ICIWRM – 2000, Proceedings of International Conference on Integrated Water Resources Management for Sustainable Development, 19 – 21 December, 2000, New Delhi, India

Optimal operation of irrigation reservoirs using stochastic dynamic programming

V. RAVIKUMAR
Assistant Professor, Agricultural Engineering College & Research Institute, Tamil Nadu Agricultural University, Kumulur, Lalgudi (Tk), Trichy (Dt)- 621 712, Tamil Nadu, India
K. VENUGOPAL
Professor, Centre for Water Resources, Anna University, Chennai - 600 025, Tamil Nadu, India

Abstract
Stochastic Dynamic Programming (SDP) is widely used in deriving optimal operating policies for reservoir. Usually inflow is represented in SDP as markovian first order in every time period or independent for every time period. One state variable is more in markovian first order inflow compared to independent inflow. In this work SDP models with three types of inflow assumptions namely inependent inflow in every time period ,first order markovian inflow in every time period and a mix of both first order markovian inflow assumption and independent inflow assumption based on the statistical significance of serial correlation values has been developed and applied to a irrigation reservoir. The mixed inflow assumption has been found to have less dimensionality problem. The results of using all the three types of inflow assumption in SDP model are compared and discussed.

INTRODUCTION

Stochastic dynamic programming (SDP) remains one of several widely used optimization techniques since it is capable of handling the stochastic nature of hydrologic variables. Steady-state SDP models are very useful in long-term planning. Excellent review articles exist in the literature (Yakowitz, 1982 and Yeh, 1985). Dudley and others (1971a, 1972; Dudley and Burt, 1973; Dudley, 1972, 1988) have published many papers on irrigation planning for single crop from single reservoir using SDP. Palmer Jones (1977) indicated that omitting the effect of serial correlation of inflow would cause a significant reduction in expected optimal benefits if the serial correlation of inflow proved significant. Bras and Cordova (1981) derived soil moisture transition probabilities analytically. However, in the case of real life systems, that method is difficult to use when compared with the usual method of deriving the transition probability matrix by simulation. Rhenals and Bras (1981) tested the effect of considering uncertainty in potential evapotranspiration as one of the state variables and found that the effect of considering this state variable was minimal.

Development of optimal water allocation models to handle multiple crops per season is an important issue in most countries, and the development of such models is currently given high priority. Development of optimal water allocation models for multiple crops with multiple seasons per year becomes still more complicated. Dudley and others (1976) have improved their early works for multi-crop systems, Vedula and Mujumdar (1992) and Mujumdar and Vedula (1992) developed a steady-state SDP model for multiple crops. They found that, for multiple crops, maximizing the expected benefits did not give desirable results, whereas minimizing the expected sum of the square of deviations of total demand for all crops from the total supply to all crops gave better results.

Ravikumar and Venugopal (1999) developed a model for optimal reservoir operation under cropping pattern uncertainty for Krishnagiri Reservoir Project in India. In this work, that model, has been tested with three kinds of inflow assumptions. One is inflow during every month is independent. Two is inflow during every month is first order markorian. Third assumption is based on finding serial correlation coefficients for each month and finding their statistical significance. In the months in which serial correlation coefficients are statistically significant, inflow for those months alone are assumed as first order markovian and for the other months, inflow is assumed as independent.

STUDY AREA

The Krishnagiri Reservoir Project (KRP), which is considered in this study, is located within the Ponnaiyar River basin in Tamil Nadu, India. The storage capacity of the reservoir is 68×10^6 m^3 and it 3,600 ha. When the reservoir was constructed, the objective was to irrigate paddy crops in the entire command area for one crop season between August and December.

After years of experience, reservoir operation has undergone a significant change. At the start of every crop season, the opening of the reservoir is decided based on a comparison of storage with critical reservoir storage (CRL). If the storage during August is more than CRL [1], water is released for irrigating the paddy crop in the entire area. It not, the opening is postponed until November even if the reservoir becomes full during September or October. The reason for that strategy is that the yield of paddy that is planted during September or October will be appreciably affected because of cool climatic conditions that prevail during the grain formation stage. If storage during November is more than CRL [2], water is supplied for raising the paddy crop. Paddy planted during August would be harvested during December. After December, if the reservoir storage is more than CRL [3], water is supplied for raising the second paddy crop.

Although the reservoir release is officially intended for paddy crop in this irrigation system area, farmers in some areas (totalling approximately 600 ha) prefer to grow sugar cane since it is more remunerative than paddy. Since sugar cane is an annual crop, it must be irrigated throughout the year. Sugar cane growers take water from the canal supply when the reservoir is opened for paddy crop irrigation. When the reservoir is closed, they rely on groundwater pumped from wells. At present, agriculture in India is more business oriented than subsistence oriented. So future reservoir operations must be undertaken in accordance with the preferences of the farmers and the need to maximize benefits. Owing to its significant size, the 600 ha area also has been included in the development of the model in this study. The proposed reservoir operation procedure, taking into account current practices, is illustrated in Figure 1.

If reservoir storage on August 1 is greater than the specified CRL (1), paddy is planted on 3,000 ha (crop B) and harvested at the end of December. If the reservoir storage on January 1,is greater than the specified CRL (3), a second paddy crop (crop D) is planted on 3,000 ha together with 600 ha of sugar cane (crop A). However, if the reservoir storage on January 1 is less than CRL (3) only crop A is planted. If the reservoir storage on August 1 is less than CRL (1), the reservoir storage on November 1 is compared with CRL (2). If the storage is greater than CRL (2), paddy is planted on 3,000 ha (crop C), followed in January by sugar cane (crop A). If the storage is less than CRL (2), then storage during January 1 is checked against CRL (3).

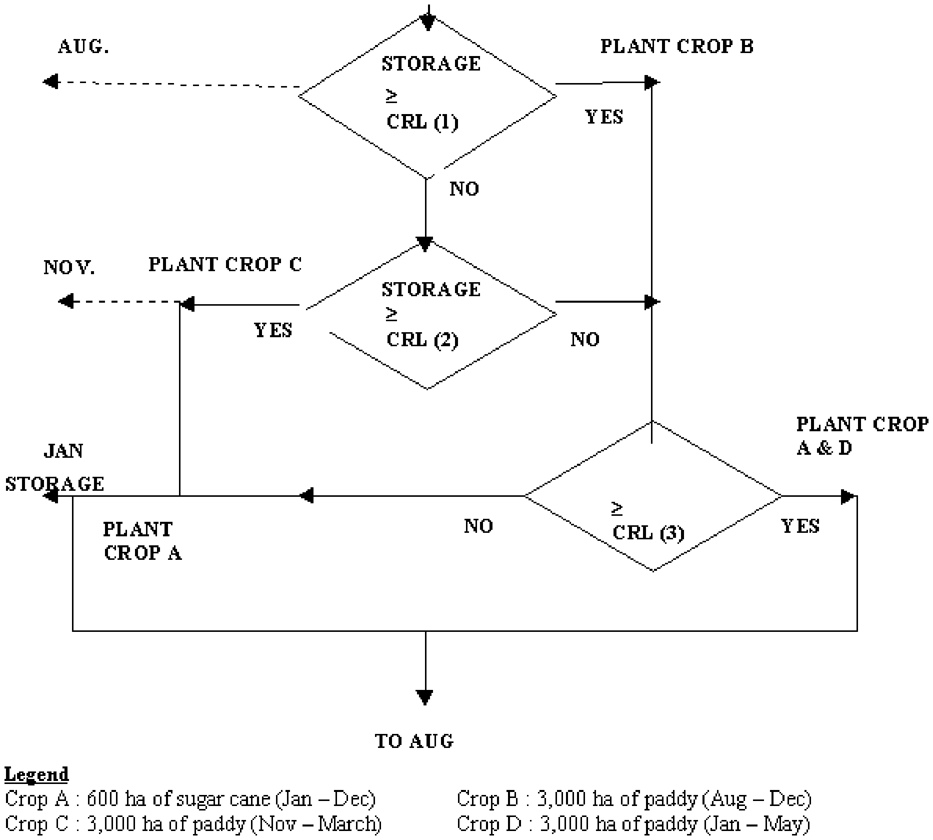

Figure 1. Proposed Reservoir Operation Practice for Cultivating Different Crops.

From the above, it is obvious that the cropping pattern varies from one year to the next. It can be seen that, at any point in time, one or more crops are under cultivation on part or all of the command area. Thus, water releases made in any period for an existing crop will also affect future water releases and crops raised. Consequently, it is necessary to carry over water from one season to the next in order to minimize deficits (deficit = release – demand). In situations such as those detailed above, steady-state SDP models are very useful in deciding on the critical storage and whether a particular crop should be planted, as well as setting an optimal release policy corresponding to the chosen set of critical storage levels.

MODEL DEVELOPMENT

State Variables

If inflow in month 't' is assumed as first order markovian, the state vector Φ^t is,

$$\Phi^t = \{S^t, Q^t, Z^t\} \tag{1a}$$

If inflow in month 't' is assumed as independent, the state vector Φ^t is,

$$\Phi^t = \{S^t, Z^t\} \tag{1b}$$

Where S^t is reservoir storage state at the start of time period t, Q^t is inflow during time period t and Z^t is the crop group existing at the start of time period t. (The meaning of the term "crop group" as used in this study is explained in a subsequent section of this paper). The reservoir inflow Q^t is considered as a stationary series and is assumed to follow the first order Markov chain model. The demand is not considered as a state variable and an average demand, obtained from simulation of the command area for 32 years of data, is used. The details of simulation model are provided in Ravikumar and Venugopal (1997).

Reservoir Storage State Transformation

Let indices i and j represent discretization indices for inflow, k and l represent discretization indices for reservoir storage at the start of time periods t and t+ l respectively. $S_k^t; S_l^{t+1}$ and $Q_i^t; Q_j^{t+1}$ are representative values of the state variables, reservoir storage and inflow for periods t and t+1 respectively. Reservoir storage state transformation is governed by

$$R_{kil}^t = S_k^t + Q_i^t - S_l^{t+1} - E_{kl}^t \tag{2}$$

Where R_{kil}^t is the release when the storage class interval at the start of time period t is k and at the start of t+1 is l and inflow class interval during time period t is i. E_{kl}^t is evaporation loss when the storage class interval at the start of the time period t is k and at the start of time period t+1 is l.

Crop Group State Transformation

If the reservoir is operated as explained above, the possibilities for cultivating different crops during any month are (Figure 2) explained below:

If crop B is not planted during August, crop A alone will exist until October. If crop C is not planted during November, crop A will exist alone until July

If crop B is planted during August, crops A and B will exist until December. If crop D is not planted crop A will exist alone until July

If crop B is not planted during August and crop C is planted during November, crops A and C will exist until March; from April, only crop A will exist until July

If crop B is not planted during August and crop C is not planted during November, and if crop D is planted during January, crops A and D will exist until May, while crop A will exist alone during June and July

If crop B is planted during August, crops A and B will exist until December; if crop D is planted during January, crops A and D will exist until May, Crop A will exist alone during June and July

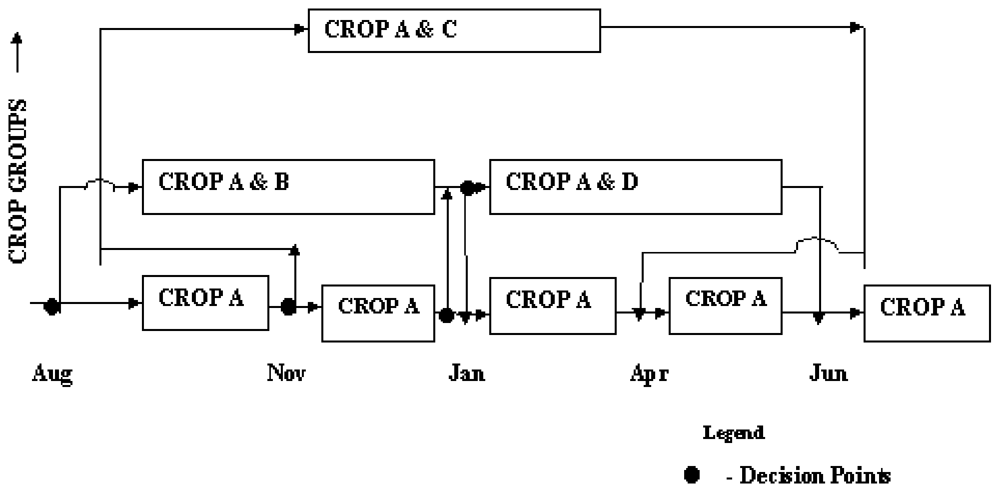

Figure 2. Crop Group Transformation.

From Figure 2 it can be observed that, from August to October, the crop group states total two (crop A alone, and crops A and B). In November and December, the crop group states total three (crop A alone, crops A and B, and crops A and C). Between January and March the crop group states total three (crop A alone, crops A and D, and crops A and C). During April and May, the crop group states total two (crop A alone, and crops A and D). During June and July the crop group states total one. Thus, the maximum number of crop group states is three in this case.

Objective Function

In choosing values of critical storage for each crop planting, the objective is to determine the optimal releases for each month with respect to the state variables. To choose from possible values of decision variable R_{kil}^{t} a measure of system performance has to be specified. Let the measure of system performance be associated with R_{kil}^{t} and the demand D_c^t for crop group Z_c^t. The system performance measure incorporated in the objective function is intended to minimize the expected sum of the square of deviation of deficits (deficit = demand – release).

Recursive Relations

The problem is formulated using backward moving dynamic programming, assuming that the reservoir operation terminates in future arbitrary year Y at period T, where T is the total number of periods in a year, Let N be the number of periods remaining until the end of year Y. Let c and d be the indices representing crop groups existing in periods t and t+1, respectively. Let $F_t^N(k,i,c)$ denote the minimum expected total value of system performance over N periods to go into the operation of the reservoir including current period t, given that reservoir storage is S_k^t, inflow is Q_i^t and crop group is Z_c^t. The recursive relation if inflow in periods 't' and 't+1' are markovian first order is as follows.

$$F_t^N(k,i,c) = \underset{\{l,d\}}{MIN}\left[\left(R_{kil}^t - D_c^t\right)^2 + \sum_j \left(P_{ij}^t \times F_{t+1}^{N-1}(l,j,d)\right)\right] \quad \forall\ k,i,c \tag{3}$$

Where {l,d} denotes feasible values of l and d. P_{ij}^t is transition probability of inflow defined as the probability that the inflow in period t+1 is in class interval j, given that inflow in period t is in class interval i.

If inflow in period 't' is independent and inflow in period 't+1' is Markovian first order, following equation should be used.

$$F_t^N(k,c) = \underset{\{l,d\}}{MIN}\left\{\sum_i \left[PQ_i^t * \left(R_{kil}^t - D_c^t\right)^2 + \sum_j P_{ij}^t \ F_{t+1}^{N-1}(l,j,d)\right]\right\} \quad \forall\, k,c \tag{4}$$

If inflow in period 't' is Markovian and inflow in period t+1 is independent, following equation should be used.

$$F_t^N(k,i,c) = \underset{\{l,d\}}{MIN}\left[\left(R_{kil}^t - D_c^t\right)^2 + F_{t+1}^{N-1}(l,d)\right] \quad \forall\, k,i,c \tag{5}$$

If inflow in period t is independent and inflow in period t+1 is also independent, following equation should be used.

$$F_t^N(k,c) = \underset{\{l,d\}}{MIN}\left\{\sum_i \left[\left(PQ_i^t * \left(R_{kil}^t - D_c^t\right)^2\right) + F_{t+1}^{N-1}(l,d)\right]\right\} \quad \forall\, k,c \tag{6}$$

Recursive equations are solved recursively until a steady-state solution is reached defining the optimal policy 1*(k,i,c,t) for all values of k,i,c, and all t. Steady state is reached when $\left\lfloor F_t^{N+T}(k,i,c) - F_t^N(k,i,c) \right\rfloor$ becomes constant for all k,i,c and t.

MODEL RESULTS

Time was separated into months. Table 1 shows some of the statistical properties of the monthly inflow. Synthetic generation of inflow was carried out to get smooth transition probability matrices. The discrete division of storage and inflow was started with coarse intervals and gradually increased. Discrete division of reservoir storage above 50 and inflow separation above 8 did not improve the results significantly. So it was limited to that level.

Table 1. Statistical Properties of Monthly Inflow into Reservoir.

Months	Mean inflow ($\times 10^6$ m^3)	Standard deviation ($\times 10^6$ m^3)	Coefficient of variation	Correlation coefficient
August	5.356	9.122	1.7	0.18
September	50.177	68.445	1.4	0.10
October	82.775	75.128	0.9	0.17
November	54.119	101.643	1.9	0.36
December	8.409	22.080	2.6	0.39
January	0.009	0.036	4.0	0.60
February	2.376	2.031	0.9	-0.09
March	1.842	2.010	1.1	0.50
April	3.106	6.419	2.1	0.17
May	14.589	34.284	2.3	0.04
June	7.445	9.165	1.2	-0.06
July	8.182	13.702	1.7	-0.11

The statistical significance of the correlation coefficient was tested at 95% level of confidence for the inflow into KRP. The correlation coefficient for the months of Novbember, December, January and March are statistically significant. So, for these months inflow was assumed as first order Markovian and for all the other months inflow was assumed as independent for the mixed inflow assumption. Though in January, the statistical significance was higher, the mean inflow during January is very low. So for that month, inflow was not discretized into 8 states and taken as only one state.

Table 2 shows the dimensionality required for all the inflow assumptions. For independent inflow assumption the total number of states is 1350, for mixed inflow assumption the total number of states is 4500 and for first order markovian inflow assumption the total number of states is 9750.

Initially, arbitrary values of CRL [1], CRL [2] and CRL [3] were used and the steady-state optimal monthly release policy was found out. Then simulation of the steady-state optimal monthly release policy was carried out using synthetically generated inflow data, and the probabilities of planting each crop and the reliability of releases for each group were obtained. CRL [1], CRL [2] and CRL [3] were then changed by a trial and error procedure until the values of the release were satisfactory.

For CRL [1] = 26.34 x 10^6m^3, CRL [2] = 33. 49 x 10^6 m^3, CRL [3] = 39.44 x 10^6 m^3, the results were found for all the three inflow assumptions.

Table 2. Dimensionality for all the inflow assumptions.

Month	Independent inflow				Mixed Inflow				Markovian Inflow			
	Reservoir storage dimension	Inflow dimension	Crop group dimension	Total number of states/month	Reservoir storage dimension	Inflow dimension	Crop group dimension	Total number of states/month	Reservoir storage dimension	Inflow dimension	Crop group dimension	Total number of states/month
Aug	50	-	2	100	50	-	2	100	50	8	2	800
Sep	50	-	2	100	50	-	2	100	50	8	2	800
Oct	50	-	2	100	50	-	2	100	50	8	3	800
Nov	50	-	3	150	50	8	3	1200	50	8	3	1200
Dec	50	-	3	150	50	8	3	1200	50	8	3	1200
Jan	50	-	3	150	50	1	3	150	50	1	3	150
Feb	50	-	3	150	50	-	3	150	50	8	3	1200
Mar	50	-	3	150	50	8	3	1200	50	8	3	1200
Apr	50	-	2	100	50	-	2	100	50	8	2	800
May	50	-	2	100	50	-	2	100	50	8	2	800
Jun	50	-	1	50	50	-	1	50	50	8	1	400
Jul	50	-	1	50	50	-	1	50	50	8	1	400
	Total number of states			1350	Total number of states			4500	Total number of states			9750

Table 3. Probability of Planting each Crop for Different Inflow Assumptions.

Crop	Probability of planting each crop		
	Independent inflow	Mixed inflow	Markovian inflow
Crop B	O.706	0.708	0.707
Crop C	0.277	0.275	0.276
Crop D	0.674	0.677	0.677
Total cropping intensity in paddy growing area	1.657	1.660	1.660

Table 3 shows the probability of planting each crop for each inflow assumption. For the paddy growing area the cropping intensity is almost same for all the three types of inflow process representation. The mixed inflow process assumption and Markovian inflow assumption improves the result by about only 1%. Table 4 shows the comparison of reliability of releases at 80% and 90% reliability levels for all the three types of inflow process assumptions. The release values also do not show any significant variation from one assumption to another. This is because of the fact that even for the three months (November, December and March) for which the inflow is considered Markovian in the mixed inflow process model, the serial correlation is comparatively less and the magnitude of the inflow is also very less except in November. So it can be concluded that for the irrigation system considered here, independent inflow assumption is enough to derive monthly operating policies using the steady state model.

Table 4. Comparison of Reliability of Releases for Different Inflow Assumptions.

Crop Group	Month	80% Reliable release			90% Reliable release			Demand ($\times10^6 m^3$)
		Independent Inflow	Mixed inflow	Markovian inflow	Independent Inflow	Mixed inflow	Markovian inflow	
Crop A and Crop B	Aug	10.52	10.52	10.52	10.509	10.496	10.491	10.520
	Sep	4.739	4.735	4.735	4.717	4.708	4.710	4.800
	Oct	3.000	3.000	3.000	3.000	3.000	3.000	3.000
	Nov	2.520	2.326	2.325	2.342	2.323	2.323	2.520
	Dec	2.323	2.325	2.325	2.323	2.323	2.323	2.400
Crop A and C	Nov	7.090	7.090	7.090	7.090	7.090	7.090	7.120
	Dec	6.800	6.800	6.800	6.800	6.800	6.800	6.800
	Jan	7.320	7.320	7.320	7.320	7.320	7.320	7.320
	Feb	7.700	7.700	7.700	7.700	7.700	7.700	7.700
	Mar	6.000	5.943	5.895	6.000	5.894	5.888	6.000
Crop A and D	Jan	7.082	7.083	7.083	7.082	7.082	7.082	7.120
	Feb	9.420	9.420	9.420	9.420	9.420	9.420	9.420
	Mar	12.50	12.50	12.50	12.469	12.466	12.476	12.50
	Apr	11.30	11.30	11.30	11.300	11.134	11.132	11.30
	May	5.560	5.560	5.560	5.437	5.426	5.424	5.560
Crop A alone	Jan	0.920	0.920	0.920	0.920	0.920	0.920	0.920
	Feb	1.276	1.271	1.271	1.265	1.263	1.183	2.100
	Mar	2.225	2.365	2.363	1.986	2.178	2.171	3.440
	Apr	3.900	3.900	3.900	3.900	3.555	3.900	3.900
	May	3.518	3.515	3.508	3.505	3.505	3.500	3.560
	Jun	4.268	4.256	4.251	4.232	4.229	4.227	4.340
	Jul	3.680	3.559	3.547	3.540	3.520	3.519	3.680
	Aug	3.000	3.000	3.000	2.970	2.970	2.958	3.000
	Sep	1.000	1.000	1.000	1.000	0.891	0.895	1.000
	Oct	0.740	0.740	0.740	0.740	0.740	0.740	0.740
	Nov	0.860	0.860	0.860	0.438	0.860	0.860	0.860
	Dec	1.164	1.164	1.540	1.163	1.163	1.000	1.540

CONCLUSION

In this work, Stochastic Dynamic Programming models were developed with three kinds of inflow assumption. One is, markovian first order in every time period, two is independent for every time period and three is a mix of markovian first order and independent inflow based on the statistical significance of serial correlation values of each period. These models were applied to a irrigation system namely Krishnagiri Reservoir Project and the results are discussed. The mixed inflow assumption used in this model has been proved to have the advantage of reduced dimensionality.

References

Bras, R.L. and J.R. Cordova, (1981), "Intra-seasonal water allocation in deficit irrigation", Water Resources Research, vol.17, No.4,pp.866-874.

Dudley, N.J., (1972), "Irrigation planning 4. Optimal interseasonal water allocation" Water Resources Research, vol. 8, No.3, pp. 586-594.

Dudley, N.J. (1988), "A single decision maker approach to irrigation reservoir and farm management decision-making", Water Resources Research, vol.24, No.5,pp. 633-640.

Dudley, N.J. and O.R. Burt, (1973), "Stochastic reservoir management and system design for irrigation", Water Resources Research, vol.9, No.3, pp. 507-522.

Dudley, N.J., D. T. Howell andW.F. Musgrave, (1971a), "Optimal intra-seasonal irrigation water allocation", Water Resources Research, vol.7, No.4, p. 770-788.

Dudley, N.J., W.F. Musgrave and D.T.Howell, (1971b), "Irrigation planning 2.Choosing optimal acreages within a season", Water Resources Research, vol.7, No.5, pp. 1051-1063.

Dudley, N.J., W.F. Musgrave and D.T. Howell, (1972), "Irrigation planning 3.The best size of irrigation area for a reservoir" Water Resources Research, vol.8, No.1,pp. 7-17.

Dudley, N.J., D. Reklis and O.R. Burt, (1976), " Reliability, trade-offs and water resources development modeling with multiple crops", Water Resources Research, vol.12, No.6,pp.1101-1108.

Loucks, D.P., J.R. Stedinger and D.A. Haith, (1981), "Water Resource Systems Planning and Analysis", Englewood Cliffs, New Jersey, Prentice Hall.

Mujumdar, P.P., and S. Vedula, (1992),"Performance evaluation of an irrigation system under some optimal operating policies", Hydrological Sciences Journal, vol.37, No.1, pp. 13-26.

Palmer Jones, R.W., (1977), "Irrigation system operating policies for manure tea in Malawi", Water Resources Research, vol. 13, No.1,pp.1-7.

Ravikumar V. and K.Venugopal (1999), "Optimal reservoir operation under cropping pattern uncertainty", Water Resources Journal, Economic and Social Commission for Asia and the Pacific, United Nations.st/escap/ser.c/201, pp.30-39

Rhenals, A.E. and R.L. Bras, (1981), "The irrigation scheduling problem and evapotranspiration uncertainty", Water Resources Research, vol. 17, No.1,pp. 1328-1338.

Vedula, S. and P.P. Mujumdar, (1992) "Optimal reservoir operation for irrigation of multiple crops", Water Resources Research, Vol. 28, No.1, pp.1-9.

Yakowitz, S., (1982), "Dynamic programming application in water resources", Water Resources Research, vol. 18, No.4,pp.673-696.

Yeh W.W.G., (1985) "Reservoir management and operation models: a state-of-the-art Review", Water Resources Research, vol.21, No.12,pp.1797-1818.