant, but with the consideration of single routing reach. The theoretical reason for the reduced outflow in the beginning of the Muskingum solution has been brought out and the needed remedial measure to avoid it is suggested. Also it has been brought out from theoretical considerstions that the maximum

value of $\Theta$ is 0.5 and its negative value is possible. The said methodology has been verified using some hypothetical problems.

## 1.0 INTRODUCTION

Flood routing is the process of tracking a flood wave as it propagates down a channel or a river. A great many different methods and procedures for solving flood routing problems have been described in engineering literature. In general, those methods that attempt a strict mathematical treatment of the many complex factors affecting flood wave movement are not easily adaptable to the practical solution of problems of routing floods as they demand on high computer resources as well as quantity and quality of input data. In order to keep the amount of computation within practical limits and to conform to limits ordinarily imposed by the type and amount of basic data available, it is generally necessary to use approximate flood routing methods that either ignore some of the factors affecting flood wave movement or are based on simplifying assumptions in regard to such factors. Approximate methods produce results at considerably less expense but are limited in generality and accuracy which is the penalty one has to pay for their simplicity and low cost of usage.

Methods of flood routing are broadly classified as empirical, hydrological, simplified hydraulics, and hydraulics. Empirical methods were generally developed from intuitive processes rather than from mathematical formulation of the problem. Their application is limited in practice for situations in which sufficient observations of inflows and outflows are available to calibrate the needed coefficients (Fread, 1981). Hydrological methods are based on some mathematical formulation of continuity equation in lumped form and, generally, a storage equation. The parameters involved in the mathematical formulation of the hydrological method are evaluated using past observations. Simplified hydraulic methods may use continuity equation either in lumped form (Hyami,1951; Harley, 1967; Cunge, 1969; and Dooge et al. 1982) or in distributed form (Thomas and Wormleaton, 1970; and NERC, 1975) in addition to simplified form of the momentum equation of St. Venants' equations. The said simplification may be obtained either by curtailing certain terms based on the consideration of order of magnitude analysis of these terms with that of bed slope, $S_o$ (Hyami, 1951; and Lighthill and Whitam, 1955) or by curtailing and replacing the terms by some appropriate approximation (Apollov et al., 1964).

It is possible to classify certain flood routing techniques under the category of both hydrological and simplified methods depending on the parameter estimation procedure. The typical example being the Muskingum method. The conventional Muskingum method introduced by McCarthy (1938) may be classified as a hydrological method wherein the parameters K and $\Theta$, respectively the travel time and the weighting coefficient are estimated based on the past observations. But the variations of the Muskingum method introduced by Cunge (1969), Dooge (1973), Koussis (1978) and Dooge et al. (1982) may fall under the category of simplified hydraulic method, wherein the parameters K and $\Theta$ are related to the channel and flow characteristics.

In practice hydrologic models are in vogue for many years. Well known among them are the Muskingum method (McCarthy, 1938), lag and route method (Meyer, 1941) and Nash model (Dooge, 1973). These methods use the parameters calibrated from the past flood records for routing floods for the purpose of forecasting or simulation. Since the flood characteristics are likely to vary from one flood to another, it would be rash to assume that the parameters determined from one set of flood observations could be used to predict the behaviour of an altogether different flood. This, in effect, limits the predictive capability of the hydrological methods to floods similar to that used in the calibration, and any attempt at extrapolation is unwarrented. This necessitates the use of simplified hydraulic models in practice which enables one to determine the parameters in terms of physical system characteristics. Such methods enables either flood analyses to be performed in area where data are not available in sufficient quantity and/or quality or do not exist at all or for studying the future behaviour of the system subject to land use changes including channel improvement. Well known examples of the simplified hydraulic models are the linear convection-diffusion method introduced by Hyami (1951), Kalinin-Milyukov method (Apollov et al., 1964), the complete linearized model (Harley, 1967), Muskingum-Cunge method (Cunge, 1969) etc.

The adoption of constant parameters simplified hydraulic models for routing a flood wave is based on the assumption of linearity and this is in contradiction with the nonlinear property of flood waves. The wide use of constant parameter simplified hydraulic models such as Kalinin-Milyukov, and Muskingum-Cunge methods in practice demonstrate that the accuracy of routing results is not severely affected. However, this aspect has not been conclusively proved. The constant parameters of these models are estimated based on the assumption that the flow variations take place around a reference discharge. This limitation produces distortion in the predicted outflow when wide variations in the flow variable are considered. Keefer and McQuivey (1974) state that if the model is linearized about a high discharge, the low flows arrived too soon and are over damped and if it is linearized around a low discharge the peaks arrive late and are underdamped.

This has led to the development of variable parameter diffusion model (NERC, 1975), variable parameter Muskingum-Cunge model (Ponce and Yevjevich, 1978), variable parameter Muskingum-Koussis model (1978) etc. The most desirable way the nonlinearity in the flood routing process may be taken into account is to use such a model that remains linear at one time level, but the linear characteristics may change from one time level to another time level. Thus the parameters involved in the modelling vary from time to time just as the flow variable involved in the phenomena. This concept has been adopted by Ponce and Yevjevich (1978), and Koussis (1978) while they applied the Muskingum method based on the diffusion analogy principle. Whereas Ponce and Yevjevich (1978) considered the variation of both K and $\Theta$, the travel time and weighting parameter respectively of the Muskingum method one time step to another, Koussis considered the variation of K only keeping $\Theta$ constant.

In this report a variable parameter simplified hydraulic flood routing model without lateral flow consideration is developed for routing flood waves in uniform rectangular channels based on the concept of varying linear characteristics from one time level to another time level. Incidently it is seen that this method is able to give physical justification for the Muskingum flood routing method in a better way than that given by Cunge (1969), Dooge (1973), Dooge et al. (1982) and Koussis (1978). This approach also disproves the theory put forwarded by Cunge (1969) and later adopted by Koussis (1978), that the attenuation property exhibited by the conventional Muskingum method is purely due to the numerical formulation of the method (Miller and Cunge, 1975).

## 2.0 REVIEW

In this section only those flood routing models which take into account the nonlinearity of the routing process by remaining in the linear domain at one time level, but varying the linear characteristics from one time level to another time level have been reviewed. It is well known that the routing process is nonlinear in nature and therefore flood routing models with variable coefficients can be expected to perform better. It has been shown by Keefer and McQuivey (1974) that if the inflow hydrograph into a channel reach is considered in several blocks with each block having its own reference or linearizing discharge then the convolution of these inflow blocks with the corresponding unit hydrographs of the channel reach developed based on the reference discharge of each block yield routed hydrographs comparable well with the observed hydrograph than that routed hydrograph obtained based on the convolution of the inflow hydrograph with the unit hydrograph corresponding to a single reference discharge for the entire inflow hydrograph. This envisages the need for adopting variable parameter routing models.

Koussis (1978) developed a variable parameter Muskingum method based on the diffusion analogy principle, using the same concept as adopted by Cunge (1969) , with constant weighting parameter $\Theta$ and varying travel time K. Koussis (1978) has found from his experience that $\Theta$ is not varying considerably with discharge, but varies with K.

Koussis varied the value of K at each time step by averaging the travel speed of the flood wave estimated at the upstream and downstream sections of the reach by introducing the correction in the rating curve at the respective sections using "Jones formula" (Henderson, 1966) as given below:

$$Q = Q_n\left(1 + \frac{1}{cSo}\frac{\partial y}{\partial t}\right)^{\frac{1}{2}} \qquad \ldots (1)$$

in which,

$Q$ = the discharge at a section during unsteady flow

$Q_n$ = the normal discharge at the same section corresponding to the flow depth y observed during unsteady flow

$c$ = the travel speed corresponding to discharge $Q$ at a section

$t$ = notation denoting time

By iteratively solving equation (1), the travel speeds at the upstream and downstream sections may be obtained corresponding to each time level of the Muskingum method solution. Koussis (1978) estimated the outflow discharge Q, using the following expression obtained by assuming linear variation of inflow over the routing time interval $\Delta t$:

$$Q_2 = C_1' I_2 + C_2' I_1 + C_3' Q_1 \qquad \ldots (2)$$

Wherein the coefficients $C_1'$, $C_2'$ and $C_3'$ are given as :

$$C_1' = 1 - \frac{K}{\Delta t}(1 - \beta)$$

$$C_2' = \frac{K}{\Delta t}(1 - \beta) - \beta \text{ and} \qquad \ldots (3)$$

$$C_3' = \beta$$

Where $\beta = e^{-\Delta t/K(1-\theta)}$

Following the same approach of Cunge (1969), Koussis estimated the parameters $\theta$ and K in terms of Channel and flow characteristics by relating the numerical diffusion with the physical diffusion. The form of the parameters so estimated are given as:

$$\theta = 1 - \frac{\Delta t/K}{\ln\left(\frac{\lambda+1 + \Delta t/K}{\lambda+1 - \Delta t/K}\right)} \qquad \ldots (4)$$

where

$$\lambda = \frac{Q_o}{BS_o c \, \Delta x}$$

$Q_o$ = Reference discharge

and $K = \Delta x/c$ ... (5)

The estimation of discharge at the outflow section requires one more iteration procedure using equation (2) besides the iteration required for the correction of rating curve at downstream section for the estimation of travel speed based on the loop rating curve. Therefore it can be realized that although the Koussis procedure is physically based, it involves tedious iterative computations.

Ponce and Yevjevich (1978) suggested a simple variable parameter method based on the Muskingum-Cunge procedure. Usually the routing time interval being fixed, and $\Delta x$ and $S_o$ are specified for each computational cell constituting of four grid points, as shown in figure (1), their method involves the determination of flood wave celerity and the unit width discharge, q for each computational cell. The values of c and q at grid point (j, n) are defined by

$$c = \left.\frac{dQ}{dA}\right|_{j,n} \quad \text{... (6)}$$

$$q = \left.\frac{Q}{B}\right|_{j,n} \quad \text{... (7)}$$

in which Q = discharge : A = flow area, and B = top width.

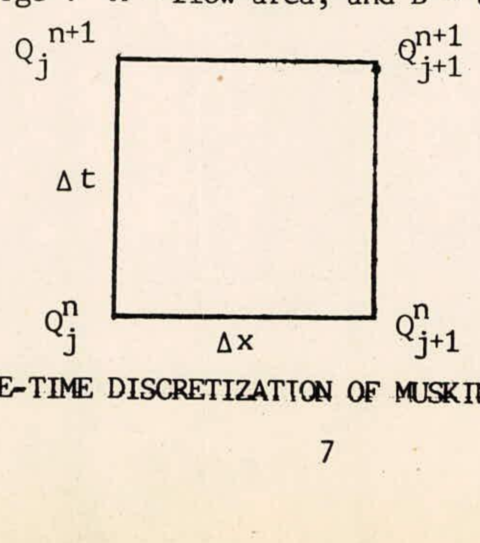


FIG.1 SPACE-TIME DISCRETIZATION OF MUSKINGUM METHOD

The following ways of determining c and q were investigated by Ponce and Yevjevich for the computation of variables $\theta$ and K of Cunge (1969) for each time level:

(1) directly by using a two point average of the values at grid points (j, n) and (j+1, n);

(2) directly by using a three point average of the values at grid points (j,n), (j+1,n) and (j,n+1); and

(3) by iteration, using a four point average calculation.

They concluded that three point and four point iterative schemes of varying c and q yield better results and both are comparable. In view of iterations involved in four point scheme, it may be considered that three point average procedure is desirable for use in practice. Besides, this method is also much simpler than the method suggested by Koussis (1978). However both Ponce and Yevjevich's (1978), and Koussis (1978) approaches for varying the parameters of the Muskingum method at each routing time level are arbitrary and not based on the mathematics of the Muskingum method solution.

The methods reviewed herein are the only methods which consider the variation of parameters from one time level to another time level by adopting the linear form of solution.

## 3.0 PROBLEM DEFINITION

It is required to develop a simplified hydraulic flood routing method for tracking flood wave movement in prismatic channels having uniform rectangular cross section. The routing procedure may adopt a linear form of solution equation with the relevant parameters varying from one time level to another time level of solution and thus taking care of approximately the non-linear behaviour of the flood wave movement.

## 4.0 METHODOLOGY

The flood routing method developed herein is a modification of the method presented earlier (NIH, 1986). The conclusion arrived using the earlier method is still valid, but not the method mentioned therein as it involves an impractical assumption. When this assumption is relaxed and modified, then it results in the improved method mentioned herein.

The mathematical analysis of this method incidently, results in the physical justification of the Muskingum method for routing floods in **channels.** The parameters K and $\theta$ of the Muskingum method have been related to channel and flow characteristics. It has been shown by analysis that the method with the original form of equations as introduced by McCarthy in 1938, is able to take care of flood wave attenuation without attributing to it any numerical characteristics of the method. This is in contradiction with the theory purported by Cunge (1969) and later adopted by Koussis (1978), as they argued that the attenuation of flood wave exhibited by the Muskingum method is solely due to the numerical diffusion property present in the method. Besides bringing out the reasons for negative flow or flow less than the steady flow in the beginning of the solution, the present theory also shows the possibility of Muskingum weighting parameter becoming negative. This is in contradiction with the general understanding that the lower limit of the weighting parameter is zero. However, like earlier theories on Muskingum method (Cunge, 1969; Dooge, 1973; and Dooge et al., 1982), this theory also shows that the maximum value of weighting parameter is 0.5. Besides, the physical reason for the wave amplification is brought out whence the weighting parameter ceeds the value of 0.5. The theory presented herein for the Muskingum flood routing method is much improved than the so far existing theories (Cunge, 1969; Dooge, 1973; Kundzewicz and Strupczewski, 1980; and Dooge et al., 1982) as the nonlinear behaviour of the flood wave movement is considered by varying the parameters of the Muskingum method at each routing time interval.

## 4.1 Physical Basis of the Proposed Theory

During steady flow in a river reach there exists a unique relationship between stage and discharge at any cross section. This situation is altered during unsteady flow, with the discharge appearing first in a cross section and at the same time the stage which corresponds to that discharge during steady flow appears at a section upstream of it. This concept has been adopted by Kalinin and Milyukov (as quoted by Miller and Cunge, 1975) to determine the 'unit length of reach' required for flood routing in river reaches. However, Kalinin-Milyukov method is less flexible since the 'unit reach length' of the channel is fixed for a given flood wave and the end section of the unit reach length may not coincide with the downstream section where the stage-discharge information is required, thus necessitating interpolation of the routed hydrographs. Besides the adoption of constant unit reach length implies that the unique relationship between discharge at the outflow section and the depth at the middle of the reach always exists during unsteady flow phenomena. This is in contradiction to the characteristics of unsteady flow phenomena in channels. In this report, it is shown that the modification of the concept of Kalinin-Milyukov method leads to a flood routing method which is devoid of such limitations mentioned above.

The concept adopted in the Kalinin-Milyukov method is that during unsteady flow in a uniform rectangular channel with linearly varying water stage along the river reach, the channel storage S in the routing reach of length $\Delta x$ is uniquely related to the mean water stage of the reach which in turn is uniquely related with the discharge observed at the outlet of the reach. Here the distance $\Delta x$ corresponds to the unit reach length.

The constant parameters of the Muskingum method have been evaluated by extending this concept that the mean water stage of the routing reach of length $\Delta x$ is uniquely related to the discharge at a section located $l$ units of length downstream of the midsection of the reach (Apollov et al., 1964). However, here $\Delta x$ need not correspond to the unit reach length as in the case of Kalinin-Milyukov method.

The above concept has been used to evaluate the variable parameters of the proposed method. The mathematical description of the method, which is different from that of Kalinin-Milyukov method, is given in the following pages with the assumptions involved.

## 4.2 Assumptions

The following assumptions have been made in developing this method:

1. The channel reach is having uniform rectangular cross section.
2. The channel bottom slope is constant over the routing reach length.
3. There is no lateral inflow or outflow from the reach.
4. The friction slope $S_f$ is constant at any instant of time over the channel routing reach.
5. During unsteady flow, there exists a one-to-one relationship at any instant of time between the stage at the middle of the routing reach and the discharge downstreeam.

## 4.3 Development of the Model

Figure (2) depicts a river reach having uniform rectangular

SECTION (1)-(1) : CORRESPONDS TO THE INFLOW POINT

SECTION (2)-(2) : CORRESPONDS TO THE OUTFLOW POINT

SECTION (3)-(3) : CORRESPONDS TO THE POINT WHERE THE DISCHARGE $Q_e$ IS UNIQUELY RELATED WITH THE STAGE AT THE MIDSECTION OF THE REACH

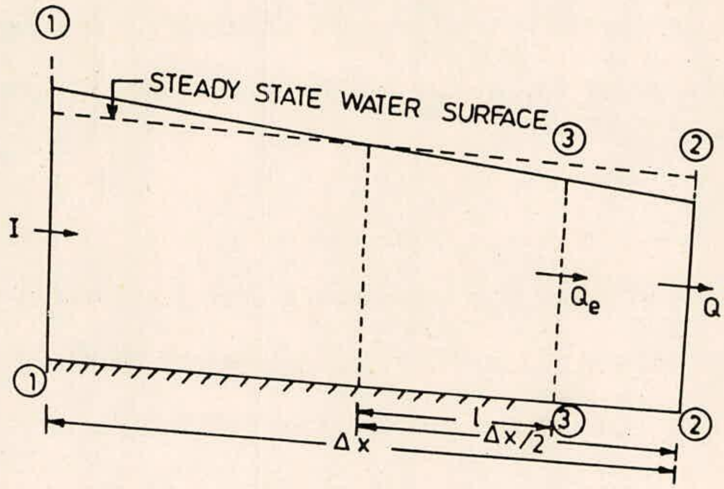


FIG.2- DEFINITION SKETCH OF THE REACH UNDER CONSIDERATION

cross section with upstream and downstream sections, where the inflow and outflow hydrographs are measured have been denoted respectively as sections (1) and (2). Let the distance between these sections be $\Delta x$.

Based on assumption (5), the water depth observed at the middle of the reach corresponds to the normal depth of that discharge which is observ d at the same instant of time $l$ units of distance downstream from the middle of the reach. Let this discharge be denoted as $Q_n$ and the section where this discharge is observed be marked as section (3). The discharge at the middle of the reach may be expressed as:

$$Q_m = A_m v_m \qquad \text{... (8)}$$

where, $A_m$ and $v_m$ are the area and velocity during unsteady flow at this section. Equation (8) may be re-written in terms of width of channel section, depth of flow at the mid section and Chezy's or Manning's roughness coefficient. First the mathematical formulation of the problem in terms of Chezy's friction law is presented followed by the formulation using Manning's friction law.

4.3.1 Mathematical formulation involving Chezy's law

Before proceeding with further mathematical operation on equation (8) using assumption (5), it is necessary to use assumption (4), in order to simplify the expression for friction slope which would be used in equation (8).

Let the expression for discharge Q at any section of the reach during unsteady flow in the reach as depicted by figure 2 be expressed as:

$$Q = Av \qquad \text{... (9)}$$

where, A is the channel cross section and v is the velocity of flow.

Applying Chezy's friction law:

$$Q = AC\sqrt{RS_f} \qquad \dots (10)$$

where,

$C$ = Chezy's constant

$R$ = A/P the hydraulic radius
$P$ = the wetted perimeter and
$S_f$ = the friction slope

Equation (10) is re-written in terms of channel width and depth of flow as:

$$Q = \frac{CB^{3/2}\, y^{3/2} \sqrt{S_f}}{(B + 2y)^{\frac{1}{2}}} \qquad \dots (11)$$

where,

$B$ = the channel width and

$y$ = the depth of flow at the considered section

The friction slope $S_f$ can be expressed as (Henderson, 1966):

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{v}{g}\frac{\partial v}{\partial x} - \frac{1}{g}\frac{\partial v}{\partial t} \qquad \dots (12)$$

where,

$S_o$ = the bed slope

$\frac{\partial y}{\partial x}$ = the water surface slope

$\frac{v}{g} \cdot \frac{\partial v}{\partial x}$ = convective acceleration slope and

$\frac{1}{g}\frac{\partial v}{\partial t}$ = local acceleration slope

$x$ = notation for distance

$g$ = the acceleration due to gravity (9.81 m/sec.$^2$)

Differentiating Q w.r.t. x

$$\frac{\partial Q}{\partial x} = \frac{CB^{3/2}\, y^{\frac{1}{2}}}{(B + 2y)^{\frac{1}{2}}} \left[ \frac{3}{2} - \frac{y}{B + 2y} \right] \frac{\partial y}{\partial x}\, S_f^{\frac{1}{2}} + \frac{CB^{3/2}\, y^{3/2}}{(B + 2y)^{1/2}} \frac{\partial}{\partial x}(S_f^{\frac{1}{2}}) \quad \ldots (13)$$

Based on assumption (4) that $S_f$ remains constant at any instant of time, the above equation reduces to :

$$\frac{\partial Q}{\partial x} = B \left[ \frac{3}{2} - \frac{y}{B+2y)} \right] v \frac{\partial y}{\partial x} \quad \ldots (14)$$

where the term $\left[ \frac{3}{2} - \frac{y}{B + 2y} \right] v$ represents the celerity of the flood wave in rectangular channels.

When $S_f$ remains constant, the slopes due to watersurface, convective acceleration, and local acceleration remain constant at any instant of time. This implies during unsteady flow, the water surface is linearly varying at any instant of time over the routing reach. Note that for wide rectangular channels equation (14) reduces to :

$$\frac{\partial Q}{\partial x} = B\left(\frac{3v}{2}\right) \frac{\partial v}{\partial x} \quad \ldots (15)$$

where the term $\frac{3v}{2}$ represents the wave celerity in wide rectangular channels.

Differentiating equation (14) yields :

$$\frac{\partial^2 Q}{\partial x^2} = B\left[\frac{3}{2} - \frac{y}{B+2y}\right] v \frac{\partial^2 y}{\partial x^2} + B\left[\frac{3}{2} - \frac{y}{B+2y}\right] \left(\frac{\partial v}{\partial x}\right) \left(\frac{\partial y}{\partial x}\right) - \left(\frac{B}{B+2y}\right)^2 v \left(\frac{\partial y}{\partial x}\right)^2 \quad \ldots (16)$$

Assuming that the multiples of above mentioned differentials are very small and may be neglected, equation (16) reduces to :

$$\frac{\partial^2 Q}{\partial x^2} = B\left[ \frac{3}{2} - \frac{y}{B+2y} \right] v . \frac{\partial^2 y}{\partial x^2} \quad \ldots (17)$$

Since $\frac{\partial y}{\partial x}$ is constant at any instant of time over the routing reach length, equation (17) reduces to :

$$\frac{\partial^2 Q}{\partial x^2} = 0 \qquad \ldots (18)$$

Equation (18) implies that at any instant of time the discharge also varies linearly over the routing reach under consideration.

<u>Evaluation of the term $\frac{v}{g}\frac{\partial v}{\partial x}$ and $\frac{1}{g}\frac{\partial v}{\partial t}$ in terms of $\frac{\partial v}{\partial x}$ :</u>

Using equation (9) and (14) one can arrive at the expression for $\frac{v}{g}\frac{\partial v}{\partial x}$ at any section in the reach in terms of the depth of flow and Froude number at that section as :

$$\frac{v}{g}\frac{\partial v}{\partial x} = \left[\frac{1}{2} - \frac{y}{B+2y}\right] F^2 \frac{\partial y}{\partial x} \qquad \ldots (19)$$

where, $F^2 = \frac{Q^2 B}{gA^3}$

Similarly the expression for $\frac{1}{g}\frac{\partial v}{\partial t}$ at any section of the reach is given as:

$$\frac{1}{g}\frac{\partial v}{\partial t} = \left[-\frac{3}{4} + 2.\left(\frac{y}{B+2y}\right) - \left(\frac{y}{B+2y}\right)^2\right]F^2 \frac{\partial y}{\partial x} \qquad \ldots (20)$$

The addition of equations (19) and (20) yield :

$$\frac{v}{g}\frac{\partial v}{\partial x} + \frac{1}{g}\frac{\partial v}{\partial t} = \left[-\frac{1}{4} + \left(\frac{y}{B+2y}\right) - \left(\frac{y}{B+2y}\right)^2\right]F^2 \frac{\partial y}{\partial x} \qquad \ldots (21)$$

Therefore the friction slope expressed by equation (12) can be modified for the routing reach under consideration using equation (21) as :

$$S_f = S_o \left\{1 - \frac{1}{S_o}\frac{\partial y}{\partial x}\left[1+F^2\left(-\frac{1}{4} + \left(\frac{y}{B+2y}\right) - \left(\frac{y}{B+2y}\right)^2\right)\right]\right\} \qquad \ldots (22)$$

For wide rectangular channels equation (22) reduces to :

$$S_f = S_o \left[1 - \frac{(1 - F^2/4)}{S_o} \frac{\partial y}{\partial x}\right] \qquad \ldots (23)$$

Now consider equation (8). The discharge at the middle of the reach may be expressed in terms of channel depth as :

$$Q_m = B_m y_m C\sqrt{\frac{B_m y_m}{B + 2y_m} \cdot S_f} \qquad \ldots (24)$$

with the suffix m denoting the middle of the section. Substituting for $S_f$ from equation (22) into equation (24), $Q_m$ is re-written as :

$$Q_m = B_m y_m C\sqrt{\frac{B_m y_m}{B+2y_m} S_o\left[1 - \frac{\frac{\partial y}{\partial x}\big|_m}{S_o}\left(1+F^2\left(-\frac{1}{4} + \left(\frac{y_m}{B+2y_m}\right) - \left(\frac{y_m}{B+2y_m}\right)^2\right)\right)\right]^{\frac{1}{2}}}$$

$$\ldots (25)$$

where $\frac{\partial y}{\partial x}\big|_m$ represent the water surface slope at mid section. Since $\frac{\partial y}{\partial x}$ remains constant over the reach, the suffix m is dropped and the water surface slope is written as $\partial y/\partial x$. Since the flow depth observed at the mid section of the reach corresponds to the normal depth of discharge $Q_n$ ,which is occurring downstream of the mid section, the term $B_m y_m C\sqrt{\frac{B_m y_m}{B_m + 2y_m} S_o}$ corresponds to the discharge $Q_n$

Therefore $$Q_m = Q_n\left\{1 - \frac{1}{S_o} \cdot \frac{\partial y}{\partial x}\left(1 + F^2\left(-\frac{1}{4} + \left(\frac{y_m}{B+2y_m}\right) - \left(\frac{y_m}{B+2y_m}\right)^2\right)\right)\right\}^{\frac{1}{2}}$$

$$\ldots (26)$$

Based on the typical values of $S_o$, $\partial y/\partial x$ in natural rivers (Henderson, 1966), it may be considered that the absolute value of the term

$$\left[\frac{1}{S_o} \frac{\partial y}{\partial x}\left(1+F^2\left(-\frac{1}{4} + \left(\frac{y_m}{B+2y_m}\right) - \left(\frac{y_m}{B+2y_m}\right)^2\right)\right)\right] < 1$$

Under such situation, equation (26) may be expanded in

Bionomial series as :

$$Q_m = Q_n \left\{ 1 - \frac{1}{2S_o} \frac{\partial y}{\partial x} [1 + F^2(- \frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2)] + \right.$$

$$\frac{1/2\ (1/2 - 1)}{\underline{|2}} [ \frac{1}{S_o} \frac{\partial y}{\partial x} (1 + F^2(- \frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2))]^2 -$$

$$\frac{\frac{1}{2}(\frac{1}{2} - 1)\ (\frac{1}{2} - 2)}{\underline{|3}} [\frac{1}{S_o} \cdot \frac{\partial y}{\partial x} (1 + F^2( - \frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2))]^3$$

$$\left. + \ldots\ldots\ldots \right\} \qquad \ldots (27)$$

Let the term $\frac{1}{S_o} \cdot \partial y/\partial x\ [1+F^2(- \frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2)] = G \quad \ldots (28)$

Equation (27) is re-written using equation (28) as :

$$Q_m = Q_n - Q_n \left\{ \frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2} - 1)}{\underline{|2}} G + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{\underline{|3}} G^2 + \ldots \right\} G \ldots (29)$$

Substituting for G from equation (28) for outside paranthesis in equation (29) yields :

$$Q_m = Q_n - Q_n \left\{ \frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2} - 1)}{\underline{|2}} G + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{\underline{|3}} G^2 + \ldots \right\}$$

$$\times [ 1 + F^2( - \frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2) ] \frac{1}{S_o} \cdot \frac{\partial y}{\partial x} \qquad \ldots (30)$$

Substituting from equation (14) for $\partial y/\partial x$, the above equation reduces to

$$Q_m = Q_n - Q_n \left\{ \frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2} - 1)}{\underline{|2}} G + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{\underline{|3}} G^2 + \ldots \right\}$$

$$\text{X} \frac{[1 + F^2( - \frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2)] \frac{\partial Q}{\partial x}}{S_o\ B\ [\frac{3}{2} - (\frac{y_m}{B+2y_m})]\ v_m} \qquad \ldots (31)$$

Therefore the term,

$$Q_n \left\{ \frac{1}{2} - \frac{\frac{1}{2}\,(\frac{1}{2} - 1)\, G}{\underline{|2}} + \frac{\frac{1}{2}\,(\frac{1}{2} - 1)(\frac{1}{2} - 2)\, G^2}{\underline{|3}} + \ldots \right\}$$

$$\times \left\{ \frac{1 + F^2\left( - \frac{1}{4} + \left(\frac{y_m}{B+2y_m}\right) - \left(\frac{y_m}{B + 2y_m}\right)^2\right)}{S_o\, B \left[ \frac{3}{2} - \left( \frac{y_m}{B+2y_m}\right)\right] v_m} \right\} \qquad \ldots (32)$$

represents the distance $l$ between the mid-section and that downstream section at which the normal discharge corresponding to the depth at mid section is observed at the same instant of time. Since the discharge is varying linearly, the discharge $Q_n$, at section (3) is computed in terms of inflow I and outflow Q as :

$$Q_n = Q + \left(\frac{1}{2} - \frac{l}{\Delta x}\right) (I - Q) \qquad \ldots (33)$$

Now applying the continuity equation,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad \ldots (34)$$

between sections (1) and (3) of Figure (1), one arrives at

$$\left.\frac{\partial Q}{\partial x}\right|_3 = - \frac{1}{\left.\frac{\partial Q}{\partial A}\right|_3} \cdot \left.\frac{\partial Q}{\partial t}\right|_3 \qquad \ldots (35)$$

Using equation (14), the above equation is re-written as :

$$\left.\frac{\partial Q}{\partial x}\right|_3 = \frac{-1}{\left[\frac{3}{2} - \left(\frac{y_3}{B + 2y_3}\right)\right] v_3} \cdot \frac{\partial}{\partial t}(Q_3) \qquad \ldots (36)$$

Since $\left.\frac{\partial Q}{\partial x}\right|_3 = \left.\frac{\partial Q}{\partial x}\right|_2$, equation (36) is modified as

$$\frac{I - Q}{\Delta x} = \frac{1}{\left[\frac{3}{2} - \left(\frac{y_3}{B+2y_3}\right)\right] v_3} \cdot \frac{\partial}{\partial t}(Q_3) \qquad \ldots (37)$$

But $Q_3$ is same as $Q_n$ as given by equation (33). Therefore equation (37) is modified as :

$$\frac{\frac{I - Q}{\Delta x}}{[\frac{3}{2} - (\frac{y_3}{B+2y_3})]v_3} = \frac{\partial}{\partial t}\ [\ Q + \ (\ \frac{1}{2} - \frac{l}{\Delta x}\ )(I-Q)\ ] \qquad \ldots (38)$$

Since I and Q varies only w.r.t. t, the partial derivative of equation (38) is changed to full derivative and the equation (38) is modified as :

$$I - Q = \frac{d}{dt}\left\{K\ [\ Q + (\frac{1}{2} - \frac{l}{\Delta x}\ )(I-Q)]\right\} \qquad \ldots (39)$$

where $l$ is given by the equation (32) and K is the travel time.

Equation (39) is in the same form as that of well known Muskingum method. The travel time K is given by the expression:

$$K = \frac{\Delta x}{(\frac{3}{2} - (\frac{y_3}{B+2y_3}))\ v_3} \qquad \ldots (40)$$

Where, $y_3$ - the flow depth at section (3)

$v_3$ - the velocity of section (3)

and the weighting factor $\theta$ is written as :

$$\theta = \frac{1}{2} - \frac{l}{\Delta x} \qquad \ldots (41)$$

substituting for $l$ from equation (32), the above equation is written as :

$$\theta = \frac{1}{2} - Q_n\left\{\tfrac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2} - 1)\ G}{\lfloor 2} + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)\ G^2}{\lfloor 3} + \ldots\ldots\ldots\right\}$$

$$\times\left\{\frac{1 + F^2(\ -\frac{1}{4} + (\frac{y_m}{B+2y_m}) - (\frac{y_m}{B+2y_m})^2)}{S_oB\ [\ \frac{3}{2} - (\frac{y_m}{B+2y_m}\ )]\ v_m\ \Delta x}\right\} \qquad \ldots (42)$$

Where $Q_n = Q_3$ , the discharge at section (3) and

$$G = \frac{\left[ 1 + F^2 \left( -\frac{1}{4} + \left(\frac{y_m}{B+2y_m}\right) - \left(\frac{y_m}{B+2y_m}\right)^2 \right) \right]}{S_o B \left[ \frac{3}{2} - \left( \frac{y_m}{B+2y_m} \right) \right] v_m} \frac{\partial Q}{\partial x} \quad \ldots (43)$$

Equations (40) and (42) are the general expressions for parameters K and $\Theta$ of the Muskingum method applied to the solution of unsteady flow which follows Chezy's friction law in rectangular channel reach of length $\Delta x$. These parameters vary with time. For wide rectangular channels equations (40) and (42) reduce to :

$$K = \frac{\Delta x}{\frac{3}{2} v_3} \quad \ldots (44)$$

$$\Theta = \frac{1}{2} - \frac{Q_n \left\{ \frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2} - 1) G}{\lfloor 2} + \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2) G^2}{\lfloor 3} + \ldots \right\} (1 - \frac{F^2}{4})}{S_o B (\frac{3}{2} v_m) \Delta x}$$

$$\ldots (45)$$

When neglecting the terms G, $G^2$,.... etc., $\Theta$ reduces to

$$\Theta = \frac{1}{2} - \frac{Q_n (1 - \frac{F^2}{4})}{2 S_o B (\frac{3}{2} v_m) \Delta x} \quad \ldots (46)$$

When the variables are fixed at reference values, then K and $\Theta$ reduces to

$$K = \frac{\Delta x}{\frac{3}{2} v_o} \quad \ldots (47)$$

$$\Theta = \frac{1}{2} - \frac{Q_o(1 - F_o^2/4)}{2 S_o B (\frac{3}{2} v_o) \Delta x} \quad \ldots (48)$$

Equation (47) and (48) respectively for K and $\Theta$ were obtained by Dooge (1973) and Dooge et al. (1982) for constant parameter Muskingum flood routing method. The deletion of the term $F_o^2/4$ also results in the expression for $\Theta$ given by Cunge (1969) and Koussis (1978), based on the concept of equating the numerical diffusion with the physical diffusion.

4.3.2 Mathematical formulation involving Manning's friction law

Proceeding in the similar manner as in the case of analysis based on Chezy's friction law, the expression for $\frac{\partial Q}{\partial x}$ using Manning's friction law is given as :

$$\frac{\partial Q}{\partial x} = B\left[\frac{5}{3} - \frac{4}{3}\left(\frac{y}{B+2y}\right)\right] v . \frac{\partial y}{\partial x} \qquad \ldots (49)$$

where the term $\left[\frac{5}{3} - \frac{4}{3}\left(\frac{y}{B+2y}\right)\right]v$ is the celerity of the flood wave in rectangular channels in which the unsteady flow is governed by Manning's friction law.

For wide rectangular channels equation (49) reduces to :

$$\frac{\partial Q}{\partial x} = B\left[\frac{5}{3} v\right]\frac{\partial y}{\partial x} \qquad \ldots (50)$$

It can be proved, as it has been done earlier for the unsteady flow governed by Chezy's law, that $\frac{\partial Q}{\partial x}$ is also varying linearly over the routing reach where-in the friction slope $S_f$ remains constant at any instant of time.

Evaluation of the terms $\frac{v}{g}\frac{\partial v}{\partial x}$ and $\frac{1}{g}\frac{\partial v}{\partial t}$ in terms of $\frac{\partial y}{\partial x}$:

Using equation (9) and (49), one can arrive at the expression for $\frac{v}{g}\frac{\partial v}{\partial x}$ at any section in the reach in terms of the depth of flow and Froude number at that section as :

$$\frac{v}{g}\frac{\partial v}{\partial x} = \left[\frac{2}{3} - \frac{4}{3}\left(\frac{y}{B+2y}\right)\right] . F^2 \frac{\partial y}{\partial x} \qquad \ldots (51)$$

Where $F'^2$ is the square of Froude number as expressed earlier.

Similarly the expression for $\frac{1}{g}\frac{\partial v}{\partial t}$ at any section of the reach is given as :

$$\frac{1}{g}\frac{\partial v}{\partial t} = \left[-\frac{10}{9} + \frac{28}{9}\left(\frac{y}{B+2y}\right) - \frac{16}{9}\cdot\left(\frac{y}{B+2y}\right)^2\right] F^2 \frac{\partial y}{\partial x} \quad \ldots (52)$$

The addition of equations (51) and (52) yield

$$\frac{v}{g}\frac{\partial v}{\partial x} + \frac{1}{g}\frac{\partial v}{\partial t} = \frac{4}{9}\left[-1+4\left(\frac{y}{B+2y}\right) - 4\left(\frac{y}{B+2y}\right)^2\right] F^2 \frac{\partial y}{\partial x} \quad \ldots (53)$$

Therefore the friction slope expressed by equation (12) can be modified for the routing reach under consideration as :

$$S_f = S_o\left\{1 - \frac{1}{S_o}\cdot\frac{\partial y}{\partial x}\left[1 + \frac{4}{9}F^2\left(-1 + 4\left(\frac{y}{B+2y}\right) - 4\left(\frac{y}{B+2y}\right)^2\right)\right]\right\} \ldots (54)$$

For wide rectangular channels equation (54) reduces to :

$$S_f = S_o\left[1 - \frac{1}{S_o}\frac{\partial y}{\partial x}\left(1 - \frac{4}{9}F^2\right)\right] \quad \ldots (55)$$

Based on the similar analysis as carried out for the case of unsteady flow following Chezy's friction law, it can be shown that the distance $l$ between the mid section and that downstream section at which the normal discharge corresponding to the flow depth of mid section is realized is given as :

$$l = Q_n\left\{\frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)\,G_m}{\lfloor 2} + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)\,G_m^2}{\lfloor 3} + \ldots\ldots\right\}$$
$$\times\left\{\frac{1 + \frac{4}{9}F^2\left[-1 + 4\left(\frac{y_m}{B+2y_m}\right) - 4\left(\frac{y_m}{B+2y_m}\right)^2\right]}{S_o B\left[\frac{5}{3} - \frac{4}{3}\left(\frac{y_m}{B+2y_m}\right)\right] v_m}\right\} \quad \ldots (56)$$

$$\text{Where } G_m = \frac{[1 + \frac{4}{9}F^2(-1 + 4(\frac{y_m}{B+2y_m}) - 4(\frac{y_m}{B+2y_m})^2]}{S_o B [\frac{5}{3} - \frac{4}{3}(\frac{y_m}{B+2y_m})] v_m} \frac{\partial Q}{\partial x} \quad \ldots (57)$$

and $y_m$ = the depth at the mid section of the routing reach.

Similar analysis as carried out earlier for the flow following Chezy's friction law, leads to the governing unsteady flow equation as :

$$I-Q = \frac{d}{dt}\left\{ K[Q + \theta\ (I - Q)]\right\} \quad \ldots (58)$$

in which,

$$K = \frac{\Delta x}{[\frac{5}{3} - \frac{4}{3}(\frac{y_3}{B+2y_3})]\ v_3} \quad \ldots (59)$$

and

$$\theta = \frac{1}{2} - Q_n\left\{\frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)}{\lfloor 2}G_m + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\lfloor 3}G_m^2 + \ldots\right\}$$

$$\frac{\left\{1 + \frac{4}{9}F^2(-1 + 4(\frac{y_m}{B+2y_m}) - 4(\frac{y_m}{B+2y_m})^2)\right\}}{S_o B[\frac{5}{3} - \frac{4}{3}(\frac{y_m}{B+2y_m})]v_m\ \Delta x} \quad \ldots (60)$$

For wide rectangular channels equations (59) and (60) reduce to:

$$K = \frac{\Delta x}{\frac{5}{3}v_3} \quad \text{and} \quad \ldots (61)$$

$$\theta = \frac{\frac{1}{2} - Q_n\left\{\frac{1}{2} - \frac{\frac{1}{2}(\frac{1}{2}-1)}{\lfloor 2}.G_m + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{\lfloor 3}G_m^2 + ..\right\}(1-\frac{4}{9}F^2)}{S_o B(\frac{5}{3}v_m)\ \Delta x.} \quad \ldots (62)$$

where,

$Q_n = Q_3$, the discharge at section (3) and

$v_m$ = the velocity at the mid section of the routing reach.

When neglecting the terms $G_m$, $G_m^2$ .... etc., $\theta$ reduces to

$$\Theta = \frac{1}{2} - \frac{Q_n \, (1 - \frac{4}{9} F^2)}{2 \, S_o B \, (\frac{5}{3} v_m) \Delta x} \qquad \text{... (63)}$$

when the variables are fixed corresponding to a reference discharge valaue, then K and $\Theta$ reduce to :

$$K = \frac{\Delta x}{\frac{5}{3} v_o} \qquad \text{... (64)}$$

$$\Theta = \frac{1}{2} - \frac{Q_o \, (1 - \frac{4}{9} F_o^2 )}{2 \, S_o B \, (\frac{5}{3} v_o) \, \Delta x} \qquad \text{... (65)}$$

The above expressions for K and $\Theta$ were obtained by Dooge et al . (1982) for the case of constant parameters Muskingum flood routing method. When the rectangular channel is not wide and after eliminating $G_m$, $G_m^2$ . etc., K and $\Theta$ reduce to :

$$K = \frac{\Delta x}{[\frac{5}{3} - \frac{4}{3} \, (\frac{y_3}{B+2y_3})] \; v_3} \qquad \text{... (66)}$$

and

$$\theta = \frac{1}{2} - \frac{y_m Q_3 \; [1 + \frac{4}{9} F^2(-1 + 4(\frac{y_m}{B+2y_m}) - 4 \, (\frac{y_m}{B+2y_m})^2) \;]}{2 \, S_o[ \, \frac{5}{3} - \frac{4}{3} \, (\frac{y_m}{B+2y_m})] \; Q_m \, \Delta x} \qquad \text{... (67)}$$

where $y_m$, $Q_m$, $Q_3$, $y_3$ , and $v_3$ are as defined earlier.

## 5.0 APPLICATION

The methodology described above was verified by applying it for the case of routing floods in rectangular channel assuming that the flow follows Manning's friction law. It was assumed that the routing parameters K and $\Theta$ can be represented in terms of channel and flow parameters by equations (66) and (67) respectively.

It was considered that the approximation involved in computing $\Theta$ using approximate $l$, the distance between the mid-section and the section downstream of it where the normal discharge corresponding to the observed depth at mid-section is realized at the same instant of time, would not affect that accuracy of routing solution based on this procedure.

### 5.1 Test Series

The best approach for verifying the suggested methodology is to use hypothetical inflow-outflow hydrographs. Accordingly a hydrograph defined by a mathematical function is routed through the given channel for a specified distance using St. Venant's equations, which govern the one-dimensional flow in open channels, and thus the "observed" outflow hydrograph at the end of the specified distance is established. Now the same inflow hydrograph is routed in the same channel using the suggested procedure for the same specified distance and the resulting routed hydrograph is compared with the corresponding St. Venant's solution. The criteria for comparison based on various characteristics of outflow hydroagraph are defined at section 5.3. The logic behind the use of hypothetical inflow outflow hydrographs for verifying such methodologies has been already established (Kundzewicz, 1986).

### 5.1.1 Inflow hydrographs

In order to get a better understanding of the suggested procedure and for the purpose of effective comparison of various outputs obtained based on this procedure, it was decided to use the same inflow hydrograph in all the test runs. The hypothetical inflow hydrograph defined by a four parameter pearson type-III distribution which is expressed by the following equation was adopted in this study :

$$Q(t) = Q_b + (Q_p - Q_b) \left(\frac{t}{t_p}\right)^{\frac{1}{(\gamma-1)}} e^{\frac{1}{(\gamma-1)}(1-t/t_p)} \qquad \ldots (68)$$

where,

| | | | |
|---|---|---|---|
| $Q_b$ | = base flow | | = 100 $m^3/S$ |
| $Q_p$ | = peak flow | | = 1000 $m^3/S$ |
| $t_p$ | = time to peak | | = 10 hours |
| $\gamma$ | = skewness factor | | = 1.15 |

This hydrograph was adopted by Weinmann (1977) based on the consideration of steepness of hydrograph and magnitude of initial flow. The hydrograph based on equation (68) is shown in all the discharge hydrograph plots presented in this report.

### 5.1.2 Channel geometry and flow resistance properties.

The rectangular channel with the width of 50m was used for all the test runs and the routing computations were carried out for a maximum reach length of 40 km. The methodology was tested on four different channel configurations which are characterised by the followi ng bed slope and friction values as given in Table - 1

Table - 1 Channel Configurations

| Channel Type | Bed Slope | n-value |
|---|---|---|
| 1 | 0.0002 | 0.04 |
| 2 | 0.0002 | 0.02 |
| 3 | 0.002 | 0.04 |
| 4 | 0.002 | 0.02 |

These configurations were earlier adopted by Weinmann (1977) possibly due to the reason that the first two contigurations represent a worst case for which the approximate routing procedure are expected to perform poorly, and the last two configurations represent the best case for which they are expected to perform well.

## 5.2 Solution Procedure

The initial parameter values for $K_o$ and $\Theta_o$ were evaluated using equations (66) and (67) respectively. Using these parameter values, the coefficients of the conventional Muskingum method were evaluated as :

$$C_1 = \frac{-K\Theta_o + \Delta t/2}{K(1 - \Theta_o) + \Delta t/2}$$

$$C_2 = \frac{K\Theta_o + \Delta t/2}{K(1 - \Theta_o) + \Delta t/2} \qquad \ldots (69)$$

$$C_3 = \frac{K(1 - \Theta_o) - \Delta t/2}{K(1 - \Theta_o) + \Delta t/2}$$

Then the discharge $Q_2$ at the outflow section corresponding to inflow $I_2$, where $I_2$ corresponds to inflow orinate at $t = \Delta t$, was evaluated as:

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1 \qquad \ldots (70)$$

Knowing $I_2$ and $Q_2$, the discharge at section (3) as depicted in figure (2) was evaluated as :

$$Q_3 = Q_2 + \Theta_o(I_2 - Q_2) \qquad \ldots (71)$$

Corresponding to this discharge, the normal depth at the middle of the reach was evaluated using Newton-Raphson method based on the normal depth-discharge relationship as :

$$Q_3 = \frac{B^{5/3} S_o^{\frac{1}{2}}}{n} \cdot \frac{y_m^{5/3}}{(B+2y_m)^{2/3}} \qquad \ldots (72)$$

Then the discharge at the middle of the reach was evaluated as :

$$Q_m = (I_2 + Q_2)/2 \qquad \ldots (73)$$

Knowing $Q_m$, $y_m$, $Q_3$ and $F^2$, the new $\Theta$ was computed using equation (67) corresponding to $Q_2$. Based on equation (49) the flow depth at section (3) was evaluated as :

$$y_3 = y_m + (Q_3 - Q_m)/[(\frac{5}{3} - \frac{4}{3} \cdot (\frac{y_m}{B+2y_m})) \frac{Q_m}{y_m}] \qquad \ldots (74)$$

The velocity $v_3$ at section (3) was computed as :

$$v_3 = \frac{Q_3}{By_3} \qquad \ldots (75)$$

Knowing $v_3$ and $y_3$, and the distance of routing reach $\Delta x$ the new travel time K was comptued using equation (66).

These revised K and $\Theta$ values were used for the next step of solution corresponding to the new input ordinate. These steps were repeated for the entire solution procedure, thus varying the values of K and $\Theta$ at every time step, but at the same time adopting the linear solution procedure. The flow depth at the outflow section corresponding to the solution $Q_2$ was computed as :

$$y_2 = y_m + (Q_2 - Q_m)/[(\frac{5}{3} - \frac{4}{3}(\frac{y_m}{B + 2y_m}))\frac{Q_m}{y_m}] \qquad \ldots (76)$$

The procedure described above correspond to the variable parameters case. Two different approaches of solution procedures were adopted for the variable paramaeters case viz,

1) Considering the entire 40 km reach as a single reach and

2) Considering it consists of number of sub-reaches. The other solution procedure corresponds to the case of adopting constant $\Theta$ and variable K, along with the consideration of 40 km reach as a single reach.

When $\frac{\partial y}{\partial x}$ was large, it was observed that a significant length of reach was required as a single reach in the case of channel type-1 for the purpose of successfully routing the hydrograph using variable parameters approach. The routing resulted in computational problem due to high negative value of $\Theta$, when the adopted routing length was less than the required minimum length.

Such situations necessitate the linear interpolation of given inflow and the routed outflow hydrographs for finding the intermediate outflow hydrographs corresponding to the reach lengths which are less than the specified required minimum length. In order to test whether such interpolations yield acceptable results, two different cases were studied with channel types 1 and 2.

With channel type 1, the minimum length of routing reach for obtaining solution with no computational problem using variable parameters routing approach was found to be 22 km. A hydrograph solution for 5 km.

reach was attempted using interpolation of the given inflow hydrograph and the routed hydrograph at 22 km.

In order to check the interpolation solution result with the direct routing solution, for a distance of 5 km, the following procedure was adopted :

For the case of **channel** type-2, the linear interpolation solution was obtained at the end of reach length of 5 km. using the given inflow hydrograph and the routed hydrograph at the end of 40 km. For comparison with this solution, the inflow hydrograph was routed for 5 km. by considering it as a single reach.

Fifteen test runs as indicated in Table-2 were made in order to have a better understanding of the proposed methodology. Runs based on different combinations of parameter variations, and number of sub-reaches considerations were made. Such combinations tested are listed in the 'Remarks' column of Table-2. In all the runs, the routing time interval $\Delta t$ was considered as 15 minutes in order to avoid any numerical error in the solution using equation (70).

## 5.3 Comparison Criteria

The following comparison criteria were adopted for checking the efficiency of the proposed method of solution in comparison with the St. Venant's solution :

### 5.3.1 The hydrograph fitting consideration

The closeness with which the proposed method of solution follows the true solution, including the closeness of shape and size of hydrograph, can be measured using the criteria of variance explained by the method. The expression for variance explained in % is given as :

Table - 2

Test Run Details

| Test Run No. | Channel Type | Required reach length in Km. | Adopted reach length in km. | No. of sub-reaches | Length of sub-reach | Remakrs |
|---|---|---|---|---|---|---|
| 1 | 1 | 40 | 40 | 1 | 40 | * |
| 2 | 1 | 40 | 40 | 1 | 40 | ** |
| 3 | 1 | 40 | 44 | 2 | 22 | *** and * |
| 4 | 2 | 40 | 40 | 1 | 40 | * |
| 5 | 2 | 40 | 40 | 1 | 40 | ** |
| 6 | 2 | 40 | 40 | 8 | 5 | * |
| 7 | 3 | 40 | 40 | 1 | 40 | * |
| 8 | 3 | 40 | 40 | 1 | 40 | ** |
| 9 | 3 | 40 | 40 | 8 | 5 | * |
| 10 | 4 | 40 | 40 | 1 | 40 | * |
| 11 | 4 | 40 | 40 | 1 | 40 | ** |
| 12 | 4 | 40 | 40 | 8 | 5 | * |
| 13 | 1 | 5 | 22 | 1 | 22 | *** and * |
| 14 | 2 | 5 | 40 | 1 | 40 | *** and * |
| 15 | 2 | 5 | 5 | 1 | 5 | * |

NOTE: * Both $\theta$ and K varying
** Only K varying and $\theta$ remaining constant
*** Solution was obtained by linear interpolation of inflow and routed outflow hydrographs

$$\text{Variance explained in \%} = \frac{(\text{Total variance} - \text{Remaining Variance})}{\text{Total variance}} \times 100 \quad \ldots (77)$$

where,

$$\text{the total variance} = \frac{1}{N} \sum_{i=1}^{N} (Q_{oi} - \bar{Q}_{oi})^2 \quad \ldots (78)$$

$$\text{the remaining variance} = \frac{1}{N} \sum_{i=1}^{N} (Q_{oi} - Q_{ci})^2 \quad \ldots (79)$$

with,

$Q_{oi}$ = the $i^{th}$ discharge observation

$\bar{Q}_{oi}$ = mean of the discharge observation

$Q_{ci}$ = the $i^{th}$ discharge computed using the proposed method

N = the total number of discharge ordinates

5.3.2 Magnitude of flood peak consideration

Relative error in peak discharge (%) is given as :

$$Q_{PE} = \frac{(Q_{pc} - Q_{po})}{Q_{po}} \times 100 \quad \ldots (80)$$

where,

$Q_{pc}$ = the computed peak outflow discharge

$Q_{po}$ = the observed peak outflow discharge

Error in peak stage (metre) is given as :

$$Y_{PE} = y_{pc} - y_{po} \quad \ldots (81)$$

where

$y_{pc}$ = computed peak stage at the outflow section

$y_{po}$ = observed peak stage at the outflow section

5.3.3 Time of peak consideration

Error in time of peak discharge (hours) is given as :

$$T_{PQE} = t(Q_{pc}) - t(Q_{po}) \quad \ldots (82)$$

where,

$t(Q_{pc})$ = time corresponding to computed peak discharge

$t(Q_{po})$ = time corresponding to observed peak discharge

Error in time of peak stage (meters) is given as :

$$T_{PYE} = t(y_{pc}) - t(Y_{po}) \qquad \ldots (83)$$

where,

$t(y_{pc})$ = time corresponding to computed peak stage at the outflow section

$t(y_{po})$ = time corresponding to observed peak stage at the outflow section

5.3.4 Conservation of mass consideration

The relative error in the flow volume in percent of the total inflow volume is expressed as :

$$EVOL = \left[\frac{\sum_{i=1}^{N} Q_{ci} - \sum_{i=1}^{N} I_i}{\sum_{i=1}^{N} I_i}\right] \times 100 \qquad \ldots (84)$$

where,

$I_i$ = the $i^{th}$ inflow discharge

## 6.0 RESULTS AND DISCUSSION

### 6.1 Results

Table - 3 presents the results of variance explained, relative errors in peak discharge, and peak stage, errors in time to peak discharges, and peak stage, and the relative error in flow volume for all 15 test runs made in this study. Figure (3) shows the inflow hydrograph, and the outflow hydrographs computed from test run nos. 1, 2 and 3 and from St. Venant's equations (the "observed" hydrograph). Figure (4) shows the corresponding computed stage hydrographs at the outflow section. Similarly figures (5), (7) and (9) respectively show the inflow hydrograph, and the outflow hydrographs computed from test run number 4-6, 7-9 and 10-12 along with the St. Venant's solutions for these runs. Figures (6), (8) and (10) respectively show the computed stage hydrographs at the outflow sections along with the concerned stage hydrographs due to St. Venant's solutions for the above mentioned runs. Figures (11),(12), (13) and (14) respectively show the variation of the travel time parameter K vs. the corresponding given inflow ordinates for test run nos. 1, 4, 7 and 10. Figures (15), (16), (17) and (18) respectively show the variation of the weighting factor $\Theta$ vs the corresponding given inflow ordinates for test run nos. 1, 4, 7 and 10. Figure (19) shows the inflow hydrograph and the computed outflow hydrograph from test run no. 13, and the corresponding St. Venant's solution. The outflow hydrograph computed for the reach length of 5 Km from test run no. 13 was obtained by interpolation of the given inflow hydrograph and the corresponding computed outflow hydrograph at 22 Km. Figure (20) shows the computed and the St. Venant's solution's stage hydrographs corresponding to test run no. 13. Note that the stage hydrograph at 5 Km. was obtained by linear interpolation of the "observed"

TABLE - 3

Comparison of Results

| Test run no. | Chananel type | Variance* explained in % | $Q_{PE}$** | $y^{**}_{PE}$(mt.) | $T^{+}_{PQE}$(hr) | $T^{+}_{PYE}$(hr) | EVOL++ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 97.28 | -6.54 | 1.38 | -0.25 | 1.50 | -0.44 |
| 2 | 1 | 87.09 | -6.14 | 1.29 | -0.75 | 1.50 | 14.36 |
| 3 | 1 | 96.50 | -12.16 | -0.11 | 0.00 | 1.00 | 2.31 |
| 4 | 2 | 99.37 | -1.92 | 0.39 | 0.00 | 0.25 | -0.49 |
| 5 | 2 | 97.84 | -2.26 | 0.38 | 0.00 | 0.25 | 5.83 |
| 6 | 2 | 99.70 | -3.16 | 0.04 | 0.00 | 0.50 | -0.63 |
| 7 | 3 | 99.26 | -1.21 | -0.03 | 0.00 | 0.00 | -0.34 |
| 8 | 3 | 99.25 | -1.21 | -0.03 | 0.00 | 0.00 | 0.05 |
| 9 | 3 | 99.98 | 0.00 | 0.00 | 0.00 | 0.00 | -0.47 |
| 10 | 4 | 99.91 | -0.30 | -0.01 | 0.00 | 0.00 | -0.25 |
| 11 | 4 | 99.91 | -0.30 | -0.01 | 0.00 | 0.00 | -0.16 |
| 12 | 4 | 99.99 | 0.00 | 0.00 | 0.00 | 0.00 | -0.29 |
| 13 | 1 | 99.14 | -4.78 | -0.73 | -0.25 | 0.50 | 0.70 |
| 14 | 2 | 99.75 | -3.23 | -0.23 | 0.00 | 0.25 | -0.07 |
| 15 | 2 | 99.98 | -1.21 | -0.16 | 0.00 | 0.75 | -0.06 |

* Reference : section 5.3.1

** Reference : section 5.3.2

\+ Reference : section 5.3.3

\++ Reference : section 5.3.4

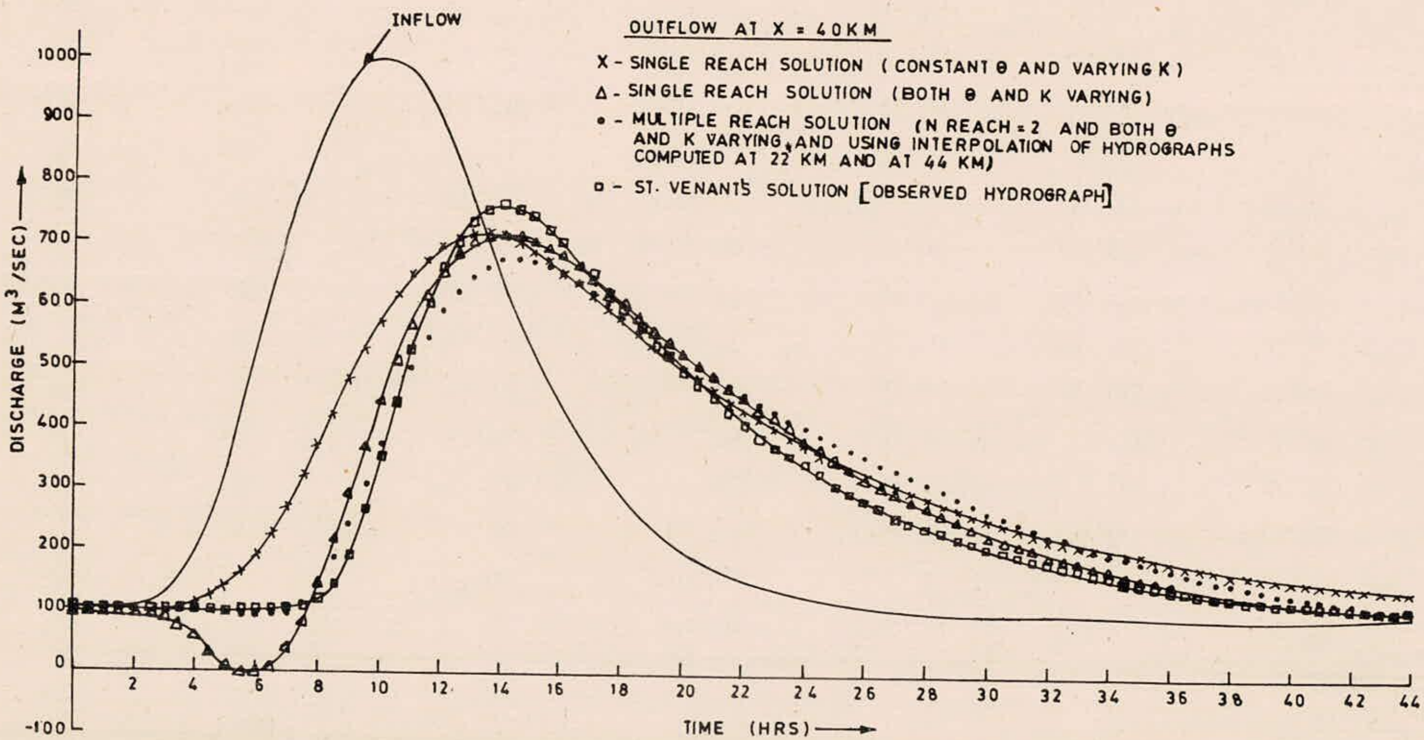


FIG. 3-OBSERVED AND COMPUTED DISCHARGE HYDROGRAPHS FOR CHANNEL TYPE-1

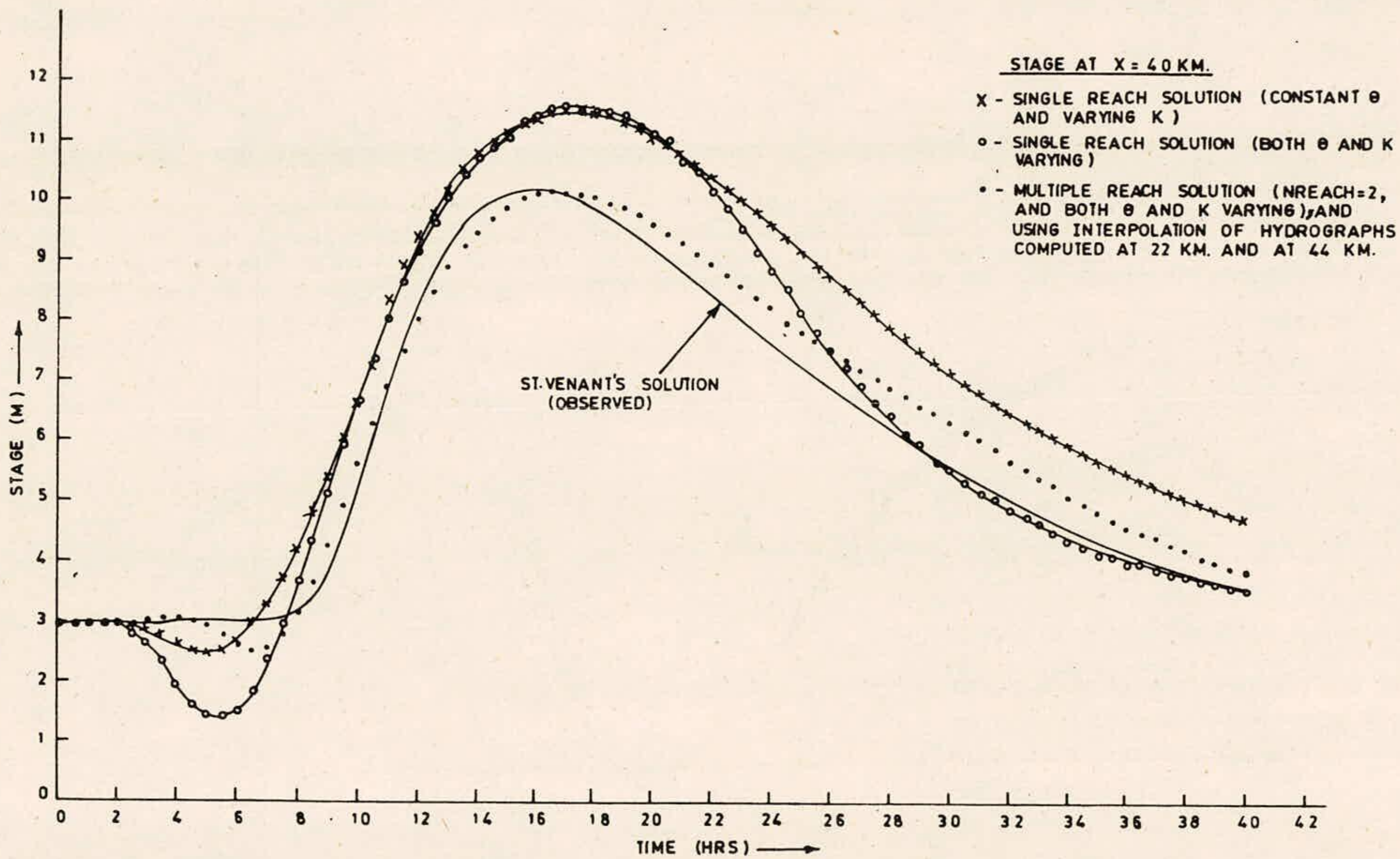


FIG. 4 - OBSERVED AND COMPUTED STAGE HYDROGRAPHS FOR CHANNEL TYPE - 1

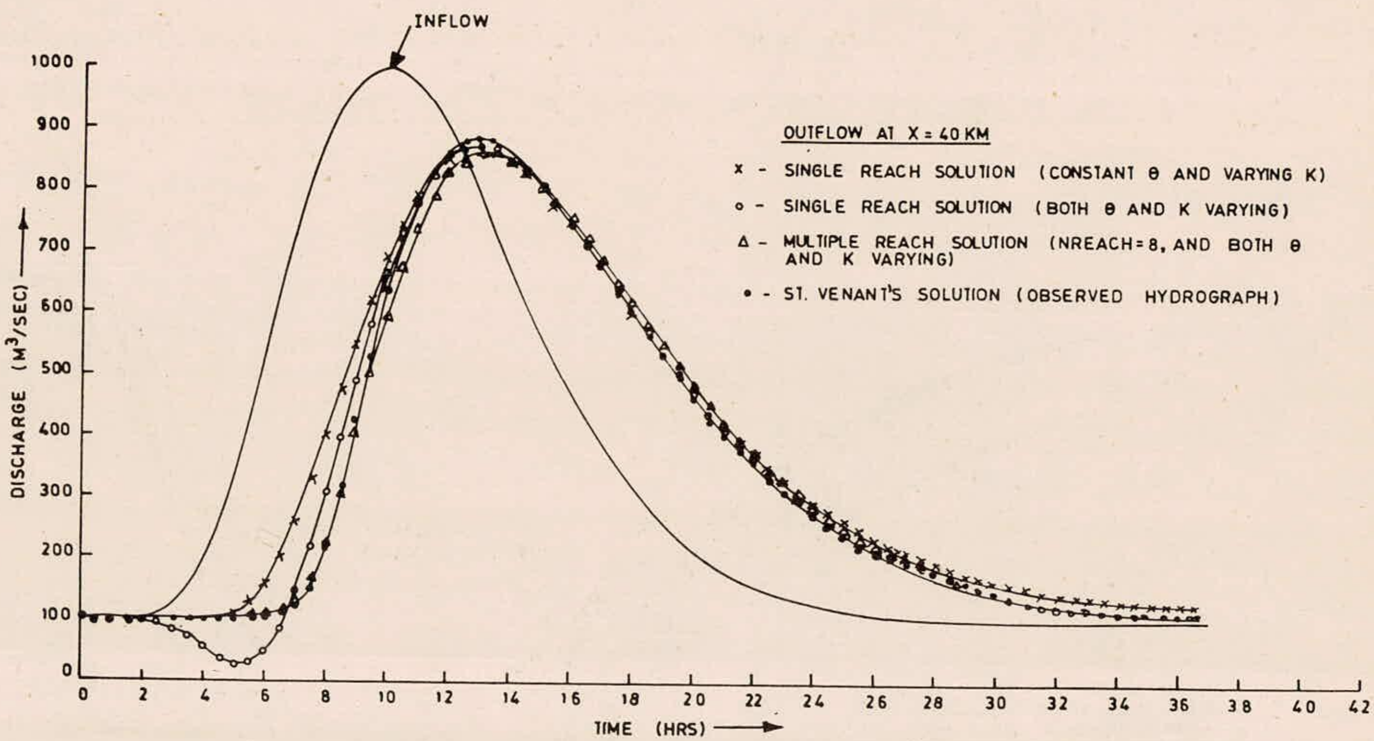


FIG. 5 - OBSERVED AND COMPUTED DISCHARGE HYDROGRAPHS FOR CHANNEL TYPE - 2

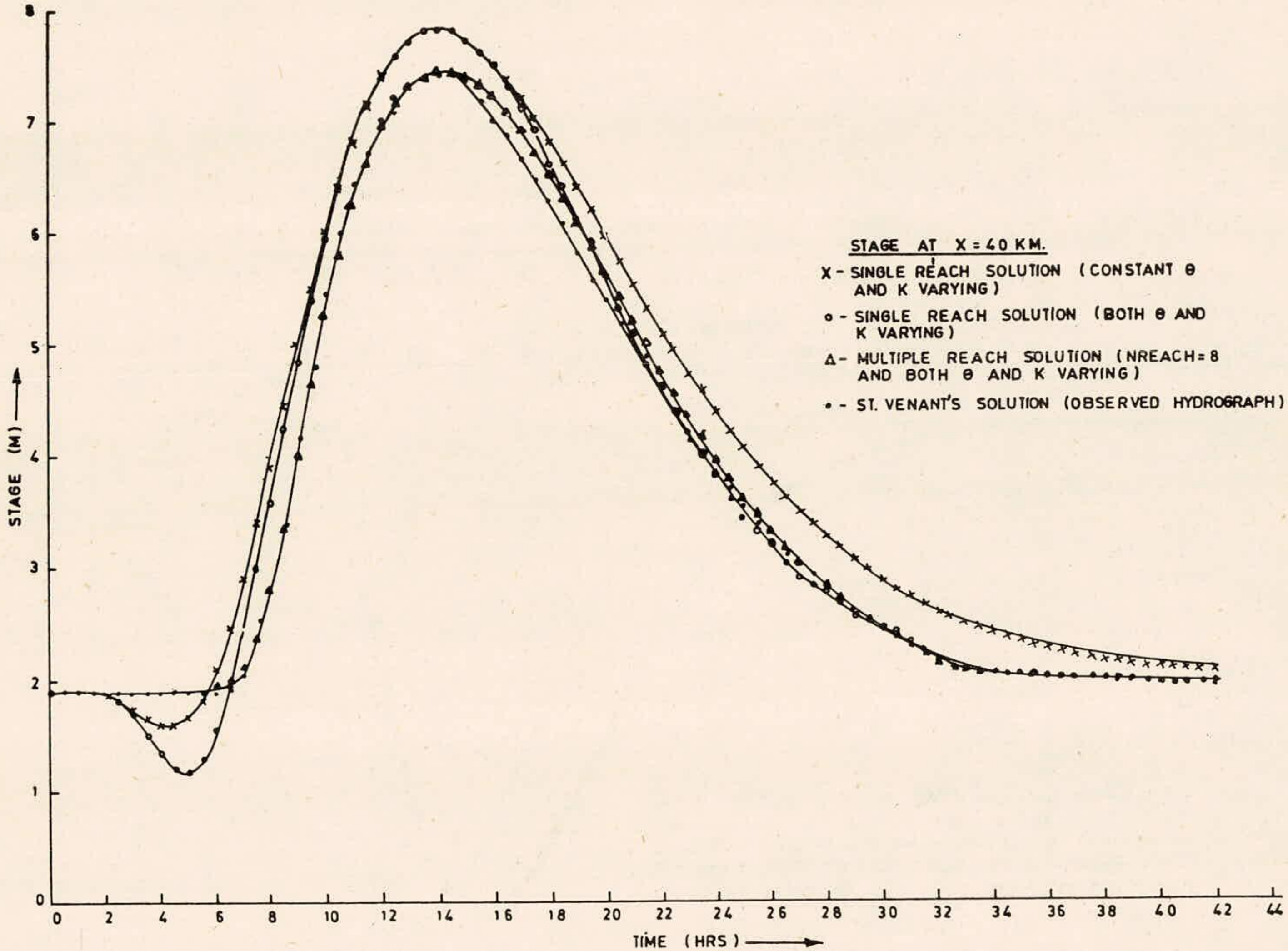


FIG. 6-OBSERVED AND COMPUTED STAGE HYDROGRAPHS FOR CHANNEL TYPE-2

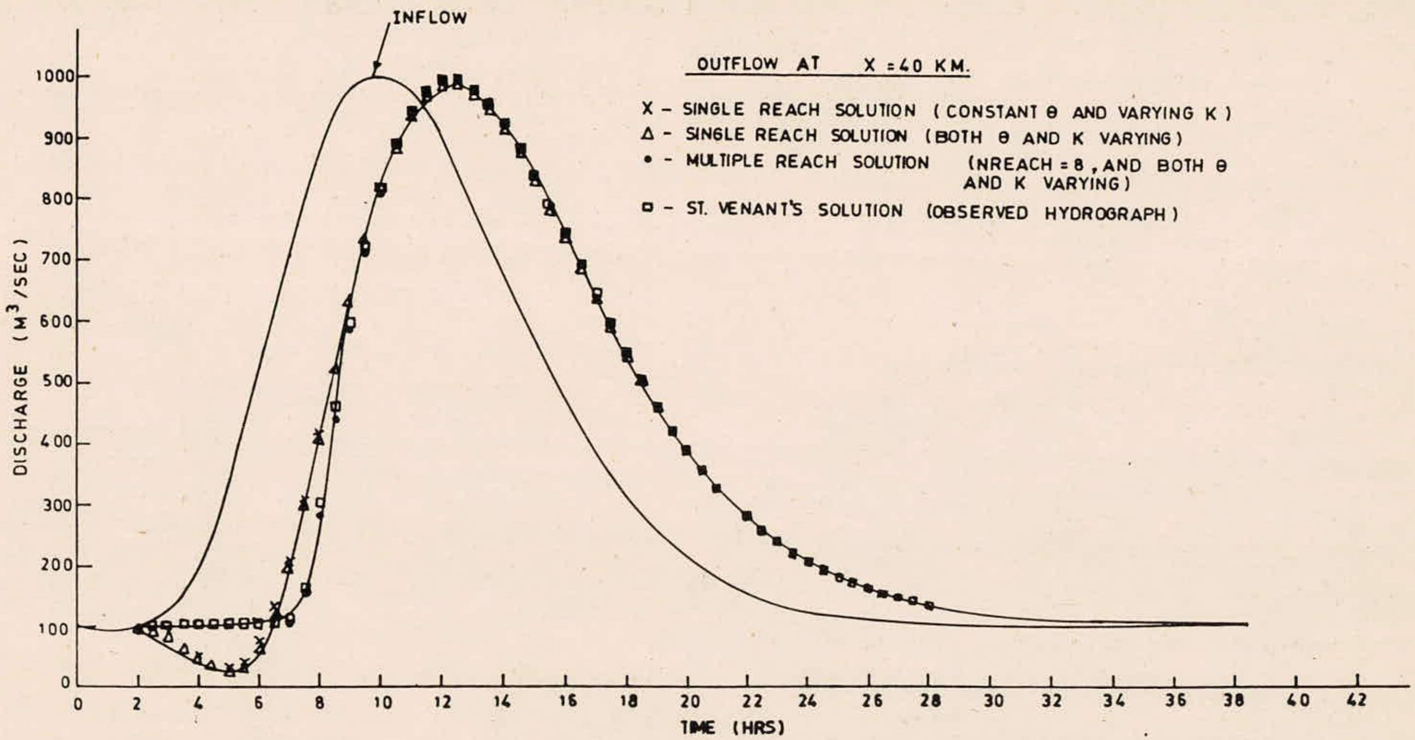


FIG. 7- OBSERVED AND COMPUTED DISCHARGE HYDROGRAPHS FOR CHANNEL TYPE -3

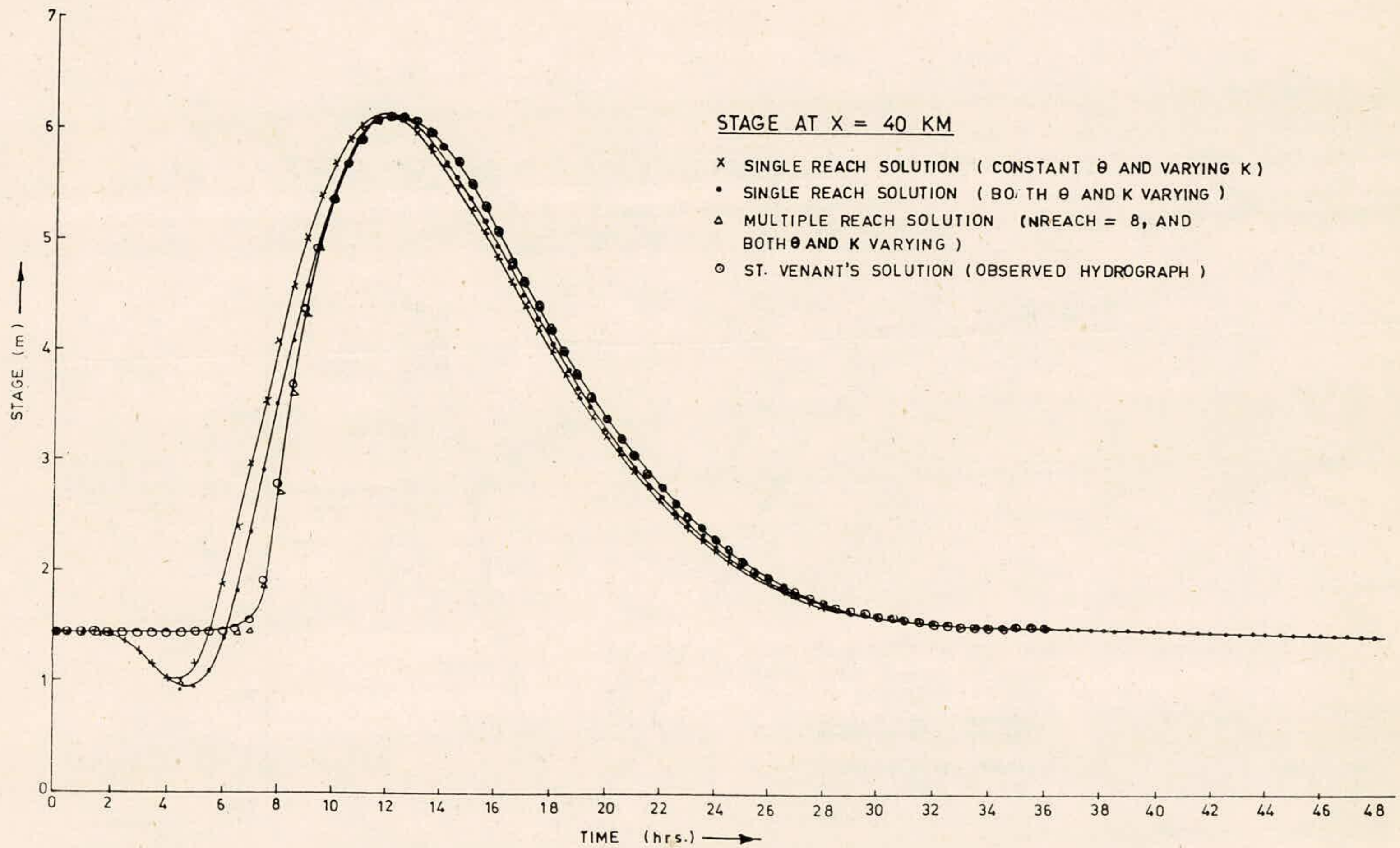


FIG. 8 - OBSERVED AND COMPUTED STAGE HYDROGRAPHS FOR CHANNEL TYPE - 3

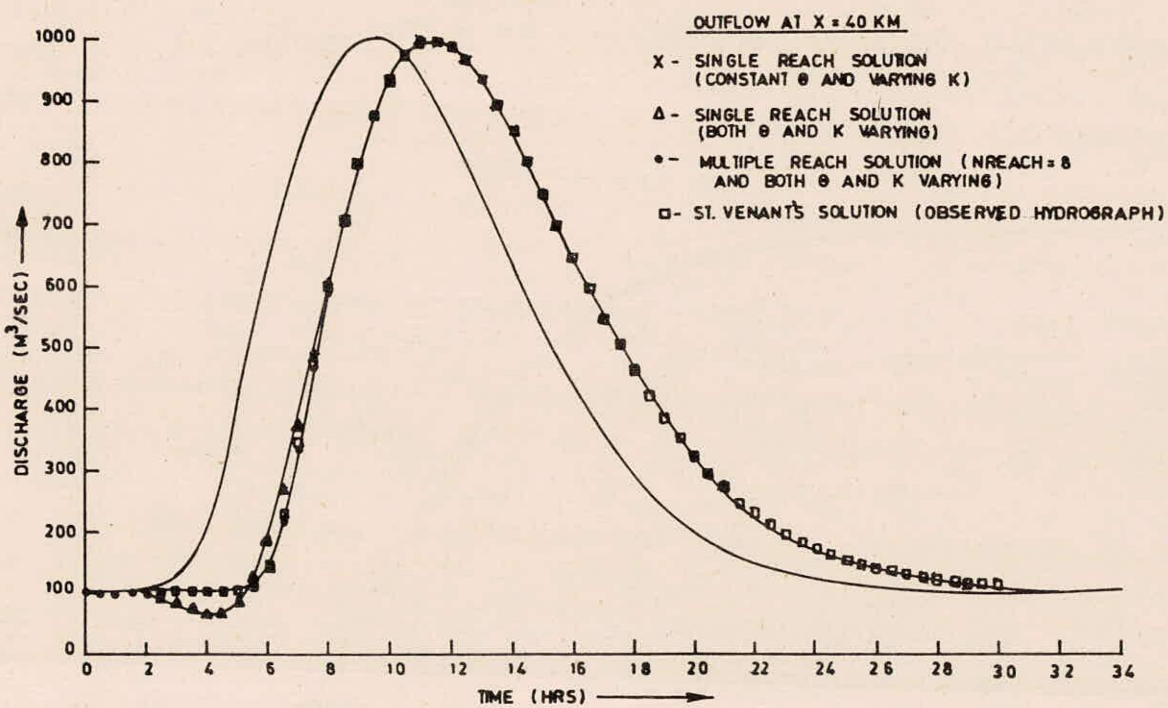


FIG. 9 - OBSERVED AND COMPUTED DISCHARGE HYDROGRAPHS FOR CHANNEL TYPE - 4

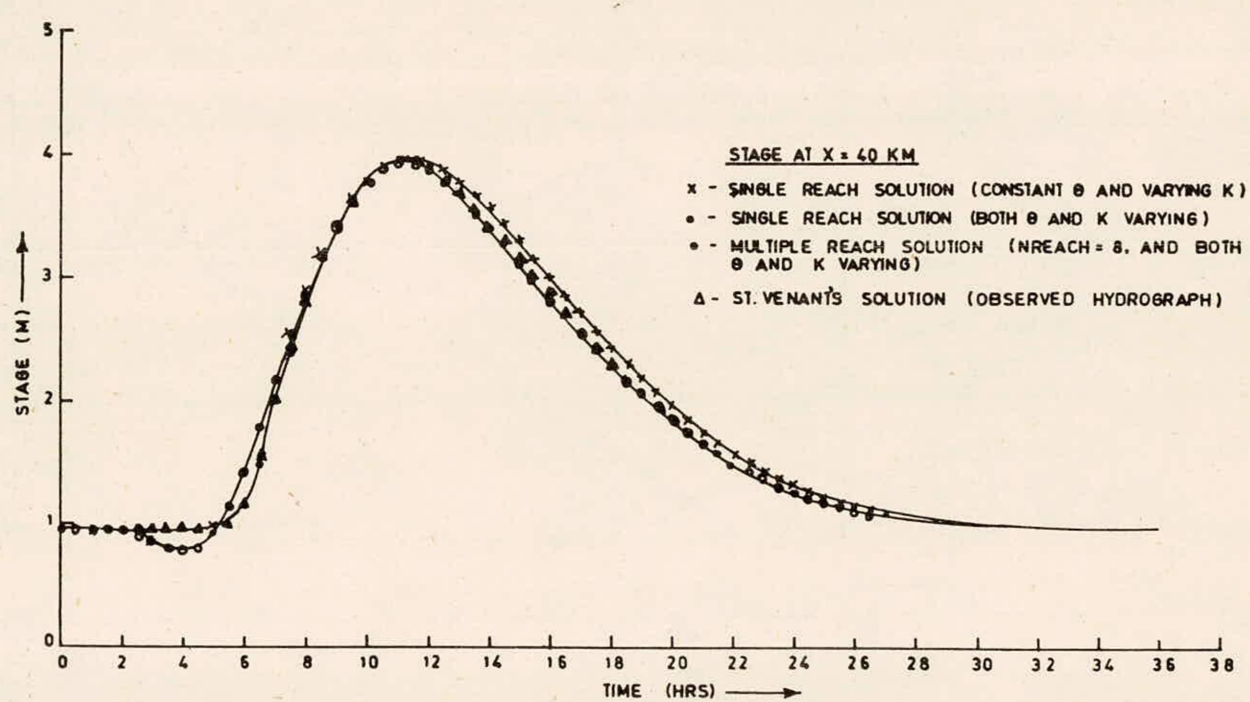


FIG.10 - OBSERVED AND COMPUTED STAGE HYDROGRAPHS FOR CHANNEL TYPE-4

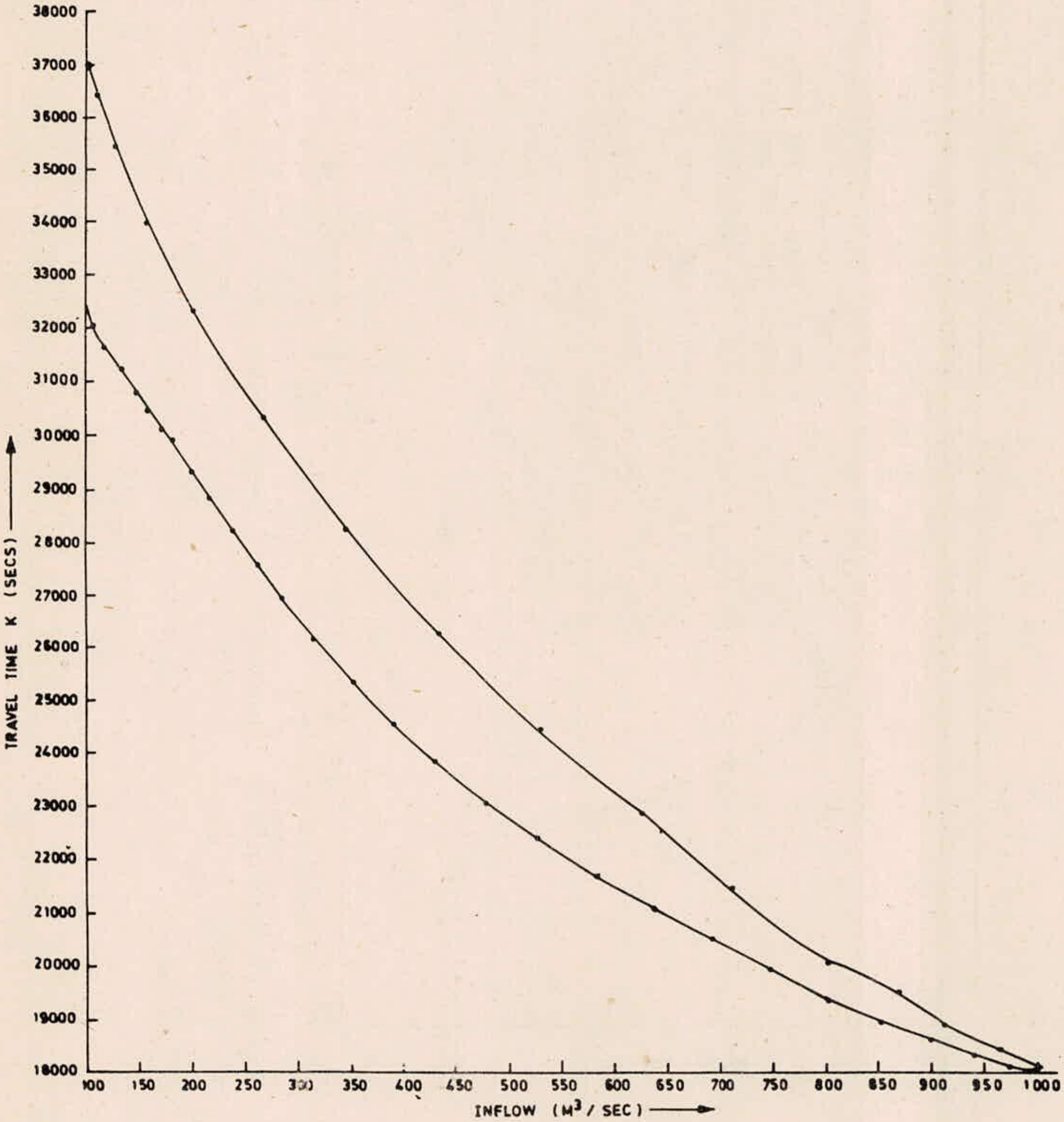


FIG. 11- VARIATION OF TRAVEL TIME WITH INFLOW FOR CHANNEL TYPE -1

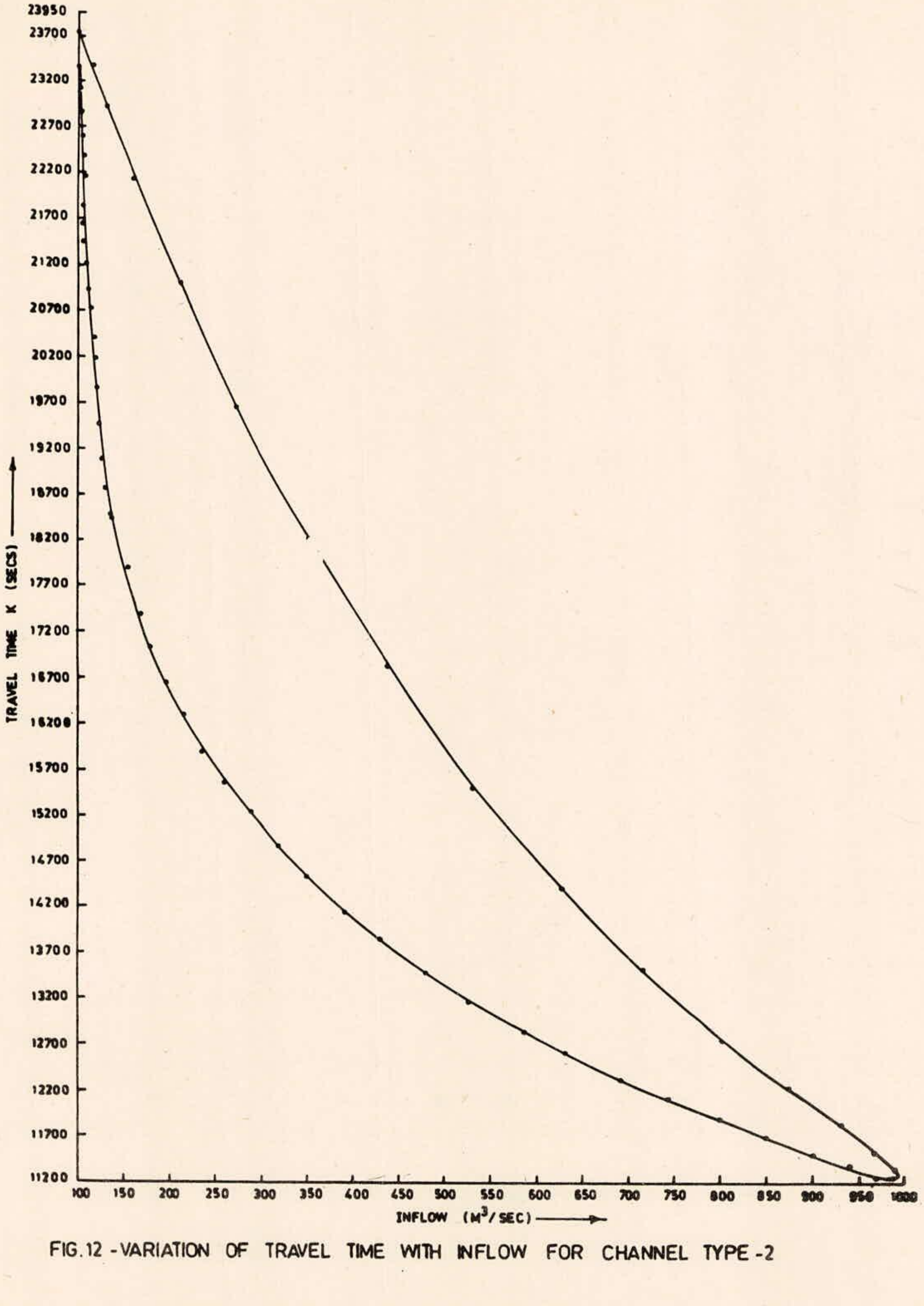


FIG. 12 - VARIATION OF TRAVEL TIME WITH INFLOW FOR CHANNEL TYPE - 2

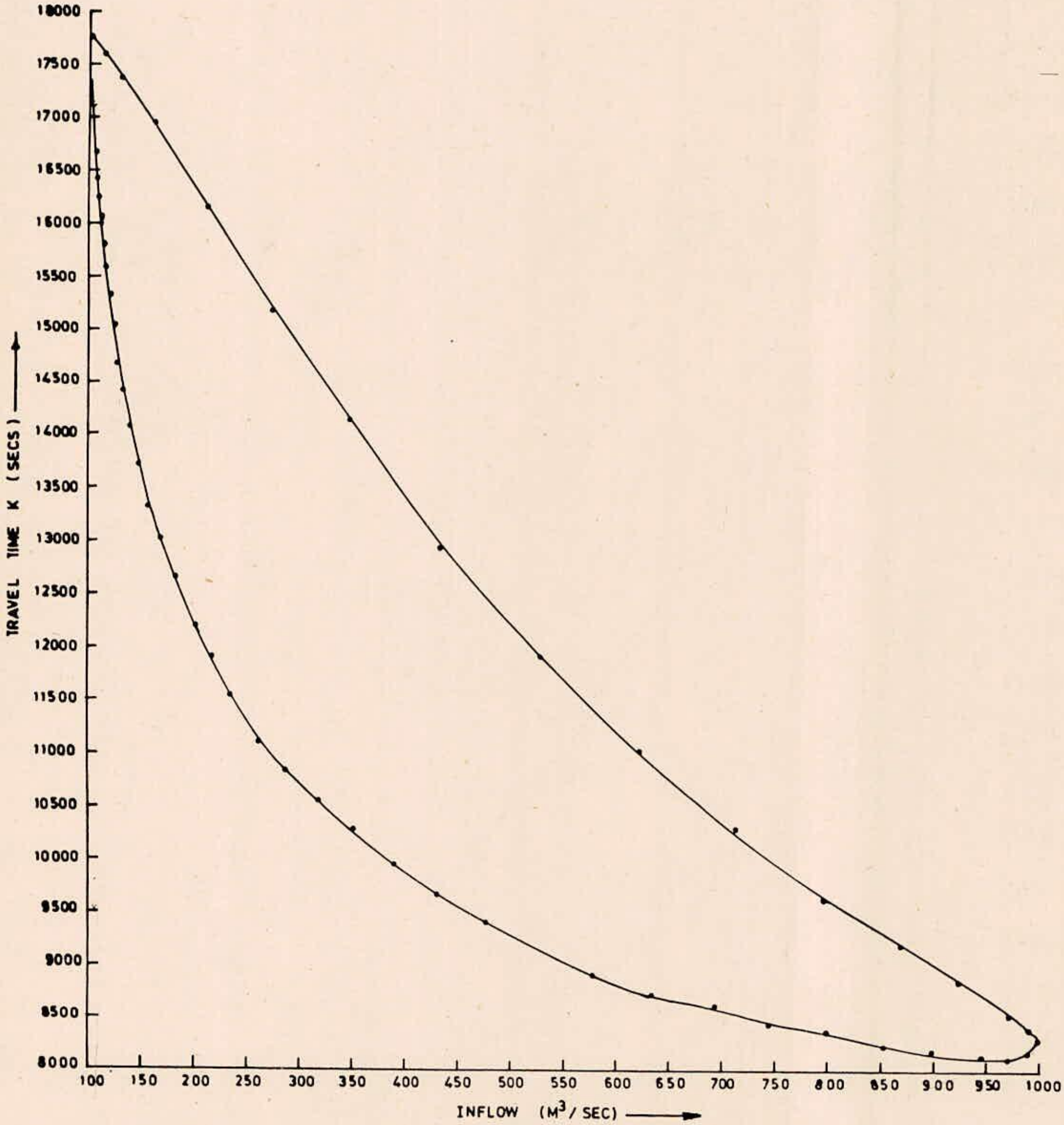


FIG. 13 - VARIATION OF TRAVEL TIME WITH INFLOW FOR CHANNEL TYPE -3

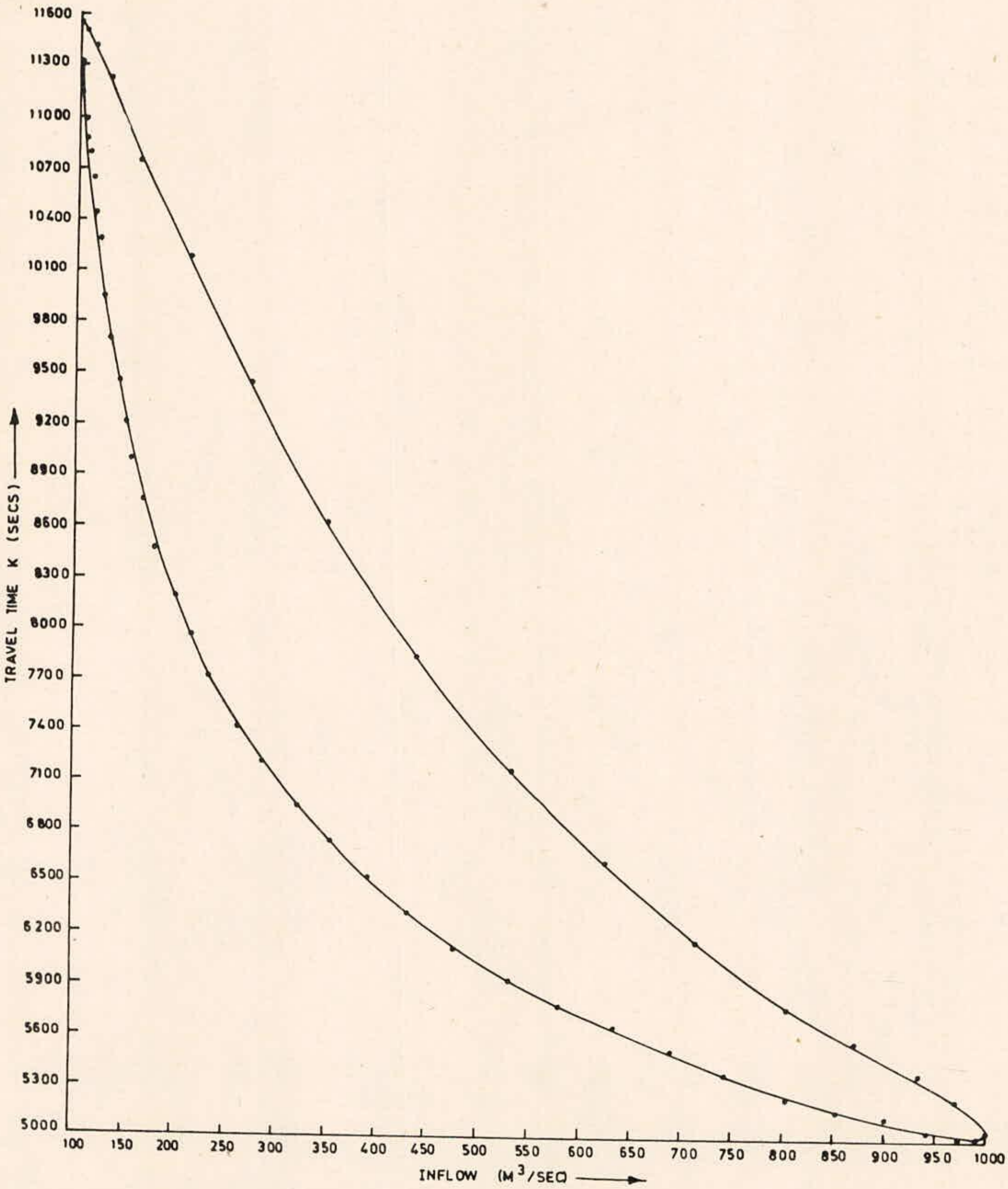


FIG. 14- VARIATION OF TRAVEL TIME WITH INFLOW FOR CHANNEL TYPE-4

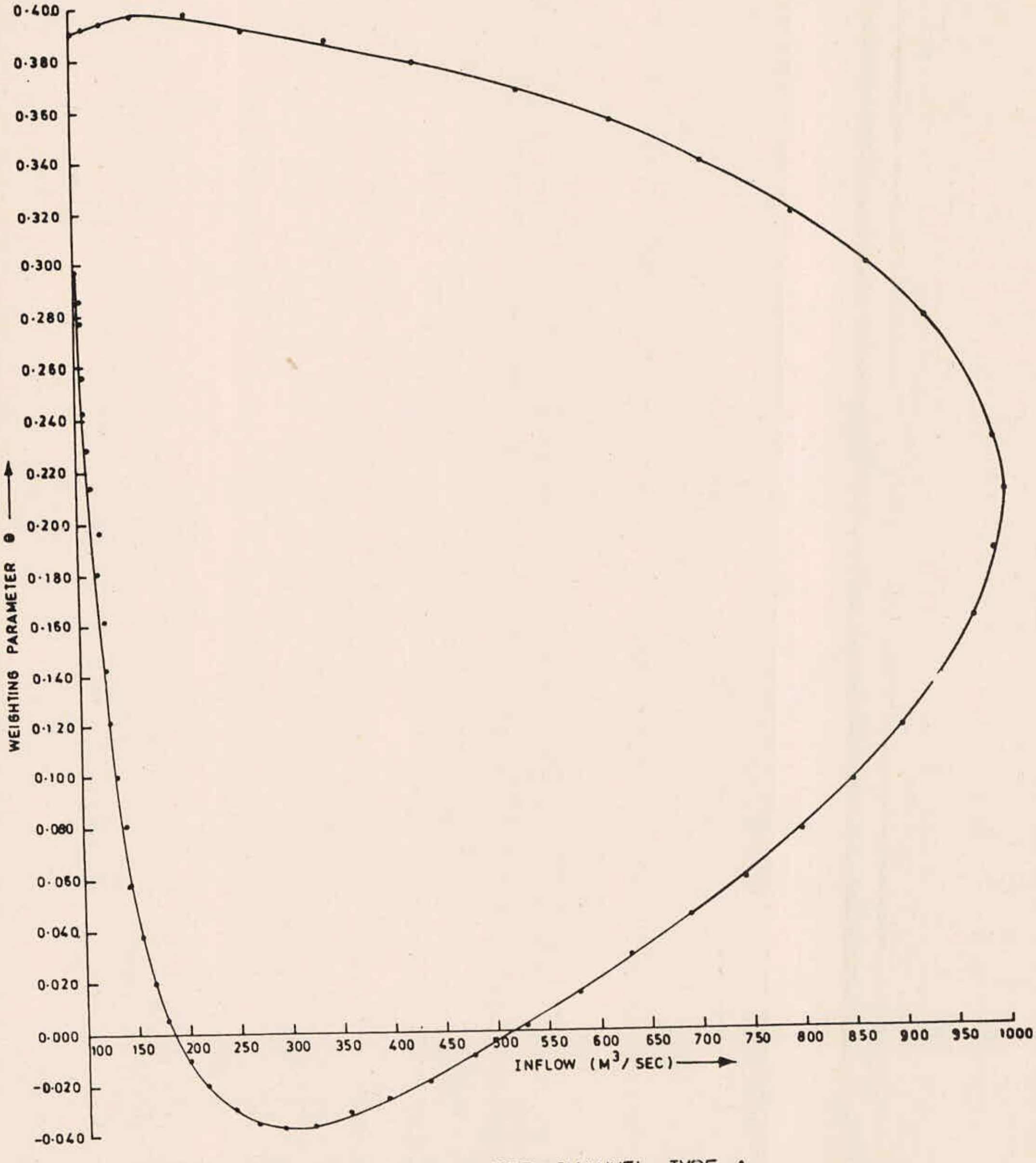


FIG. 15- VARIATION OF θ WITH INFLOW FOR CHANNEL TYPE-1

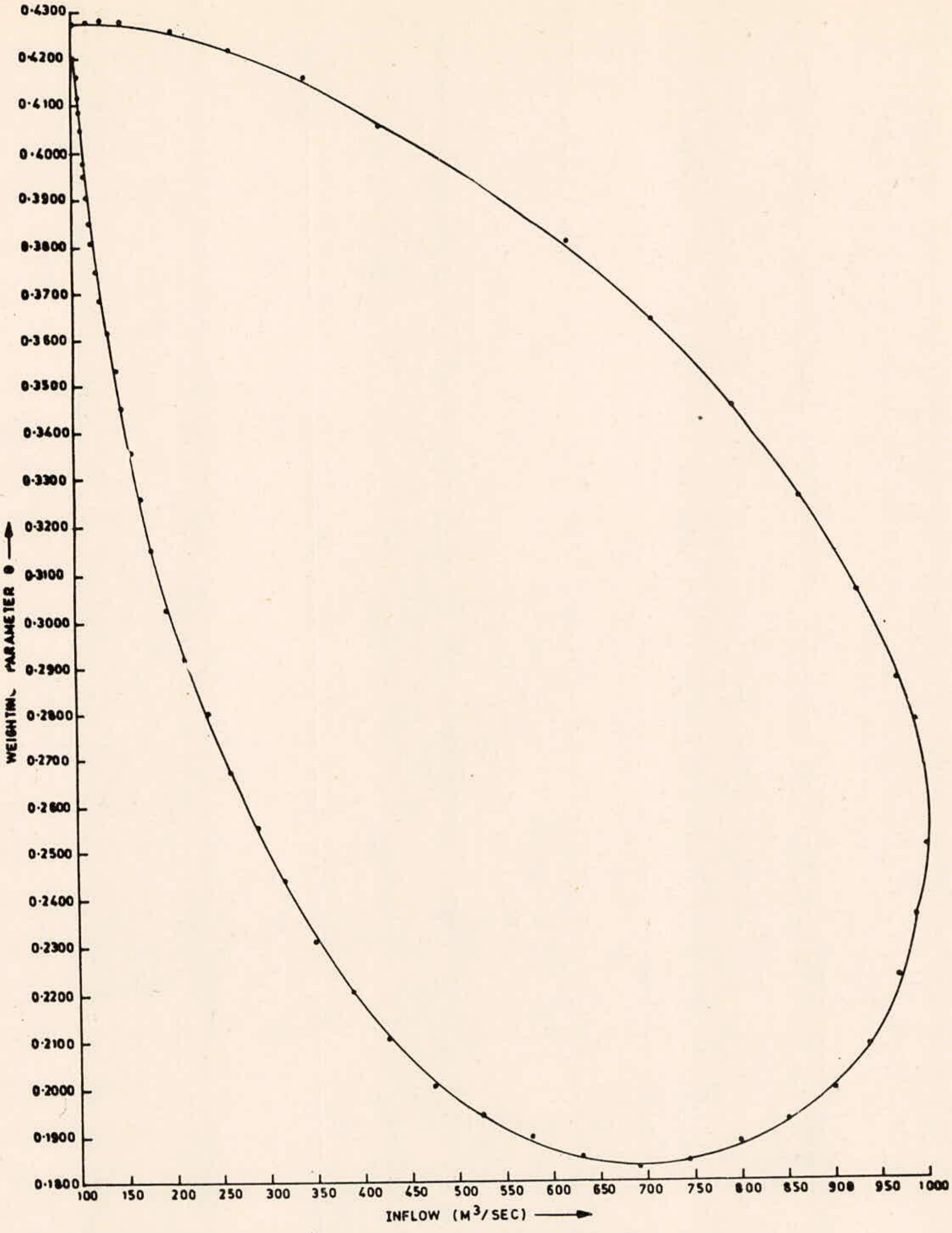


FIG.16 - VARIATION OF θ WITH INFLOW FOR CHANNEL TYPE - 2

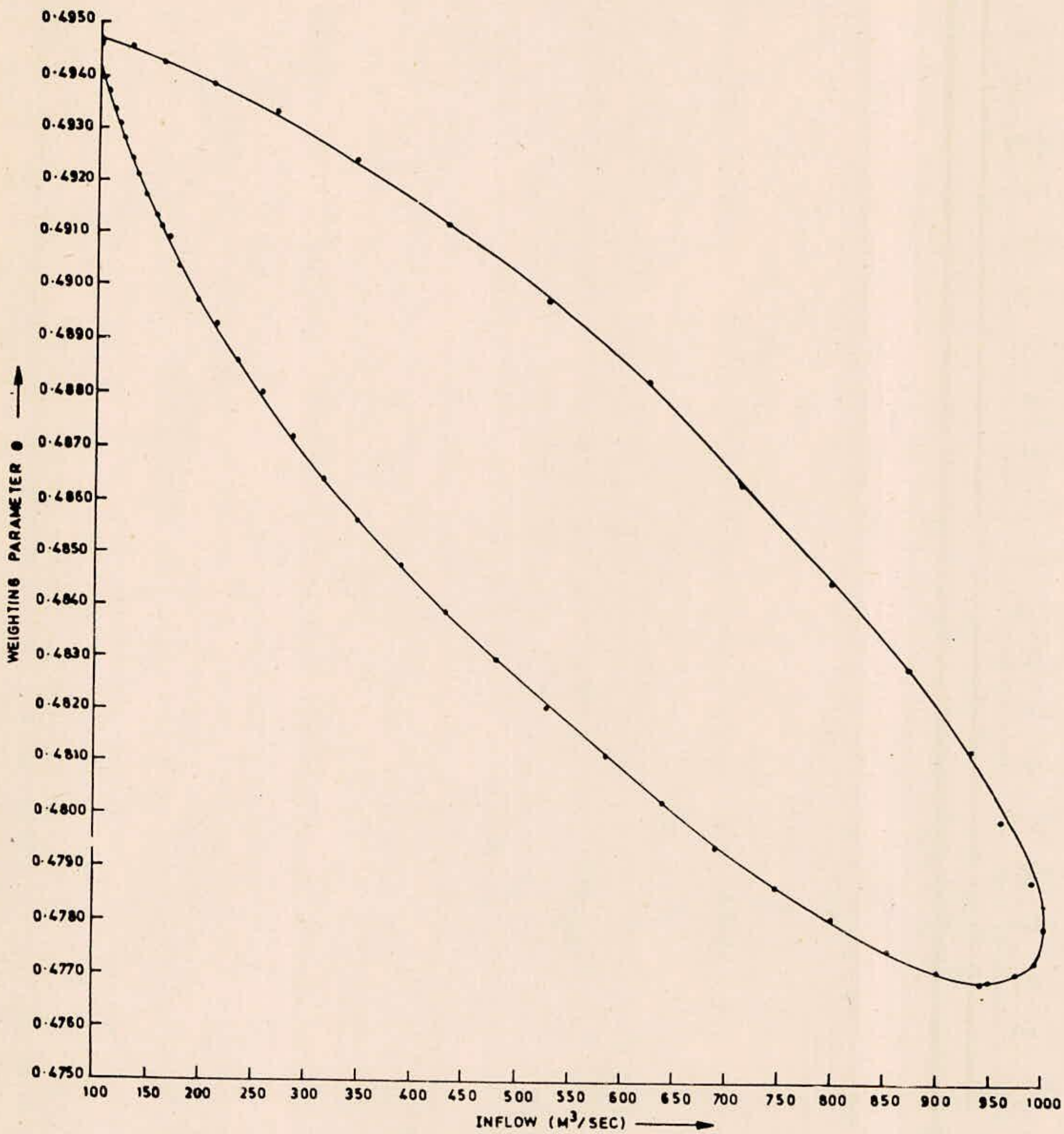


FIG.17 - VARIATION OF θ WITH INFLOW FOR CHANNEL TYPE - 3

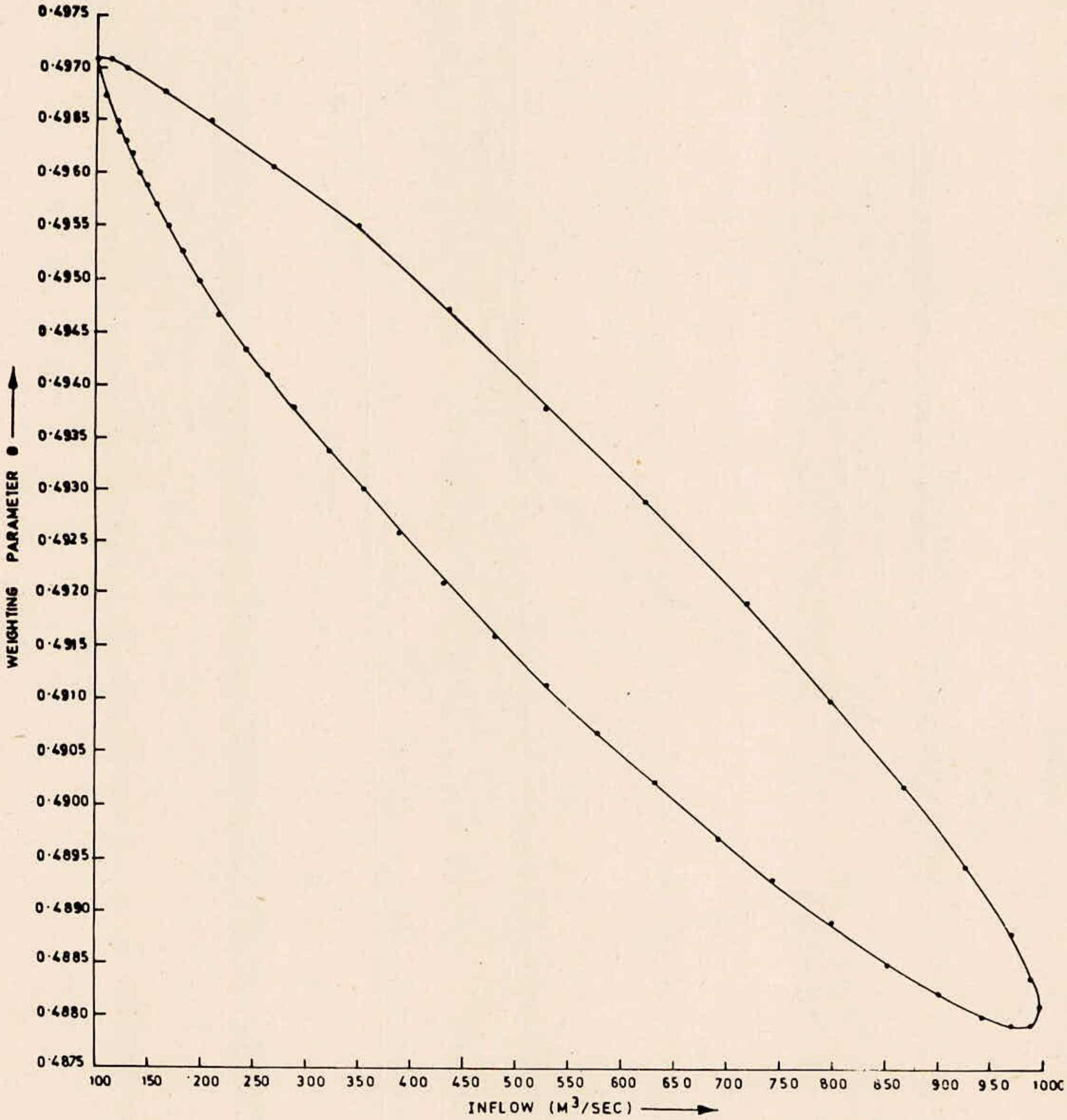


FIG. 18 - VARIATION OF θ WITH INFLOW FOR CHANNEL TYPE - 4

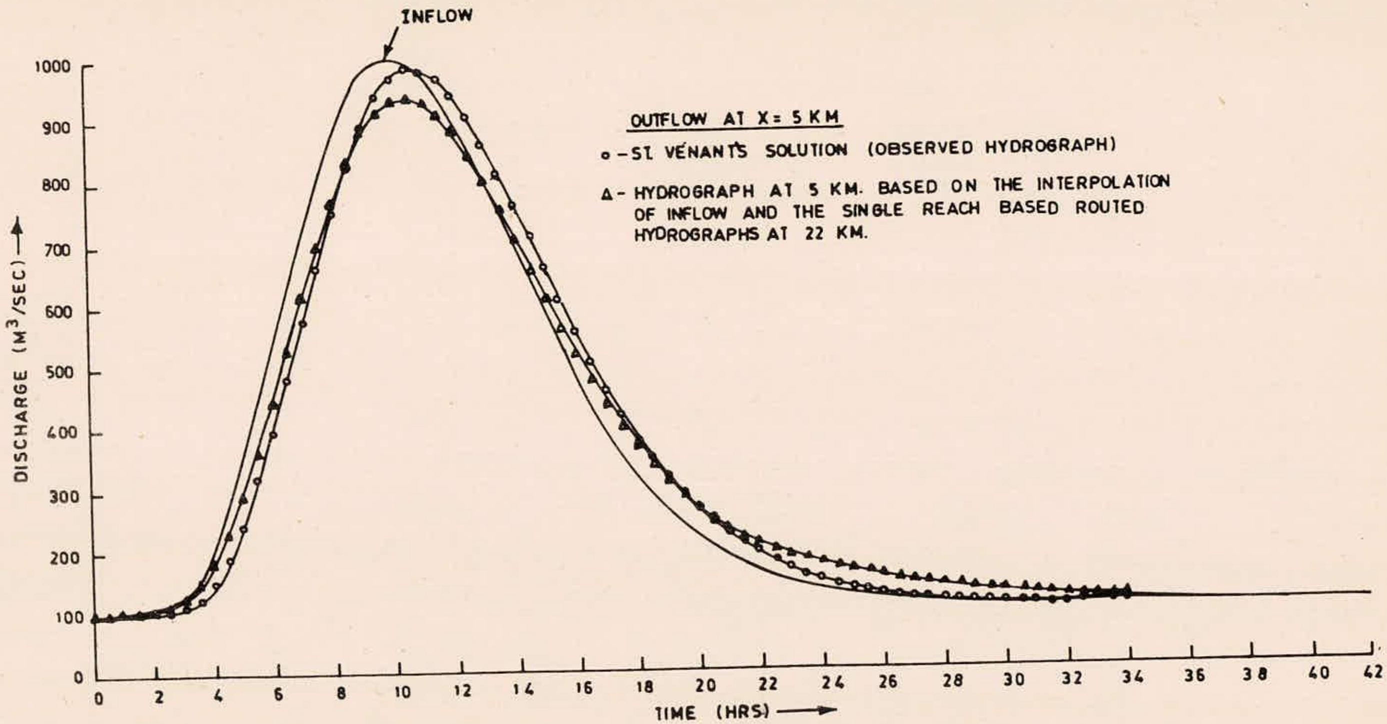


FIG. 19 ROUTING BASED ON INTERPOLATION OF HYDROGRAPHS (FOR CHANNEL TYPE - 1)

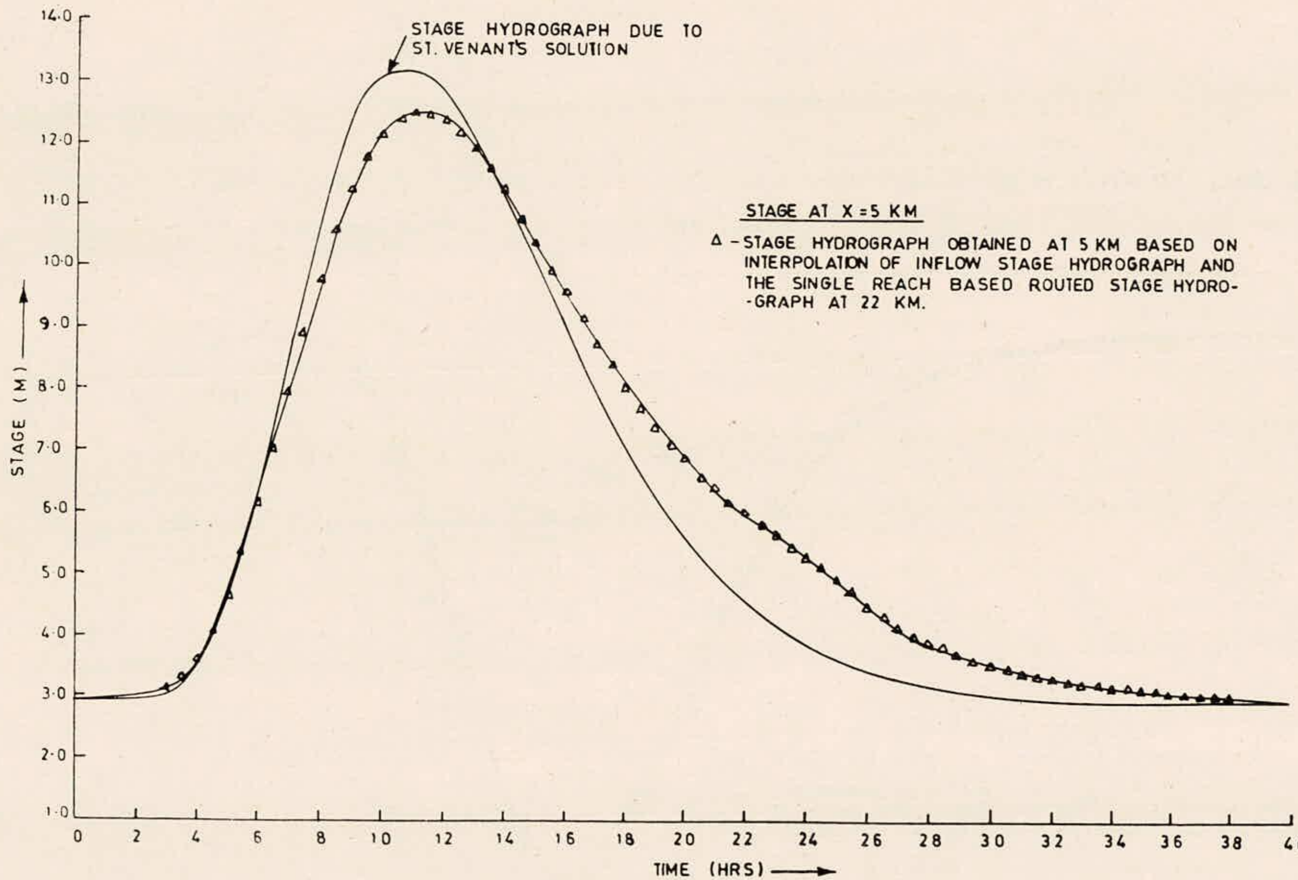


FIG 20 - STAGE HYDROGRAPH BASED ON INTERPOLATION (FOR CHANNEL TYPE - 1)

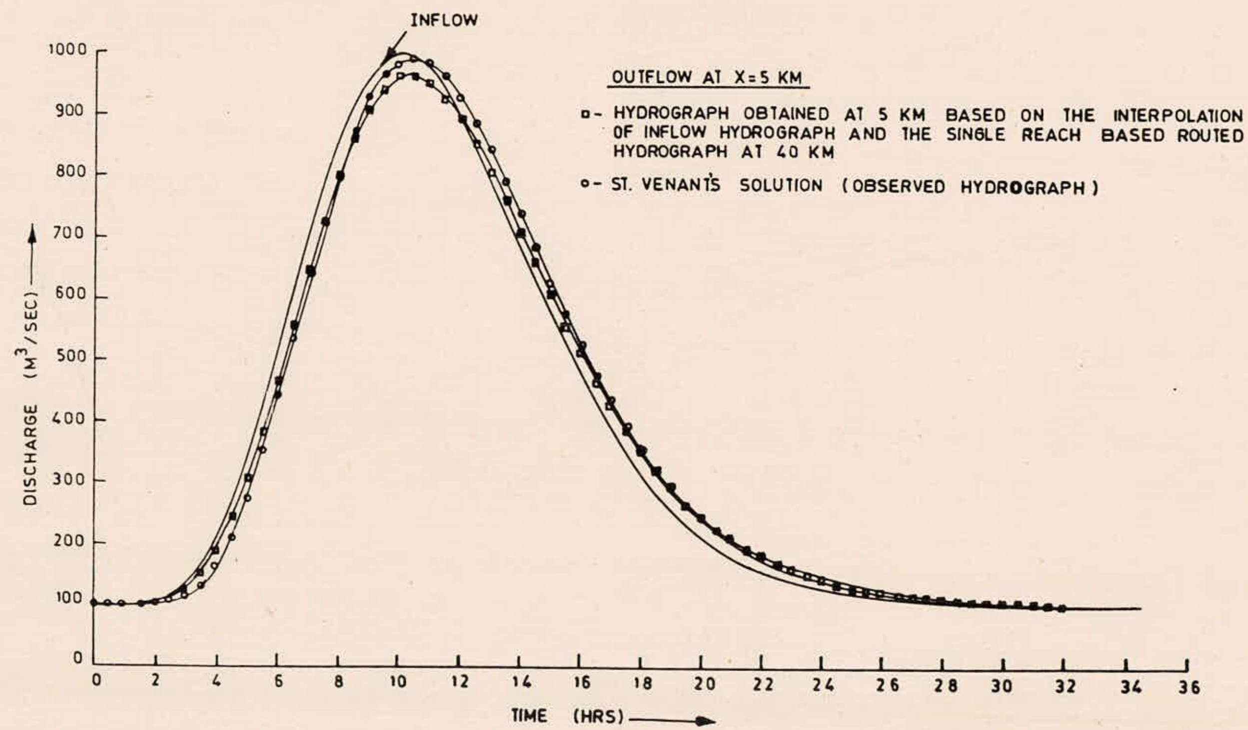


FIG. 21 - ROUTING BASED ON INTERPOLATION OF HYDROGRAPHS (FOR CHANNEL TYPE -2)

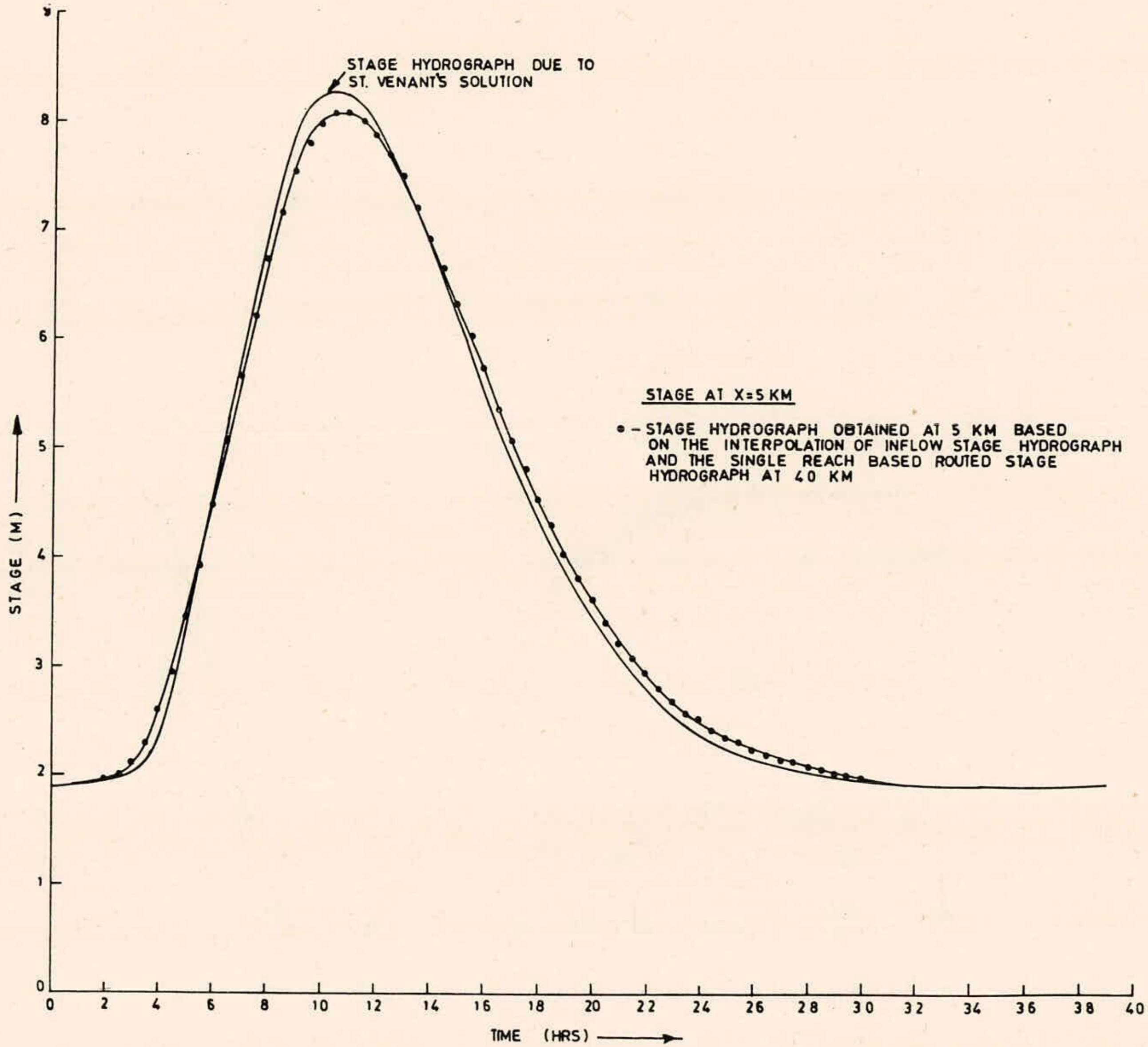


FIG.22 - STAGE HYDROGRAPH BASED ON INTERPOLATION (FOR CHANNEL TYPE-2)

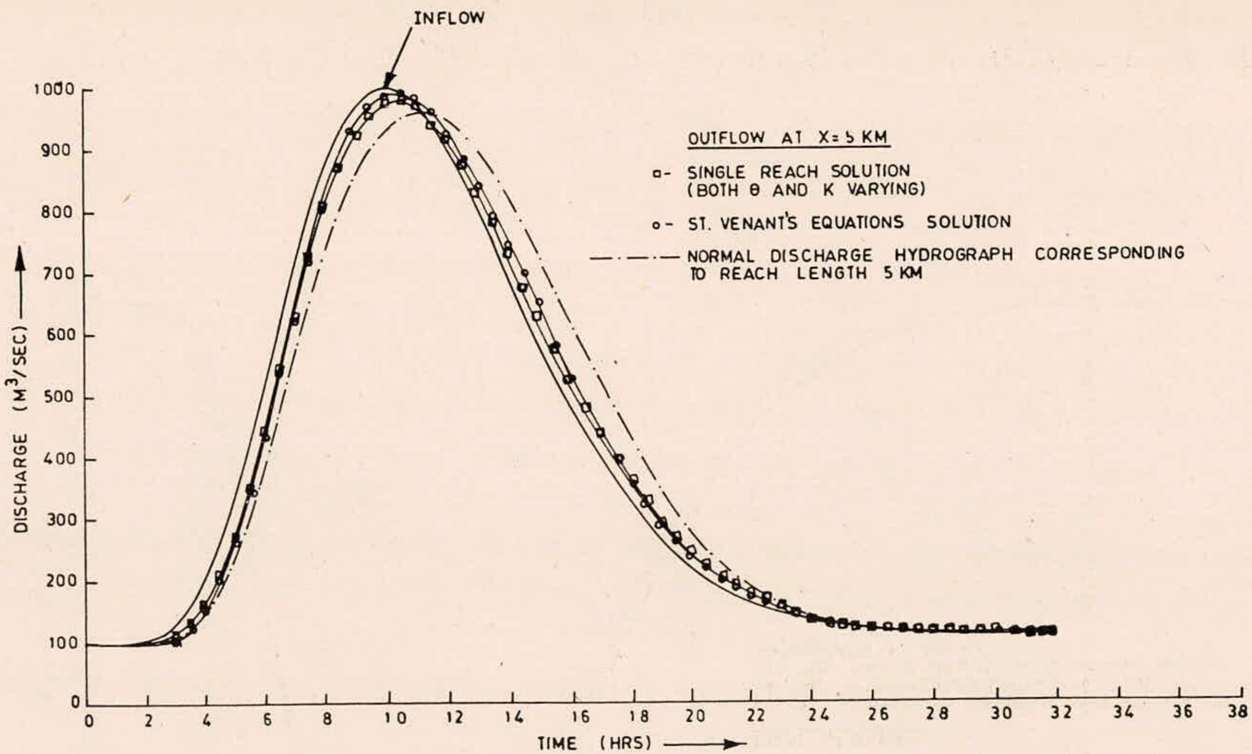


FIG. 23 - SOLUTION USING DIRECT ROUTING OF INFLOW HYDROGRAPH (FOR CHANNEL TYPE - 2)

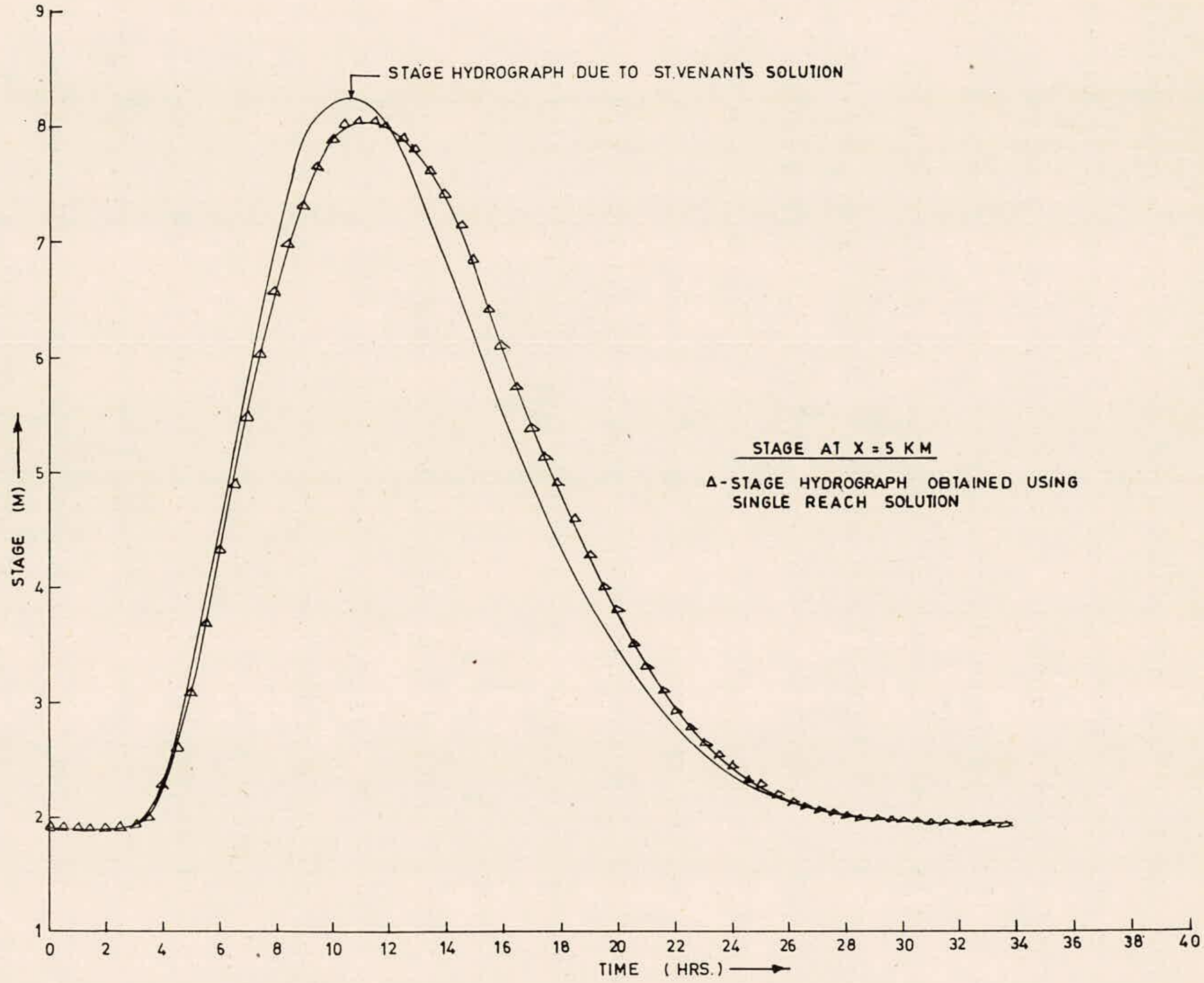


FIG. 24- COMPUTED STAGE HYDROGRAPH DUE TO DIRECT ROUTING (FOR CHANNEL TYPE-2)

stage hydrograph at the input section and that of computed hydrograph at 22 Km. Figure (21) shows the inflow hydrograph and the hydrograph obtained based on the interpolation of the same inflow hydrograph and the corresponding single reach based routed hydrograph at 40 Km. along with the St. Venant's solution. These results belong to test run no. 14. The stage hydrograph computed in the similar manner corresponding to test run no. 14 is shown in Figure (22) along with the corresponding St. Venant's solution. Figure (23) shows the inflow hydrograph, and the computed outflow hydrograph and the St. Venant's equations solution corresponding to test run no. 15. The normal flow hydrograph obtained from this method, representing the weighted discharge is also shown in the same plot. Figure (24) shows the stage hydrographs due to St. Venant's solution and due to the present method corresponding to test run no. 15.

## 6.2 Discussions

### 6.2.1 On the results of test run nos. (1), (2) and (3)

Based on the consideration of variance explained, it can be seen from Table - 3 and verified from figure (3), that the hydrograph computed corresponding to test run no. 1 is able to reproduce the St. Venant's solution more closely than the solutions of test run nos. 2 and 3, except at the beginning of routing. The computed hydrograph dips in the beginning as observed by many researchers (Nash, 1959; Venetis, 1969; and Dooge, 1973) in the case of Muskingum flood routing method. The reasoning for this dip is explained at a later stage.

The hydrograph of test run no. 2, corresponding to the case of constant $\Theta$ and varying K does not reproduce the St. Venant's solution satisfactorily. The constant $\Theta$ value estimated for this test run was

-0.0497 and it was obtained using the expression given by equation (67) after freezing all the flow variable with reference to the reference discharge $Q_o$ which was computed as (Price, 1973):

$$Q_o = \frac{I_p + Q_p}{2} \quad \dots (85)$$

where

$I_p$ = the inflow hydrograph peak

$Q_p$ = the outflow hydrograph peak

The reasoning for the weighting parameter becoming negative is given later. Note that the value of $Q_p$ required for the comaputation of $Q_o$ was unknown corresponding to the constant $\Theta$ value and it was approximately considered as the peak value of the hydrograph obtained by routing the given inflow hydrograph for the same reach length using varying K and that $\Theta$ value which was computed from equation (67) based on initial flow conditions. It may be noted from Table - 3 that conservation of mass principle is grossly violated in this case when compared with the cases of test run nos. 1 and 3. However both test run nos. 1 and 2 reproduce equally well, the other characteristics of hydrographs such as error in peak flow and stage value, and the errors in time to peak discharges and stages.

Test run no. 3 which corresponds to the 2 sub-reaches solution, reproduces the St. Venant's solution both from the consideration of closeness and conservation of mass much better than the case of test run no. 2, but slightly poorer than the case of test run no.1. No dip at the beginning of the solution was found in this case unlike the case of test no.1 . Although the peak flow was slightly underestimated in this case

(672 $m^3$/sec when compared with 714 $m^3$/sec of test run no. 1), the other hydrograph characteristics were closely reproduced especially the peak stage. Attempts made to increase the number of sub-reaches more than 2 for routing using the suggested method resulted in computational problem due to high negative value of $\Theta$. From the overall considerations of results presented in Table - 3 for these three runs, it may be inferred that the routing solution obtained from the two sub-reaches consideration may be preferable than the other two cases especially for flood forecasting purposes. If particular importance is not attached to the slight under estimation of peak flow and increase in total outflow volume over that of inflow, then the results of test no. 3 may be considered as the best both for the purpose of flood forecasting and design flood estimation.

6.2.2 On the results of test run nos. (4), (5) and (6)

As seen from Table - 3, the variance explained by the solution approaches of test nos. 4 and 6 were greater than 99%. Similarly in both cases, the conservation of mass was well maintained ($<|0.63\%|$). However the multiple reach solution with 8 sub-reaches and $\Delta x$ = 5 Km., belonging to test run no. 6, performed well when compared with the results of test run no. 4 in reproducing the stage hydrograph. Note that the peak stage was differing from the true solution only by 0.04 m. when compared with 0.39 m of test run no.4. The variance explained by the solution procedure of test run no. 5 is less than that of the other two cases, although the difference is not significant. However from the consideration of conservation of mass, this test case performed poorly than the other two cases. In this aspect, the performance was similar to that of test run no. 2 which also used the varying K and constant $\Theta$ solution approach in arriving at the routed hydrograph at 40 Km. Therefore the routing of steep rising inflow hydrographs such as in the cases of test run no. (2) and (5) in verv flat streams using constant $\Theta$ and varying K based solution approach may not yield appropriate results. However further studies are required to arrive at any definite conclusion about this statement.

From the over all considerations of test run nos. (4) , (5) and (6), one may prefer again the multiple reach based solution allowing both the parameters K and $\Theta$ to vary.

6.2.3 On the results of test run nos. (7), (8) and (9), and (10), (11) and (12)

In all these runs the variance explained by the different solution approaches were greater than 99% with the absolute maximum error in the conservation of mass being 0.47 %. All the other hydrograph characteristics were very well reproduced. These test runs results indicated that there was no significant difference between the results of variable parameters solution approach in which both $\Theta$ and K varying, the solution approach based on the variation of K only keeping $\Theta$ constant, and number of sub-reaches solution approach considering the variation of both $\Theta$ and K. As will be discussed later, that there exists no significicnt variation of $\Theta$ values corresponding to the given inflow ordinates for the test run nos. (7) and (9) of channel type-3, and test run nos. (10) and (12) of channel type-4. In these cases the value of $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ was nearer to zero indicating that the flood wave is of kinematic in nature. This

inference has been verified by figures (7) and (9) as there was very little attenuation of flood peaks in these cases. It may be inferred from the closeness of the solutions shown by figures (7) and (9) that the method suggested herein may be used for kinematic routing of flood wave in long reaches in a single step routing.

6.2.4 On the results of test run no. (13)

This test on channel type-1 was conducted with the aim of assessing the validity of the inference arrived through equation (18) that the discharge at any instant of time along the routing reach varies linearly. As mentioned in section 5.2, the minimum length of routing reach required for this case was 22 Km. for the successful routing using the solution approach based on the variation of both $\theta$ and K. The hydrograph may be obtained at any section of the channel reach within 22 Km. only by linear interpolation of the inflow hydrograph and the routed hydrograph available at 22 km. using this solution approach. The hydrograph so obtained in test run no. (13) at 5 Km. from the inflow section compares well with the corresponding St. Venant's solution for the discharge hydrograph, as shown in figure (19). However the interpolated stage hydrograph at this section, as shown in figure (20), based on the inflow stage hydrograph and the corresponding computed outflow stage hydrograph at 22 km. using this procedure , shows a poor comparison with the St. Venant's solution especially after the peak stage. This difference may be considered significant at such a short reach of 5 km. However as discussed in section 6.2.1 based on test run no. 3. and shown by figure (4), the solution for the stage hydrograph at 40 km. for the same channel and based on the procedure of two sub-reach solution and subsequent interpolation was found comparable with the St. Venant's solution. This only suggests that for channel type-1, the assumption of linear variation of water surface as implied through assumption (4) may be more valid at a point located long distance from the inflow section and when the solution at this distance is obtained through multiple reach routing consideration. This is important as the single reach solution based stage hydrograph at 40 km. for the same channel was also poorly estimated when compared with the corresponding St. venant's solution hydrograph as shown in figure (4). A further discussion on this aspect will be covered at a later stage while discussing the importance of the factor $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ , the dimensionless water surface slope.

6.2.5 On the results of test run nos. (14) and (15)

These tests conducted on channel type-2 were aimed at the verification of interpolation solution obtained at 5 km. distance from the inflow point using the hydrographs at the inflow section and the computed outflow hydrographs obtained based on single reach routing solution at 40 km. The verification was made by comparing the interpolation solution at 5 km. with the corresponding direct routing solution based on the same solution approach. It can be seen from table-3 that the results of these runs are comparable to each other and also they are well comparable with the St.venant's solution. The same may be verified from figures (21), (22), (23) and (24). It may be inferred from the results of test run nos. (14) and (15) that the unsteady flow solution required at any section of the reach may be obtained by linear interpolation of the inflow hydrograph and the resulting routed outflow hydrograph of long reach obtained in a single step solution. Detailed studies may be required to verify on this aspect. This interpolation approach replaces the number of tedious routing computations for short reaches. The basic difference between test nos. (13), (14) and (15) is with reference to the value of Manning's roughness coefficient of the channel. While $n = 0.04$ for test run no. 13, it was

0.02 for the latter test runs. Therefore the possible reason for the success of direct routing of inflow hydrograph using the variable parameter approach in channel type-2 for a short distance of 5 km, when compared with the required minimum direct routing length of 22 km of test run no. (13) for channel type-1, may be attributed to the reduction in the roughness coefficient which indirectly causes the reduction in the magnitude of water surface slope, $\frac{\partial y}{\partial x}$ and thus making it possible, without creating computational problem due to negative value of $\theta$ to use short routing reaches for direct routing when both $\Theta$ and K are varying.

6.2.6 On the variations of K and $\theta$

Variation of K :

Figures (11), (12), (13) and (14) show the variations of the travel time parameter K at each routing time level with reference to the corresponding time level inflow ordinates for the cases of test run nos. (1), (4), (7) and (10). The purpose of relating K with the inflow hydrograph ordinates is to assess the real variation of K for all channel configurations studied standing on a common platform such as the inflow hydrograph which is not influenced by the outflow information based on this method. Note that in all these cases the reach length $\Delta x$ was fixed as 40 km. It can be seen from these figures that for all the cases the travel time corresponding to the same inflow discharge decreases as the order of channel type increase which implies that the velocity increases with the increase in the order of channel types. The reduction in the magnitude of K in the case of channel type-2 when compared with chananel type-1 is solely due to reduction in Manning's roughness coefficient to 0.02 in case of channel type-2, when compared with the corresponding value of 0.04 in the case of channel type-1. As indicated by the figures (13) and (14) the increase in the bed slope also causes increase in the velocity. Therefore this discussion confirms that the physics of the open channel flow, i.e. the decrease in roughness coefficient or increase in bed slope or both cause increase in the velocity of flow, is closely followed by the methodology presented herein.

Variation of $\Theta$:

Figures (15), (16), (17) and (18) show the variations of the weighting parameter $\Theta$ at each routing time level with reference to the corresponding time level inflow ordinates for the cases of test run nos. (1), (4), (7) and (10). Before discussing these results, it is necessary to look into the aspects of the variation of $\theta$ from the physical point of view.

As given earlier, the weighting parameter $\Theta$ is expressed as :

$$\Theta = \frac{1}{2} - \frac{l}{\Delta x} \qquad \dots (41)$$

With reference to figure (2), $\theta$ represents the non-dimensional distance between sections (3) and (2). Using equation (41) , the variation of $\theta$ ca.. be studied.

When section (3) lies between the mid section and the outflow section of the routing reach, $0<\Theta< 0.5$. When section (3) coincides with section (2), then $\Theta = 0$ as in the Kalnin-Milyukov method.

However if the routing reach length is such that section (2) is located ahead of section (3), in which case $l > \frac{\Delta x}{2}$ , the value of $\Theta < 0$. When such a situation occurs during the routing process using this procedure, the outflow discharge magnitude would be greater than the normal discharge $Q_3$ as observed at section (3).

This situation was experienced in test run no. (15) in which the $\theta$ values corresponding to each time level of routing was negative and thus the outflow discharge was greater than the normal discharge $Q_3$ at all the time levels of routing. Figure (23) shows the discharge hydrograph results of test run no. 15, in which the single reach solution obtained by varying both $\Theta$ and K is plotted along with the St.Venant's solution. The corresponding normal discharge hydrograph is also shown therein. It can be seen from this plot, that the outflow discharge hydrograph is observed ahead of normal discharge hydrograph confirming the interpretations based on equation (41). It was observed that $-2.2763<\Theta<-0.0692$ for this case. Althoguh the possibility of $\Theta< 0$ was indicated by Dooge (1973) , the argument in favour of $\Theta$ becoming negative from physical point of view has been put forwarded by Strupczewski and Kundzewicz (1980). Note that the value $\Theta< 0$ does not have any meaning in the case of Muskingum Cunge method as it is considered as the numerical weighting factor with $0<\Theta< 1$. From the point of view of numerical mathematics as generally understood for the flood routing application $\Theta \nless 0$. Therefore the reasoning given herein for $\Theta < 0$ makes the present theory more attrative than any other theories presented so far on the Muskingum flood routing method.

When section (3) coincides with the mid section, i.e., $l = 0$ and this leads to $\Theta = \frac{1}{2}$ . This represents the situation in which the normal discharge coincides with the normal depth at the mid section of the reach and thus leading to the Kinematic flood wave movement.

The situation wherein $\Theta > 0.5$, implies the location of section (3) upstream of mid section of the routing reach and based on the physical basis of the model, i.e., the discharge precedes the corresponding steady flow stage in unsteady flow situation, the change of direction of flow could be realized. Accordingly, the computed hydrograph at the outflow section i.e., at section (2) would be the amplification of the inflow hydrograph. Explanations on the basis of various considerations are also available for $\Theta > 0.5$ by other researchers (Cunge, 1969; Dooge, 1973; and Strupczwski and Kundzewicz, 1980).

It can be seen from figures (15) and (16) which belong to test run nos. (1) and (4) respectively, that the variation of $\Theta$ w.r.t. inflow ordinates are wider, with $\Theta<0$ occurring in test run no. (1). However for test run nos. (7) and (10) , the variation of $\Theta$ was not very much and their values were also found to be nearer to 0.5. These variations are brought out in figures (17) and (18).

It can be inferred from these variations that when the term $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ is small and its variation is not significant then the value of $\Theta$ is nearer to 0.5 and its variation is not significant. But when the magnaitude of $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ is large and varies much, it causes wider variations in the value of $\Theta$ including the possibility of $\Theta$ values becoming negative as shown in figure (15). An understanding of these variations as explained above can be obtained from equations (30) and (31). The term $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ is inversely proportional to the velocity of flow and therefore, higher magnitude of $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ implies lower magnitude of velocity, $v_m$ which in multiplication with $S_o$ results in the higher value of $l$ the distance between the mid section and section (3) of the routing reach. Thus the magnitude of $\Theta$ will be much less than 0.5. When $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ is nearer to zero, there is increase in the velocity and this causes decrease in the value of $l$ . Thus the magnitude of $\Theta$ will be nearer to 0.5. The typical values of the term $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ as calculated using this methodology for test runs nos. (1), (4), (7) and (10) have been tabulated below.

Table - 4

Typical Values of $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$

| Test Run No. | Channel Type | Length of Reach | No. of Reaches | Magnitude of $\frac{1}{S_o} \cdot \frac{\partial y}{\partial x}$ Minimum | Maximum | Remarks |
|---|---|---|---|---|---|---|
| 1 | 1 | 40 km | 1 | -0.8188 | 0.6199 | single reach |
| 3 | 1 | 44 km | 2 | -0.9140 | 1.1250 | first reach |
| | | | | -0.8827 | 0.3463 | second reach |
| 4 | 2 | 40 km | 1 | -0.4566 | 0.2800 | single reach |
| 6 | 2 | 40 km | 8 | -0.5243 | 0.3949 | first reach |
| | | | | -0.5369 | 0.3600 | second reach |
| | | | | -0.5483 | 0.3324 | third reach |
| | | | | -0.5580 | 0.3095 | fourth reach |
| | | | | -0.5656 | 0.2904 | fifth reach |
| | | | | -0.5713 | 0.2740 | sixth reach |
| | | | | -0.5767 | 0.2598 | seventh reach |
| | | | | -0.5811 | 0.2471 | eight reach |
| 7 | 3 | 40 km | 1 | -0.0349 | 0.0151 | single reach |
| 10 | 4 | 40 km | 1 | -0.0156 | 0.0064 | single reach |

It can be seen from Table - 4 that the typical value of $\frac{1}{S_o} \frac{\partial y}{\partial x} > 1$ for test run no. (3) and for this situation the binomial series expansion is strictly not valid even though the results obtained are not very poor from the true values. Further, it can be seen as the order of channel type increases, the typical values of $\frac{1}{S_o} \frac{\partial y}{\partial x}$ become less and less indicating that the attenuation causing factors do not have any role to play in the routing process.

It was observed while discussing the results of test run no. (3) in section 6.2.1 that the two sub-reach solution with both $\Theta$ and K varying resulted in the stage hydrograph much closer to that of St. Venant's solution when compared with the case of single reach solution with both $\Theta$ and K varying. This is due to the assumption of linear variation of water surface is closely followed in two sub-reaches solution case than in the case of single reach solution. Therefore to follow the assumption of line r variation of $\frac{\partial y}{\partial x}$, it is necessary to sub-divide the reaches into smaller reaches. At this juncture one may arise the question that why the discharge hydrograph of test run no. (3) was not properly estimated in the case of two sub-reaches solution when compared with the discharge hydrograph of single reach solution. The reason may be attributed to the magnitude of $\frac{1}{S_o} \frac{\partial y}{\partial x} > 1$ as observed in the first reach of the two sub-reaches solution, thus invalidating the solution of discharge hydrograph from the first reach. When this hydrograph is routed along the sub-reach, the resulting hydrograph is pooorly estimated than the single reach solution. From these discussions one can infer that the assumption of linear variation of discharge is more valid than the assumption of linear variation of flow depth for a longer routing reach.

6.2.7 On the cause of dip in the beginning of solution

This physically based routing method enables to ascertain the cause of negative or reduced or dip in the beginning of solution of the Muskingum flood routing method in the following manner :

The governing unsteady flow equation of the Muskingum method is given as :

$$I - Q = \frac{d}{dt}[K(\theta I + (I-\theta)Q] \qquad \ldots (58)$$

Multiplying both sides of equation (58), by $(1-\theta)$ gives :

$$I-(\theta I + (1-\theta)\,Q) = \frac{d}{dt}\,[K(1-\theta)\ (\theta I + (1-\theta)Q] \qquad \ldots (86)$$

But the expression $\theta I + (1-\theta)Q$ is same as $Q_3$, the normal discharge. Therefore equation (86) is re-written as :

$$I-Q_3 = \frac{d}{dt}[K(1-\theta)Q_3] \qquad \ldots (87)$$

The solution of equation (87) assuming K and $\theta$ to be constant, yields :

$$Q_3 = \frac{e^{-t/K(1-\theta)}}{K(1-\theta)} \int_o^t Ie^{\tau/K(1-\theta)}\, d\tau + I_o e^{-t/K(1-\theta)}$$

when $I = I_o$ at $t = 0$ ... (88)

$$Q_3 = \frac{e^{-t/K(1-\theta)}}{K(1-\theta)} \int_o^t Ie^{\tau/K(1-\theta)}\, d\tau$$ when $I=0$ at $t=0$ ... (89)

Equation (88) and (89) indicate that at section (3), $Q_3 = I_o$ and $Q_3 = 0$ respectively when $t = 0$. Since the discharge varies linearly along the reach from $t = 0$ onwards, this leads to a discharge less than $I_o$ or 0 at section (2) when it is located downstream of section (3) for which case $0 \leq \theta < 0.5$.

The discharge at section (2) would be always greater than the initial steady flow if it is located upstream of section (3) for which case $\theta<0$ . The above inference arrived based on constant $\theta$ and K is also valid for variable K and $\theta$. Note that when $l$ is small and section (2) is located for away downstream of section (3), then such a situation leads to dip or negative flow in the beginning of routing. The larger distance between sections (2) and (3) is due to longer reach considered for routing. This aspect has been brought out by the results of test run nos. (1), (4), (7) and (10) wherein the routing was carried out by considering 40 km. length of the channel as a single reach. The respective discharge and stage hydrographs plotted in figures (3) - (9) show the dip in the beginning of the solution.

The magnitude and duration of this dip depends on the magnitude of the term $\frac{1}{S_o} \frac{\partial y}{\partial x}$ as explicitly brought out by equation (3). When the magnitude of the term is high, then the magnitude and duration of the dip increases. This inference can be verified from the typical values of $\frac{1}{S_o} \frac{\partial y}{\partial x}$ given in Table-4 for runs (1), (4), (7) and (10), and from the respective stage and discharge hydrographs given in figures (3) - (10) . The hydrograph solutions obtained for the above mentioned runs and for the same length of reach, after dividing it into sub-reaches are also depicted in figures (3) - (10). These solutions indicate no dip in the beginning of routing and thus confirm the above inference arrived regarding the formation of dip and its elimination.

## 7.0 CONCLUSIONS

1. A variable parameter simplified hydraulic method has been developed for routing floods in channel reaches having uniform rectangular cross section and constant bed slope.

2. The governing equations of this method which describe the flood wave movement in channels are same as that of Muskingum flood routing method introduced by McCarthy (1938), and it has been demonstrated using this method that these equations can directly account for flood wave attenuation without attributing to it the numerical property of the method as stated by Cunge (1969) . Therefore this method gives a new insight into the theoretical aspects of the Muskingum flood routing method.

3. The parameters $\Theta$ and K of the Muskingum method have been related to the channel and flow characteristics.

4. he nonlinear behaviour of flood wave movement in channels having uniform rectangular cross section may be modelled using this method by varying the parameters $\Theta$ and K at every routing time level, but still adopting the linear form of solution equation.

5. There exists a minimum routing reach length for which this method with both $\Theta$ and K varying can be applied successfully without experiencing computational problem due to high negative value of $\Theta$ .

6. The flood routing solution in reaches having length less than the above mentioned minimum reach length, can be obtained by

linear interpolation of discharge and stage hydrographs of given inflow hydrographs and the corresponding computed outflow hydrographs obtained at the location of minimum reach length using this variable parameters method.

7. In general, the method in which both $\Theta$ and K varying along with multiple routing reaches consideration is able to reproduce the true solution much closer than the method in which both $\Theta$ and K varying, but with the consideration of single routing reach.

8. In general, the method in which both $\Theta$ and K varying is able to reproduce the true solution much closer than the method in which only K varying and $\Theta$ remaining constant.

9. However when the relative water surface slope $\frac{1}{S_o} \frac{\partial y}{\partial x}$ is very small, there is no difference between the solutions obtained using the method in which both $\Theta$ and K varying, and the method in which only K varying and $\Theta$ remaining constant.

10. As there is no standard definition of "small" and "larg applicable with regard to the magnitude of the relative water surface slope $\frac{1}{S_o} \frac{\partial y}{\partial x}$, it is always desirable to use this routing method with the consideration of multiple routing reaches, and both parameters $\Theta$ and K varying in each reach routing.

11. The higher the absolute magnitude of the relative water surface slope $\frac{1}{S_o} \frac{\partial y}{\partial x}$, the higher the values of travel time K and their variation,

12. The higher the absolute magnitude of the relative water surface slope $\frac{1}{S_o} \frac{\partial y}{\partial x}$, the higher the variation of weighting parameter $\theta$.

13. The weighting parameter $\Theta$ would be negative when section (2) is located upstream of section (3) at any instant of time during routing.

14. The cause of reduced outflow in the beginning of routing solution of the Muskingum method is due to the linear variation of discharge considered by the method over the routing reach and due to longer routing reach length $\Delta x$ considered for routing.

15. The magnitude and duration of reduced outflow is directly proportional to the magnitude of the relative water surface slope $\frac{1}{S_o} \frac{\partial y}{\partial x}$, and the length of routing reach $\Delta x$.

16. To avoid this reduced outflow theoretically, the routing reach should be divided in such a manner that section (2) is located upstream of section (3) for each considered sub-reach.

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