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DESIGN FLOOD ESTIMATION FOR NARMADA SAGAR PROJECT USING PARTIAL DURATION SERIES

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ABSTRACT

The report highlights the details of the study for the estimation of flood magnitudes for different return periods for Narmada Sagar dam(India) using partial duration series. The data of hourly stages and discharges at Mortakka located at 40 km downstream of proposed dam site have been used.

The comparison of the efficiencies of annual and partial flood series has been given on the basis of exact theoretical approach and approximate theoretical approach. On the basis of exact theoretical approaoch it is seen that the partial flood series estimate of T year flood Q(T) always(for any value of λ) has a smaller sampling variance than that of the annual flood series for the return period T less than 11 years. For the whole range of return periods the partial flood series estimate of Q(T) has a smaller sampling variance than that of the annual flood series, if the average number of peaks per year (λ) is at least 1.65.

On the basis of approximate theoretical approach, it is seen that in the range of λ studied (1.0 to 2.437) the sampling variance of annual flood series is lesser than that of partial flood series.

1.0 INTRODUCTION

The basic requirement of flood frequency analysis is to know the probability with which a flow is equalled or exceeded during the stated design life of a particular project. The three approaches which are generally used for frequency analysis are based on the analysis of (i) time series (ii) the peaks over a threshold or partial duration series, and (iii) the annual maximum series.

In the time series model, the flow hydrograph is considered as a time series of daily flows. The time series of mean daily flows closely represents instantaneous peak flows on large catchments but on small flashy catchments this would not be necessarily so, since the flood peaks would be somewhat smoothed out by daily averaging. A time series model may be written as the sum of deterministic and stochastic components. The deterministic component includes trend and periodicity. This type of model allows both estimation of parameters and model formulation to proceed together through the three components beginning with trend and finishing with the stochastic effect. Examples of the use of such a model on United States data for mean daily flows have been given by Quimpo (1967), and on British data by Hall and O'Connell(1972).

The partial duration series model concerns the distribution of the number and magnitude of peak flows that exceed a threshold. Such peak flows are said to constitute a partial duration series. The threshold level may be raised or lowered so as to involve a desirable number of peaks per year (λ). Most of the models proposed in literature for partial duration series assume Poisson distribution for number of exceedances and exponential distributions for magnitude of exceedances.

Annual maximum model is a special case of time series model in which the unit of time is one year and the flow representing that time is the highest flow during the year. In practice, this time series is statistical rather than stochastic, since there is no dependence between successive peaks which may be considered as identically and independently distributed. Annual maximum approach has gained popularity because of application of theory of extremes by Gumbel (1941-45).

The classical dilemma in flood frequency analysis is whether to use annual maximum model or partial duration series model. The most frequent objection for the use of annual maximum model is regarding its use of only one largest flood for each year. In certain cases, the second largest flood in a year which the annual flood series approach neglects, may outrank many annual floods of other years. The maximum annual discharges in dry years of some rivers in arid or semi arid regions may be so small that calling them floods may be somewhat misleading. Another shortcoming of annual flood series approach is that only a small number of floods is considered. The estimate of higher moments like coefficient of skewness of historical flood series will not be reliable in case of annual flood series with small sample size.

On the other hand, partial duration series model contains more floods than annual maximum model and as such the estimate of parameters of annual flood distribution from the partial flood series would be subjected to lesser uncertainty. Secondly, the theoretical expressions for annual flood distribution obtained through characteristics of partial floods have physical relevance and often are exact distributions rather than asymptotic (Viraphol et.al 1978).

2.0 REVIEW

The earliest approaches for the estimation of future floods in a drainage basin were based upon simple empirical formulae involving the correlation of past peak discharges with various parameters. The most popular of these parameters were the area, width and length of the basin.

Statistical methods were introduced in hydrology about sixty years ago. The main objective of these methods is to fit theoretical distributions to flood data. The mean, the standard deviation and the coefficient of skewness of the flood magnitudes are used to fit the parameters of the distribution function. The work on partial duration series model started with the theory of Langbein (1949). Langbein's theory relates recurrence interval of annual maximum series and partial duration series as per the following equation.

$$T_a = 1/[1-\exp(1-1/T_p)] \qquad ...(1)$$

where,

T_a is the recurrence interval of annual maximum series and

T_p is the recurrence interval of partial duration series.

Borgman (1961) gives a simplified technique for computing the probability that a near extreme occurrence of a physical phenomenon will exceed a selected type. Further Borgman (1963) discussed the return period concept with other risk criteria such as (i) encountered probability (ii) distribution of waiting time (iii) distribution of total damage (iv) probability of zero damage, and (v) mean total damage.

Shane and Lynn (1964) developed a probability model based on the time independent Poisson process and theory of sums of a random number of random variables for the use in analysis of flood data.

The first attempt to develop a theory by Todorovic (1970), Todorovic and Zelenhasic (1970) was based on streamflow partial duration series. The series of flows in a partial duration series within an arbitrary but fixed time interval is represented by a random number of random variables. The time dependent Poisson process was used to describe the distribution of the random number of exceedances. It is applied to streamflow by further assuming that the individual exceedances form a sequence of identically independent random variables which are represented by an exponential distribution. Todorovic and Zelenhasic (1970) applied this model to 72 year record of the Susquehanna river at Wilkes-Barre, Pensylvania. Todorovic and Rousselle (1971) extended the work of Todorovic and Zelenhasic (1970) by realizing that for a time interval equal to a year the assumption for exceedances being identically distributed is unrealistic, since different storm types can produce different flood characteristics from one season to another. Accordingly, they derived a distribution function for the largest flood peak for the case where two or more different exceedance distribution functions occur within a time interval. The results were applied to the 72 year record of the Greenbrier river at Alderson, West Virginia. Todorovic (1971) used the above method together with the mathematical assumptions of Todorovic and Zelenhasic (1970) to derive another important property of the extreme flood, namely, its time of occurrence within a selected time interval. The expression for the time of occurrence of the extreme flood obtained by Todorovic (1970) is exact.

Todorovic and Woolhiser (1972) applied the above theory to two rivers of United States. Gupta, Duckstein and Peebles (1976) extended the work of Todorovic and Woolhiser(1972) and developed the expression for the joint distribution function of the largest flood peak and its time of occurrence. They derived distribution function of the time of occurrence of the largest flood for the Mississippi river., St.Paul, Minnisota, and Licking river, Catawaba, Kentucky.

Todorovic (1978a) presented stochastic models of extreme flows and their application to design. Todorovic (1978b) discussed the two approaches i.e. annual flood series and partial duration series for flood frequency analysis. Viraphol and Yevjevich (1978) estimated the probability distribution of maximum annual flood peak by using a combination of probability distributions of the number and the magnitude of flood peaks., that exceeded a selected truncation level. They compared the efficiency of annual flood series model and partial duration series model based on the sampling variance of the estimates. By using the generated samples of daily flows the efficiency of estimated annual flood peaks of given return periods was also investigated by using both the annual and the partial flood peak series.

The models available for partial duration series differ from each other only in the way in which the number of peaks over the threshold each year is treated. These models vary from the simplest in which a constant number of exceedances λ is assumed to occur each year to one where the rate of occurrence of peaks and distribution of peak magnitudes vary with season in a year. A good description of these models has been given in Flood Studies Report, Vol.1, NERC, 1975. Detailed review of partial duration series models has been given by Seth and Goel, 1985.

The practitioners often remain in dilemma whether to use annual maximum model or partial duration series model as the recurrence intervals calculated from two approaches are not mutually comparable. Langbein (1949) gave a solution to this problem by deriving a relationship between the two recurrence intervals i.e. recurrence intervals given by partial duration series and annual maximum series. Chow (1950) discussed Langbein's formula and pointed out that the difference between T_a and T_p evaluated by relative difference $(T_a - T_p)/T_p$ is less than 5% for $T_p \gg 10$ years and greater than 10% for $T_p \leqslant 5$ years. In ordinary engineering practice a five percent difference is tolerable and that the two methods give essentially identical results for intervals greater than about ten years. The development in the field of partial duration series freezed to some extent after Chow's discussion. But it again gained popularity with the work of Todorovic (1967 onwards) for the development of more and more sophisticated models for partial duration series and Cunnan (1973). Cunnan(1973) gave a method for comparing the statistical efficiency of the estimate of T year flood by two approaches. The present study is an attempt in the direction of comparing statistical efficiency of the estimate of T year flood by two approaches.

3.0 STATEMENT OF THE PROBLEM

The objective of the present study is to estimate flood magnitudes for different return periods (100, 500, 1000 and 1000 years) using partial duration series approach. The results of frequency analysis using annual peak discharge series have been presented earlier (Goel and Seth, 1985). The efficiencies of annual and partial flood series models in the estimation of T year flood have also been compared.

4.0 DESCRIPTION OF STUDY AREA

River Narmada is one of the major rivers of India. This rises in the Amarkantak plateau of Maikala range in the Shadol district of Madhya Pradesh at an elevation of 1057 meters above sea level. Narmada Sagar dam is a major project on river Narmada in Madhya Pradesh proposed to be located upstream of Mortakka site at about $22^{o}10'$ latitude and $76^{o}10'$ longitude. The total cost of project is expected to be Rs.1393 crores (about 1200 million US Dollars, CBIP 1984). The dam is expected to be completed by 1994 A.D. and canal system by 2004 A.D. The catchment area of river Narmada upto Mortakka is 67170 km^2 and that upto Narmada Sagar is 61642 km^2.

5.0 AVAILABILITY OF DATA

The data of hourly stages and daily discharges was available from 1951-1982. The stage-discharge relationship was developed specially for the higher range of stages. The information obtained from river cross section was also used in identifying the realistic values of parameters of the relationship. The stage-discharge relationship, thus developed is as follows:

$$Q = 317.18\ (H - 155.2)^{1.768} \qquad ...(2)$$

where

Q Discharge in m^3/sec

H Stage in meters above mean sea level

The above stage discharge relationship has been used to convert peak stages to corresponding discharges.

6.0 METHODOLOGY

6.1 Model Used

The model used in present study acknowledges variation between years in the number of peaks but ignores variation between seasons. The number of peaks in a year is considered to be random variable with mean λ . The distribution of the number of exceedances in a year is assumed as Poissonian and the distribution of the magnitude of the exceedances as exponential.

6.1.1 Derivation of flood magnitudes for the model

Since, the distribution function for magnitude of exceedances [H(X)] is exponential, as such

$$H(X) = 1 - \exp(-X/\beta) \qquad ...(3)$$

Since, the distribution of the number of exceedances $[P(E_k)]$ is Poissonian, therefore,

$$P(E_k) = e^{-\lambda} \lambda^{k} / k! \qquad ...(4)$$

The distribution of largest exceedance in a year will be given by

$$F(X) = \sum_{k=0}^{\infty} [(H(X))^{k} P(E_k)] \qquad ...(5)$$

by putting the values of H(X) and $P(E_k)$ in equation (4)

$$F(X) = \sum_{k=0}^{\infty} [(1-e^{-X/\beta})^{k} e^{-\lambda} \lambda^{k} / k!] \qquad ...(6)$$

which in the limits becomes

$$F(X) = e^{-\lambda e^{-X/\beta}} \qquad ...(7)$$

The relationship between the distribution function of the largest exceedance and the return period is

$$T = 1/[1-F(X)] \qquad ...(8)$$

by eliminating F(X) from equation 7 and 8 the flood exceedance for a given period is expressed by

$$X = \hat{\beta} \ln \lambda + \hat{\beta} y (T) \quad ...(9)$$

where

$$y(T) = -\ln [-\ln(1 - \frac{1}{T})]$$

$$X = Q(T) - q_o$$

So $$Q(T) = \hat{q}_o + \hat{\beta} \ln \lambda + \hat{\beta} y(T) \quad ...(10)$$

where

$\hat{Q}(T)$ Annual flood magnitude for a given return period T.

q_o Truncation level (estimated)

$\hat{\beta}$ parameter estimated from partial duration series

$\hat{\lambda}$ Average number of peaks per year

6.2 Preparation of Partial Duration Series

There are two methods of abstracting data from the discharge record depending upon whether threshold is fixed first or number of floods to be extracted from the data is fixed first.

The method in which the threshold flood q_o is a fixed quantity, the number and magnitudes of floods are used to estimate λ and β. If N is the number of years(also fixed) sufficient estimates of λ and β are

$$\hat{\lambda} = M/N \quad ...(11)$$

$$\hat{\beta} = \sum_{i=1}^{M} (q_i/M) - q_o \quad ...(12)$$

and $$\hat{Q}(T) = q_o + \hat{\beta} \ln \hat{\lambda} + \hat{\beta} \ln (T) \quad ...(13)$$

In the second method, fixed number of floods (M) are extracted from the data. This means that the average number of floods per annum in the sample is fixed before extraction and that q_o and β are initially unknown and must be estimated from the sample.

It is therefore, required to estimate jointly the location and scale parameters q_o and β of an exponential distribution from a sample of fixed size M. The maximum likelihood estimate of q_o and β (after correction for bias) are:

$$\hat{q}_o = (Mq_{min} - \bar{q})/M-1) \qquad ...(14)$$

and

$$\hat{\beta} = M(\bar{q}-q_{min})/(M-1) \qquad ...(15)$$

The estimated T year flood is

$$Q(T) = \hat{q}_o + \hat{\beta}\ [\ln\lambda + \ln(T)] \qquad ...(16)$$

In the method adopted for this study the threshold level for peak stages was kept as 161.54 meter as first approximation, keeping in view the stage corresponding to minimum peak discharge value in annual maximum series being 162.68 meters. All the peak stages above 161.54 were compiled and converted to corresponding peak discharge values.

Two neighbouring peaks were included only if (a) the flow between them dropped to less than two thirds of the earlier two and the time between the peaks exceeded $3T_p$ (231 hours) where T_p is the average time to peak of the first five hydrographs on the record. Average T_p in case of Narmada river upto Mortakka is 77 hours. Total 78 peaks which satisfied above mentioned criterion were selected for further study. The peak discharges with date and time are given in Table 1.

Lag one autocorrelation coefficient, given by the following equation

$$r_1 = \frac{\sum_{t=1}^{M-1} (Q_t-\bar{Q})(Q_{t+1} - \bar{Q})}{\sum_{t=1}^{M} (Q_t - \bar{Q})^2} \qquad ...(17)$$

was calculated for the series and it comes out to be 0.0441, which falls within the 95% tolerance limits to test the hypothesis of zero autocorrelation. The limits are -0.234 and 0.208.

For other values of λ , the number of exceedances were fixed first.

TABLE 1

PARTIAL DURATION SERIES WITH DATE AND TIME

S.N.	Date	Time of day (hrs)	Stage (m)	Discharge (m^3/sec)
1	9.8.51	8	162.68	11127
2	4.8.52	8	163.59	13631
3	28.8.52	8	162.07	9573
4	1953		165.48	19521
5	10.9.54	7	162.54	10761
6	23.9.54	24	169.25	33915
7	18.8.55	18	162.03	9475
8	1.9.55	1	165.84	20746
9	14.9.55	8	163.49	13345
10	3.10.55	23	163.15	12392
11	15.7.56	3	163.00	11982
12	31.7.56	15	161.85	9038
13	11.8.56	10	162.73	11258
14	23.8.57	21	167.03	25023
15	1.9.58	15	161.66	8586
16	11.9.58	15	163.37	13005
17	25.9.58	17	163.19	12503
18	13.7.59	17	168.40	30372
19	8.8.59	6	163.62	13717
20	3.9.59	17	168.22	29644
21	15.9.59	7	165.48	19521
22	5.8.60	10	164.65	16822
23	19.8.60	1	165.78	20540
24	19.7.61	8	162.64	11022
25	8.8.61	17	162.37	10324
26	25.8.61	2	165.75	20437
27	6.9.61	4	163.68	13890
28	16.9.61	1	173.73	55323
29	27.9.61	21	163.37	13005
30	12.10.61	24	162.54	10761
31	6.9.62	21	163.25	12669
32	20.9.62	6	168.70	31604
33	15.8.63	16	163.68	13890
34	26.8.63	15	161.57	8376
35	5.9.63	23	164.53	16446
36	16.9.63	4	164.43	16135
37	13.8.64	1	166.60	23438
38	28.8.64	13	161.73	8751
39	29.7.65	7	165.20	18591
40	3.9.65	22	161.88	9110
41	2.8.66	16	162.76	11338
42	3.8.67	17	163.28	12753
43	22.8.67	17	162.28	10096
44	31.8.67	24	165.53	19690
45	5.8.68	22	168.70	31604
46	5.8.69	7	167.79	27935

47	18.8.69	8	162.00	9401
48	23.9.69	24	161.63	8516
49	4.7.70	8	163.77	14152
50	19.8.70	9	161.84	9014
51	6.9.70	4	170.99	41691
52	5.7.71	15	163.49	13345
53	30.7.71	8	162.46	10555
54	7.9.71	12	165.05	18101
55	18.8.72	12	172.27	47851
56	2.9.72	8	165.96	21162
57	16.7.72	8	168.10	29163
58	23.7.73	8	167.58	27117
59	31.8.73	2	173.49	54063
60	20.8.74	16	169.86	36562
61	14.8.75	19	167.00	24911
62	25.8.75	8	163.86	14416
63	11.9.75	9	169.10	33278
64	6.8.76	5	163.13	12337
65	29.8.76	2	164.93	17713
66	14.9.76	11	162.22	9946
67	8.8.77	5	166.36	22572
68	31.8.77	20	164.71	17011
69	15.9.77	1	166.85	24354
70	14.7.78	2	162.72	11232
71	17.8.78	5	166.40	22716
72	29.8.78	23	168.20	29564
73	10.8.79	12	167.35	26232
74	6.8.80	24	163.60	13659
75	30.8.80	16	166.41	22751
76	9.7.81	7	162.03	9475
77	10.8.81	10	167.20	25662
78	1982	.	164.58	16602

The variations of values of λ , β and (q_o + β ln λ) with truncation level q_o for the partial flood series are given in Table 2. The values of u and α for Gumbel distribution are also given in the bottom of table for the sake of comparison.

TABLE - 2

VARIATIONS OF VALUES λ, β AND ($q_o + \beta \ln \lambda$) WITH TRUNCATION LEVEL (q_o) FOR THE PARTIAL FLOOD SERIES

	Truncation level q_o											
q_o	18265.75	16164.84	13114.56	12537.78	12193.68	11141.83	10946.65	10387.93	9784.73	9250.1	8870.06	8376
λ	1.0	1.187	1.5	1.593	1.687	1.781	1.875	2.0	2.093	2.218	2.31	2.437
β	10407.7	10683.7	11061.0	10975.7	10709.18	11181.1	10820.5	10692.1	10804.7	10712.8	10651.2	10583.1
$q_o + \beta \ln\lambda$	18265.7	17996.3	17599.4	17648.2	17794.0	17595.3	17748.5	17799.1	17765.0	17784.0	17787.8	17803.0

$u = 20705.05, \quad \alpha = 9061.48$

7.0 ANALYSIS

To compare the efficiency of flood peaks of given return periods using annual and partial duration series, the sampling variance of Q(T) obtained from both flood series have been compared. It was shown by Zelenhasic (1970) that partial duration series gives double exponential or Gumbel distribution for annual floods when we assume Poisson distribution for number of exceedances and exponential distribution for magnitude of exceedances. This theoretical finding is used in the comparison of sampling variances of Q(T) obtained from the two flood series, assuming the Gumbel distribution for annual maximum series.

7.1 Flood Magnitudes and their Sampling Variances for Given Return Periods from Annual Flood Series Using Gumbel Distribution

For Gumbel distribution the probability density function is given by

$$f(X) = \frac{1}{\alpha} e^{[-((x-u)/\alpha).e^{-((x-u)/\alpha)}]} \quad \ldots (18)$$

and the cumulative density function by

$$f(X) = e^{-e^{-(x-u)/\alpha}} \quad \ldots (19)$$

In equation (18) and (19) u is the locationparameter and α is the scale parameter.

The mean, variance and skewness are

$$\mu = u + 0.5772\alpha \quad \ldots (20)$$

$$\sigma^2 = \pi^2\alpha^2/6$$

$$C_s = 1.139 \quad \ldots (22)$$

The parameters u and α are obtained by the method of maximum likelihood. (Jenkins,1969,NERC 1975).

The estimate of T year flood denoted by $\hat{Q}(T)_a$ is given by

$$\hat{Q}(T)_a = \hat{u} + \hat{\alpha}\, y(T) \qquad \ldots(23)$$

in which $y(T) = -\ln(-\ln(\frac{T-1}{T}))$ and T is the return period.

The sampling variance of $\hat{Q}(T)_a$ is

$$\mathrm{Var}\,(\hat{Q}(T)_a) = \mathrm{Var}\,\hat{u} + 2\,\mathrm{Cov}(\hat{u},\hat{\alpha}\, y(T)) + \mathrm{Var}\,[\hat{\alpha}\, y(T)] \qquad \ldots(24)$$

The variance-covariance matrix of the maximum likelihood estimates of u and α (Kimball, 1946) is

$$\begin{bmatrix} \mathrm{Var}\,\hat{u} & \mathrm{Cov}(\hat{u},\hat{\alpha}) \\ \mathrm{Cov}(\hat{u},\hat{\alpha}) & \mathrm{Var}\,(\hat{\alpha}) \end{bmatrix} = \frac{\alpha^2}{N} \begin{bmatrix} 1 + \frac{6}{\pi^2}(1-\gamma)^2 & \frac{6}{\pi^2}(1-\gamma) \\ \frac{6}{\pi^2}(1-\gamma) & 6/\pi^2 \end{bmatrix}$$

$$= \frac{\alpha^2}{N} \begin{bmatrix} 1.11 & 0.26 \\ 0.26 & 0.61 \end{bmatrix}$$

Therefore $\mathrm{Var}(\hat{u}) = 1.11\,\alpha^2/N$, $\mathrm{Var}\,(\hat{\alpha}) = 0.61\,\alpha^2/N$ and $\mathrm{Cov}(\hat{u}\hat{\alpha}) = 0.26\alpha^2/N$. By substituting these in equation (24)

$$\mathrm{Var}\,(Q(T)_a = \frac{\alpha^2}{N}[1.11 + 0.52\, y\,(T) + 0.61\, y^2(T)] \qquad \ldots(25)$$

7.2 Sampling Variance of Flood Magnitudes for given Return Periods From Partial Flood Series by Using Combination of Poisson and Exponential Distributions.

Sampling variance of Q(T) for partial duration series is mainly based on the work by Cunnane (1973). The sampling variance of $Q(T)_p$ is given by the following equation (NERC, 1975 pp.195)

$$\mathrm{Var}\,(\hat{Q}(T)_p) = \frac{\beta^2}{N\lambda}\left[\frac{(1-\ln\lambda\ -\ln T)^2}{N\lambda\ -1} + (\ln\lambda + \ln T)^2\right] \qquad \ldots(26)$$

7.3 Comparison of Efficiencies

The following two approaches have been used to compare the sampling variance of Q(T) of annual and partial flood series:

7.3.1 Exact theoretical approach

Let R_1 be the ratio of the sampling variances of $\hat{Q}(T)_a$ and

$\hat{Q}(T)_p$ obtained theoretically from annual and partial flood series respectively. Hence for a given return period(T), R_1 will be

$$R_1 = \frac{\text{Sampling variance of } \hat{Q}(T)a}{\text{Sampling variance of } \hat{Q}(T)_p} \qquad ...(27)$$

$$= \frac{\frac{\alpha^2}{N}[\,1.11 + 0.52\ y\ (T) + 0.61\ y^2(T)\,]}{\frac{\beta^2}{N\lambda}[\,\frac{1-\ln\lambda\ -\ln\ T)^2}{\lambda} + (\,\ln\lambda + \ln\ T)^2]} \qquad ...(28)$$

under the assumption that $Q(T)_a = Q(T)_p$; α will be equal to β .

so

$$R_1 = \frac{\lambda\ [\,1.11 + 0.52\ y(T) + 0.61 y^2(T)]}{[\frac{(1-\ln\lambda\ - \ln\ T)^2}{N\lambda\ \ -1} + (\ln\lambda + \ln\ T)^2]} \qquad ...(29)$$

7.3.2. Approximate theoretical approach

In this approach, the parameters u and α are estimated from the annual flood series and parameters q_o and β from the corresponding partial flood series. Let R_2 denote the ratio of sampling variances of $\hat{Q}(T)_a$ and $\hat{Q}(T)_p$, then

$$R_2 = \frac{\lambda\alpha^2\ \ [\,1.11 + 0.52\ y\ (T) + 0.61\ y^2(T)]}{\beta^2\ [\frac{(1-\ln\lambda\ - \ln\ T)^2}{N\lambda\ \ -1} + (\ln\lambda + \ln\ T)^2]} \qquad ...\ (30)$$

The difference between R_1 and R_2 is that the difference between α and β is taken into consideration in computing R_2.

8.0 RESULTS

8.1 Annual Maximum Series Using Gumbel Distribution

The annual peak stage and discharge series is given in table 3 along with statistical parameters. Using equation (20) and (21) the values of location parameter(u) and scale parameter (α) for the Gumbel distribution come out to be 20705.05 and 9061.48 respectively. 100,500,1000 and 10000 years return period floods are 62389, 77009, 83295 & 104163 cumecs. The sampling variances corresponding to these are 4.21×10^7, 7.15×10^7, 8.67×10^7 and 1.47×10^8 respectively.

8.2 Partial Duration Series Using Combination of Poisson and Exponential Distribution

The variation of values of λ, β and $q_o + \beta \ln \lambda$ with truncation level level q_o for the partial flood series is given in Table 2.

For q_o = 8376 cumecs the values of λ and β are 2.437 and 10583.098 respectively. Using equation (10) the 100, 500, 1000 and 10000 years return period floods are 66487, 83562, 90903 and 115277 cumecs respectively which are slightly higher than the estimates of annual maximum flood series.

Using exact theoretical approach i.e. $\alpha = \beta$ the sampling variances, of 100, 500, 1000 and 10000 years return period floods are 3.2×10^7, 5.36×10^7, 6.46×10^7 and 1.08×10^8 respectively.

Using approximate theoretical approach i.e. $\beta \neq \alpha$ & β = 10583.098 the sampling variances are 4.37×10^7, 7.32×10^7, 8.82x10 and 1.48×10^8. These results have been tabulated in Table 4.

8.2.1 Exact theoretical approach

Equation (29) shows how the ratio of sampling variances obtained by the exact theoretical approach varies with the return period T. For a

TABLE 3

ANNUAL PEAK STAGE AND DISCHARGE SERIES AT MORTAKKA STAGE
STAGE DISCHARGE RELATIONSHIP: $Q=317.18*(H-155.2)^{1.768}$

Year	Peak stage above MSL (m)	Peak Discharge (m^3/sec.)
1951	162.68	11127
1952	163.59	13631
1953	165.48	19521
1954	169.25	33915
1955	165.84	20746
1956	163.00	11982
1957	167.03	25023
1958	163.37	13005
1959	168.40	30372
1960	165.78	20540
1961	173.73	55323
1962	168.70	31604
1963	164.43	16135
1964	166.60	23438
1965	165.20	18591
1966	162.76	11338
1967	165.53	19690
1968	168.70	31604
1969	167.79	27935
1970	170.99	41691
1971	165.05	18101
1972	172.27	47851
1973	173.49	54063
1974	169.86	36562
1975	169.10	33278
1976	164.93	17713
1977	166.85	24354
1978	168.20	29564
1979	167.35	26232

1980	166.41	22751
1981	167.20	25662
1982	164.58	16602

Mean = 25935.344 cumecs

Standard deviation = 11615.904 cumecs

Coeff.of skewness = 1.044

TABLE - 4

RESULTS OF FREQUENCY ANALYSIS

	Recurrence Interval in Years			
	100	500	1000	10000
Estimated peak flow in cumecs from partial duration series using Poisson and Exponential Distributions	66487	83562	90903	115277
Sampling Variance using exact theoretical approach	3.2×10^{7}	5.36×10^{7}	6.46×10^{7}	1.08×10^{8}
Sampling variance using approximate theoretical approach	4.37×10^{7}	7.32×10^{7}	8.82×10^{7}	1.48×10^{8}
Estimated peak flow in cumecs from annual maximum series using Gumbel distribution	62389	77009	83295	104164
Sampling variance	4.2×10^{7}	7.15×10^{7}	8.67×10^{7}	1.47×10^{8}

given value of λ , the relationship between the ratio R_1 and the return period T expressed as the Gumbel reduced variate y(T) can be derived. The results of these relationships for the range of λ =0.8 to 10.0 are shown in Figure 1. It is evident from the figure that the partial flood series estimate of Q(T) always has a smaller sampling variance than that of the annual flood series for the return period T less than 11 years. For the whole range of return periods the partial flood series estimate of Q(T) has a smaller sampling variance than that of the annual flood series if the value of λ is atleast 1.65.

8.2.2 Approximate theoretical approach

The values of u and α for annual flood series and $q_o + \beta \ln \lambda$ and β for partial duration series are tabulated in Table 2. It is clear from the table that u is slightly higher than the $q_o + \beta \ln\lambda$ for the range λ = 1.0 to 2.437. β is slightly higher than α.

By substituting the estimates of α, λ and β for each threshold level into equation (30) relationships between R_2 and T are obtained for various λ values. (Fig.2). By comparing Fig.1 & Fig 2,(relationships between ratio of Var [Q(T)] and T, based on exact theoretical approach(Fig.1) and approximate theoretical approach (Fig.2)) R_2 is always lower than R_1. The value of R_2 increases with λ . For λ =2.437 the ratio R_2 comes closer to unity. In the range of λ studied (1.0 to 2.437) the sampling variance of annual flood series is less than that of partial flood series.

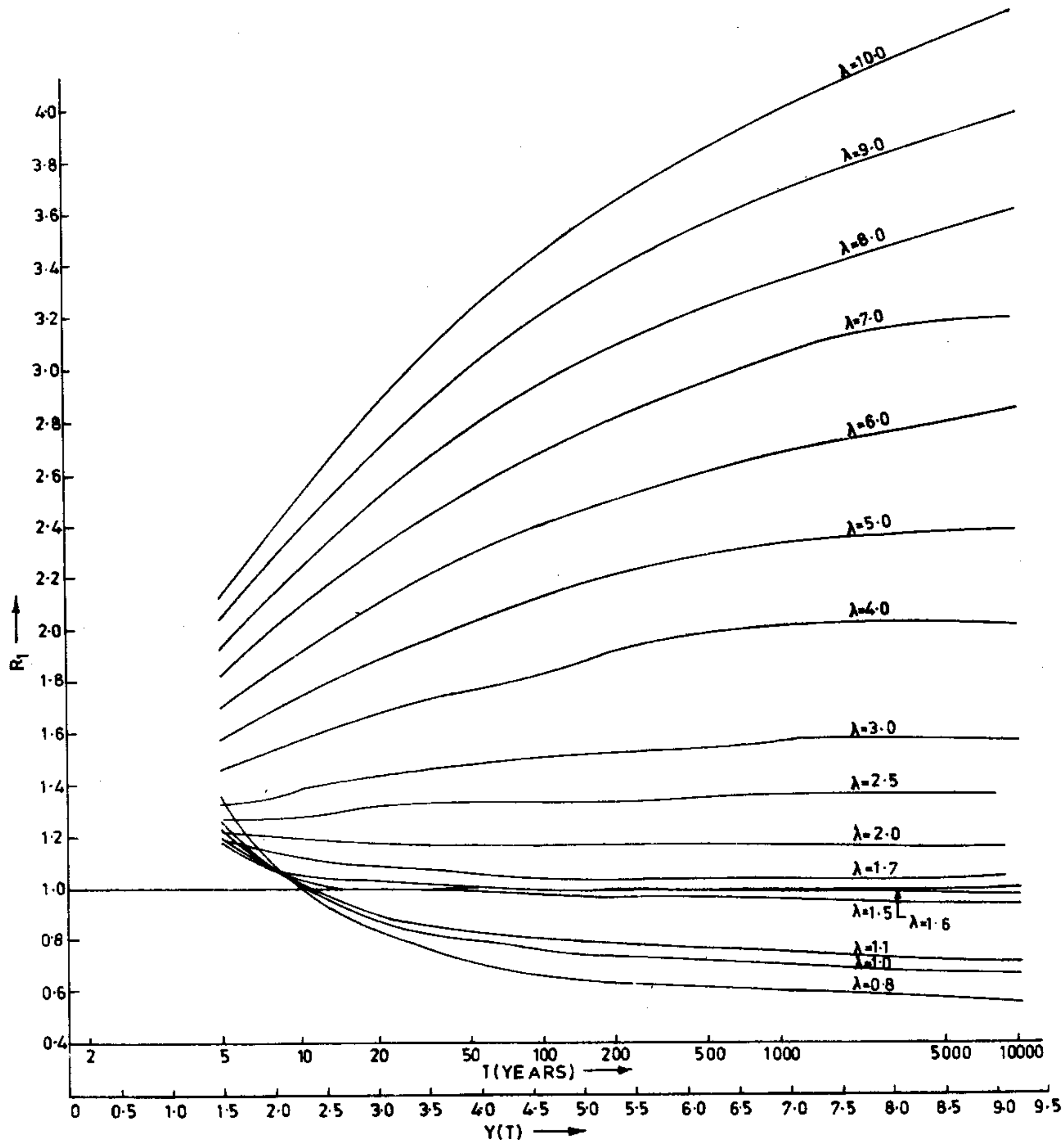

FIGURE 1- RELATIONSHIP BETWEEN RATIO R_1 OF SAMPLING VARIANCES $Q(T)_a$ AND $Q(T)_p$ BASED ON EXACT THEORETICAL APPROACH AND RETURN PERIOD T FOR GIVEN λ

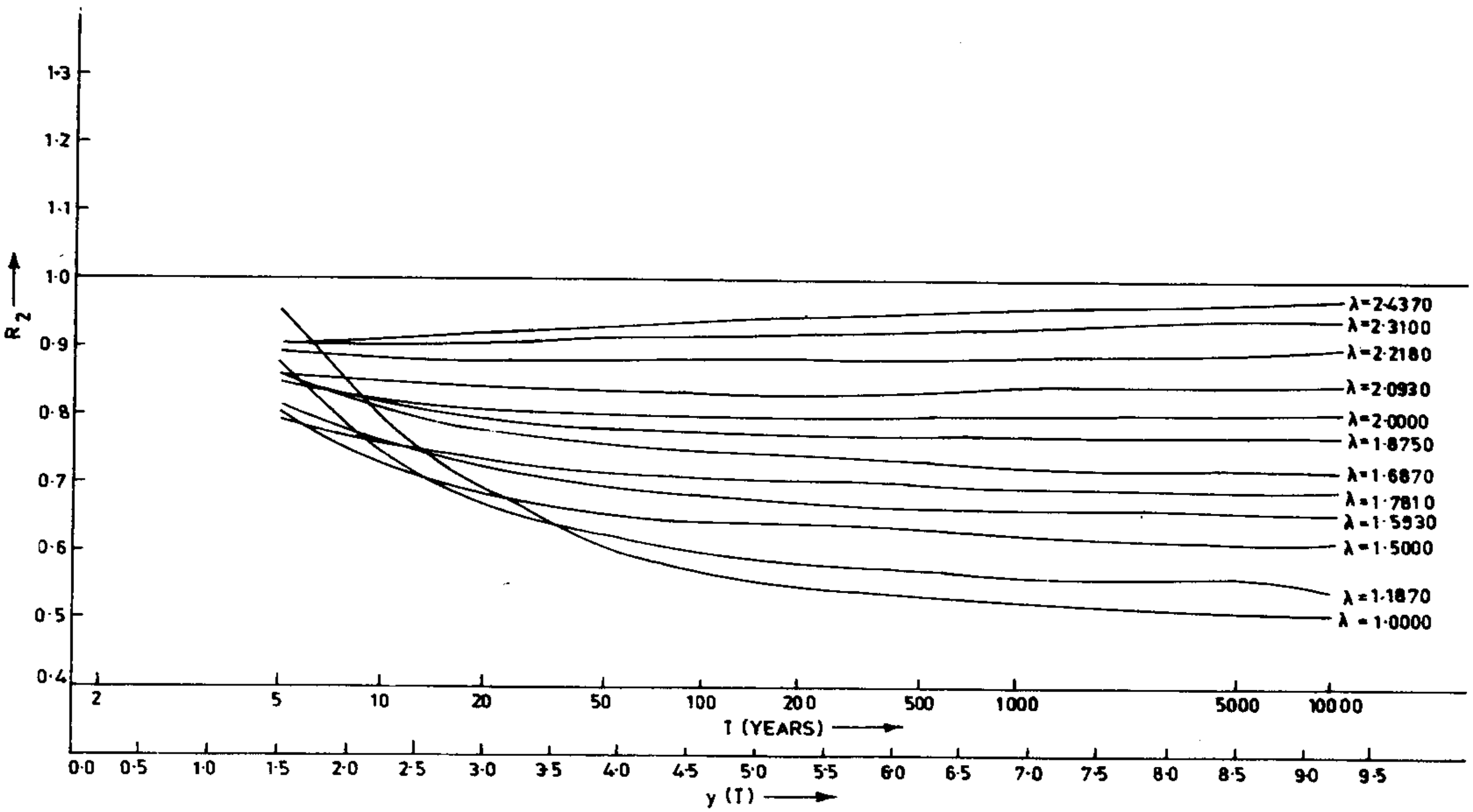

FIGURE 2. RELATIONSHIP BETWEEN RATIO R_2 OF SAMPLING VARIANCES $Q(T)_a$ AND $Q(T)_p$ BASED ON APPROXIMATE THEORETICAL APPROACH AND THE RETURN PERIOD T FOR GIVEN λ

9.0 CONCLUSIONS

(A) For Narmadasagar dam the 100,500,1000 and 10000 years return period floods are 66540, 83573, 90909 and 115277 cumecs respectively using partial duration series, which are slightly higher than corresponding estimates of annual flood series assuming that annual flood series follow Gumbel distribution.

(B) On the basis of exact theoretical approach it is seen that the partial flood series estimates of Q(T) always have a smaller sampling variance than that of the annual flood series for the return period T less than 11 years. For the whole range of return periods the partial flood series estimate of Q(T) has a smaller sampling variance than that of the annual flood series, if the value of is atleast 1.65.

(C) On the basis of approximate theoretical approach (Computed estimates of α and β are taken) it is observed that in the range of λ studied (1.0 to 2.437) the sampling variance of partial flood series is more than the sampling variance of annual flood series. However, this difference reduces when λ increases. For λ = 2.437, the sampling variances of the two series are comparable.

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