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# Scenario analysis in water resources systems optimization under uncertainty conditions

**GIOVANNI M. SECHI and PAOLA ZUDDAS**
Dipartimento di Ingegneria del Territorio - Sezione di Ingegneria Idraulica, Università degli Studi di Cagliari, Piazza D'Armi – 09123 Cagliari, Italy

## Abstract

In this paper we present a methodology to perform water system optimization under climatic and hydrological uncertainty, in order to reach a sufficient degree of systems reliability to determine the risk of significant water deficiencies. Different generation techniques are compared to set up and analyze a number of scenarios. Uncertainty is modeled by a scenario-tree in a multistage environment, which includes different possible configurations of inflows in a wide time-horizon. The mathematical model structure representing the multiperiod optimization stochastic problem allows to handle a huge number of variables and constraints. The solutions of the optimization process on the scenario-tree are post-processed in order to reach a "robust" solution. The aim is to identify trends and essential features on which to base a robust decision policy.

## INTRODUCTION

Water resources management problems with a multiperiod feature are associated to mathematical optimization models that handle thousands of constraints and variables depending on the level of adherence required to reach a significant representation of the system (*Loucks et al.*, 1981; *Yeh*, 1985)

Moreover these models are typically characterized by a level of uncertainty about the value of hydrological exogenous inflows and demand patterns. On the other hand inadequate values assigned to them could invalidate the results of the study. When the statistical information on data estimation is not enough to support a stochastic model or when probabilistic rules are not available, an alternative approach could be in practice that of setting up the scenario analysis technique.

In this paper we present a general-purpose scenario-modeling framework to solve water system optimization problems under input data uncertainty, as an alternative to the traditional stochastic approach in order to reach a "robust" decision policy that should minimize the risk of wrong decisions. This approach leads to a huge model that includes a network sub-model for each scenario plus linking constraints, and must be treated with specialized resolution techniques. In the proposed approach, the problem is to be expanded on a set of scenario sub-problems, each of which corresponding to a possible configuration of the data series. By studying the global-scenarios optimal solution, one could discover similarities and trends that should quantify the risk of management operations. Each scenario can be weighted to represent the "importance" assigned to the running configuration. Sometimes the weights can be viewed as the probability of occurrence of the examined scenario. A "robust-barycentrical" optimization solution can be obtained by a procedure that minimizes the distance between sub-problems optima.

The model is usually defined in a dynamic planning horizon in which management decisions have to be made either sequentially, by adopting a predefined scenario independently, or by following different scenarios in a "scenario-tree" context as the data characteristics change as described in the next paragraph. The scenarios aggregation into a tree must be performed following the basic "implementable principle" or “principle of progressive hedging”: *“If two different scenarios are identical up to stage r on the basis of the information available about them at stage r, then the variables must be identical up to stage r”.* (*Rockafellar and Wets,* 1991).

This condition guarantees that the solution obtained by the model in a period is independent of the information that is not yet available; in other words model evolution is only based on the information available at the moment, a time when the future configuration may diversify.

## SCENARIO-TREE GENERATION

As previously mentioned, in water management problems uncertainty can be referred to cost-benefits or demands-supplies data. One of the main goals in this field is to reach a configuration that should guarantee an adequate level of water system reliability and provide management criteria to be adopted by the Water Authorities. In previous papers (*Cao et al*.,1988; *Sechi and Zuddas,* 1993; *Sechi and Zuddas,* 1998) it was stated that the usual optimization techniques do not allow a detailed modeling of the system, and that they must be accompanied by a simulation testing process starting from the solution obtained in the optimization phase. An interactive process however may be set up between the optimization phase and the simulation testing phase, which should limit the recourse to the latter burdensome computational procedure. This is a typical problem in which the representation of the level of uncertainty by probabilistic rules is inadequate, and the data uncertainty can be modeled usefully by scenario analysis.

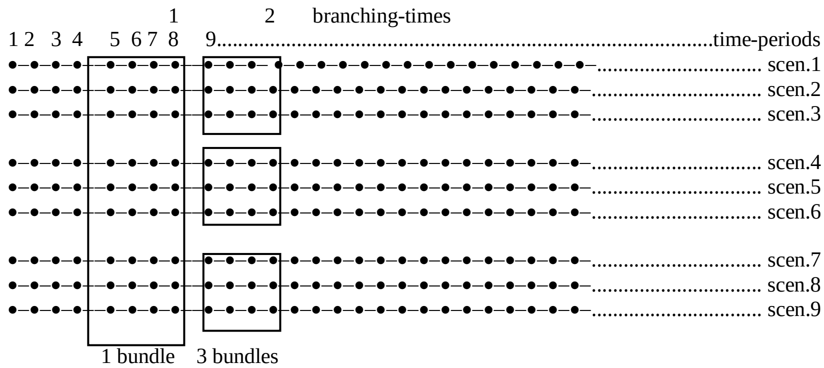


**Figure 1a. Set G of nine scenarios as a sequence of nine parallel series.**

The data pre-processor that builds scenario sequences follows procedures that have been developed in the *WARSYP Project* (2000). Therefore the scenarios are to be viewed as a set ***G*** of synthetic hydrological series obtained from historical samples. As an example, Figure 1a shows a set of nine scenarios as a sequence of nine parallel series. Each scenario *g* is expanded on a time-horizon of a number of periods.

In this paper two approaches have been performed to generate the series: one refers to the Monte Carlo (MC) generation scheme, the other to Neural Network (NN) techniques. MC generation requires a preliminary definition of time-period clusters on hydrological data, and synthetically consists of the following steps: 1) random generation of meteorological characterization at each cluster; 2) generation of hydrological data from predefined sets of clusters; 3) addition of noise components to improve the statistical fitness. (*Onnis et al.,* 1999; *Lorrai and Sechi,* 1996)

The NN approach has been developed using both the classical Multilayer Perceptron scheme and the Locally Recurrent NN scheme. In any case, a first sensitivity analysis was carried out to evaluate the best fitting NN configuration, the number of nodes in the layers, and the number of iterations to be used in model training. Subsequently (in the testing phase) the hydrological series were generated.

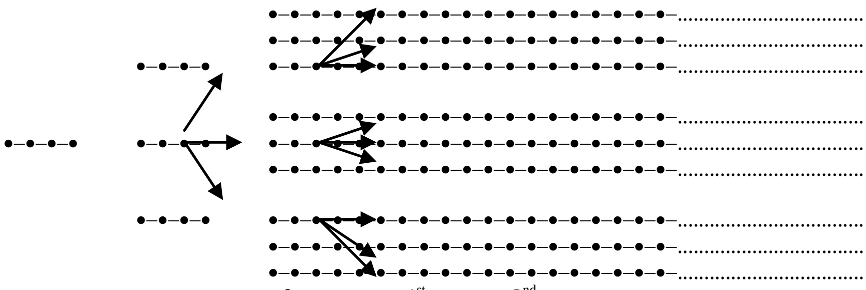


**Figure 1b. Scenario-tree construction.**

When the set of synthetic hydrological sequences has been generated, the "principle of progressive hedging" is performed by bundling the sequences to build the scenario-tree as described in the following steps:

**Branching**: Identify branching-times $\tau$ as time-periods in which to apply bundles on parallel sequences, while identifying the stages in which to divide the scenario horizon.

**Bundling**: Identify the number, $\beta_\tau$, of bundles at each branching-time.

**Grouping**: Identify groups, $\Gamma_\tau$ , of scenarios to include in each bundle.

In this way a number of stages are defined, where stage *0* corresponds to the initial hydrological characterization of the system up to the first branch time-period. In the scenario-tree this represents the root. In stage *1* a number, $\beta_1$ , of different possible hydrological configurations can occur, in stage *2* a number, $\beta_1 * \beta_2$ , can occur, and so on and so forth.

In this way the graph "explodes" in size, on increasing the possible branches, and each root-to-leaf path represents a particular scenario. Figure 1b shows an example of the scenario-tree derived from the parallel sequences. The figure represents a tree with two branches: the first branching-time is the 4$^{th}$ time-period, the second is the 8$^{th}$ period. One bundle is operated at the first branch and three bundles are operated at second branch. The zero bundle includes all scenarios: in the $1^{st}$ stage 3 bundles are generated including 3 scenarios in each group, while in the $2^{nd}$ stage the 9 scenarios run until they reach the end of the time-horizon.

Once scenarios have been generated, some general checks must be performed to test their statistical properties. Among others: stationary test on mean and variance in order to check process changes over time; independence test, to look for possible relations or for a trend among subsequent stages; time and space-correlation test; etc.

## DETERMINISTIC AND STOCHASTIC MATHEMATICAL MODEL

In the past years, water resources optimization problems in a deterministic framework have been extensively studied (*Sechi and Zuddas*, 1996). A high level of solution accuracy was reached thanks to the recourse to specialized techniques that exploit algebraic structures of related mathematical models. On the other hand, though this kind of model is not suitable to describe water management optimization problems under uncertainty conditions, it may be used as an underlying model in generating the stochastic mathematical model extended to the scenario-tree. As a reference, in the deterministic framework, we consider the usual linear programming (LP) model that can be adopted as a starting point to evaluate the performances of the physical system without taking uncertainties into account. As extensively illustrated in previous papers (*Sechi and Zuddas*, 1998), the deterministic model can be expressed as follows in a compact form:

$$\begin{array}{ll} \min & \boldsymbol{c}\,\boldsymbol{x} \\ s.t. & \boldsymbol{A}\,\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0} \end{array} \qquad (1)$$

The stochastic model reflects uncertainty in the rhs-vector, $\boldsymbol{b}$, and in cost, $\boldsymbol{c}$, for each scenario, $g$, extracted from the predefined set, **G**, of all scenarios. Moreover, as previously described, the decision-maker can assign weights, $\boldsymbol{w_g}$ , to each scenario. A possible approach to obtain a solution in the scenario optimization framework is to represent the problem by a new linear programming model, where the constraints matrix exhibits the special structure of a block-diagonal matrix, $\boldsymbol{A_g}$ , with complicating constraints on a subset, $\boldsymbol{x^*}$ , of $\boldsymbol{x}$.

In terms of scenarios, these additional constraints define the congruity of the model, and are expressed as requirements of equal interstage flow transfers in all scenarios between two consecutive stages. The stochastic model will have the following structure:

$$\begin{array}{lll} \min & \Sigma_g\, \boldsymbol{w}_g\, \boldsymbol{c}_g\, \boldsymbol{x}_g & \\ s.t. & \boldsymbol{A}_g\, \boldsymbol{x}_g = \boldsymbol{b}_g & \forall g \in \boldsymbol{G} \\ & \boldsymbol{x}_g \geq \boldsymbol{0} & \\ & \boldsymbol{x}^* \in \boldsymbol{S} & \end{array} \qquad (2)$$

where $\boldsymbol{x}^* \in \boldsymbol{S}$ represents the linking constraints on interstage flows.

This kind of model can be solved by decomposition methods such as Benders decomposition techniques, which exploit the special structure of constraints. When the size of the problem becomes huge, it is possible to resort to parallel computing. As a matter of fact, a block in the block-diagonal part of the matrix constraints corresponds to each scenario, and the links among the scenarios are the complicating constraints. In this way we can perform a parallel process on each scenario with the master problem on the linking constraints.

In this phase of the study, the stochastic problem is solved by adopting high efficient computer codes based on the state of the art of linear programming, such as those implementing interior point methods that give good quality results even in very large-sized models.(*Vanderbei*, 1988)

The resolution approach can therefore be described as a three-phase algorithm.

In the first phase the scenario-tree is generated, and the optimization L.P. problem is represented as a collection of deterministic optimization problems, one for each node of the tree plus the linking constraints.

In the second phase the entire problem is solved by a highly efficient computer code, such as *CPLEX* (1995) or *MP-XPRESS* (1999). At the end of this phase we obtain a solution configuration on all scenarios, i.e. on all the paths from the leaves to the root of the tree.

In the third phase the proposed solution is obtained as the best configuration in terms of minimizing the risk of wrong decision, and finding a sort of "barycentrical" solution, $\xi$, between all scenarios. Different techniques can be used to reach such a "robust" solution. Some set the objective function in problem (2) as a penalization function on the weighted difference between the solution, $\xi$, to be adopted and the optimal solution in each scenario, $\boldsymbol{x}_g$, (*Dembo*, 1991, *Rockafellar and Wets*, 1991). Other techniques are based on a post-processor on the solution obtained from problem (2) (*WARSYP - UC Reports*, 2000). The former give a good level of adherence to the implementable policy but destroy the linear structure of the objective function to include penalization. Though a number of improvements have been proposed for this approach (*Alvarez et al*., 1994), the accuracy of the global solution decreases dramatically on increasing the number of variables. The latter seem very promising even in the event of a huge number of variables and constraints. Early results have been obtained carrying out weighted averages of the solutions in the scenario-tree.( *WARSYP- UC Reports*, 2000) An extended testing phase is in progress to improve the rules of solution handling of the stochastic problem to evaluate the global solution.

As regards the definition of weights, one can use the results of the independent scenario optimizations, considering a regressive relation between scenario inputs and output node deficits, in order to obtain a function that should allow to assign weights to subproblems related to different scenarios. Moreover, a tuning phase is crucial in this kind of approach since it allows the decision-maker to increase adherence and to update the decision process. Different rules could be adopted: e.g., at the zero-stage the root can be optimized independently or ignored as single scenario and considered as a stage belonging to others scenario. In the attribution of weights different techniques could be adopted too: e.g., weights on stages, weights on scenarios, weights both on scenarios and stages, etc.. Since, as is obvious, different results correspond to different techniques, the latter must be tuned on the features of the examined problem.

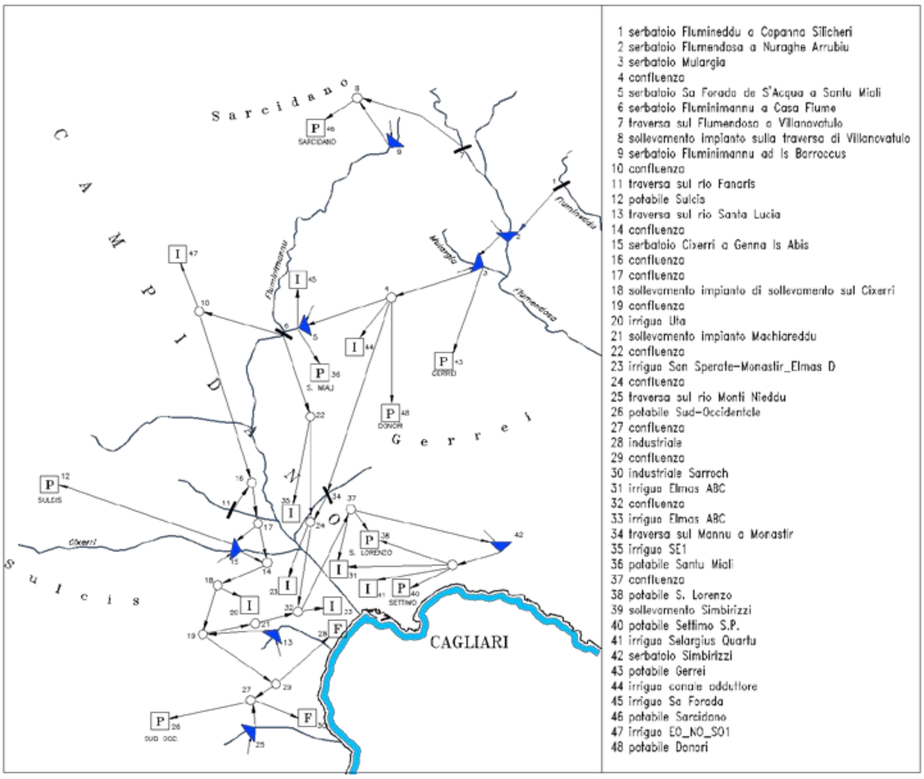


**Figure 2. Flumendosa Campidano system sketch.**

## TEST CASE

Scenario analysis was performed on the Flumendosa-Campidano system sketched in Figure 2. A correct evaluation of the system performances and requirements became increasingly urgent as the system managers were obliged to face the serious resource deficits caused by the drought events of the past decade. Different hydrological and demand scenarios therefore must be considered to obtain system optimization. A synthetic series has been generated with both MC and NN techniques, starting from a database of a time-horizon of 75 years, corresponding to 900 monthly time-periods. A set of 30 scenarios was then submitted to statistical validation and selected. Stochastic optimization was performed on a scenario-tree of 3 stages up to 30 leaves. Since each scenario involves about 3,000 variables, the stochastic problem is supported by a model of several thousand variables and constraints. A graphical interface, which was used to aid the testing phase, allowed the extensive experimentation reported in (*Sechi and Zuddas, 2000).*

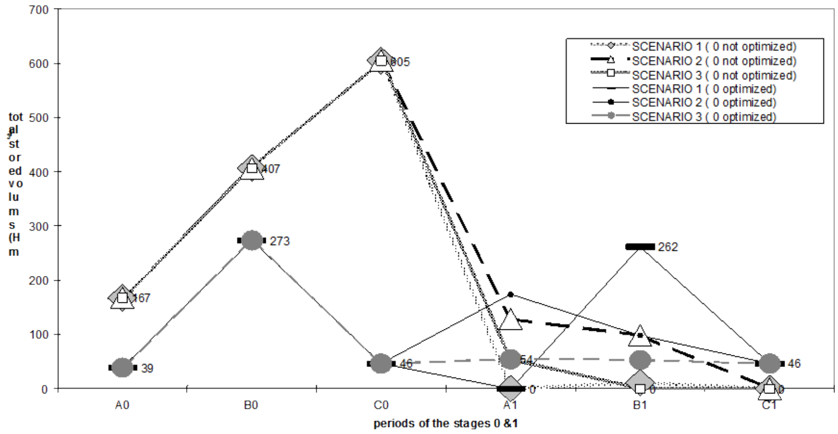


**Figure 3. Scenario optimization results (weights defined for scenario and stages).**

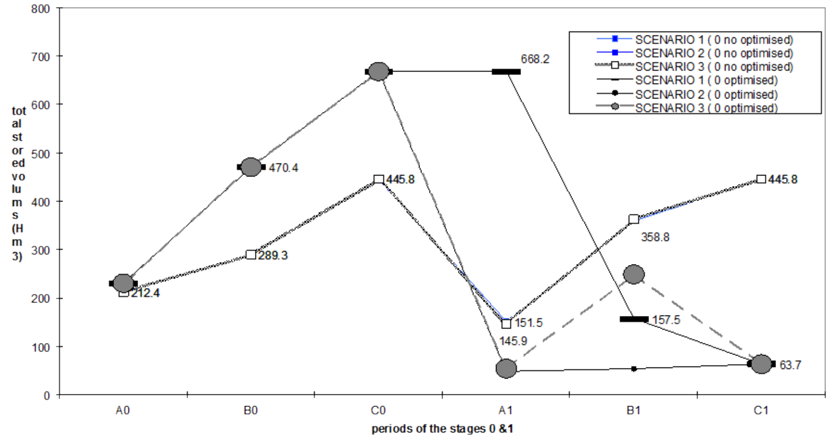


**Figure 4. Scenario optimization results (weights defined only for stages).**

As an example, Figures 3&4 show the behavior of optimization results in stored reservoir volumes, corresponding to different choices in weighting the stages in the tuning phase.

## CONCLUSIONS AND FUTURE PROSPECTS

This paper was aimed to give a contribution to the mathematical optimization of water resources systems, when the role of uncertainty is particularly important. In such a problem, which involves social, economical, political, and physical events, no probabilistic description of the unknown elements is available, either because a substantial statistical base is lacking or because it is impossible to derive a probabilistic law from conceptual considerations.

In our experience, scenario analysis could be an alternative approach to stochastic optimization or to deterministic optimization techniques (mathematical programming), even in problems that manage a huge quantity of data extended to long periods.

The numerical results obtained so far clearly show that it is a good choice to extend the study to a long time-horizon, while maintaining the stochastic model in a linear context instead of disturbing linearity, and consequently reducing the temporal and spatial dimension of the study thus losing solution accuracy.

This type of analysis is accompanied by an assessment of the disaggregation possibilities of multiperiod problems. It is clear that by operating in a single subproblem dimensionally, the operative size of the model can be reduced dramatically. This would allow a parallel treatment of the discretization periods rather than process the entire problem. In this way we can reach a strategy on how the different solutions can be faced and consolidated in an overall decision policy.

In the next phase of the algorithm, which is in progress, a level of uncertainty has been introduced on a wider class of crucial data to obtain general rules on the formulation of scenario analysis problems.

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