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STUDY OF PARAMETERS AFFECTING BASE-FLOW

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LIST OF SYMBOLS

A	parameter
a	constant
B	parameter
B'	constant discharge
b	constant
F	area of watershed
f	transformed time
H, H_1	saturated thickness/initial saturated thickness
h, h_2, h_4	height of water table above the datum
h_1, h_3	height of water level in the river above the datum
K	coefficient of permeability
K'	constant
K_r	$\exp(-\alpha)$, recession constant
L'	length of main river
L	length of the aquifer
m	an integer
m'	constant exponent
N	an integer
n	an integer
n'	constant exponent
Q	discharge at time t
$Q_r(N)$	flow rate at time step N
Q_0	initial discharge at time t=0
Q_1, Q_2	initial discharges for two components at time t=0
T	transmissivity of the aquifer
T'	transformed time

t	time
V'	groundwater runoff volume
V	storage volume at time t
α	recession constant/parameter
α_1 α_2	recession constants for two components
β	recession constant/parameter
u	recession constant
σ	recession constant
π	constant
ϕ	storage coefficient of the aquifer

LIST OF FIGURES

ABSTRACT

Baseflow computation plays an important role for assessment of lean flow in a river. During passage of flood in a river the river stage rises which leads to recharging of the aquifer. Besides the aquifer also gets recharged by rainfall. The changes in river stages are abrupt whereas the changes in the water level in an aquifer at some distance away from the river are slow. In the present report the baseflow has been computed for generally observed river stage and groundwater table hydrographs. It is assumed that the changes in river stage occurs during the monsoon period during which the water table rises exponentially. Beyond monsoon the water table decays in a exponential manner and the river stage remains constant more or less. In the present analysis the baseflow has been computed for such variations.

One dimensional Boussinesq's equation has been solved and Duhamal's approach has been applied to find the aquifer response for varying river stages. The methodology provided can be used to predict the baseflow for any pattern of changes in the river stage and the water table. It is found from the study that the baseflow will be sustained for longer periods by aquifers having higher T and ϕ values. Also it is found that the baseflow does not follow an expenential decay curve.

1.0 INTRODUCTION

Water reaching an outlet to the surface, at any region of a watershed and originating from groundwater is called baseflow or groundwater runoff. The ideal baseflow is defined as the flow to the stream from depletion of the unconfined aquifer when the factors such as evapotranspiration, leakage upward from them, recharge from rain or irrigation water and pumpage or artificial recharge are not operative (Singh, 1968). The base flow supply of river runoff is formed by the main body of groundwater being drained by the river valley, as well as by the descending source of constant supply (Chebotarev, 1966). Baseflow is one of the basic sources of ground water supply to rivers and represents a natural regulator of surface runoff. Together with artesian supply, baseflow supply ensures minimum runoff in rivers during the summer and winter which are usually low water seasons. It is responsible not only for the seasonal, but also for the annual and long term fluctuations in runoff.

The amount of groundwater runoff depends on a series of variable factors that could be classificed as rapidly and slowly varying factors. From the first group precipitation depth and intensity, atmospheric pressure, temperature, moisture deficiency, wind velocity, air humidity etc. are important, and to the second group belong: climate, area of watershed, geological conditions of the soil, topography, drainage density, depth of river valley, hydraulic slope and vegetation cover. Slowly varying factors cause by their very nature only slow changes in the groundwater runoff which are noticeable only after a very long period.

Assuming uniform climatic and geological conditions within a given watershed, there may be established some relationship between the groundwater runoff (V') and the length of the main river of the basin. We may write

$$V' = AL^{\alpha} \qquad \ldots(i)$$

The basin area may be expressed as:

$$F = BL^{\beta} \qquad \ldots(ii)$$

where L' = length of main river,

A, B, α = parameters

For a given length, we find from equation (ii) that

$$L = \left(\frac{F}{B}\right)^{\frac{1}{\beta}} \qquad \ldots(iii)$$

Inserting equation (iii) into equation (i) we obtain

$$V' = \frac{A}{B^{\alpha/\beta}} \cdot F^{\alpha/\beta} \qquad \ldots(iv)$$

The exponent α should be larger than β since it characterizes a three-dimensional magnitude, whereas β refers only to a two-dimensional value. Consequently the ratio $\alpha/\beta \geqslant 1$ and this points to an increase in the low-water runoff with the increase in area of the watershed F. The higher the ratio α/β , the more pronounced will this relationship be. According to Chebotarev (1966), the discharge of groundwater runoff is proportional to the basin area to the first power, i.e. $\alpha/\beta = 1$. For a maountain river the area of the watershed loses its importance as a determining factor because the area of a watershed has a definite relationship with the altitude.

Depth of precipitation, velocity of overland flow, and type and location of geological strata are related to the nature of the relief in the watershed. The drainage density determines the frequency of stream channels: the larger the density, the smaller is the amount of groundwater flow per unit length of river. The depth of river valley or the depth of cut into the strata is an important factor of groundwater runoff: the slope and thickness of the water-bearing layer which supplies to groundwater table, increase with the increase in the depth of the river valley. This is confirmed by Norvatov (1966) who assumed equality between the modulus of low-water runoff and the modulus of groundwater runoff(which is inexact).

The vegetation cover, particularly forests exert a regulating effect on runoff, increasing the groundwater flow and decreasing flood flow.

Fig.1 is a schematic illustration of the type of baseflow hydrograph that results from a hydrologic event of sufficient magniture to exert a basin-wide influence on the water table. Baseflow rates must lie between D_{max}, the maximum possible baseflow, which would occur under conditions of a fully saturated basin, and D_{min}, the minimum likely baseflow which would occur under conditions of the lowest recorded water table configuration.

The streamflow hydrographs reflects two very different types of contributions from the watershed. The peaks, which are delivered to the stream by overland flow and subsurface stormflow, and sometimes by groundwater flow, are the result of a fast response to short term changes in the subsurface flow systems in hill slopes adjacent to channels. Secondly, the baseflow, which is delivered to the stream by deeper groundwater flow, is the result of a slow response to long term changes in the regional groundwater flow systems. If a stream discharge is plotted on a logarithmic scale with time, the recession portion of the baseflow curve very often takes the form of a straight line or a series of straight lines. The equation that describes a

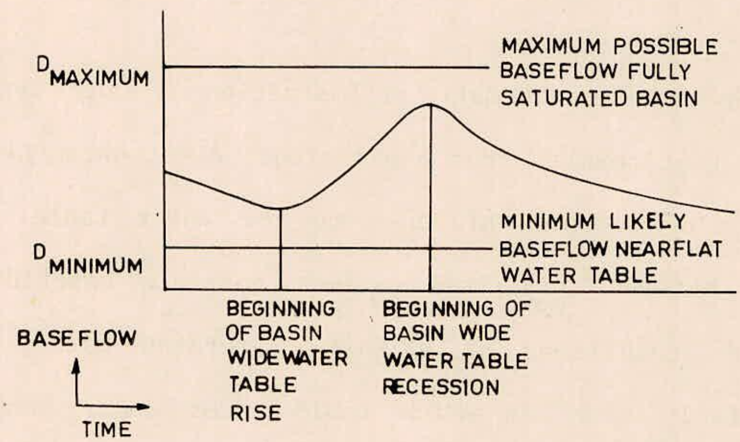

FIGURE 1. SCHEMATIC DIAGRAM OF BASEFLOW HYDROGRAPH

straight line recession on a semilogarithmic plot is given by

$$Q = Q_o e^{-\alpha t} \qquad \ldots(v)$$

where Q_o is the baseflow at time t=0 and Q is the baseflow at a later time t and α is the recession constant.

In the present study it has been aimed to find out whether such relation as given in equation (v) really exists or not and also to find out the factors that influence the constant α such as transmissivity, storage coefficient etc.

2.0 REVIEW

Many authors have discussed particular aspects of baseflow, but few have undertaken comprehensive reviews of the broader features. Horton(1933b) in a classic paper not only discussed his own contributions but also reviewed earlier work on baseflow in the united States and Europe. In an investigation of rainfall and runoff, Hoyt et al.(1936) described methods for determining baseflow and included an extensive list of references to baseflow for the period 1893 through 1934. An annotated bibliography on parametric hydrology covers the period 1921 through 1967 (Dickinson et al.,1967). The report containing the bibliography also has a brief review of hydrograph separation methods.

Fairly complete discussions of baseflow equations, mathematical derivations, and applications are given in recent French works by Schoeller (1962), Roch (1963), and Castany (1967). A compilation of a number of equations used in baseflow studies and a discussion of methods of hydrograph analysis and application have been presented by Toebes and Strang (1964). In a study of stream connected aquifer systems, Spiegel (1962) reviews some of the early work on baseflow and gives an extensive mathematical treatment with emphasis on leaky aquifer theory.

Modern interest in baseflow recession can be traced back at least to the 1840's and the law of Dausse'(Dausse, 1842), which states, as interpreted by Horton (1933a), that "there is no accretion to the water table as long as water losses exceed the rainfall". The nature of the hydrologic cycle was becoming well understood, and much of the basic mathematics and some methods of hydrograph analysis were known by the early 1900's when Maillet (1902, 1905) began publishing the results of studies on the Vanne, a major supply of water for Paris. He obtained recession curves for various 'sources' and fitted equations to them.

Problems similar to those in France were beginning to arise at about the same time in the eastern United States. Vermeule (1894) and Horton (1903) began investigations of runoff and low flow in New Jersey and New York, respectively. Vermeule took essentially a hydrologic budget approach to developing his 'Diagram Showing Ground-water Flow of Various Streams for a Given Depletion', which is a form of what would now be called a storage-discharge curve. Horton analysed hydrographs and obtained recession curves for a number of streams. He also began to consider the mathematical aspects of base-flow, but he did not begin to publish on this phase of his work until 1914. In contrast to the French, very little mathematical application of development, with the exception of Horton's work, came from America during this early period. The reason may well be that the Americans were dealing with streams fed by shallow, unconsolidated aquifers that responded to summer rains. On the other hand, the French had smooth long term recession curves from streams fed by extensive aquifers that were relatively unaffected by summer rains.

Early work in Great Britain does not appear to have lagged very far behind that in France and the United States, and it appears to have been in response to similar problems. Beardmore (1862) realized that summer low flows were supplied by groundwater, and Hall(1918) presented a paper on components of the hydrograph and methods of hydrograph separation. For much the same reasons comparable efforts were also under way elsewhere in Europe and Japan (Horton, 1933b, Forchheimer, 1930, Iwasaki, 1934, Roessel,1950).

The basic differential equation governing flow in an aquifer was presented by Boussinesq (1877). The equation is nonlinear and difficult to solve exactly. Boussinesq linearized the equation by making simplifying assumptions, and the result was a form of the heat flow or diffusion equation that can be solved more readily. His linear solution is used widely in baseflow work either as equation 1 or in the alternative forms (1a) or (1b).

$$Q = Q_o \exp(-\alpha t) \qquad \text{...(1)}$$

$$Q = Q_o K_r^t \qquad \text{...(1a)}$$

$$Q = Q_o (10)^{-\beta t} \qquad \text{...(1b)}$$

In a series of papers published during 1903 and 1904, of which only the most comprehensive is cited herein, Boussinesq (1904) further developed his linear solution and introduced one nonlinear solution for the case where a stream is located on a horizontal, impermeable lower boundary with an initial curvilinear water table and zero water-level elevation in the stream.

$$Q = Q_o/(1+\sigma t)^2 \qquad \text{...(2)}$$

This equation has been used for many studies in Europe, especially for spring discharge, but rarely in the United States.

The first applications of equations 1 and 2 appear to be by Maillet. He published a series of papers in 1902 and 1903, of which only one is cited herein, and a book in 1905. In the book, Maillet (1905) demonstrated the applicability of equations 1 and 2 and gave a number of other cases for different boundary conditions based on his own and Boussinesq's work. He discussed steady-state flow, problems of stability of flow, influence of basin size and geometry, and the effects of antecedent precipitation. Also, he made an early application of the correlation method of finding recession curves.

One difficulty with many recession curves obtained from hydrograph is that although they are nonlinear they do not fit equation 2. Maillet (1905, and In Boussinesq, 1904) coped with this problem by assuming two components or sources of baseflow one constant and one declining either as

$$Q = (Q_o - B')/(1 + \sigma t)^2 + B' \qquad \ldots(3)$$

or

$$Q = (Q_o - B')\exp(-\alpha t) + B' \qquad \ldots(4)$$

but Boussinesq (1904) showed that a recession fitted by equation 3 could be given equally well by

$$Q = Q_1 \exp(-\alpha_1 t) + Q_2 \exp(-\alpha_2 t) \qquad \ldots(5)$$

Equations 3,4, and 5 show that a nonlinear recession curve can be decomposed into or obtained from combinations of linear or linear and nonlinear curves. Furthermore, the same nonlinear curve may be obtained from various combinations. Equations 4 and 5 are ofcourse examples of the principle of superposition of linear solutions, which is particularly useful because of the relative ease of manipulation of exponentials. For example, Barnes (1939, 1944) has separated hydrographs into the three linear components of baseflow, interflow, and direct runoff. Dooge(1960) and Kraijenhoff van de Leur (1958) have shown the advantages of using linear solutions to approximate nonlinear systems.

Storage volume can be obtained by integration of equations 1 through 5 between specific time limits. The results led Maillet (1905) to suggest that storage volume was a function of discharge. Horton (1935, 1936b, 1937) and Langbein (1938) have shown that a general relationship for channel storage is

$$Q = K'V^{n'} \qquad \text{...(6)}$$

Coutagne (1948) and Denisov (1961) assumed that such a relation should hold for baseflow and combined equation 6 with a simple inflow-outflow equation for periods of no recharge.

$$dV/dt + Q = 0 \qquad \text{...(7)}$$

For n'=1, the result is equation 1, and for n' $\neq$1, the result is a general nonlinear equation

$$Q = Q_o(1+\mu t)^{n'/(1-n')}, n' \neq 1 \qquad \text{...(8)}$$

Schoellar (1962), Coutagne (1948), and Denisov (1961) gave solutions for a number of values of n. One feature worth noting is that equations derived by Cooper and Rorabaugh (1963) and Rorabaugh (1964) for bank storage in an aquifer with infinite distance to the valley wall can be reduced to equation 8 for n' = -1 when other time terms drop out.

Equation 8 may be put in a form used for drainage of soil moisture (Richards et al.,1956), and more recently for unsaturated drainage in groundwater recharge and baseflow (Nixon and Lawless, 1960, Hewlett and Hibbert, 1963)

$$Q = aT^b \qquad \text{...(8a)}$$

Another nonlinear relationship was proposed by Horton (1933b, 1935), who believed that any one phreatic basin would have a linear response (in fact, he considered this to be a law), but that two or more contributing sub-basins would give a nonlinear curve. Horton suggested that two exponential curves could be added together as in equation 5, or that an equation of the form

$$Q = Q_o \exp(-\alpha f^{m'}) \qquad \text{...(9)}$$

could be used. Equation 9 is sometimes referred to as the Horton double exponential. Some writers have referred to equation 9 as empirical, but it can be derived from equation1 by a simple time transformation (Hall, 1968).

More elegant or more complete solutions of the Boussinesq differential equation have been derived in recent years, mainly by workers interested in drainage and bank storage. One advantage of the recent efforts, although they are to some extent repetitive of earlier efforts, has been the attention devoted to assessing the effects of simplifying assumptions (Brutsaert and Ibrahim, 1966, Butler, 1967, Cooper and Rorabaugh, 1963, Guyon, 1966, Maasland and Bittinger, 1963, Rorabaugh, 1964, Singh, 1968, Van Schilfgaarde, 1965, van Schilfgaarde et al.,1956, Werner, 1957, Werner and Sundquist, 1951).

Efforts to obtain and apply baseflow recessions are complicated by problems arising from the assumptions used in the mathematical development and from difficulties in interpreting the stream hydrograph. The equations are derived for flow from a single source or storage component, generally of unit width, under conditions of no recharge. Furthermore, the storage component is filled and allowed to drain without interruption or change. The real stream hydrograph, on the other hand, is an integrated curve of prior hydrologic events, as stated by Kraijenhoff van de Leur (1958).

Problems arise with all of the assumptions, but perhaps the most troublesome assumptions are that discharge comes only from one source and that there is no recharge during recession. Horton (1914) recognized that sources other than groundwater including lakes, marshes, snow and ice, and stream channel and bank storage could supply baseflow. In a detailed study of small basins Hursh and Brater (1941) pointed out that the various possible sources could have regular characteristic responses and should thus contri/- bute to the hydrograph in a determinable manner. If the responses were not regular, then hydrograph separation would be much more difficult. Brater (1940) also suggested that a quick stream rise could cause water to flow back into the aquifer, thereby creating a period of negative groundwater flow. Work by Todd (1954, 1955), Rorabaugh (1964), and Cooper and Rorabaugh (1963) not only has confirmed Brater's concept but has shown that considerable time may be required for the resulting bank storage to drain. In fact, a large part of baseflow may be supplied by bank storage (Kunkle, 1962, 1968, Meyboom, 1961).

Precipitation on stream channels, as well as direct runoff and interflow, affects channel storage (Hursh and Brater, 1941). However, during periods of minor recharge channel storage should be a function mainly of seepage inflow along the channel. Meinzer et al.,(1936) utilized this as a method of determining influent seepage between gaging stations.

Losses of streamflow by evapotranspiration, by underflow beneath the gaging station, by vertical leakage through semipermeable layers, or by groundwater moving through aquifers that discharge outside the basin, present difficulties in interpretation. The same is true ofcourse for groundwater inflow from another basin. Underflow and groundwater movement generally have been coped with by field investigations. Singh (1968) has discussed the effect on hydrographs where water leaks upward through a semipermeable layer. Evapotranspiration losses have been considered in more detail, most workers being concerned with the effect of evapotranspiration on the stream hydrograph (Miller, 1965, Singh, 1968, Troxell, 1936, Croft, 1948). Riggs (1953) and Whelan (1950) have demonstrated the value of obtaining recession curves for various times of the year as a method of assessing evapotranspiration losses. Langbein (1942) has used baseflow recessions to compute evapotranspiration losses. Another approach has been the use of seasonal fluctuations of the hydrograph to calculate daily withdrawals by evapotranspiration (Reigner, 1966, Tschinkel 1963).

The matter of whether an aquifer or other source of baseflow has a linear or nonlinear response must also be resolved. Riggs (1964) has shown, however, that combinations of two linear sources such as a large artesian aquifer with long response time and a water table aquifer with short response will yield nonlinear recession curves. On the other hand, it can also be shown (Hall, 1968) that one nonlinear recession curve may be fitted by at least four nonlinear equations such as (3),(5),(8) and (9). Furthermore, nonlinearity may be caused by factors not accounted for in the mathematics. Riggs (1964) and Ineson and Downing (1964) have studied the relationships between baseflow and groundwater. They conclude that nonlinearity can be a function of factors such as carry-over storage from a prior period of recharge, multiple sources, variations in areal pattern of recharge, channel, bank and flood plain storage, and evapotranspiration. These same authors also discuss the difficulties of determining whether what is observed on the hydrograph is baseflow, to say nothing of determining whether it is exclusively from groundwater.

Another problem, particularly in humid or subhumid areas, is that recharge may occur frequently. The major consequences depending on hydrologic and geologic conditions are that baseflow may be fed by pulses of recharge or by drainage of soil moisture. Roessel (1950) has shown that pulses of recharge induce a nonlinear response from an aquifer. Work by Hewlett (1961) and Hewlett and Hibbert (1963, 1967) indicates that in mountain watersheds in humid areas baseflow is supplied in part by soil moisture, which appears to drain in a nonlinear fashion according to equation 8a. Therefore, the stream hydrograph would probably be nonlinear too. Their work seems to cast doubts on the traditional separation of baseflow and interflow. Hewlett (1961) also suggests that the area supplying baseflow is not constant but is expanding or shrinking in response to the interactions between recharge, soil moisture, and precipitation. Therefore, baseflow as commonly defined may occur, strictly speaking, only in arid or semiarid areas, or where aquifers are relatively unaffected by precipitation during the growing season.

Hydrologists have long been aware that if baseflow is supplied by groundwater, then a relationship should exist between stream discharge and groundwater levels (Pochet, 1905). Ideally, analysis of baseflow recessions could yield a groundwater depletion curve for the drainage basin. Thomson (1921) made an early application of equation 4 to the recession of groundwater levels in an area where nearly all flow was in the subsurface. Harrold (1934) observed a good relationship between recession in a stream and water levels in a nearby well. The possible effect of maximum water level , before recession begins on the stream hydrograph was considered by Horton (1936a). Hursh and Brater (1941) attempted to relate baseflow to water level fluctuations in a small basin, and Merriam (1956) developed a relationship for a very large basin. Clark (1956) obtained a good relationship for dry weather flows, and he concluded that groundwater discharge was nearly constant and that variations in streamflow were due to changes in stream level and evapotranspiration close to the stream. Detailed treatments of the fluctuations of groundwater levels have been given by Jacob (1943) and Tison (1965).

Another application of baseflow recession has been the attempt to determine the relations between hydrologic and geologic parameters in a drainage basin. Such studies may also involve low flow forecasting, but usually the emphasis is on hydrologic or geomorphic interpretations. A consideration of the recession constants for the various baseflow equations shows that they are a function of transmissivity, specific yield or coefficient of storage, and a characteristic lenght (normally the distance from stream bank to valley wall). Langbein (1960) has indicated that the recession constant is a function of drainage density, and Carlston (1963, 1966) has attemted to correlate minimum flows with drainage density. Most attempts to relate these factors in real basins seem to have been unsatisfactory or inconclusive.

Studies of problems arising from multiple sources, localized and regional aquifer systems, connection between stream and aquifer or unusual climatic conditions have been made by Curtis (1966), Dingman (1966), Kilpatrick (1964), McGuinness et al.(1961), and Renard et al.(1964). Efforts to study the hydrogeology of drainage basins by use of recessions fitted to equation 1 have been made by Farvolden (1963) and Knisel (1963).

Some workers have preferred to use flow duration and frequency analysis rather than recessions. Cross (1949) showed that a flow in cubic feet per second per square mile that is exceeded 90% of the time was a reasonable criterion for dry weather flow in Ohio.

As the result of his observations, Cross also put the whole problem of hydrogeologic interpretations into perspective when he stated: 'It is concluded that streamflow records provide useful inferences to groundwater geology, but the converse is not true'. Applications similar to Cross and in some cases including frequency analysis for various geologic conditions have been made by Schneider (1957, 1965), LaSala (1967), Thomas (1966), and Hely and Olmsted (1963).

Studies by Lenz and Sawyer (1944), Durum (1953), Langbein and Dawdy (1964), and Gunnerson (1967) have indicated that good correlations may be obtained between stream discharge and chemical content of water. None of these authors was concerned directly with baseflow but their results suggest that chemical content could be used to find the amount of groundwater in base flow or to determine if base flow is from groundwater. Kunkle (1965, 1968) has used conductivity to estimate groundwater contribution to base flow, and Toler (1965a, 1965b) has used conductivity to determine quantity of baseflow from two different sources, as well as to determine total groundwater contribution.

The study of stream aquifer interaction with various boundary and initial conditions has done by many investigators. Singh (1969), Pinder and Saun (1971) have analysed the flow from an unconfined aquifer to drainage ditches corresponding to certain sequence of infiltration inputs constant within time. Skaggs (1975) has investigated the flow in an unconfined aquifer between two drains for time invariant evapotranspiration. the response of an unconfined aquifer bounded by two rivers, to time variant-- recharge and change in river stage has been found by Chandra et al (1979), using the solution of Boussinesq's equation for one dimensional seepage corresponding to uniform time invariant recharge and a step rise in river stage by making use of Duhamal's approach. The recharge rate and one of the two river stages are assumed to be exponentially decaying function of time. The variation of base flow with time resulting from recharge and river stage fluctuation are presented separately for different lengths of the aquifer and for different coefficients of transmissivity. The base flow due to simultaneous change in river stage and unsteady recharge can be found out from these results by algebraic addition.

The free-aquifer base flow curves have been derived by Singh (1969) using the Boussinesq equation. The following assumptions have been made during the analysis:

i) The aquifer is homogeneous and isotropic, it overlies a horizontal impervious layer.

ii) Inclination of the water table is low.

iii) The hydraulic gradient equals the slope of the water table and is invarient with depth at any given section.

iv) There is no recharge to or depletion from the water table because of infiltration, evapotranspiration, and leakage, etc.

Dimensionless baseflow curves for fully penetrating as well as partially penetrating streams have been obtained using a finite difference solution for initial elliptical as well as parabolic water table profile curves. It has been found that baseflow curves for an initial elliptic water table profile yield a little higher discharge than the initial parabolic profile curves. However, they became practically parallel for the dimensionless time greater than 0.3. Thus, the baseflow recession rates are only slightly affected by the assumption of different initial water profiles.

In the analysis the idealized baseflow curves are modified by variation in evapotranspiration, leakage downward into the underying artesian aquifers or leakage upward from them into the overlying free aquifer, and recharge from infiltration and deep percolation of rain water etc. With increase in dimensionless height of the water table or reduction in stream entrenchment, the baseflow recession steepens, and the magnitude of baseflow decreases.

Boundary conditions at the outlet end, such as height of the stream bed above the impervious layer, variation in stream stage for essentially a baseflow regime, and the relative magnitude of this variation as compared with the height of the stream bed are found to have an pronounced effect on the baseflow curves, which may vary from a straight line to pronounced curvature when plotted on a semi-logarithmic paper. The effect of downward leakage into the underlying aquifer and evapotranspiration can render the stream influent and thus dry it up in extended rainless periods.

Flow and head variations in stationary linear stream-aquifer systems have been obtained through application of the convolution equation (Hall, 1972). Four highly idealized cases involving finite and semi-finite aquifers with and without semipervious stream banks, are considered. Equations for the instantaneous unit impulse response function, the unit step response function, and the derivative of the unit step response function have been presented for each case. Head fluctuations in the aquifer due to an arbitrary varying flood pulse have been obtained for the cases involving a finite aquifer with and without a semipervious stream bank. Flow in and out of the aquifer at the stream bank has been determined. Head variations, and to a lesser extend flow variations, are apparently relatively insensitive to variations in aquifer diffusivity. This insensitivity suggests that perhaps less emphasis be placed on evaluation of transmissivity from a determination of diffusivity (unless coefficient of storage is known, Hall, 1972) and more attention be given to groundwater contribution to streamflow.

3.0 STATEMENT OF THE PROBLEM

A schematic diagram of a flow domain bounded by a river at one end is shown in Fig.2. An observation well is located on the right side at a distance L from the river. The river stage changes with time. The river stage and water table hydrographs are as shown in Figure 2.

It is required to find the following:

i) The exchange of flow that takes place between river and the aquifer at different time.

   a) when the aquifer is of infinite length

   b) When the aquifer is of finite length bounded by two rivers and

   c) when the aquifer is of finite length bounded by a river on one side and a no-flow boundary on the other.

ii) To find the validity of the commonly assumed baseflow equation $Q = Q_o e^{-\alpha t}$

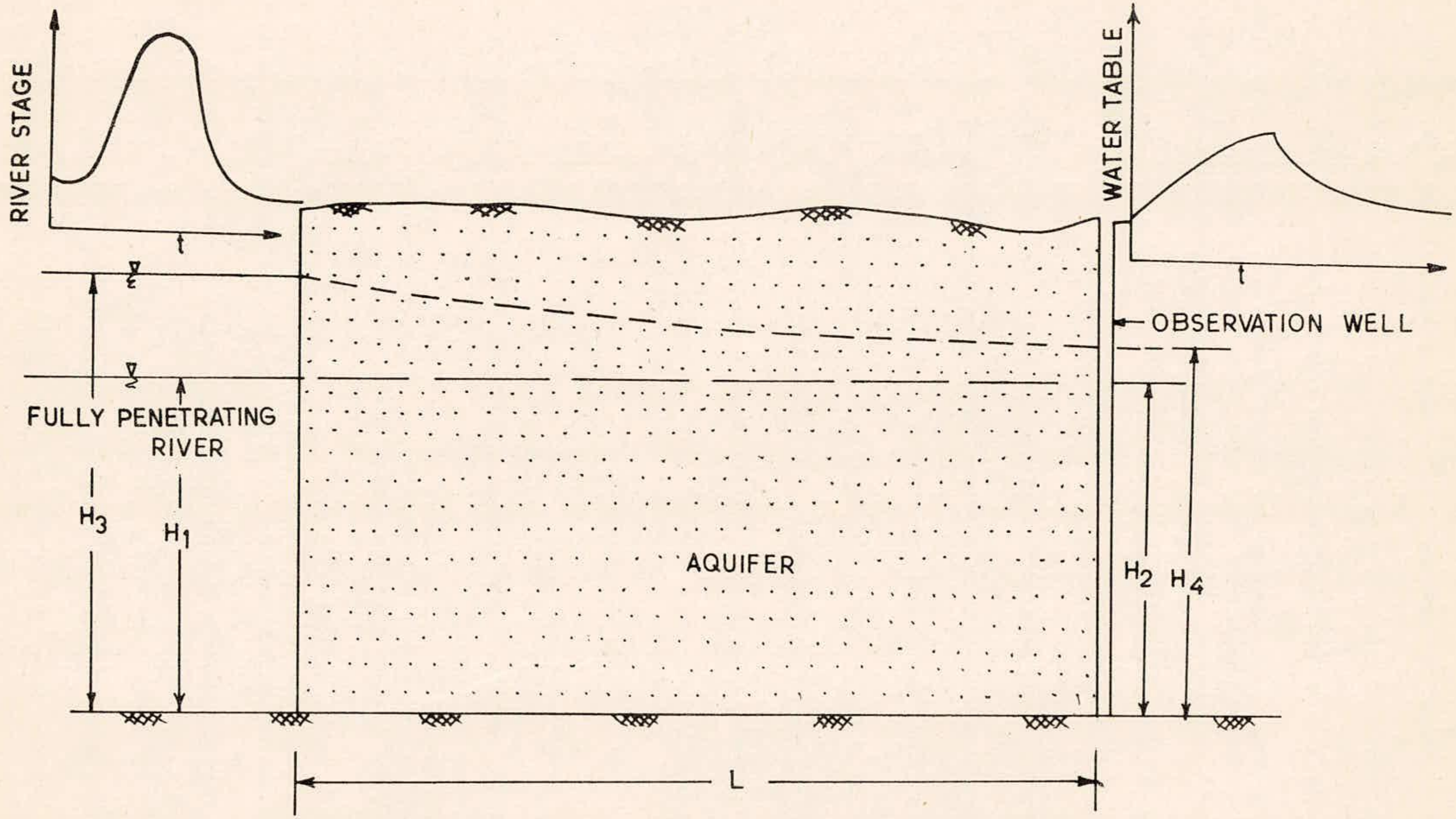

FIGURE 2. SCHEMATIC DIAGRAM OF PHYSICAL FLOW DOMAIN

4.0 METHODOLOGY

For a one dimensional flow situation it may be assumed that the equipotential lines are vertical hence the observation well can be replaced by a fully penetrating river whose hydrograph is same as that of the observation well. If there is a no-flow boundary at distance L from the river the flow can be simulated by -- assuming an aquifer of length 2L having identical boundary conditions on either side. The following assumptions are made in the analysis:

a) Aquifer is homogeneous and isotropic

b) River is fully penetrating the aquifer

c) The fluctuation in the water table due to change in river stage is negligible as compared to the saturated thickness of the aquifer

d) There is no recharge to or discharge from the aquifer

d) Dupuit-Forchhmier assumptions holds good.

The Boussinesq's equation describing one dimensional seepage in a homogeneous isotropic unconfined aquifer is given by

$$\frac{K}{\phi} \frac{\partial}{\partial x}\left(h \frac{\partial h}{\partial x}\right) = \frac{\partial h}{\partial t} \qquad ..(10)$$

in which

h = height of water table above the datum that is at the impermeable boundary,

ϕ = effective or drainable porosity, and

K = coefficient of permeability.

It is intended to solve the above differential equation for the following boundary and initial conditions:

Boundary condition;

$h = h_3$ at $x = 0$

and $h = h_4$ at $x = L$ ...(11)

h_3 and h_4 may change with time.

Initial condition;

$$h = [h_1^2 \ (1-\frac{x}{L}) + h_2^2 \ (\frac{x}{L})]^{1/2}, \quad 0 < x < L \qquad t \to 0 \qquad \ldots(12)$$

By using Dhumal's approach, solution to Boussinesq equation for unsteady river stages on either side can be obtained from the solution corresponding to step-rise in the river stages on either side.

The solution to Boussinesq's equation for the initial condition given by equation (12) and for the boundary conditions specified by equation (11), can be obtained (Carslaw and Jaeger, 1959) by making use of the substitution $u = h^2$ and it is found to be

$$h^2 = h_1^2 \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \frac{n\pi x}{L} \, e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}}$$

$$- h_2^2 \sum_{n=1}^{\infty} (-1)^n \frac{2}{n\pi} \sin \frac{n\pi x}{L} \, e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}}$$

$$+ h_3^2 \left[1 - \frac{x}{L} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \operatorname{Sin} \frac{n\pi x}{L} \, e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}}\right]$$

$$+ h_4^2 \left[\frac{x}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (-1)^n \sin \frac{n\pi x}{L} \, e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}}\right] \qquad \ldots(13)$$

It is aimed to find the exchange of flow between the river and aquifer which can be obtained from the relation

$$Q_r \ (t) = -Kh \frac{\partial h}{\partial x} \Big|_{x=0} \qquad \ldots(14)$$

$Q_r(t)$ +ve means the aquifer is being recharged.

$Q_r(t)$ = -ve means the flow is from the aquifer to the river .

Differentiating equation (13)

$$2h \frac{\partial h}{\partial x} = h_1^2 \sum_{n=1}^{\infty} \frac{2}{n\pi} \cos \frac{n\pi x}{L} \left(\frac{n\pi}{L}\right) e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}}$$

$$- h_2^2 \sum_{n=1}^{\infty} (-1)^n \frac{2}{n\pi} \cos \frac{n\pi x}{L} \left(\frac{n\pi}{L}\right) e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}}$$

$$+ h_3^2 \left[-\frac{1}{L} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \cos \frac{n\pi x}{L} \left(\frac{n\pi}{L}\right) e^{-\frac{n^2 \pi^2 Tt}{\phi L^2}} \right]$$

$$+ h_4^2 \left[\frac{1}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} (-1)^n \cos \frac{n\pi x}{L} \left(\frac{n\pi}{L} \right) e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right] \quad \ldots(15)$$

For $x = 0$,

$$h \frac{\partial h}{\partial x} \bigg|_{x=0} = \frac{h_1^2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 Tt}{\phi L^2}}$$

$$- \frac{h_2^2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 Tt}{\phi L^2}}$$

$$+ \frac{h_3^2}{2} \left[-\frac{1}{L} - \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right]$$

$$+ \frac{h_4^2}{2} \left[\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right] \quad \ldots(16)$$

Obtaining the value of $h \frac{\partial h}{\partial x}$ at $x=0$ from equation (16) and substituting it in equation (14) the flow rate entering into the aquifer is found to be

$$Q_r(t) = - \left[\frac{Kh_1^2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right] + \left[\frac{Kh_2^2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right]$$
$$+ \left[\frac{Kh_3^2}{2} \left(\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right) \right] - \left[\frac{Kh_4^2}{2} \left(\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 Tt}{\phi L^2}} \right) \right] \quad \ldots(17)$$

In equation (17) terms in the first and the second square brackets refer to responses attributed to the intial conditions (noise) and in the third and in the fourth brackets refer to the boundary conditions.

For varying river stages on either sides (i.e., at $x=0$, $x=L$) the expressions for flow is derived as follows:

Let the time be descretised by uniform time steps, let the river stages be assumed to be constant within a time step but vary from step to step. Let the time t is divided into N time steps then the recharge $Q_r(N)$ at the end of time step N can be written as

$$Q_r(N) = \sum_{\gamma=1}^{N} [h_3^2(\gamma) - h_3^2(\gamma-1)] \int_{\gamma-1}^{\gamma} \frac{K}{2} \left[\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 T(N-Z)}{\phi L^2}}\right] dZ$$

$$+ \sum_{\gamma=1}^{N} [h_4^2(\gamma) - h_4^2(\gamma-1)] \int_{\gamma-1}^{\gamma} -\frac{K}{2} \left[\frac{1}{L} + \frac{2}{L}\sum_{n=1}^{\infty}(-1)^n e^{-\frac{n^2\pi^2 T(N-Z)}{\phi L^2}}\right] dZ$$

$$-\left[\frac{Kh_1^2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 TN}{\phi L^2}}\right] + \left[\frac{Kh_2^2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 TN}{\phi L^2}}\right] \quad \ldots(18)$$

Let $Z-\gamma+1=v$, and $Z=v+\gamma-1$

Substituting the value of Z in equation (18)

$$Q_r(N) = -\left[\frac{Kh_1^2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 TN}{\phi L^2}}\right] + \left[\frac{Kh_2^2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 TN}{\phi L^2}}\right] +$$

$$\sum_{\gamma=1}^{N} [h_3^2(\gamma) - h_3^2(\gamma-1)] \int_{0}^{1} \frac{K}{2} \left[\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 T(N-\gamma+1-v)}{\phi L^2}}\right] dv$$

$$+ \sum_{\gamma=1}^{N} [h_4^2(\gamma) - h_4^2(\gamma-1)] \int_{0}^{1} -\frac{K}{2} \left[\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 T(N-\gamma+1-v)}{\phi L^2}}\right] dv$$

$$\ldots(19)$$

$$\text{Let } \partial_3(m) = \int_{0}^{1} \frac{K}{2} \left[\frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 T(m-Z)}{\phi L^2}}\right] dZ$$

$$= \frac{K}{2L} + \frac{K}{L} \int_{0}^{1} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 Tm}{\phi L^2}} e^{\frac{n^2\pi^2 TZ}{\phi L^2}} dZ \quad \ldots(20)$$

$$\text{Let } -\frac{n^2\pi^2 T(m-Z)}{\phi L^2} = U$$

$$dZ = du \frac{\phi L^2}{n^2\pi^2 T}$$

Substituting Z by the new variable U in equation (20) and integrating

$$\partial_3(m) = \frac{K}{2L} + \frac{K}{L} \int_{-\frac{n^2\pi^2 Tm}{\phi L^2}}^{-\frac{n^2\pi^2 T(m-1)}{\phi L^2}} \sum_{n=1}^{\infty} e^{u} \frac{\phi L^2}{n^2\pi^2 T} du$$

$$= \frac{K}{2L} + \frac{K}{L} \sum_{n=1}^{\infty} \frac{\phi L^2}{n^2\pi^2 T} [e^{-\frac{n^2\pi^2 T(m-1)}{\phi L^2}} - e^{-\frac{n^2\pi^2 Tm}{\phi L^2}}] \qquad \ldots(21)$$

Similarly

$$\partial_4(m) = -[\frac{K}{2L} + \frac{K}{L} \sum_{n=1}^{\infty} (-1)^n \frac{\phi L^2}{n^2\pi^2 T} [e^{-\frac{n^2\pi^2 T(m-1)}{\phi L^2}} - e^{-\frac{n^2\pi^2 Tm}{\phi L^2}}]$$

$$\ldots(22)$$

The final expression for flow rate can be written as

$$Q_r(N) = -\frac{Kh_1^2}{L} \sum_{n=1}^{\infty} e^{-\frac{n^2\pi^2 TN}{\phi L^2}} + \frac{Kh_2^2}{L} \sum_{n=1}^{\infty} (-1)^n e^{-\frac{n^2\pi^2 TN}{\phi L^2}}$$

$$+ \sum_{\gamma=1}^{N} [h_3^2(\gamma) - h_3^2(\gamma-1)]\partial_3(N-\gamma+1) + \sum_{\gamma=1}^{N} [h_4^2(\gamma) - h_4^2(\gamma-1)]\partial_4(N-\gamma+1)$$

$$\ldots(23)$$

5.0 RESULTS AND DISCUSSIONS

The discrete kernel coefficients $\partial_3(.)$ and $\partial_4(.)$ are generated for known values of aquifer parameter K, and ϕ , saturated thickness H and aquifer length L. H_1 being the initial saturated thickness. The transmissivity T has been assumed to be equal to KH_1. Using these discrete kernel coefficients the response of the aquifer to various boundary conditions(change in river stages) have been obtained.

The results of the present study are compared with the analytical solution pertaining to flow from an infinite aquifer to a fully penetrating river due to unit step draw-down in the river stage. For a fully penetrating river, the flow from an aquifer of infinite length to the river per unit length in response to a unit step drawdown is given by the expression

$$Q_r = -\frac{T}{\sqrt{\pi(\frac{T}{\phi})t}}$$

The results obtained from the present study for T = $1001 m^2$/15days, ϕ = 0.1 are present in Table 1. These results compare well with those obtained using the above equation.

Hall (1972) has presented numerical results for flow from an aquifer bounded by two fully penetrating rivers. He has assumed that the stages in one nof the river remains invariant with time. In the present study the same boundary conditions are imposed and the flow quantities are determined. The results obtained from the present study are presented in Fig.3. These results compare with those of Hall.

The exchange of flow between the river and an aquifer of length L/2 bounded by a no flow boundary in response to a unit pulse excitation is presented in Figure 4. The flow from the aquifer of

TABLE 1

FLOW RATE DUE TO UNIT STEP RISE IN THE RIVER STAGE AS WELL AS WATER TABLE FOR $T=1001 m^2$/day, $\phi=0.1$, K=1 m/day, L = 5000m, $H_1=H_2=1000$m, TIME STEP=1 day

S.No.	TIME STEP	FLOW RATE (m^3/day)
1	1	-5.641818
2	2	-3.989388
3	3	-3.257375
4	4	-2.820909
5	5	-2.523174
6	6	-2.303324
7	7	-2.132407
8	8	-1.994694
9	9	-1.880645
10	10	-1.784152
11	11	-1.701112
12	12	-1.628688
13	13	-1.564801
14	14	-1.507871
15	15	-1.456741
16	16	-1.410455
17	17	-1.368335
18	18	-1.329815
19	19	-1.294350
20	20	-1.261587
21	21	-1.231188
22	22	-1.202858

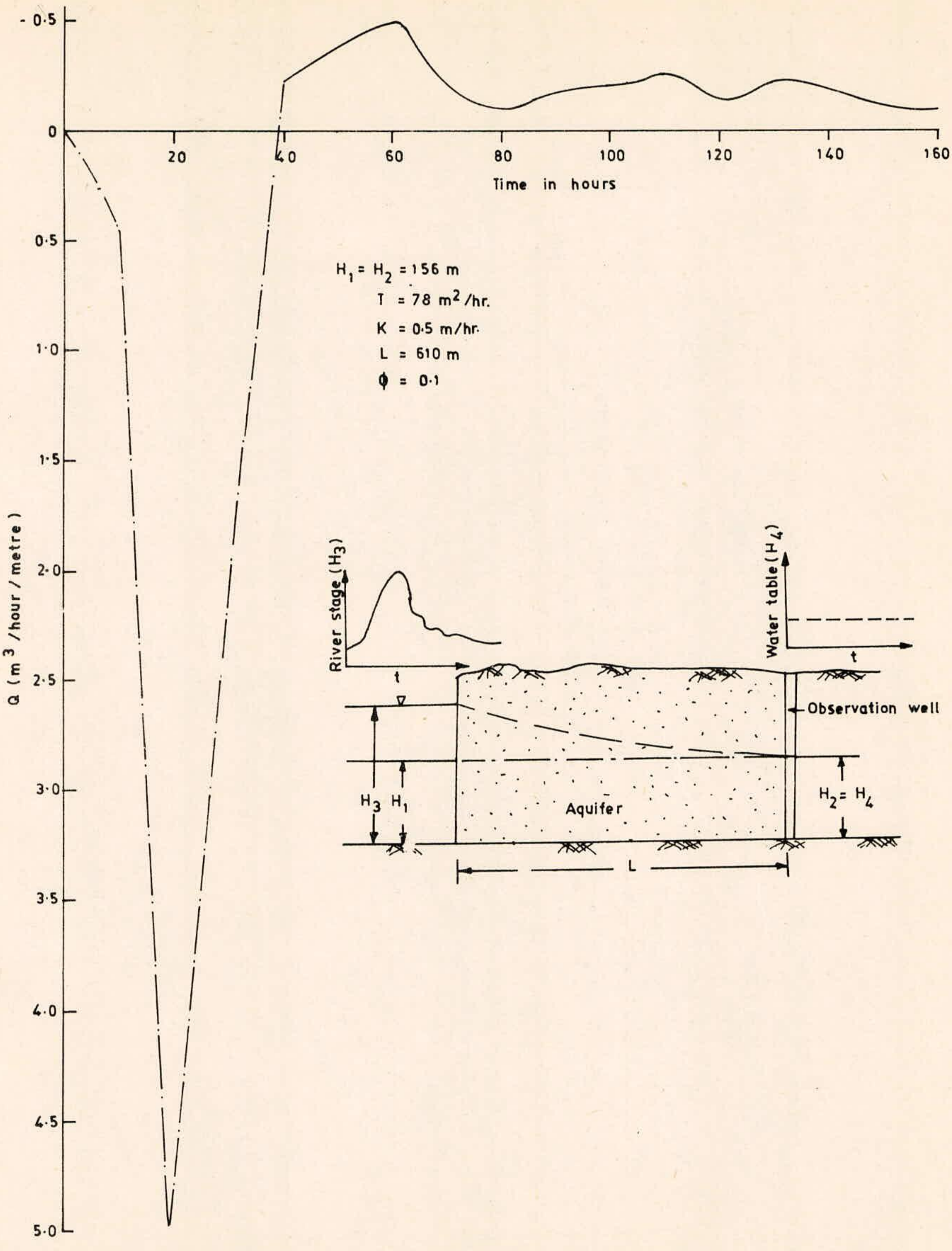

FIGURE 3. VARIATION OF Q WITH TIME DUE TO STREAM STAGE VARIATION

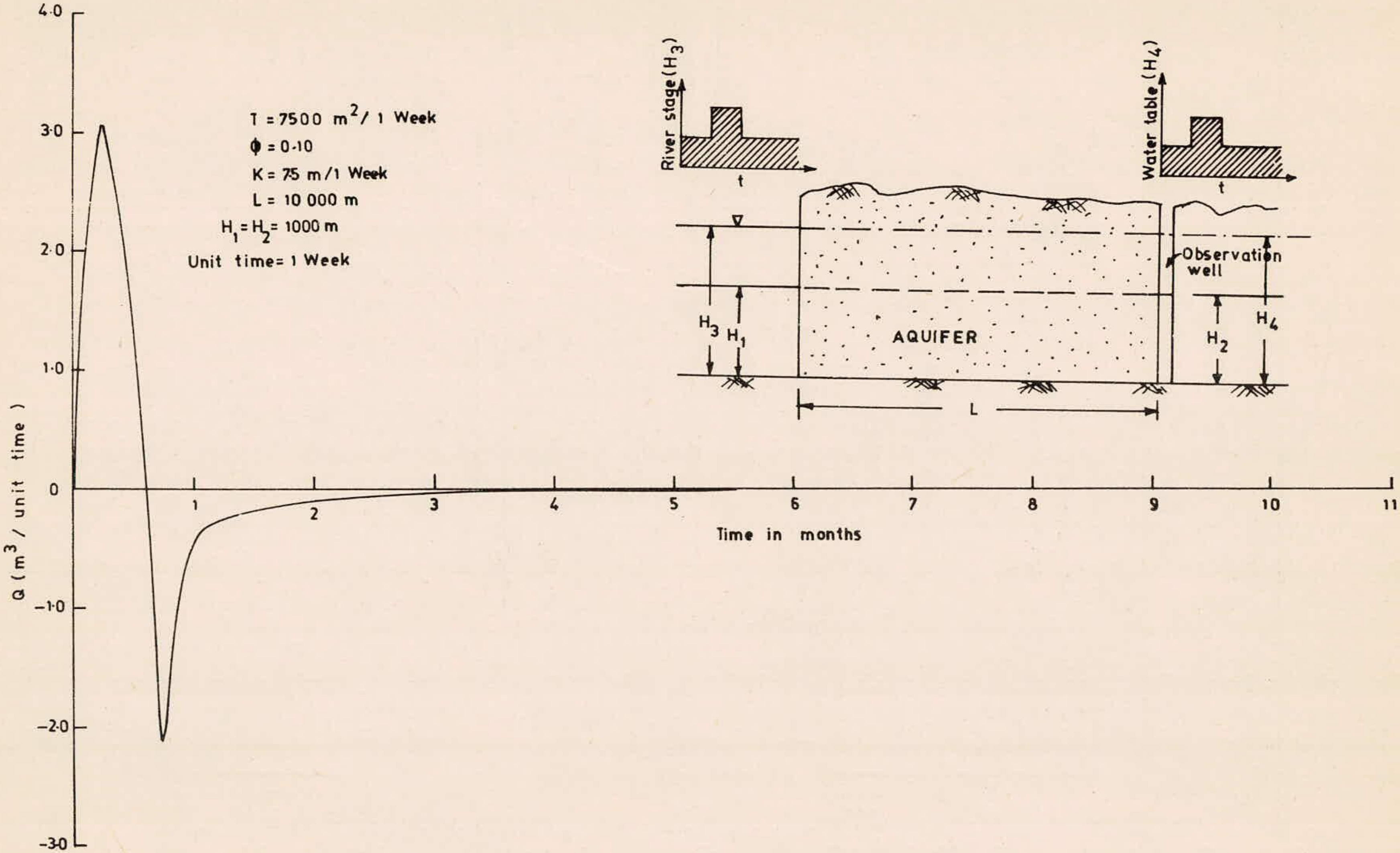

FIGURE 4. VARIATION OF Q WITH TIME DUE TO UNIT PULSE EXCITATION

finite length bounded by a no flow boundary has been simulated by assuming identical boundary conditions on either side of an aquifer of lengh L. The unit period has been assumed to be 1 week. This means the rise in river stage is 1m which continues for one week and then the river stage drops to the original level.

Contribution from an aquifer to a river, when only the water table changes in the aquifer is shown in Fig.5 for various length of the aquifer. It could be seen that the shape of the base-flow contribution has resemblance with the water table stage hydrograph. The shape of base-flow hydrograph in comparison to water table hydrographs gets damped with increase in aquifer length. In Figure 6, the results of aquifer contributions at various times for a commonly observed river stage and water table change are presented for different values of T. The results have been plotted on semilog graph. It can be seen that, the variation of log Q with time t does not follow a linear path. Therefore, the exponential relation $Q = Q_o e^{-\alpha t}$ is not observed. It can be seen from the equation $Q = \frac{T}{\sqrt{\pi (T/\phi) t}}$, that $\log Q = \log \frac{T}{\sqrt{\pi (T/\phi)}} - \frac{1}{2} \log t$. Therefore in the absence of no other development, the base-flow in response to a single change in river stage follows a linear relation in log-log plot. That the relation $Q = Q_o e^{-\alpha t}$ is not followed has also been stated by Singh (1969).

The results presented in Fig.6 are for various values of transmissivity, with increasing transmissivity the baseflow contribution is sustained for a longer period.

Variation of Q with time for a specific transmissivity value and for various values of storage coefficient are presneted in Fig.7. It can be seen with higher storage coefficient the base-flow sustaining for longer period. The contribution of aquifer at different time for

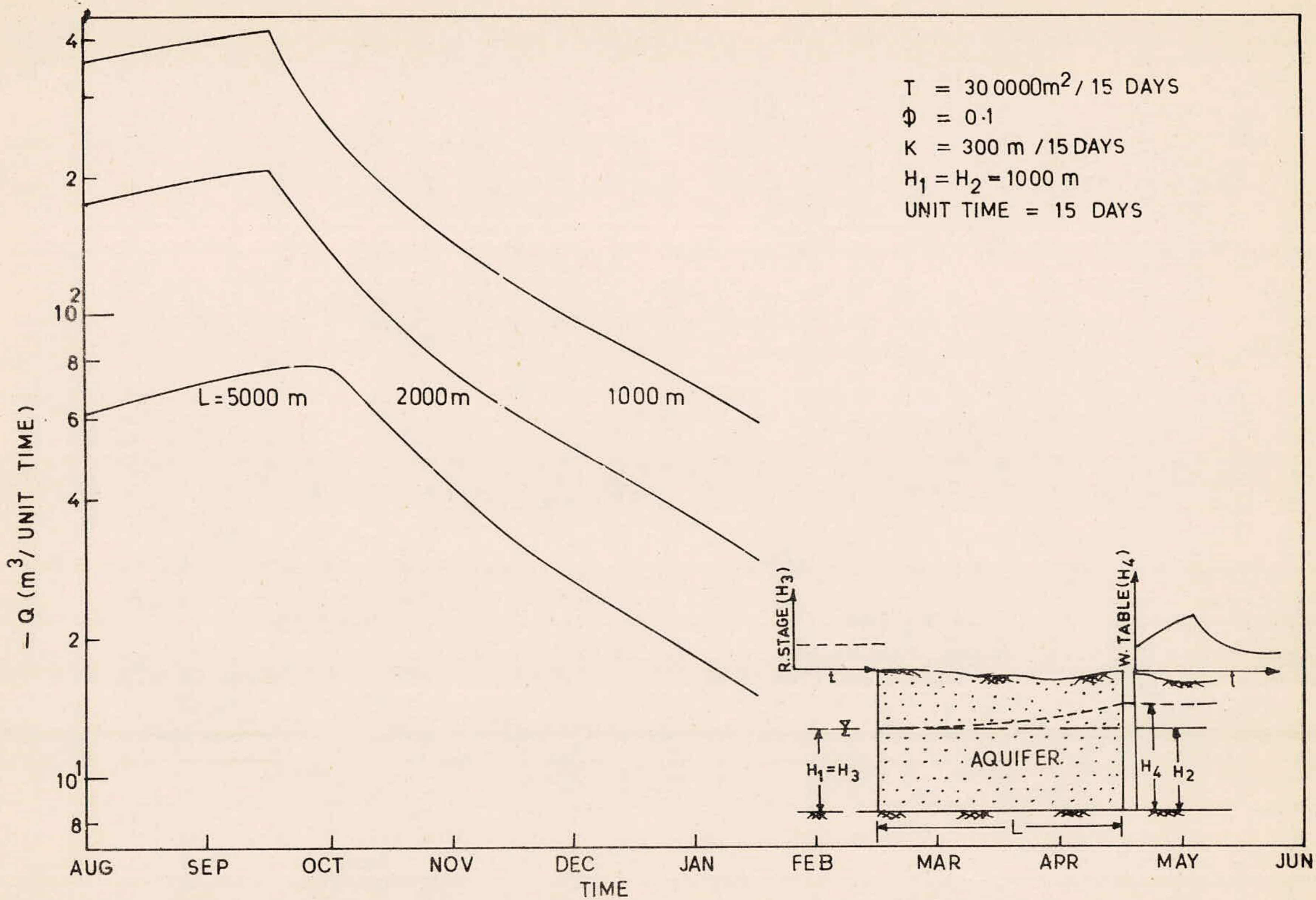

FIGURE 5. VARIATION OF Q WITH TIME DUE TO CHANGES IN WATER TABLE

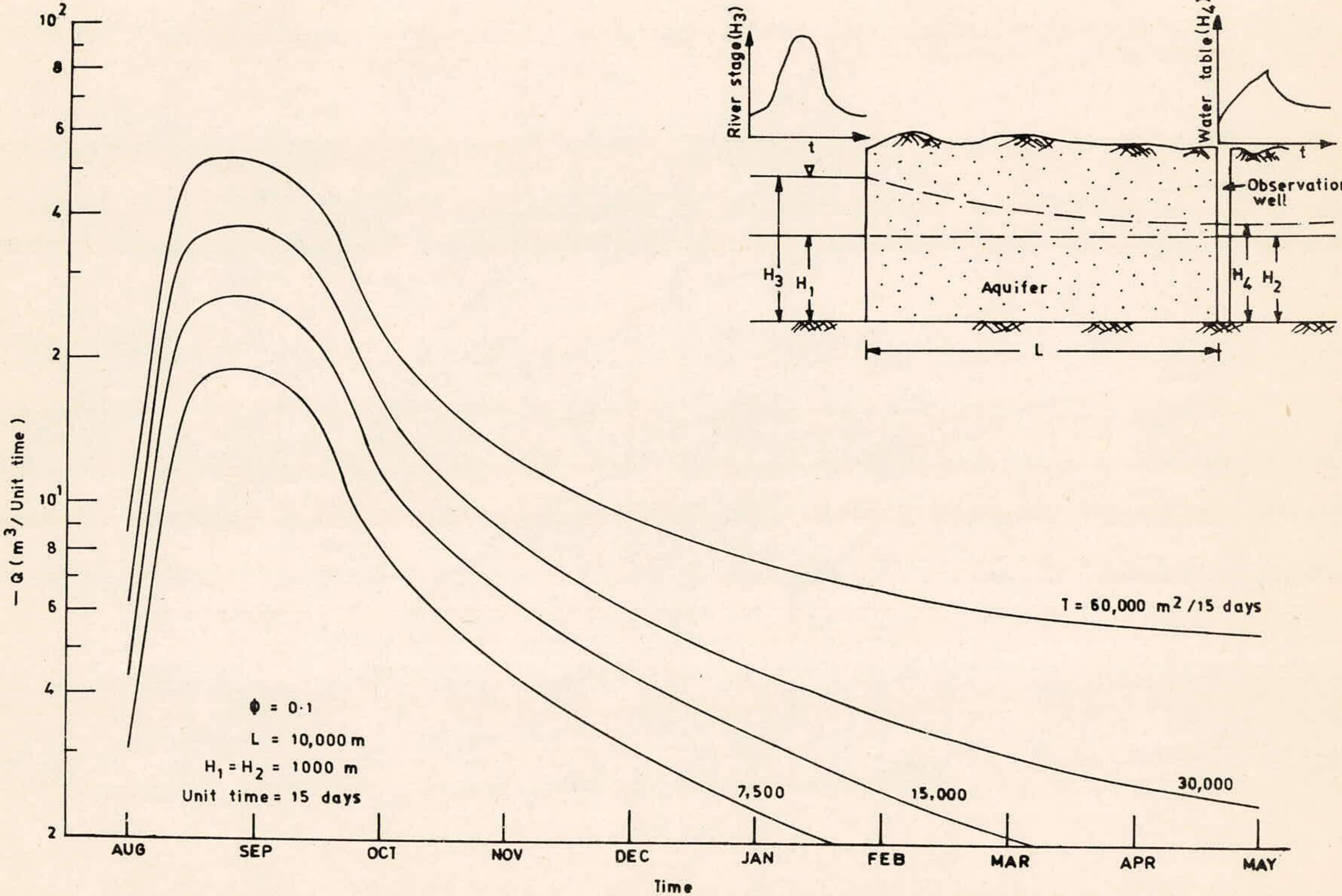

FIGURE 6. VARIATION OF Q WITH TIME FOR DIFFERENT VALUES OF T

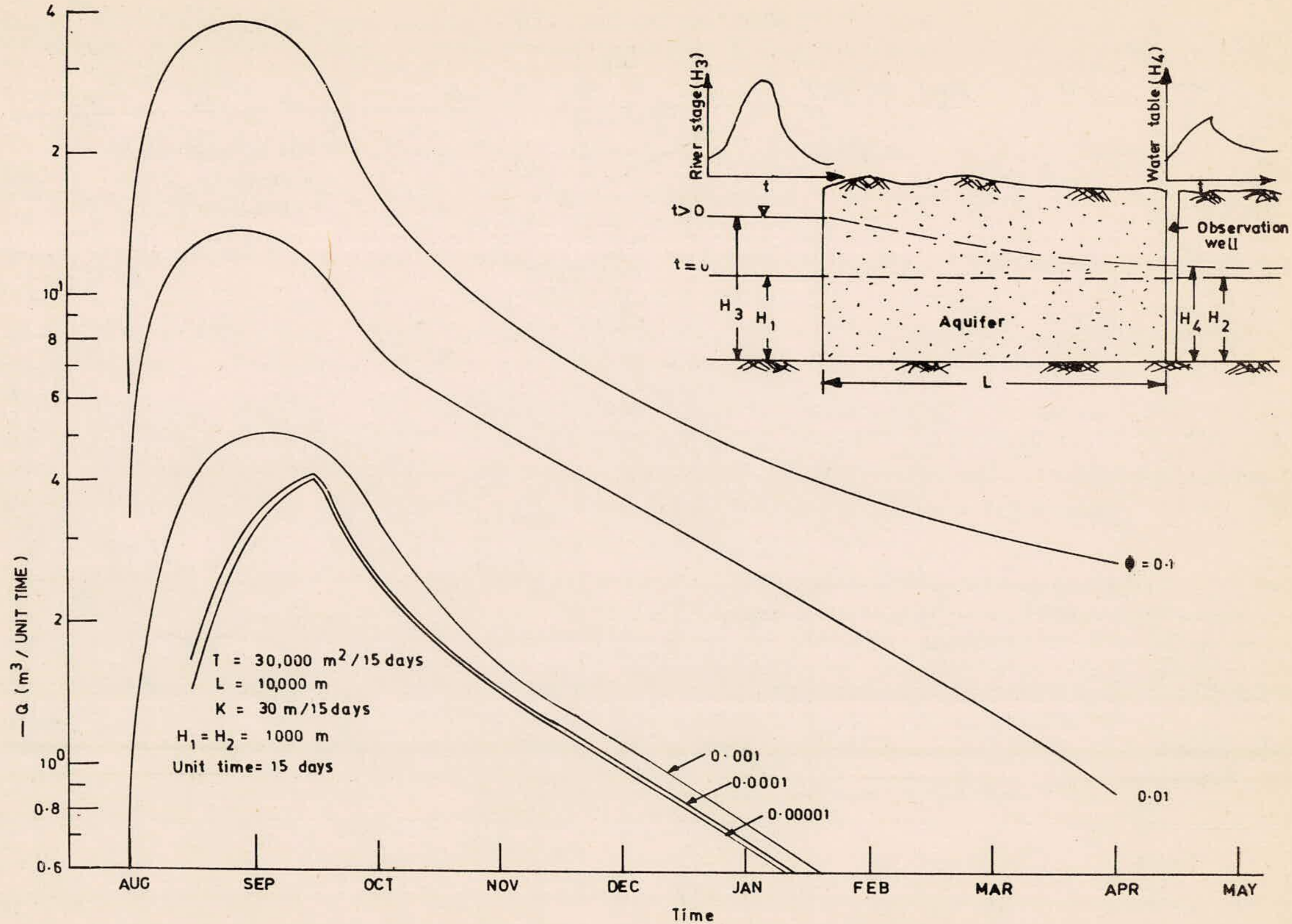

FIGURE 7. VARIATION OF Q WITH TIME FOR DIFFERENT VALUES OF Ø

same water table fluctuation observed at different distances are presented in Fig.8. In Fig.9 the contribution of aquifer due to same river stage hydrograph for different length of aquifer bounded by a no flow boundary are presented. It could be seen from Fig.9, that with higher length of aquifer, longer is the duration of sustained flow.

A typical variation of cummulative discharge with time for specific river stage and water table hydrograph is presented in Fig.10 for various transmissivity values. Cummulative discharges increases with increase in transmissivity..

In Fig.11, variation of cummulative flow with storage volume present in the aquifer has been presented for a specific river stage changes and water table hydrograph. The relation is found to be linear.

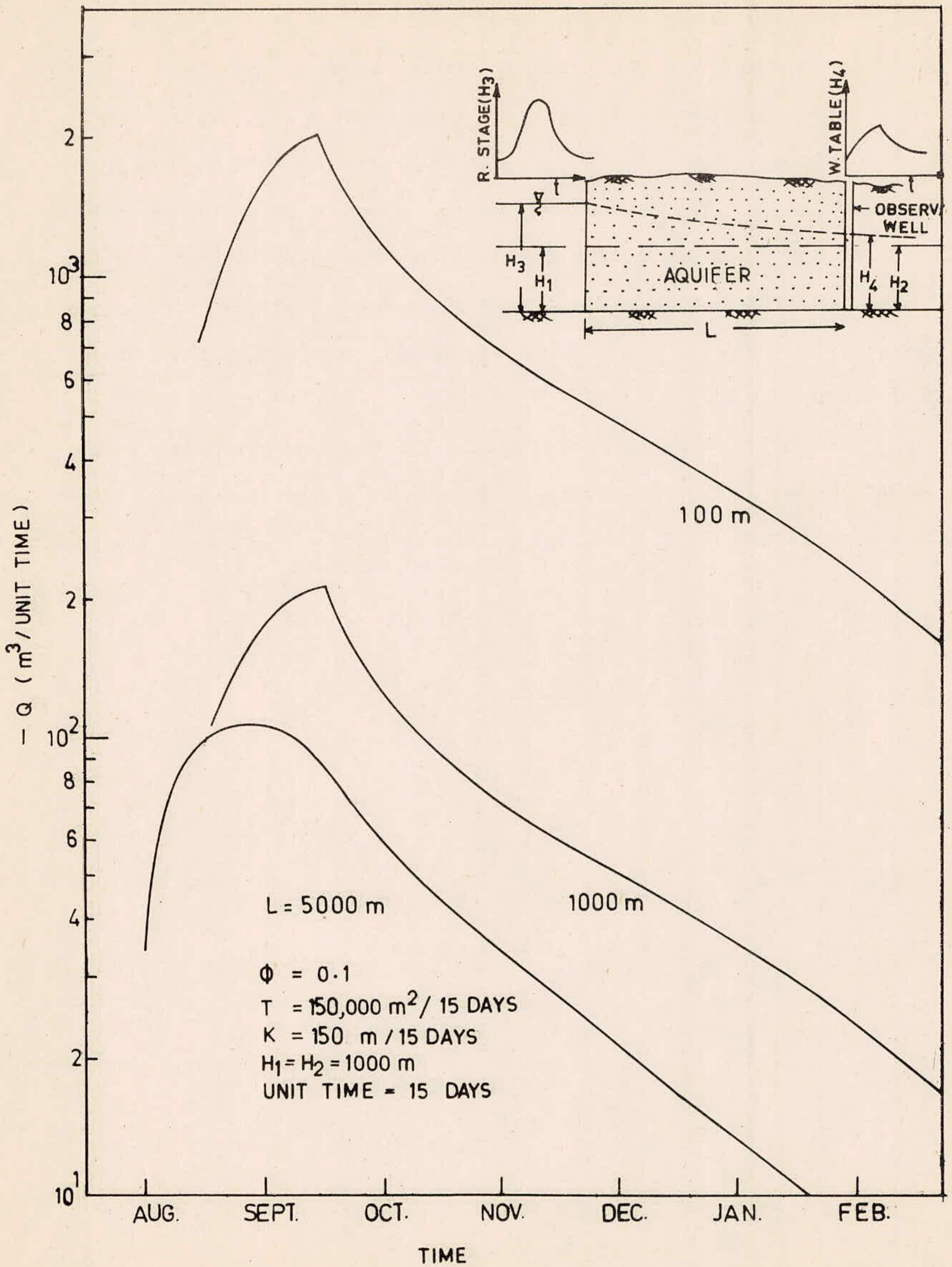

FIGURE 8. VARIATION OF Q WITH TIME FOR DIFFERENT VALUES OF L

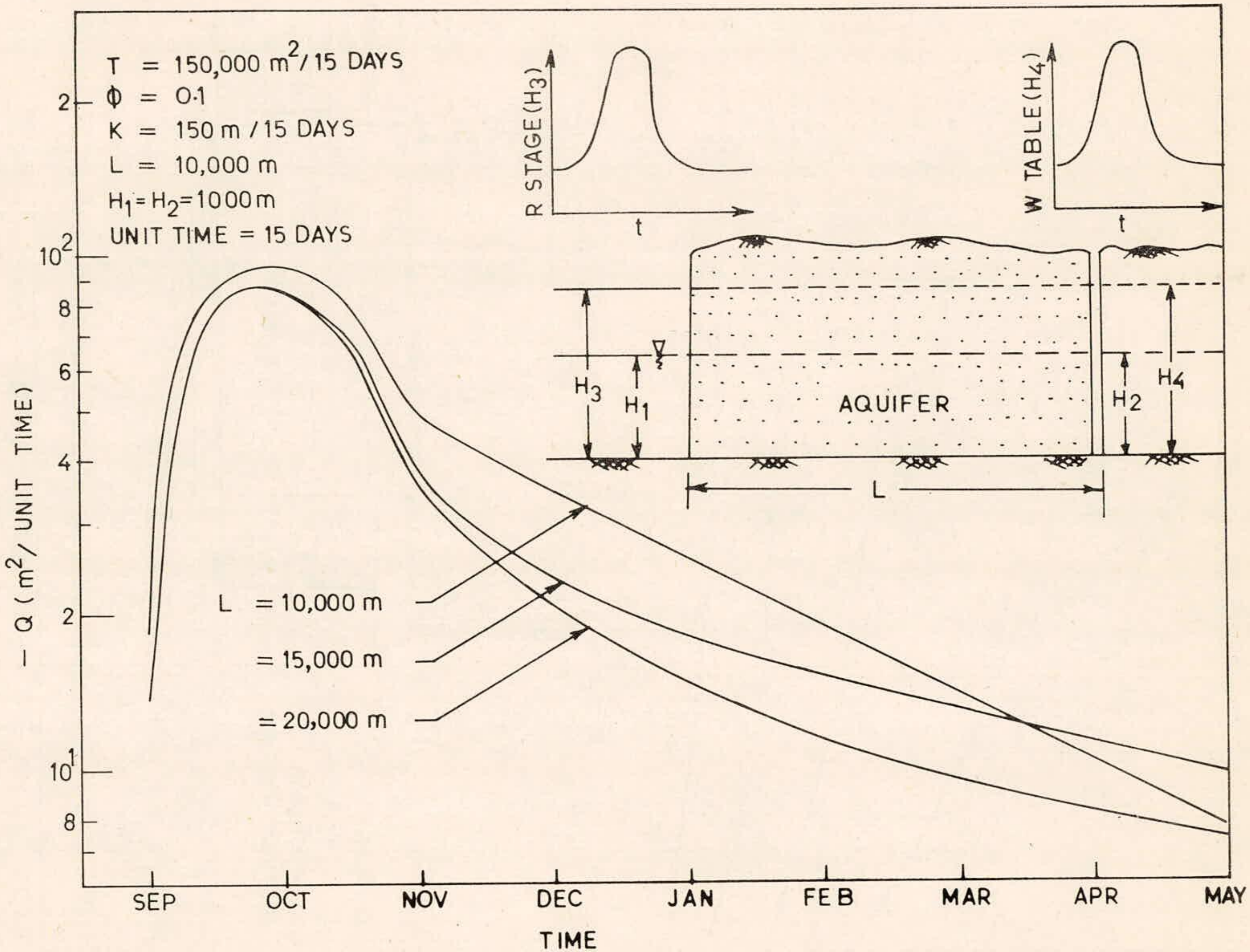

FIGURE 9. VARIATION OF Q WITH TIME FOR DIFFERENT VALUES OF L BOUNDED BY NOFLOW BOUNDARY

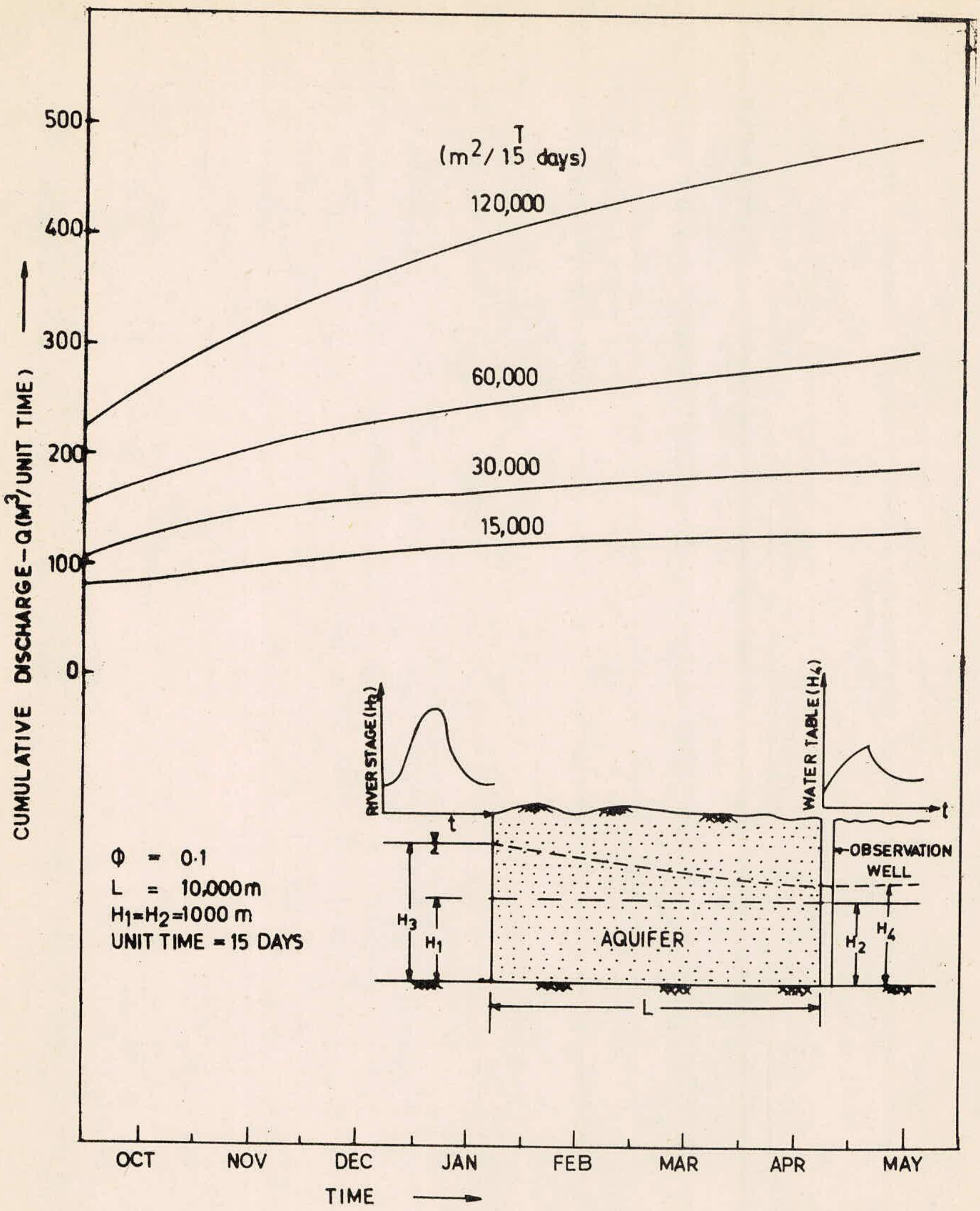

FIGURE 10. VARIATION OF CUMULATIVE DISCHARGE WITH TIME FOR DIFFERENT VALUES OF T

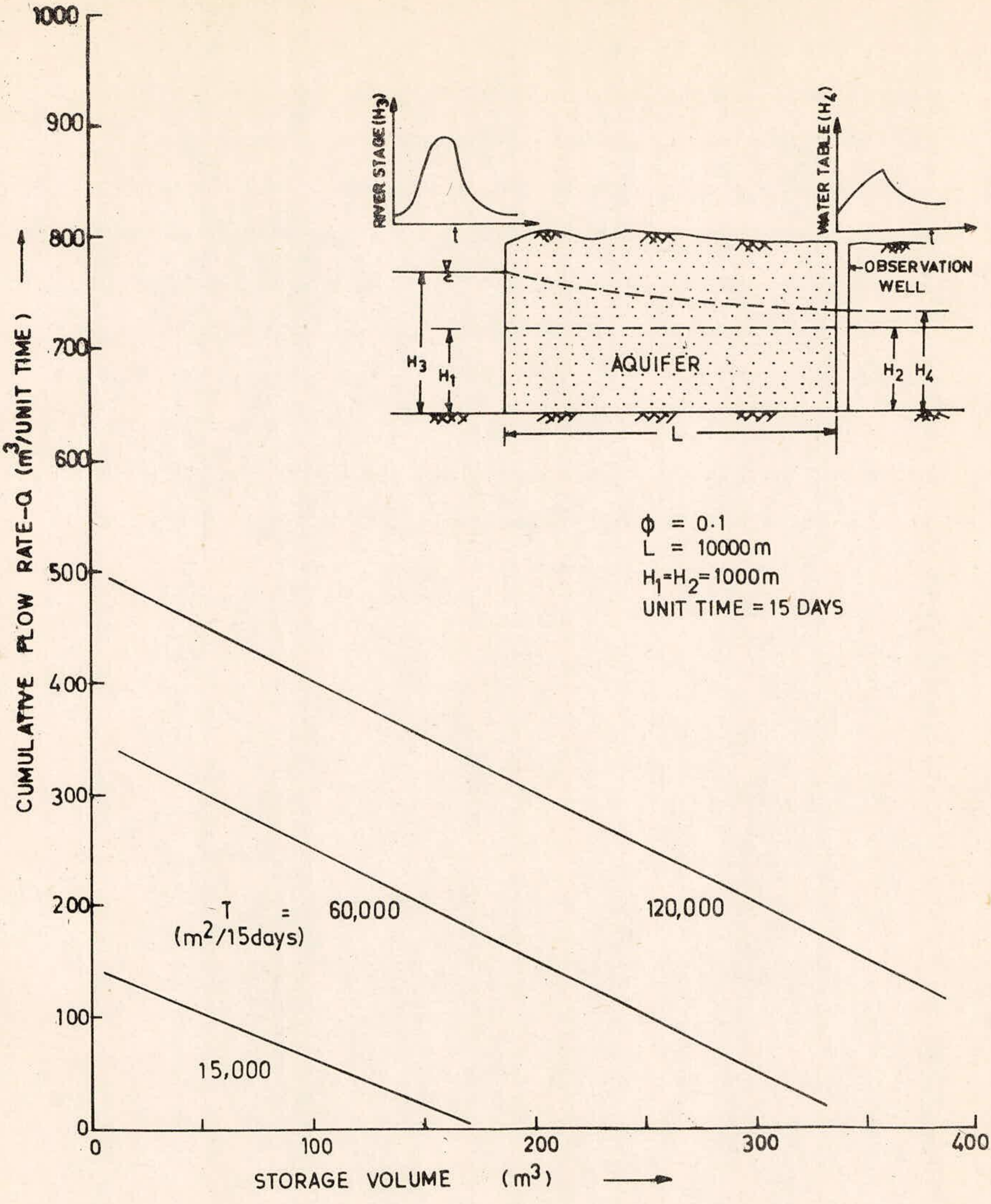

FIGURE 11. VARIATION OF CUMULATIVE FLOWRATE WITH STORAGE VOLUME FOR DIFFERENT VALUES OF T

6.0 CONCLUSIONS

One dimensional Bousinesq's equation has been solved for varying river stages on either side of an aquifer. An analytical approach has been described to study effect of transmissivity, storage coefficient, and length of aquifer on baseflow contribution from an aquifer to a fully penetrating river. It is found from the study that the baseflow equation does not fallow an exponential decay curve. This finding is in confirmation to the findings of earlier investigations. The aquifers having higher T and ϕ will contribute baseflow at a higher rate for longer time. In the study a generally encountered river stage fluctuation and water table changes have been considered and the exchange of flow between river and aquifer has been studied for various values of T, ϕ and finite length of the aquife.

TABLE 2

COORDINATES OF THE RIVER STAGE AND WATER TABLE HYDROGRAPHS USED IN THE PRESENT ANALYSIS [UNIT TIME - 15 DAYS]

S.No.	UNIT TIME	RIVER HYDROGRAPH (IN METERS)	WATER TABLE HYDROGRAPH (IN METERS)
1	1	1000	1000.08
2	2	1001	1000.45
3	3	1002	1000.69
4	4	1002.5	1000.85
5	5	1002.25	1000.97
6	6	1002	1001.07
7	7	1001.5	1001.16
8	8	1000.75	1001.23
9	9	1000.25	1001.30
10	10	1000	1001.36
11	11	1000	1000.79
12	12	1000	1000.59
13	13	1000	1000.47
14	14	1000	1000.39
15	15	1000	1000.33
16	16	1000	1000.27
17	17	1000	1000.23
18	18	1000	1000.19
19	19	1000	1000.15
20	20	1000	1000.12
21	21	1000	1000.09
22	22	1000	1000.07
23	23	1000	1000.04
24	24	1000	1000.02

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