TR - 3

STORAGE IN CONFINED AQUIFER WITH FLOWING ARTESIAN WELL

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LIST OF SYMBOLS

a	distance from the flowing well to the no flow boundary
b	distance to which the excitation propagated in a confined aquifer of infinite areal extent
H_1	height of the initial piezometric surface from a datum
H_2	height of the flowing well's threshold measured from the same datum
I	time step
J_o	Bessel's function of first kind zero order
J_1	Bessel's function of first kind first order
$K(N)$	Unit step response at the end of time step N
N	time step
Q	Constant pumping rate
$Q(t)$	Variable well discharge at time t
$Q(I)$	well discharge during time step I
r	radial distance from the flowing well
s	drawdown
s_o	drawdown at flowing well
$s(r,t)$	drawdown at radial distance r at time t
t	time
T	transmissivity
Y_o	Bessel's function of second kind and zero order
ϕ	storativity
$\delta_r(.)$	discrete kernel for drawdown or technological function pertaining to radial distance r.
α_m	$(\alpha_m a)$ is the m^{th} zero of J_1

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ABSTRACT

Flowing wells are uncommon occurrences resulting from erratic geological process. The discharge of a flowing well depends on the difference between the elevations of the piezometric head in the aquifer in the vicinity of the flowing well and elevation of the flowing well's threshold. As the flowing well's discharge is derived from water stored in the aquifer the piezometric head in the aquifer gradually declines and with it the flowing well's discharge reduces.

Using discrete kernel approach an analytical solution has been obtained to determine temporal variation of discharge of a flowing well in a confined aquifer of finite areal extent. The quantities of water that remains in the aquifer storage at any time after the onset of flow, which will be subsequently drained by the flowing well, have been quantitied. Type curves have been prepared to enable determination of aquifer parameters such as : storage coefficient transmissivity, distance of the no flow boundary from the flowing well and the initial hydraulic head.

1.0 INTRODUCTION

When a permeable bed sandwitched in between impermeable strata is warped into synclinal fold with the permeable bed exposed at the surface along an out crop, conditions favourable for flowing well may develop. Recharge due to precipitation to the aquifer may take place along the out crop and the permeable layer may contain water under artesian condition. In such a case when a well is sunk to the permeable bed a flowing well can be obtained. Flowing wells are uncommon occurrence resulting from erratic geological process. Discharge characteristics of a flowing well and certain spring are comparable. An artesian aquifer is drained by the flowing well. The discharge of the flowing well depends on the difference between the elevations of the piezometric head in the aquifer in the vicinity of the flowing well and the elevation of the flowing well's threshold (point A in Fig.1). The flowing well's discharge is derived from water stored in the aquifer. Hence piezometric head in the aquifer gradually declines and with it the flowing well's discharge reduces. If the aquifer has finite areal extent the discharge of the flowing well would reduce to zero.

In the present report a solution for temporal variation of discharge of a flowing well in an aquifer of finite areal extent has been presented. Using the solution it is possible to predict the time when the well discharge would become insignificant.

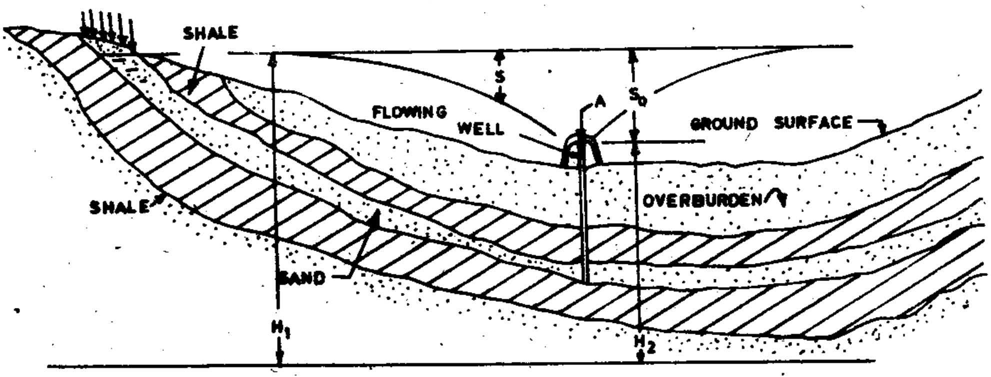

Fig.1 SECTION SHOWING A FLOWING WELL

2.0 REVIEW

Many investigators have analysed the unsteady flow associated with a flowing well. Notable among them are Nicholson, Smith , Goldstein ,Carslaw and Jaeger(vide Glover, 1974). Assuming that upto a radial distance b from the centre of the flowing well the excitation has not propagated, the solution of the differential equation

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t} \qquad ...(1)$$

for the initial condition: s=0 when t=0 for $r>r_w$ and for the boundary condition: $s=s_0$ at $r=r_w$ for $t>0$ has been found to be (vide Glover, 1974)

$$s = s_0\left[1-\sum_{n=1}^{n=\infty} A_n U_o(\beta_n r) e^{\frac{-k^2(\beta_n b)^2}{4}\left(\frac{4Tt}{\phi r_w^2}\right)}\right] \qquad ...(2)$$

$$\text{where } A_n = \frac{\left(\frac{2k}{\beta_n b}\right) U_o'(\beta_n r_w)}{[U_o(\beta_n b)]^2 - k^2[U_o'(\beta_n r_w)]^2} \qquad ...(3)$$

and $U_o(\beta_n r) = J_o(\beta_n r_w)\, Y_o(\beta_n r) - Y_o(\beta_n r_w)\, J_o(\beta_n r)$

$k = r_w/b$ ...(4)

and $(\beta_n b)$ are the roots of equation $U_o'(\beta_n) = 0$

where $U_o'(\beta_n r) = \frac{dU_o(\beta_n r)}{d(\beta_n r)}$

J_o and Y_o are respectively Bessel's function of first and second kind of zero order. The flow from the well is given by

$$Q^*(t) = 2\pi T\, s_o G\left(\frac{\sqrt{4Tt/\phi}}{r_w}\right) \qquad ...(5)$$

where

$$G\left(\frac{\sqrt{4Tt/\phi}}{r_w}\right) \sum_{n=1}^{n=\infty} A_n(\beta_n r_w)\, U_o'(\beta_n r_w) e^{\frac{-k^2(\beta_n b)^2}{4}\frac{4Tt}{\phi r_w^2}} \qquad ...(6)$$

When the excitation propagates to the farthest boundary the above solution will be no longer valid for aquifer of finite areal extent.

Analysis of unsteady flow to a well in an aquifer of finite areal extent has been done by Muskat (1937) and Kuiper (1972). Differential equation (1) has been solved for the following boundary and initial conditions:

$$\left.\frac{\partial s}{\partial r}\right|_{r=a} = 0$$

$$\lim_{r \to 0} r\frac{\partial s}{\partial r} = -\frac{Q}{2\pi t}, \text{ and}$$

$$s(r, 0) = 0,$$

in which 'a' is the radial distance to the imprermeable boundary, Q is the constant pumping rate. The solution that appears in Muskat is

$$s = -\frac{Q}{2\pi T}\left[\frac{3}{4} + \ln\left(\frac{r}{a}\right) - \frac{1}{2}\left\{\left(\frac{r}{a}\right)^2 + \frac{4\,Tt}{\phi\,a^2}\right\}\right.$$

$$\left. + 2\sum_{m=1}^{\infty}\left\{\alpha_m J_o(\alpha_m a)\right\}^{-2} J_o(\alpha_m r).\exp\left\{-(\alpha_m^2)\frac{Tt}{\phi\,a^2}\right\}\right] \quad ..(7)$$

$(\alpha_m a)$ values m=1, 2, 3, ... are zeros of J_1, the Bessel's function of first kind and of first order. $(\alpha_m a)$ values have been tabulated for values of m upto 20 (Abramowitz and Stegun, 1970). $\alpha_m a$ values for higher values of m can be evaluated using the following formula of McMahon's expansions for large zeros:

$$(\alpha_m a) \simeq \beta - \frac{\mu-1}{\beta} - \frac{4(\mu-1)(7\mu-31)}{3(8\beta)^3} - \frac{32(\mu-1)(83\mu^2-982\mu+3779)}{15(8\beta)^5}$$

$$-\frac{64(\mu-1)(6949\mu^3-153855\mu^2+1585743\mu-6277237)}{105(8\beta)^7} \quad ..(8)$$

in which,

$\beta = (m + \frac{1}{4})\pi$, and

In case of a flowing well the discharge rate, Q , varies with time and drawdown at the well point is constant and equal to the difference between the initial piezometric level and level of the flowing well's threshold. The solution given by Muskat can be used to solve the unsteady flow to a flowing well in an aquifer of finite areal extent.

3.0 PROBLEM DEFINITION AND METHODOLOGY

3.1 Statement of the Problem

Figure 1 shows a schematic cross section of a flowing well in a confined aquifer of finit areal extent. The level of the flowing well's threshold is at a height H_2 above the datum. Before the well was sunk, the piezometric level was at a height H_1. The time is reckoned from the instant the well is sunk and starts flowing. It is required to determine (i) discharge of the flowing well at various time (ii) temporal and spatial variation of drawdown in piezometric surface and (iii) quantities of water that remain in the storage of the aquifer.

3.2 Methodology

The following assumptions have been made in the analysis:

(i) The time parameter is discrete. Within each time step, the discharge of the flowing well is constant but it varies from step to step.

(ii) Though the aquifer near the outcrop is unconfined, the entire aquifer has been assumed to be confined and the position of the no flow boundary is assumed to be fixed.

The solution of differential equation (1) needs to satisfy the following initial and boundary conditions for the flowing well under consideration:

$$s(r,o) = o$$

$$s(r_w, t) = H_1 - H_2$$

$$\left.\frac{\partial s}{\partial r}\right|_{r=a} = o$$

Let K(t) be the drawdown in piezometric surface of a confined aquifer of finite areal extent at a radial distance r from the flowing well due to a unit step excitation. Expression for K(t) can be obtained from equation(7) substituting Q by 1. Let $\delta_r(N)$ be the response of the aquifer at the end of time step N due to a unit pulse excitation. $\delta_r(N)$ is recognised as discrete kernel (Morel Seytoux, 1975) and it has the following relation with unit step response:

$$\delta_r(N) = K(N) - K(N-1) \qquad ...(9)$$

Substituting for K(N) and K(N-1) in equation (9) and simplifying, the following expression for discrete kernel for drawdown for a confined aquifer of finite areal extent is obtained:

$$\delta_r(N) = \frac{N}{\pi \phi a^2} - \frac{1}{\pi T} \sum_{m=1}^{\infty} \{(\alpha_m a)\, J_o(\alpha_m a)\}^{-2} J_o(\alpha_m r) . \exp\{\frac{-(\alpha_m)^2\, TN}{\phi}\}$$

$$- \frac{(N-1)}{\pi \phi a^2} + \frac{1}{\pi T} \{ \sum_{m=1}^{\infty} \{(\alpha_m a)\, J_o(\alpha_m a)\}^{-2} J_o(\alpha_m r) . \exp\{\frac{-(\alpha_m)^2\, T(N-1)}{\phi}\}\}, \; N > 1$$

For N = 1, $\delta_r(1)$ is given by ...(10)

$$\delta_r(1) = -\frac{1}{2\pi T} [\frac{3}{4} + \ln(\frac{r}{a}) - \frac{1}{2}\{(\frac{r}{a})^2 + \frac{4T}{\phi a^2}\}$$

$$+ 2 \sum_{m=1}^{\infty} \{(\alpha_m a)\, J_o(\alpha_m a)\}^{-2} J_o(\alpha_m r) . \exp\{-(\alpha_m)^2 \frac{T}{\phi}\}] \qquad ...(11)$$

Let $Q(\gamma)$, $\gamma = 1, 1, 2, 3, \ldots, I$, be the discharges of the flowing well during various time steps. The drawdown in piezometric surface at the end of time step I at any point in the aquifer is governed by the discharges of the flowing well upto time step I. The relation between drawdown at flowing well and the well discharge is

$$s(r_w, I) = \sum_{\gamma=1}^{I} Q(\gamma)\, \delta_{rw}(I-\gamma+1) \qquad ...(12)$$

The discrete kernel coefficient $\delta_{rw}(.)$ can be obtained from equations (10) and (11) replacing r by r_w.

Since the drawdown at the flowing well at any time step is $H_1 - H_2$

therefore,

$$H_1 - H_2 = \sum_{\gamma=1}^{I} Q(\gamma)\, \delta_{rw} (I - \gamma + 1) \qquad ...(13)$$

Equation (13) can be written as

$$H_1 - H_2 = \sum_{\gamma=1}^{I-1} Q(\gamma)\delta_{rw}(I-\gamma+1) + Q(I)\delta(1)$$

Thus, the discharge of the flowing well during time step I is given by

$$Q(I) = \frac{1}{\delta(1)} [H_1 - H_2 - \sum_{\gamma=1}^{I-1} Q(\gamma)\delta_{rw}(I-\gamma+1)] \qquad ...(14)$$

Q(I) can be found in succession starting frfom time step 1.

In particular for time step 1, $Q(1) = \frac{1}{\delta(1)} [H_1 - H_2]$. Once Q(I) values are known the drawdown at any point can be evaluated using the relation

$$s(r, I) = \sum_{\gamma=1}^{I} Q(\gamma)\ \delta_r\ (I-\gamma+1)$$

4.0 RESULTS

The discrete kernel coefficients $\delta_r(I)$ are generated for r equal to r_w and for other radial distances at which drawdown calculations are sought, for a known set of aquifer parameters T, ϕ and a. The large zeros ($\alpha_m a$) of the Bessel function $J_1(.)$ required for the evaluation of the discrete kernels have been obtained making use of equation (8). The first one hundred zeros (m=100) have been considered for evaluation of the discrete kernels. A plot of $\delta_r(I)$ versus I is presented in Fig.2 for r equal to 0.1 0.5a and a. Since the discrete kernels are response of the aquifer to a unit withdrawal during the first time period, and the aquifer has a finite radius equal to 'a', at large time step I, $\delta_r(I)$ tends to the limiting value $\frac{1}{\pi a^2 \phi}$. After generating the discrete kernel coefficients, the discharges of the flowing well Q(I) have been solved in succession starting from time step 1. The variation of dimensionless discharge rate $Q(t)/(T(H_1-H_2))$ with non dimensional time factor $\phi r_w^2/4Tt$ is shown in Fig.3 for a/r_w ranging from 0.5×10^4 to 5.0×10^4. It can be seen in the figure that at large $\phi r_w^2/4Tt$ i.e. in the beginning when the well starts flowing the curves merge with each other indicating that the presence of no flow boundary has not affected the response. The discharge of the flowing well decreases with increasing time and reduces to a negligible quantity. Larger the value of 'a' longer is the life of the well. If a/r_w increases from 1×10^4 to 2×10^4 the non dimensional time $4Tt/\phi r_w^2$ corresponding $Q(t)/(T(H_1-H_2))=0.1$ increases from 3.4×10^9 to 15.8×10^9. If a/r_w increases to 5×10^4, the corresponding non dimensional time is 95.2×10^9. The variation of the dimensionless discharge with the dimensionless time is shown in Fig.4 in a log-log paper.

The variation of $\frac{s(r,t)}{H_1-H_2}$ with $\frac{\phi r^2}{4Tt}$ is shown in Figs 5(a) and 5(b) for $a/r_w = 1.0\times10^4$ and 5.0×10^4 respectively for various values of r/a.

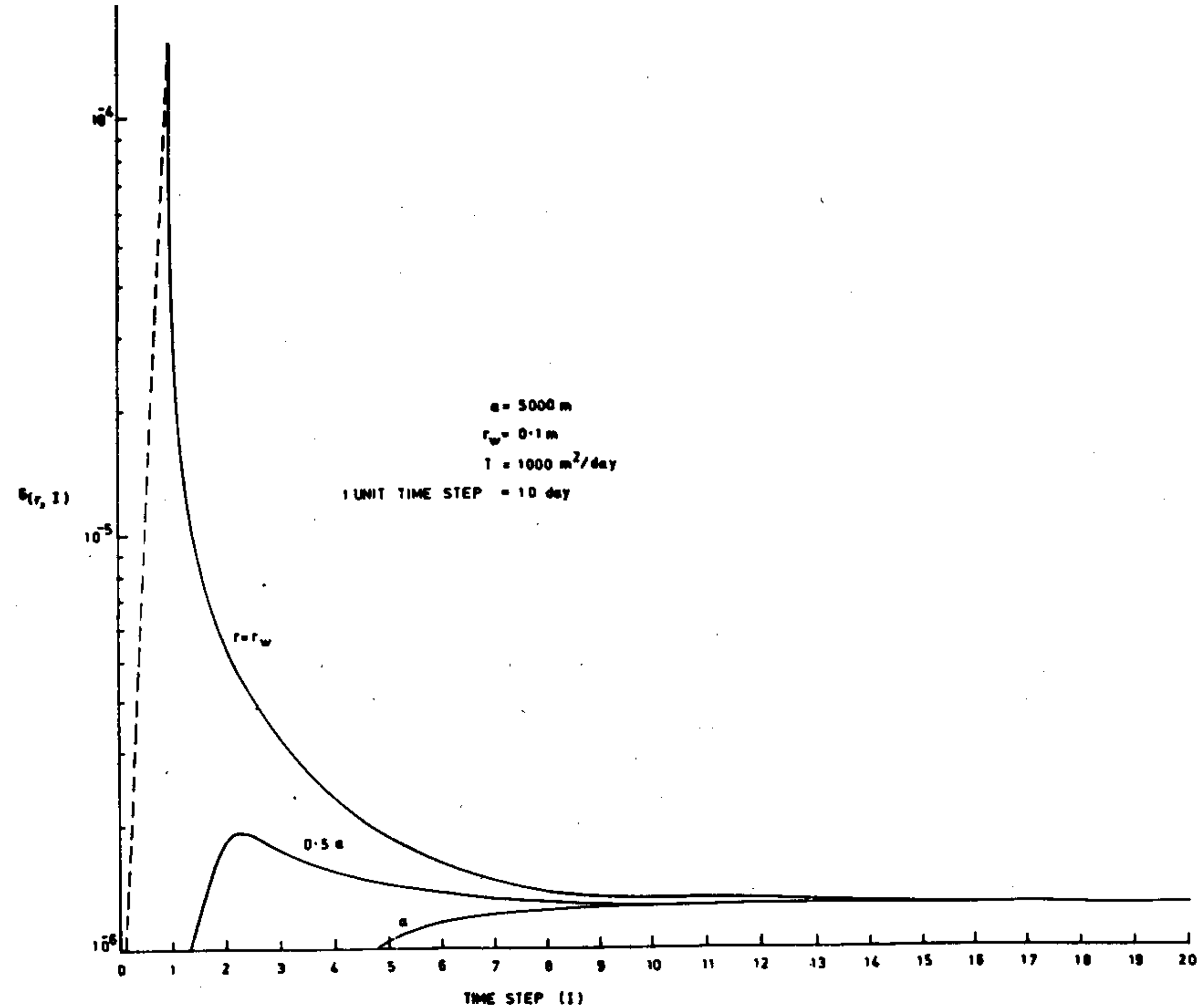

Fig 2DISCRETE KERNEL COEFFICIENT ∂_r (I) AT DIFFERENT TIME STEP I

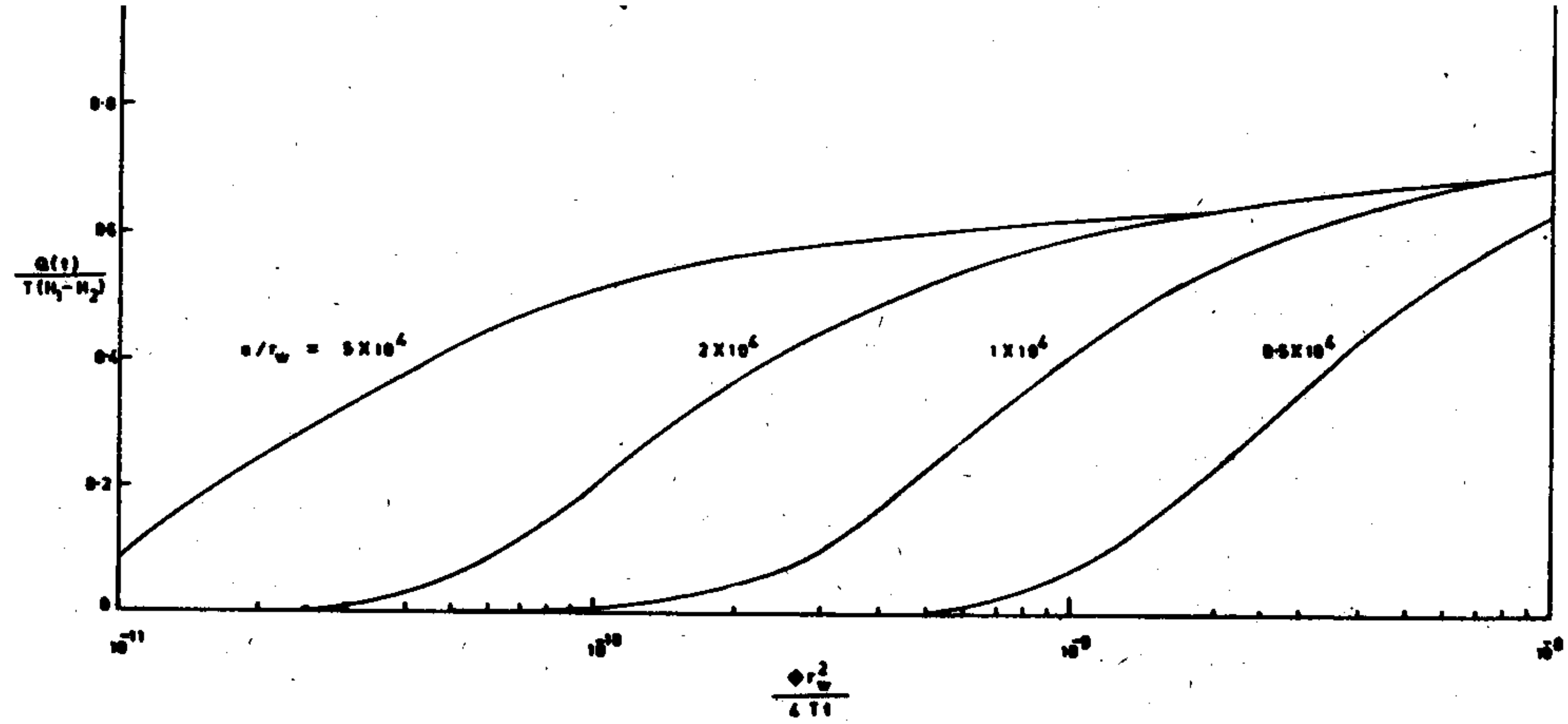

Fig.3 VARIATION OF DIMENSIONLESS DISCHARGE QUANTITY $Q(t)/T(H_1-H_2)$ WITH DIMENSIONLESS TIME PARAMETER $(\phi r^2_w/4Tt)$ PLOTTED IN A SEMILOG PAPER

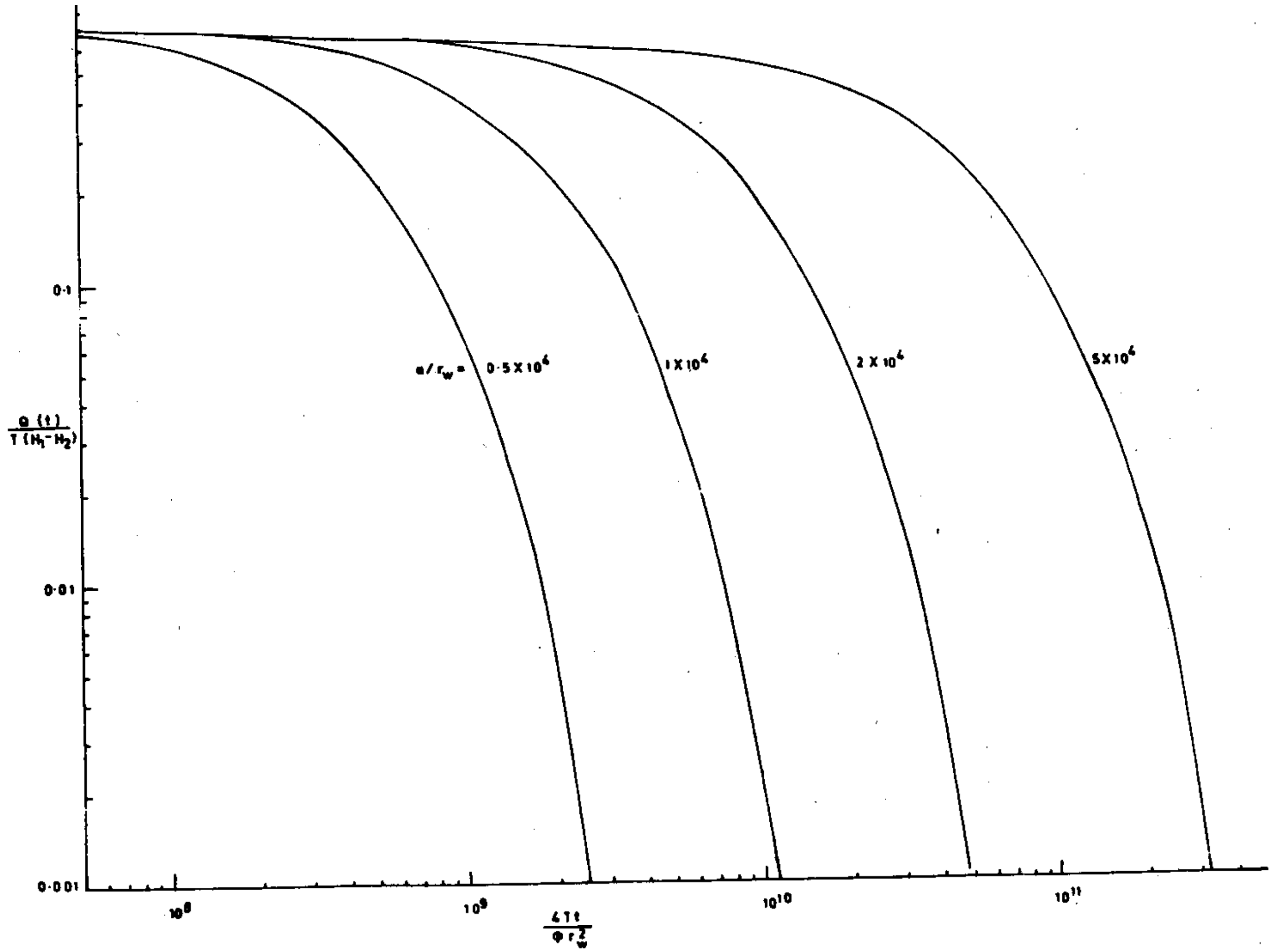

Fig 4 VARIATION OF $Q(t)/T(H_1-H_2)$ WITH $4Tt/\phi r_w^2$ PLOTTED IN A DOUBLE LOG PAPER

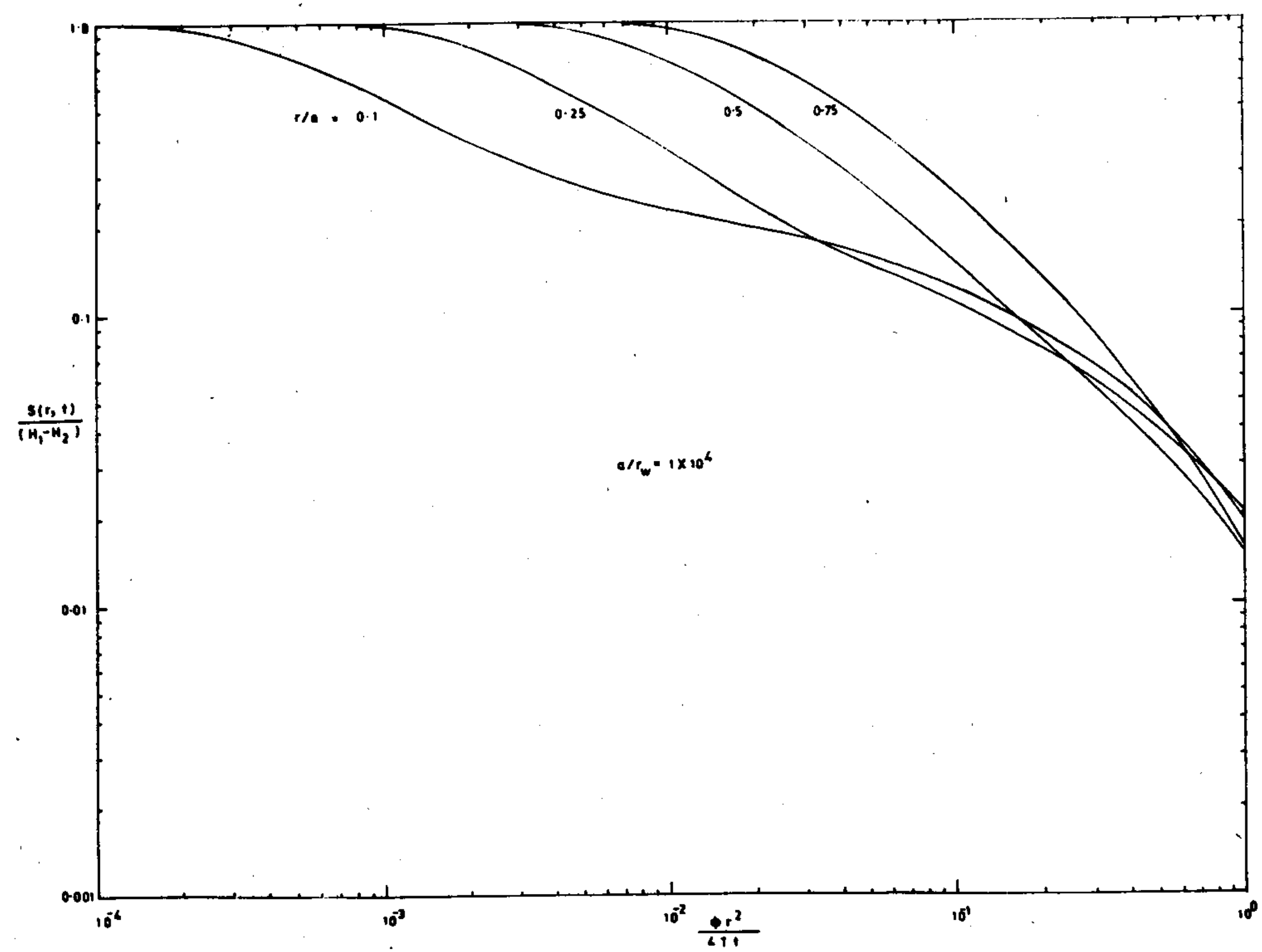

Fig 5(a) VARIATION OF $s(r, t)/(H_1-H_2)$ WITH $\phi\ r^2/4Tt$ FOR DIFFERENT VALUES OF r/a

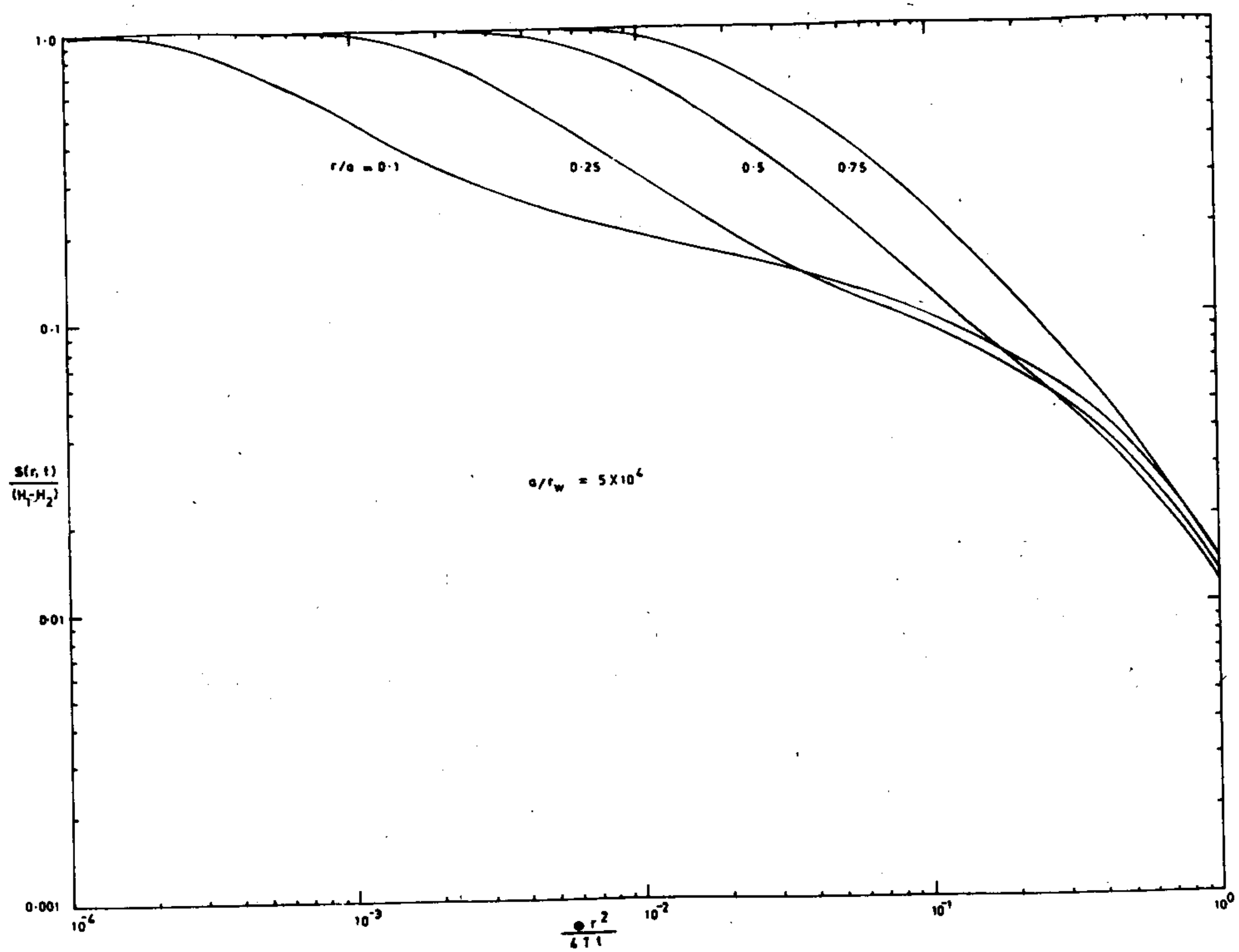

Fig 5(b) VARIATION OF $s(r, t)/(H_1-H_2)$ WITH $\phi r^2/4Tt$ FOR DIFFERENT VALUES OF r/a

$\frac{s(r,t)}{H_1 - H_2}$ can be regarded as the well function for a flowing well in an aquifer of finite areal extent. The type curve presented in Figs.5(a) and 5(b) can be used to find the parameters (H_1-H_2), a, T/ϕ and r_w. If the drawdown in piezometric surface recorded at an observation well is plotted in a log-log paper which has the same scale as that of the type curve presented in Fig.5 and if this graph could be matched with any of these curves it is possible to estimate the parameters H_1-H_2, a, r_w and T/ϕ. If the discharge of the flowing well is measured and its variation with time is plotted in a double log paper T and ϕ can be estimated by matching this graph with the type curve presented in Fig.4.

The total quantity of water that can be drained by a flowing well is equal to $\pi a^2 \phi (H_1-H_2)$. In Fig.6 the variation of the ratio cumulative discharge to total discharge of the flowing well with time is presented. This graph also presents the quantity of water that remains in the aquifer storage to be drained.

A computer program developed for predicting the discharge of the flowing well and drawdown at any point in the aquifer is given in Appendix-I.

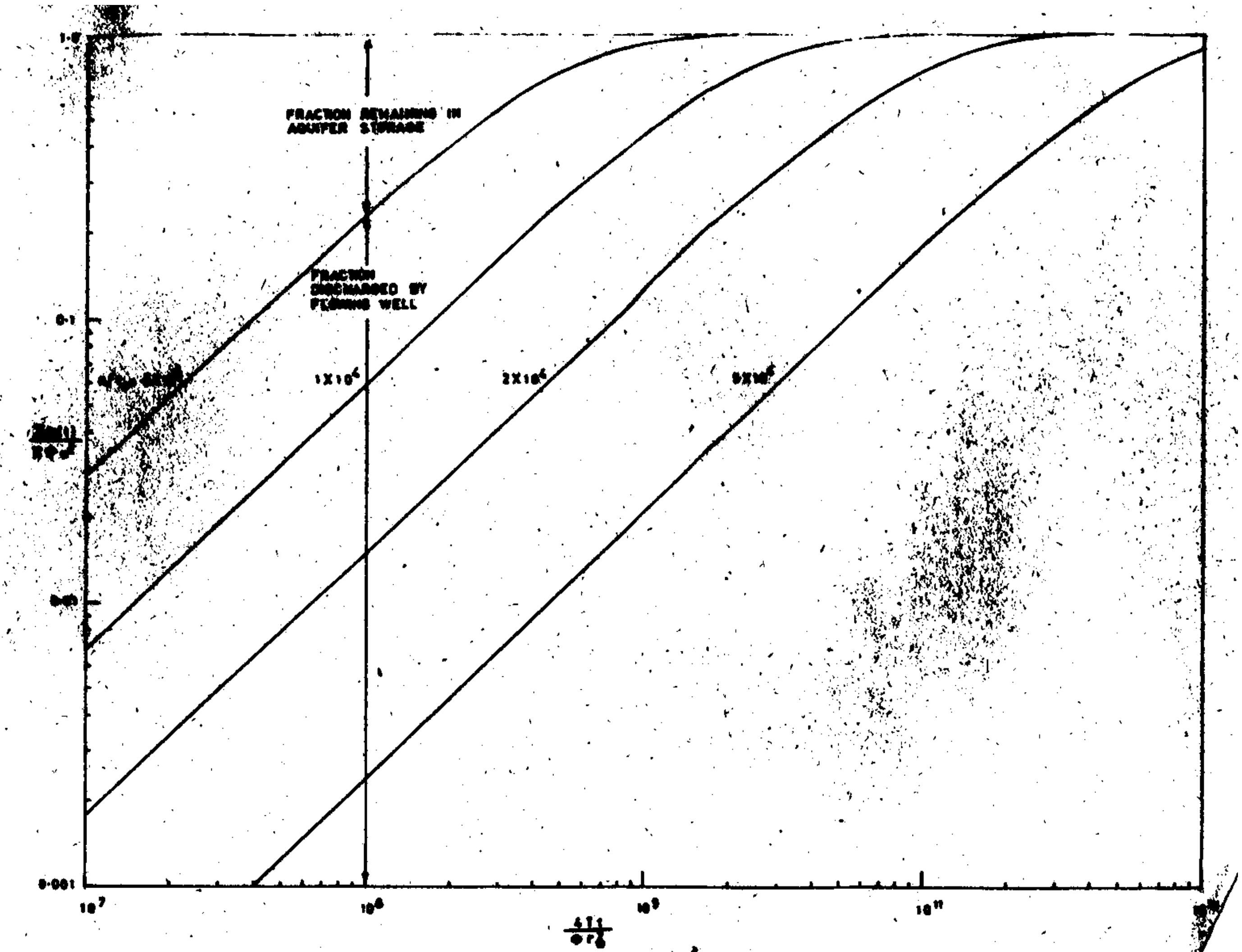

Fig 6 VARIATION OF $\Sigma Q(t)/\phi\pi a^2$ WITH $4Tt/\phi r_w^2$

5.0 CONCLUSIONS

A solution of unsteady flow to a flowing well in a confined aquifer of finite areal extent has been obtained using Muskat's basic solution of unsteady flow to a well in a confined aquifer of finite areal extent. The solution has been obtained discretising the time span and using technological functions. The temporal variation of the flowing well's discharge has been predicted. Pertinent types curves for prediction of aquifer parameters, the distance to the no flow boundary, and the head difference which causes the flow have been presented. The quantity of water that remains in the aquifer storage at any time has been assessed.

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APPENDIX I: A PROGRAM FOR CALCULATION OF DISCHARGE OF A FLOWING WELL AND DRAWDOWN AT ANY POINT IN THE AQUIFER

```
DIMENSION ALFNA(100),DELRW(500),DELRA(500),HRW(500),HRA(500),
1URW(500),URA(500),WFRA(500),WFRW(500),Q(500),QQ(500),SQQ(500)
2,DELR1(500),DELR2(500),DELR3(500),DELR4(500),WFR1(500),WFR2(500)
3,WFR3(500),WFR4(500),HR1(500),HR2(500),HR3(500)+HR4(500),UR1(500)
4,UR2(500),UR3(500),UR4(500)
OPEN(UNIT=1,FILE='FLOW10.DAT',STATUS='OLD')
OPEN(UNIT=2,FILE='FLOW10.OUT',STATUS='NEW')
READ(1,1),HO,RW,RA,T,PHI,NTIME
FORMAT(5F10.5,I3)
HO=H1-H2
RW=RADIUS OF THE WELL
RA=RADIAL DISTANCE TO THE NO FLOW BOUNDARY
T=TRANSMISSIVITY OF THE AQUIFER
PHI=STORAGE COEFFICIENT
NTIME=NUMBER OF TIME STEP UP TO WHICH CALULATION ARE TO BE MADE
WRITE(2,3)
FORMAT(9X,'HO',10X,'RW',15X,'RA',15X,'T',15X,'PHI',15X,'NTIME')
WRITE(2,2),HO,RW,RA,T,PHI,NTIME
FORMAT(5E16.7,8X,I5)
PAI=3.14159265


CHECKING OF SUBROUTINE FOR BESSEL'S FUNCTION


X=0.
CALL JO(X,VALUE)
TYPE*,X,VALUE
X=1.
CALL JO(X,VALUE)
TYPE*,X,VALUE
X=14.9
CALL JO(X,VALUE)
TYPE*,X,VALUE


GENERATION OF DISCRETE KERNEL IS BEING DONE


ALFNA(1)=3.83171
ALFNA(2)=7.01559
ALFNA(3)=10.17347
ALFNA(4)=13.32369
ALFNA(5)=16.47063
ALFNA(6)=19.61586
ALFNA(7)=22.76008
ALFNA(8)=25.90367
```

```
      ALFNA(9)=29.04683
      ALFNA(10)=32.18968
      ALFNA(11)=35.33231
      ALFNA(12)=38.47477
      ALFNA(13)=41.61709
      ALFNA(14)=44.75932
      ALFNA(15)=47.90146
      ALFNA(16)=51.04354
      ALFNA(17)=54.18555
      ALFNA(18)=57.32753
      ALFNA(19)=60.46946
      ALFNA(20)=63.61136
C
C
C     CHECKING OF ZEROS OF BESSEL'S FUNCTION OF 1ST ORDER &1ST KIND
C
C     S=20.
C     CALL ZERO(S,RES)
C     TYPE*,S,RES,ALFNA(20)
C
C
C
C
C     ALFNA(21) TO ALFNA(100) ARE GENERATED
C
C
      DO 25 I=21,100
      S=I
      CALL ZERO(S,RES)
25    ALFNA(I)=RES
      DO 200 N=1,NTIME
      AN=N
      SUMRW=0.
      SUMRA=0.
      SUMR1=0.
      SUMR2=0.
      SUMR3=0.
      SUMR4=0.
      DO 100 M=1,100
      X=ALFNA(M)/RA*RW
      CALL J0(X,VALUE)
      TERM1=VALUE
      X=ALFNA(M)
      CALL J0(X,VALUE)
      TERM4=ALFNA(M)**2*VALUE
      TERM2=(VALUE*ALFNA(M))**2
      X=ALFNA(M)*.1
      CALL J0(X,VALUE)
      TERMR1=VALUE
      X=ALFNA(M)*.25
```

```
      CALL JO(X,VALUE)
      TERMR2=VALUE
      X=ALFNA(M)*.5
      CALL JO(X,VALUE)
      TERMR3=VALUE
      X=ALFNA(M)*.75
      CALL JO(X,VALUE)
      TERMR4=VALUE
      TERM3=EXP(-ALFNA(M)**2*T*AN/(PHI*RA*RA))
      SUMRW=SUMRW+TERM1/TERM2*TERM3
      SUMRA=SUMRA+TERM3/TERM4
      SUMR1=SUMR1+TERMR1/TERM2*TERM3
      SUMR2=SUMR2+TERMR2/TERM2*TERM3
      SUMR3=SUMR3+TERMR3/TERM2*TERM3
      SUMR4=SUMR4+TERMR4/TERM2*TERM3
100   CONTINUE
      HRW(N)=.75+ALOG(RW/RA)-.5*((RW/RA)**2+4.*AN*T/(PHI*RA*RA))
     1+2.*SUMRW
      HR1(N)=.75+ALOG(.1)-.5*(.1**2+4.*AN*T/(PHI*RA*RA))+2.*SUMR1
      HR2(N)=.75+ALOG(.25)-.5*(.25**2+4.*AN*T/(PHI*RA*RA))+2.*SUMR2
      HR3(N)=.75+ALOG(.5)-.5*(.5**2+4.*AN*T/(PHI*RA*RA))+2.*SUMR3
      HR4(N)=.75+ALOG(.75)-.5*(.75**2+4.*AN*T/(PHI*RA*RA))+2.*SUMR4
      T1=ALOG(.75)
      T2=.5*(.75**2+4.*AN*T/(PHI*RA*RA))
      T3=2.*SUMR4
      HRW(N)=-HRW(N)/(2.*PAI*T)
      HR1(N)=-HR1(N)/(2.*PAI*T)
      HR2(N)=-HR2(N)/(2.*PAI*T)
      HR3(N)=-HR3(N)/(2.*PAI*T)
      HR4(N)=-HR4(N)/(2.*PAI*T)
      URW(N)=RW*RW*PHI/(4.*T*AN)
      URA(N)=RA*RA*PHI/(4.*T*AN)
      UR1(N)=(RA*.1)**2*PHI/(4.*T*AN)
      UR2(N)=(RA*.25)**2*PHI/(4.*T*AN)
      UR3(N)=(RA*.5)**2*PHI/(4.*T*AN)
      UR4(N)=(RA*.75)**2*PHI/(4.*T*AN)
      HRA(N)=.75-.5*(1.+4.*AN*T/(PHI*RA*RA))+2.*SUMRA
      T1=4.*AN*T/(PHI*RA*RA)
      HRA(N)=-HRA(N)/(2.*PAI*T)
200   CONTINUE
      DELRA(1)=HRA(1)
      IF (DELRA(1).LT.0.)  GO TO 26
      GO TO 27
26    DELRA(1)=0.
27    CONTINUE
      DELR1(1)=HR1(1)
      DELR2(1)=HR2(1)
      DELR3(1)=HR3(1)
      DELR4(1)=HR4(1)
      DELRW(1)=HRW(1)
```

```
      DO 300 I=2,NTIME
      DELRA(I)=HRA(I)-HRA(I-1)
      DELRW(I)=HRW(I)-HRW(I-1)
      DELR1(I)=HR1(I)-HR1(I-1)
      DELR2(I)=HR2(I)-HR2(I-1)
      DELR3(I)=HR3(I)-HR3(I-1)
      DELR4(I)=HR4(I)-HR4(I-1)
300   CONTINUE
      WRITE(2,11)
11    FORMAT(4X,'DELRW(N)',2X,'DELR1(N)',2X,'DELR2(N)',2X,'DELR3(N)'
     1,2X,'DELR4(N)',2X,'DELRA',5X,'URW(N)',5X,'UR1(N)',4X,'UR2(N)'
     2,5X,'UR3(N)',4X,'UR4(N)',4X,'URA(N)',4X,'N')
      DO 400 N=1,20
400   WRITE(2,10)DELRW(N),DELR1(N),DELR2(N),DELR3(N),DELR4(N),DELRA(N)
     1,URW(N),UR1(N),UR2(N),UR3(N),UR4(N),URA(N),N
10    FORMAT(2X,12E10.3,I5)
C
C
C     GENERATION OF DISCRETE KERNEL IS OVER
C
C
C     DICHARGE OF FLOWING WELL IS BEING CALCULATED
C
      Q(1)=HO/DELRW(1)
      QQ(1)=Q(1)/(HO*T)
      SQQ(1)=QQ(1)
C
C
C     QQ(J) IS DIMENSIONLESS DICHARGE DURING TIME STEP J
C     SQQ(J) IS CUMULATIVE DIMENSIONLESS DICHARGE UP TO TIME STEF J
C
C
      DO 500 N=2,NTIME
      ASUM=0.
      DO 501 NGAMA=1,N-1
501   ASUM=ASUM+Q(NGAMA)*DELRW(N-NGAMA+1)
      Q(N)=HO/DELRW(1)-ASUM/DELRW(1)
      QQ(N)=Q(N)/(HO*T)
500   SQQ(N)=SQQ(N-1)+QQ(N)
C
C
C     CALCULATION OF FLOWING WELL'S DICHARGE IS OVER
C
C
C     DRAWDOWN AND WELL FUNCTION ARE BEING CALCULATED
C
      DO 506 N=1,NTIME
      SWFR1=0.
      SWFR2=0.
      SWFR3=0.
```

```
      SWFR4=0.
      SWFRW=0.
      SWFRA=0.
      DO 507 NGAMA=1,N
      SWFRW=SWFRW+DELRW(N-NGAMA+1)*Q(NGAMA)
      SWFR1=SWFR1+DELR1(N-NGAMA+1)*Q(NGAMA)
      SWFR2=SWFR2+DELR2(N-NGAMA+1)*Q(NGAMA)
      SWFR3=SWFR3+DELR3(N-NGAMA+1)*Q(NGAMA)
      SWFR4=SWFR4+DELR4(N-NGAMA+1)*Q(NGAMA)
      SWFRA=SWFRA+DELRA(N-NGAMA+1)*Q(NGAMA)
507   CONTINUE
      WFRW(N)=SWFRW/HO
      WFR1(N)=SWFR1/HO
      WFR2(N)=SWFR2/HO
      WFR3(N)=SWFR3/HO
      WFR4(N)=SWFR4/HO
      WFRA(N)=SWFRA/HO
506   CONTINUE
C
C
C     CALCULATION OF DRAWDOWN  AND WELL FUNCTION IS OVER
C
C
      WRITE(2,14)
14    FORMAT(9X,'Q(N)',9X,'QQ(N)',10X,'SQQ(N)',10X,'WFRW(N)',8X,'N',
      15X,'URW(N)',10X,'WFRA(N)',9X,'URA(N)')
      DO 514,N=20,NTIME,20
      WRITE(2,503),Q(N),QQ(N),SQQ(N),WFRW(N),N,URW(N),WFRA(N),URA(N)
503   FORMAT(4E16.7,I5,3E16.7)
514   CONTINUE
      WRITE(2,515)
515   FORMAT(8X,'WFR1(N)',8X,'UR1(N)',10X,'WFR2(N)',8X,'UR2(N)',10X,
      1'WFR3(N)',10X,'UR3(N)',10X,'WFR4(N)',10X,'UR4(N)')
      DO 505,N=20,NTIME,20
      WRITE(2,516),WFR1(N),UR1(N),WFR2(N),UR2(N),WFR3(N),UR3(N),WFR4(N),
      1UR4(N)
516   FORMAT(2X,8E16.7)
505   CONTINUE
C     VOLUME=TOTAL QUANTITY OF WATER THAT WILL FLOW FROM THE AQUIFER
C     SUM5= TOTAL QUANTITY DICHARGED  AT THE  END OF NTIME
      VOLUME=PAI*RA*RA*HO*PHI
      SUM5=0.
      DO 15 I=1,NTIME
15    SUM5=SUM5+Q(I)
      WRITE(2,16)
16    FORMAT(10X,'VOLUME',10X,'TOTAL OUTFLOW','NTIME')
      WRITE(2,17),VOLUME,SUM5,NTIME
17    FORMAT(E16.7,5X,E16.7,I5)
      STOP
      END
```

```
      SUBROUTINE JO(X,VALUE)
      IF(X.GT.3.) GO TO 100
      VALUE=1.-2.2499997*(X/3.)**2+1.2656208*(X/3.)**4-.3163866*(X/3.)**6
     1+.0444479*(X/3.)**8-.0039444*(X/3.)**10+.00021*(X/3.)**12
      GO TO 200
100   THETA=X-.78539816-.04166397*(3./X)-.00003954*(3./X)**2+.00262573
     1*(3./X)**3-.00054125*(3./X)**4-.00029333*(3./X)**5+.00013558*
     2(3./X)**6
      FO=.79788456-.00000077*(3./X)-.00552740*(3./X)**2-.00009512*
     1(3./X)**3+.00137237*(3./X)**4-.00072805*(3./X)**5+.00014476*
     2(3./X)**6
      VALUE=X**(-.5)*FO*COS(THETA)
200   CONTINUE
      RETURN
      END
      SUBROUTINE ZERO(S,RES)
      U=4.
      PAI=3.14159265
      B=(S+.25)*PAI
      RES=B-(U-1.)/(8.*B)-4.*(U-1.)*(7.*U-31.)/(3.*(8.*B)**3)-
     132.*(U-1.)*(83.*U*U-982.*U+3779.)/(15.*(8.*B)**5)-
     264.*(U-1.)*(6949.*U**3-153855.*U**2+1585743.*U-6277237.)/(105.*
     3(8.*B)**7)
      RETURN
      END
```

HO	RW	RA	T	PHI	NTIME
0.1000000E+01	0.1000000E+00	0.1000000E+04	0.8000000E+02	0.1000000E-02	100

DELRW(N)	DELR1(N)	DELR2(N)	DELR3(N)	DELR4(N)	DELRA	URW(N)	UR1(N)	UR2(N)	UR3(N)	UR4(N)	URA(N)	N
0.166E-01	0.290E-02	0.124E-02	0.320E-03	0.702E-04	0.236E-04	0.313E-07	0.313E-01	0.195E+00	0.781E+00	0.176E+01	0.313E+01	1
0.692E-03	0.677E-03	0.604E-03	0.409E-03	0.240E-03	0.180E-03	0.156E-07	0.156E-01	0.977E-01	0.391E+00	0.879E+00	0.156E+01	2
0.429E-03	0.425E-03	0.405E-03	0.348E-03	0.295E-03	0.274E-03	0.104E-07	0.104E-01	0.651E-01	0.260E+00	0.586E+00	0.104E+01	3
0.352E-03	0.351E-03	0.345E-03	0.328E-03	0.311E-03	0.305E-03	0.781E-08	0.781E-02	0.488E-01	0.195E+00	0.439E+00	0.781E+00	4
0.329E-03	0.328E-03	0.327E-03	0.321E-03	0.316E-03	0.314E-03	0.625E-08	0.625E-02	0.391E-01	0.156E+00	0.352E+00	0.625E+00	5
0.322E-03	0.321E-03	0.321E-03	0.319E-03	0.318E-03	0.317E-03	0.521E-08	0.521E-02	0.326E-01	0.130E+00	0.293E+00	0.521E+00	6
0.319E-03	0.319E-03	0.319E-03	0.319E-03	0.318E-03	0.318E-03	0.446E-08	0.446E-02	0.279E-01	0.112E+00	0.251E+00	0.446E+00	7
0.319E-03	0.319E-03	0.319E-03	0.318E-03	0.318E-03	0.318E-03	0.391E-08	0.391E-02	0.244E-01	0.977E-01	0.220E+00	0.391E+00	8
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.347E-08	0.347E-02	0.217E-01	0.868E-01	0.195E+00	0.347E+00	9
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.313E-08	0.313E-02	0.195E-01	0.781E-01	0.176E+00	0.313E+00	10
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.284E-08	0.284E-02	0.178E-01	0.710E-01	0.160E+00	0.284E+00	11
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.260E-08	0.260E-02	0.163E-01	0.651E-01	0.146E+00	0.260E+00	12
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.240E-08	0.240E-02	0.150E-01	0.601E-01	0.135E+00	0.240E+00	13
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.223E-08	0.223E-02	0.140E-01	0.558E-01	0.126E+00	0.223E+00	14
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.208E-08	0.208E-02	0.130E-01	0.521E-01	0.117E+00	0.208E+00	15
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.195E-08	0.195E-02	0.122E-01	0.488E-01	0.110E+00	0.195E+00	16
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.184E-08	0.184E-02	0.115E-01	0.460E-01	0.103E+00	0.184E+00	17
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.174E-08	0.174E-02	0.109E-01	0.434E-01	0.977E-01	0.174E+00	18
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.164E-08	0.164E-02	0.103E-01	0.411E-01	0.925E-01	0.164E+00	19
0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.318E-03	0.156E-08	0.156E-02	0.977E-02	0.391E-01	0.879E-01	0.156E+00	20

Q(N)	QQ(N)	SQQ(N)	WFRW(N)	N	URW(N)	WFRA(N)	URA(N)
0.4078828E+02	0.5098535E+00	0.1228698E+02	0.1000000E+01	20	0.1562500E-08	0.2923692E+00	0.1562500E+00
0.2805066E+02	0.3506333E+00	0.2071337E+02	0.1000000E+01	40	0.7812501E-09	0.5133524E+00	0.7812501E-01
0.1929083E+02	0.2411354E+00	0.2650832E+02	0.1000000E+01	60	0.5208334E-09	0.6653256E+00	0.5208334E-01
0.1326658E+02	0.1658322E+00	0.3049359E+02	0.1000000E+01	80	0.3906251E-09	0.7698398E+00	0.3906250E-01
0.9123631E+01	0.1140454E+00	0.3323431E+02	0.1000000E+01	100	0.3125001E-09	0.8417157E+00	0.3125000E-01

WFR1(N)	UR1(N)	WFR2(N)	UR2(N)	WFR3(N)	UR3(N)	WFR4(N)	UR4(N)
0.4398484E+00	0.1562500E-02	0.3675136E+00	0.9765626E-02	0.3185682E+00	0.3906250E-01	0.2980668E+00	0.8789063E-01
0.614775[illegible]E+00	0.7812500E-03	0.5650302E+00	0.4882813E-02	0.5313697E+00	0.1953125E-01	0.5172707E+00	0.4394531E-01
0.7350761E+00	0.5208334E-03	0.7008653E+00	0.3255208E-02	0.6777164E+00	0.1302083E-01	0.6680202E+00	0.2929688E-01
0.8178079E+00	0.3906250E-03	0.7942808E+00	0.2441406E-02	0.7783611E+00	0.9765626E-02	0.7716928E+00	0.2197266E-01
0.8747040E+00	0.3125000E-03	0.8585241E+00	0.1953125E-02	0.8475760E+00	0.7812501E-02	0.8429902E+00	0.1757813E-01

VOLUME	TOTAL OUTFLOWNTIME	
0.3141593E+04	0.2658744E+04	100