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A simple 5 - parameter conceptual daily rainfall-runoff model

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Abstract
To use sufficiently fine data interval to catch the complete temporal variations of observed flows is impossible and impractical. It is important to see that the number of parameters of the model is not so large that modelling the process becomes cumbersome. Utilising easy accessibility to computational facilities like PC's and the acceptability of concepts of physics to describe the land phase of hydrological cycle in space and time daily rainfall - runoff modelling is being made easy and simple. Also for most of the catchments, daily data records are available for both rainfall and flow.

In this study, a simple 5-parameter model, based on the concept of probability distributed method is applied to simulate the daily runoff over a 9-year period of 1989 to 1997 at Tammavaram on the Gundlakamma River in Prakasam district of Andhra Pradesh. A Fortran 77 program is used to undertake automatic optimization of the model to simulate the observed flows using an appropriate objective function. From the calibration and validation of the modelling study, it is found that the model, though a five-parameter one, could respond properly to the rainfall and resulted in a reasonable efficiency of 72.14% in calibration and 68.25% in validation.

INTRODUCTION

Hydrologists are concerned with developing a proper relationship between the rainfall over a catchment and the resulting runoff at the catchment outlet. Since 1990's, some new techniques are being applied more widely in solving analytical and numerical engineering problems on the readily accessible personal computers. Utilizing the concepts of physics to describe the land phase of hydrological cycle in space and time and the computing facilities available, hydrological modelling is being made easy and simple. Conceptual modelling is one way of undertaking hydrological modelling wherein mathematical representation of physical process is employed with specific inputs to derive the output. At the same time it is important to see that the number of parameters of the model is not so large that modelling the process becomes cumbersome. To make the modelling quick, effective and reasonably efficient conceptual models with a few parameters are needed. "Fewer parameters" means quicker optimization and simpler application to any basin.

The hydrology of a drainage basin, from precipitation through to the stream discharge at the point of interest can be conceived as a series of inter-linked processes of inflows, storages and outflows. In conceptual modelling the catchment processes are described mathematically, and storages are considered as reservoirs for which water budgets are kept. Many conceptual catchment models have been developed over the past. Dawdy and O'Donnel (1965) described the structure and operation of a conceptual model. Nash and Sutcliffe (1970) discussed the principle of river flow forecasting through conceptual models. Blackie and Eeles (1985) discussed in detail about lumped catchment models and parameter optimization for hydrological forecasting.

Since a near perfect simulation of observed flows may not be as important as obtaining a response hydrograph that has general features as the observed flows complex models with large number of parameters can be relegated for simple few parameter models. Also, relaxing the goodness of fit criteria may enable much more simple models to be used (Bonvoisin & Boorman, 1992). A reasonable number of model parameters are probably around 3 to 5 which should allow adequate simulation, enable fairly confident parameter estimates to be made, ensure no parameters are redundant, provide parameters that have readily understandable functions and hence parameter values that describe major catchment effects.

In this study, a simple 5-parameter model, based on the concept of probability distributed method as proposed by Moore (1985) is applied to simulate the daily runoff over a 9 year period of 1989 to 1997 at Tammavaram in Prakasam district of Andhra Pradesh on the Gundlakamma river.

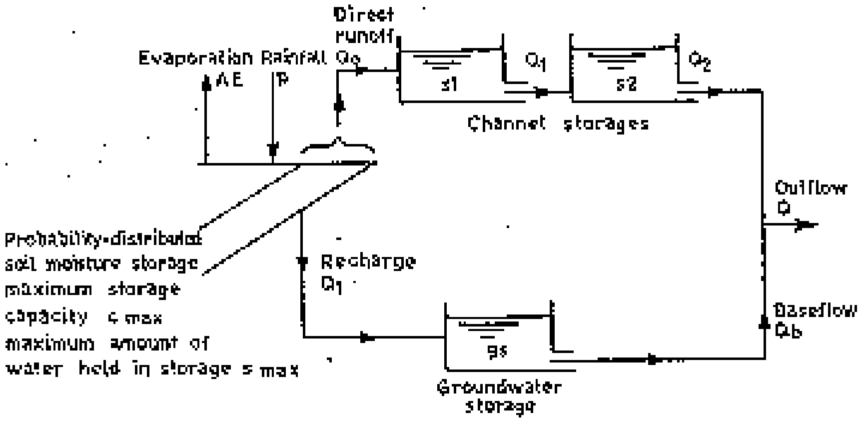

Figure 1. Schematic diagram of the 5 parameter model.

MODEL DESCRIPTION

The 5-parameter model used in this study was discussed in detail by Houghton-carr and Arnell (1994) and is briefly described here. The schematic diagram of the model is shown in Fig.1. The model is based on Moore's probability distributed technique (Moore, 1985) and has a soil moisture store with a capacity varying across the basin and a groundwater store. Vijayakumar (1995) applied 5 such simple daily rainfall-runoff models for the Nagavali and for the Sarada catchments along the East Coast of India and observed the performance of the 5-parameter probability distributed model as efficient among the five models. In the model distribution of the soil capacity, C, is represented by the reflected power (or pareto) distribution.

$$F(c) = 1 – 1(1-C/Cmax)b \text{ for } 0 \leq C \leq Cmax. \quad (1)$$

Where 'Cmax' is the maximum storage capacity at any point within the basin and 'b' is a dimensionless parameter, which defines the degree of spatial heterogeneity. The maximum amount of water that can be held in storage in the basin, 'Smax', for the reflected power distribution is

$$Smax = \int_0^{C\max} (1-F(c)).dc \quad (2)$$

$$= Cmax/(b+1) \quad (3)$$

In the model, precipitation is added to the soil moisture and excess precipitation becomes direct runoff which is routed through two cascading linear reservoirs as direct runoff. Evapotranspiration from the soil moisture store occurs at a rate proportional to store contents, as does drainage from the soil moisture store. Baseflow occurs from the groundwater store and is added to the direct runoff and becomes the catchment outflow.

The model has five parameters. The maximum storage capacity at any point within the basin 'Cmax'; the average maximum amount of water that could be held in storage over the whole basin 'Smax', a soil drainage coefficient 'Kb', a groundwater discharge coefficient 'Grout' and a channel routing coefficient 'Srout'. The model formulation and the accounting procedure is discussed in detail by Houghton-carr and Arnell (1994) and is briefly presented here:

Actual evapotranspiration (AE) is derived from potential evapotranspiration (PE) as

$$AEt= PEt \{1-e (-6.68 St-1/Smax)\} \quad (4)$$

Drainage to the Groundwater store is

$$Qi = Kb(St-1/Smax) \quad (5)$$

If rainfall P is less than AE and Qi there is no direct runoff. Otherwise direct runoff does occur. The critical capacity at the end of previous time step 'Cc' below which all the soil moisture goes to storage is calculated from the reflected power (pareto) distribution as

$$Cct-1 = Cmax\{1-(1-St/Smax)(1/(b+1)\} \quad (6)$$

Hence critical capacity at the end of the present time step is

$$Cct = Cct-1 + (Pt – AEt – Qit) \quad (7)$$

If Cct is less than Cmax, direct runoff is

$$Qot = (Pt-AEt-Qit) – Smax\{1-Cct-1/Cmax)b+1 - (1-Cct/Cmax)b+1\} \quad (8)$$

If Cct is greater than Cmax, direct runoff is

$$Qot = (Pt\text{-}AEt\text{-}Qit) – (Smax – St\text{-}1) \quad (9)$$

and the soil moisture store is full to Smax.

Baseflow (Qb) from groundwater storage gs is

$$Qbt = Grout\ (St\text{-}1/100) \quad (10)$$

Direct runoff through two cascading reservoir of storage Ss may be routed as

$$Q = Srout\ (Ss) \quad (11)$$

Adding the baseflow ‘Qb’ and direct runoff ‘Q’ results in the modelled catchment runoff, which can be compared with the observed runoff at the point of interest.

Optimization

With a model, for any given set of parameter values, one can estimate modelled flows using input data like rainfall, evaporation etc.,. The job of the modelling is to recommend the best set of parameters that will closely simulate the observed flows at the particular point of interest on the stream. It can be accomplished either by trial and error or by automatic optimization of the parameter set. There are many criteria to undertake modelling. One technique is by plotting both observed and modeled flows and selecting the parameters, which give visually better fit. Another one is a numerical technique, in which the parameters are subjected to automatic optimization to achieve a mathematically best fit, indicated by an objective function. Hence the objective function is regarded as a tool to aid fitting and assess the model. If proper limits for parameter set are chosen, this criteria generally results in visually better fit too. In daily rainfall – runoff modelling the following two functions are used as fitting criteria as error functions.

The first objective function is to maximize the sum of the squares of difference of the observed and simulated daily flow during the entire period of simulation i.e.

$$\text{Minimize } Obj1 = \sum (Qobs – Qsim)^2 \quad 12)$$

Which may give a good fit for long periods of low flows.

To undertake automatic optimization Rosenbrock (1960) optimization procedure was invoked to minimize the objective function. The objective function may be used to compare the results from calibration and validation data sets.

Objective function values as explained above are not comparable across different catchments, since they are not normalized. Hence, a suitable technique like Nash-Sutcliffe (1970) efficiency criteria may be used to undertake normalization.

$$Efficiency = Obj1 / \sum (Qobs\text{-}Qbar)^2 \quad (13)$$

Where Obj1 is objective function one as defined above. The denominator is the sum of the square of differences of the daily observed and mean of daily observed flow over the period of modelling. Since the objective function one is minimized in the optimization criteria the equation 13 gives maximum efficiency. A perfect agreement between the observed and simulated flows yields an efficiency of 1.0, whilst a negative efficiency represents a lack of agreement, worse than if the simulated flows were replaced with the observed mean daily flows.

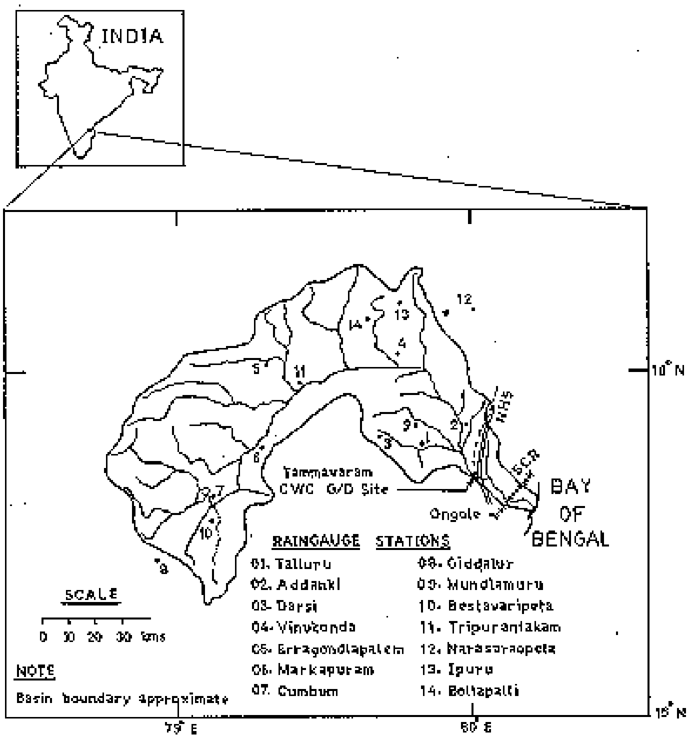

Figure 2. Location map of Gundlakamma river basin.

STUDY AREA

The Gundlakamma river basin is a medium sized basin along the east coast of India. It has a main stream of about 264 kms length taking its origin from the Nallamalai hills in Nallamalai forest near Gundla-brahmeswaram village in Andhra Pradesh, India, with an altitude of 680 metres and at a longitude of 76046’E and latitude of 15040’N in Nandyal taluka of Kurnool district. The location map of the basin is given at Fig. 2. The basin's total catchment area is about 8195 km^2, including the area drained by its tributaries Jampaleru, Venumuleru, Mekaleru, Teegaleru, Duvvaleru, Rallavagu, Konduleru, Pasupaleru, Konkeru, Chilakaleru, Voleru etc.,. The Gundlakamma basin is bounded by Vogeru vagu, Romperu on East side, by Nallamalai hill range on the western side, Krishna basin on the northern side and Musi river basin on the southern side and the stream flows eastwards into Bay of Bengal.

The Climate of coastal part of the study area may be broadly classified under tropical coastal type and rest is of steppe type. According to Koppen the climate is tropical Savannah in upper part and dry season in high sun period in the rest of the area. The daily mean temperature is about 27.5 0^{C}. Mean maximum temperature is around 32.5 0^{C}. Mean minimum temperature is about 22.5 0^{C}. Highest maximum temperature is about 47 0^{C} and lowest minimum temperature is about 14 0C. The mean diurnal range of temperature is about 10 0^{C}.

A rainfall of 25 mm between January and February; about 75 mm between March to May; about 400 mm between June to September and about 300 mm between October to December is experienced in the area. Annual rainy days are about 40, Southwest Monsoon is set during 1st week of June and retreats by middle of November, normally. The annual runoff is about 200 mm. Also the area under study gets heavy rainfall due to cyclones forming in Bay of Bengal and travelling inland over the catchemnt area.

Table 1. Average daily mean monthly evaporation data (mm/day) at Darsi.

Month	Jan.	Feb.	March	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Evaporation (mm)	3.2	4.4	5.6	5.8	9.6	9.9	7.1	5.3	5.0	3.3	3.2	2.7

DATA

As mentioned earlier, the modelling study is a data driven application. The more the recorded data on various observations such as rainfall, flows and evaporation, the more realistic the success of modelling. Daily rainfall from 10 stations in Prakasam district and 4 stations in Guntur district totalling to 14 stations from 1st January 1989 to 30th November 1997 is used in consonance with the availability of flow records at Tammavaram site of Central Water Commission. The location of raingauge stations used in the study is shown at Fig.2. The mean daily flow data is measured using current meter at Tammavaram site. Due to reasons unknown, only gauge readings are available over some days and some time during extraordinary floods. In such cases estimated flow from stage discharge relations are substituted. The catchment area up to the site is about 7831 sq. km. Efforts were made to collect evaporation data, representative of the basin. The agriculture research station, ANGRAU at Darsi has provided the Pan-evaporation data for about 3 years period (table 1). This information is used to estimate average daily mean monthly Pan- evaporation for use in modelling study.

ANALYSIS & RESULTS

In this study an attempt is made to simulate the daily rainfall-runoff at Tammavaram CWC Gauge-Discharge site on the Gundlakamma River in Prakasam district of Andhra Pradesh. The period of study is from 1989 to 1997. Data of the first 5 year period of 1st January, 1989 to 31st December, 1993 is used for calibration of the model and the data of later 4 year period of 1st January, 1994 to 30th November, 1997 is used for validation of the calibrated model. The analysis and results are briefly presented here.

Analysis

Rainfall data for 14 stations within and near the catchment is used to derive the basin average rainfall using the Theissen method. This procedure distorts the original rainfall intensity pattern if the rainfall distribution is not uniform over the entire catchment. The effect of storms uniformly distributed over the catchment can be simulated properly. Thus, the spatial averaging brings in certain error into the modelled flows if storms are not uniformly distributed. It is noticed that there are some events that recorded high rainfall in some parts and low rainfall at some other parts in the basin. Such local storm events of heavy intense rainfall on the lower reaches of catchment resulted in high-observed discharges, which were difficult to be simulated properly with the lumping of data, as the storms are not uniformly distributed over the catchment.

Average daily flow data as recorded at the Tammavaram CWC G/D site is used in the study. The discharge data was analyzed and processed as for some days gauge data was only recorded. This has been undertaken mainly using rating curve information. It is noticed that during 24th and 25th September, 1997 discharges as high as 2348 cumecs and 2697 cumecs were measured which are not in consonance with the basin average rainfall of 45mm and 86 mm. As stated earlier, this is due to local high intense rainfall events within the catchment and is difficult for any lumped model to simulate perfectly. Compared to calibration data set, validation data set has more such events. It is to be mentioned here that such events have an effect on the value of objective functions and thus on efficiency of the model.

The daily mean monthly evaporation is worked out from the observed data from December 1995 to August 1998, provided by Agricultural research station of ANGRAU at Darsi with in the study area. Daily mean monthly evaporation data with a Pan Coefficient of 0.7 is used in the model to evaluate the potential evaporation, which in turn is used to estimate actual evapotranspiration as explained earlier. This daily mean monthly data is used for the entire period of modelling. It is assumed that this information of evaporation at Darsi station represents the whole basin.

Table2. Best Model Parameter Set for Gundlakamma at Tammavaram.

Parameter	Parameter Name	Value
1	Cmax	394.4653
2	Smax	260.8219
3	Kb	1.388142
4	Grout	1.158575
5	Srout	0.597486

Calibration

As the modelling procedure requires information on catchment data, rainfall, flow and evaporation data, respective data files were prepared for the 5-year period. The number of parameters to be optimized in the model and the maximum number of iterations of optimization are also to be specified. On successful completion of the optimization run the program gives the objective function value, the efficiency value, the modelled daily flows and the monthly flows. The program may be extended to validation part of the study by specifying the period of validation in the catchment file to obtain the results for validation data set. In the present case the optimization resulted in the best parameter set as listed at Table 2 for the first objective function.

The objective function obj-1 resulted is 382.0793 and the efficiency is 72.14%. The modelled runoff over the entire 5-year period of calibration is presented at Fig.3. The modelled and observed monthly runoff for calibration period is shown as Bar Chart in Fig. 5. As 1989 is first year of the run and is used for warming of the model simulated flows are different from the observed flows. The efficiency and the response of the modelled runoff to the rainfall as observed visually from the figures plotted suggested that the model could simulate the flows reasonably well. The peaks, however could not be simulated satisfactorily.

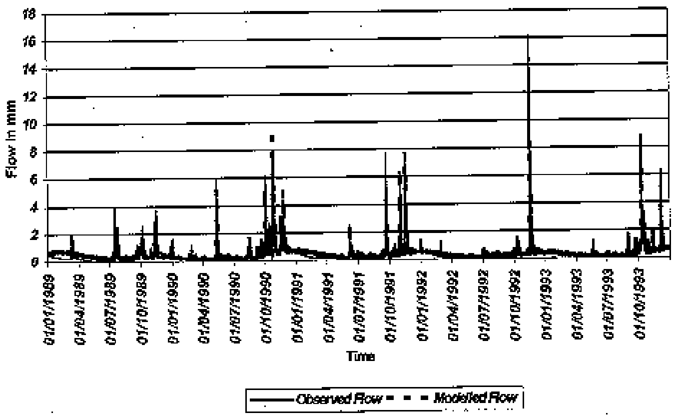

Figure 3. Gundlakamma daily hydrograph – calibration.

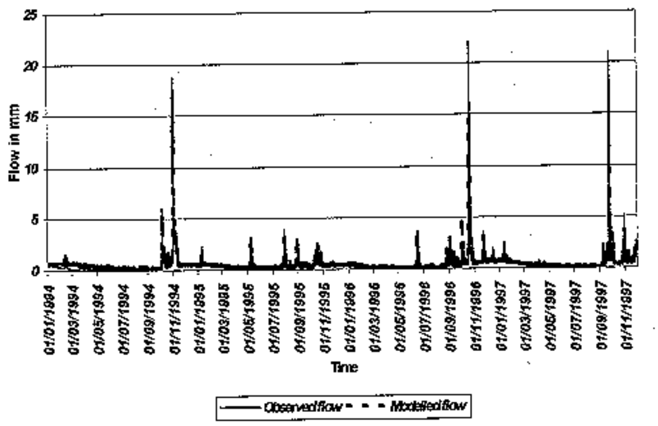

Figure 4. Gundlakamma daily hydrograph – validation.

Validation

With the parameter set optimized from the calibration run, the program is extended to validate the model using data set for the 4 year period from 1994 to 1997 with the respective data files for rainfall, evaporation and flow. The validation resulted in an efficiency of 68.25%, which is close to the efficiency obtained for the calibration run. The model response to the rainfall pattern in general is proper as observed from the modelled and observed runoff plots. The model could not simulate the peak flows properly may be due to error involved in lumping of the rainfall, which is common with many lumped models. The modelled and observed daily runoff for the entire validation period is plotted at Fig. 4. The modelled and observed monthly runoff values for validation period is shown as bar chart at Fig. 6. The peak monthly flows of monsoon season at Tammavaram site are modelled reasonably well by the model over both calibration and validation periods. The observed flows of October to December are on higher side than the modelled flows. In contrast during validation period the observed monsoon flows of September 1995 to November 1995 are on lower side than the modelled flows. This may be because the year appears to be a drought year and the moisture distribution might not have been properly taken care of by the model parameters appropriately. Otherwise the simulation seems to be close to the observed data.

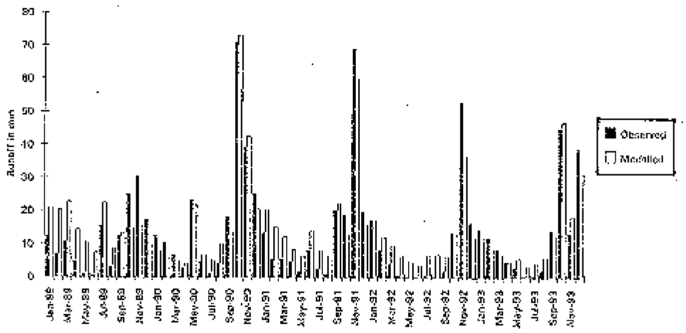

Figure 5. Modelled and observed monthly runoff for calibration.

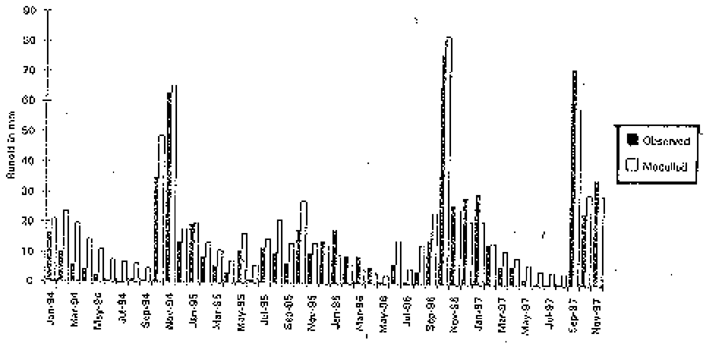

Figure 6. Modelled and observed monthly runoff for validation.

Modelled non-monsoon flows are consistently on higher side over both calibration and validation periods. This reflects that the expected flows are not being realized due to diversions upstream of the site or in other words due to man's influence. Otherwise over all the model performance is reasonably well from the few parameters and lumping nature of the 5-parameter model used in the study.

CONCLUSIONS AND RECOMMENDATIONS

From the calibration and validation of the modelling study, it is found that the model, though a five parameter one, could respond properly to the rainfall and resulted in an acceptably good efficiency of 72.14% in calibration and 68.25% in validation. As observed in other modelling studies on the Nagavali and the Sarada rivers (Vijayakumar, 1995), the model could simulate the flows acceptably well in the Gundlakamma basin too. The optimized parameter values are also in conformity with the parameter set obtained in the other studies on the Sarada and the Nagavali.

This study wherein a 5-parameter simple conceptual model was applied on the Gundlakamma may be extended by applying it to few more basins in the region to standardize the different parameters for different river basins in the region. Also, a slight variation of the model by reducing the number of parameter to 3 or 4 may be attempted to further simplify the model.

It is recommended that the model with the modelled parameter set can be used for various purposes like daily flow generation, hydrologic design, flood forecasting etc., over the basin, provided the obtained efficiency is acceptable. To simulate the peak flows more accurately, modelling study may be undertaken with much finer rainfall data, say hourly, and corresponding flow data of individual events.

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