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CAUSE OF NEGATIVE OUTFLOW IN MUSKINGUM METHOD

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LIST OF SYMBOLS

A	area at section(3)
B	channel width
c	Chezy's coefficient
C	wave celerity for any discharge
C_Q	wave celerity corresponding to the discharge Q
C_{Q_e}	wave celerity corresponding to the discharge Q_e
C_o, C_1, C_2	coefficients of the conventional Muskingum method
I	inflow at time t
I_o or I(o)	inflow when t= 0
K	the average travel time of flood wave described by the Muskingum method
Q_o	Outflow when t=0. Also the reference discharge.
R	the hydraulic radius
S	storage of the reach at time t
S_f	the energy slope
S_o	bed slope
t	notation for time
V	velocity at section (3)
x	notation for channel length
y	depth of flow
Y_e	normal depth corresponding to discharge Q_e
θ	the Muskingum weighting factor
τ	the response delay time as envisaged by Gill
Δt	the routing time interval
Δx	reach length under consideration
$\delta(0_+)$	Dirac-delta function applied at the origin

LIST OF FIGURES

ABSTRACT

The Muskingum method is a widely used method for routing floods in rivers and channels. The applicability of the method has increased after Cunge related the parameters of the method, based on the conventional difference scheme, with the channel and flow characteristics using the principle of diffusion analogy. Since Cunge's study a number of papers and reports have been added to the literature of Muskingum method. However, one of the disturbing fact of the Muskingum method is the formation of negative or reduced outflow in the beginning of the solution. Various remedial measures like skipping the negative or reduced outflow zone, finding lower bound of reach length so that the magnitude of the defect is reduced, accepting this defect considering that it is small enough and short lived etc. have been suggested in the literature. Some researchers have even suggested not to use Muskingum method for field applications due to the presence of this defect. Another researcher's suggestion for the amendment of the method, through initial conditions, in order to correct this defect was met with severe criticisms and it led to many controversies. Inspite of nearly fifty years of widespread usage, the potential for controversy regarding Muskingum method has not been fully explored. Until the reason for this anomaly is identified, the Muskingum method will suffer from a lack of credibility. This can only hamper its wide acceptance for practical channel routing applications. Taking this into consideration, it is attempted in this note to explore the theoretical basis for the formation of negative or reduced outflow in the beginning of the Muskingum solution.

1.0 INTRODUCTION

Since its development in the thirties by McCarthy (1938), the Muskingum method of flood routing has been extensively used in river engineering practice. It is widely used, because it is a simple method which can be applied without much complication as far as the procedural details are concerned. The recent development by Cunge (1969) made it possible to link the Muskingum method, which used to be treated as an empirical method, with hydrodynamics and thus enable to compute the parameters not based on the observed hydrographs, but on the channel and flow characteristics. This improvement in the Muskingum method, generally referred, as Muskingum-Cunge method, has enhanced the predictive capability of the method, while remaining within the computational frame work of the conventional Muskingum method. Later Koussis (1976) also related the parameters of the method in a similar manner as Cunge, but considering linear variation of inflow over the routing time interval.

Inspite of its simplicity, wide applicability and improvements by Cunge (1969) and Koussis (1976), the Muskingum method has the defect of producing negative or reduced outflow ordinates in the beginning of the routed hydrograph. The presence of such defect has been explicitly brought out by Venetis (1969) when he derived the Instantanecus Unit Hydrograph(IUH) of the Muskingum method. Figure 1 depicts the IUH of the Muskingum channel reach. It can be seen that there is negative Dirac-delta function at the origin having a magnitude of

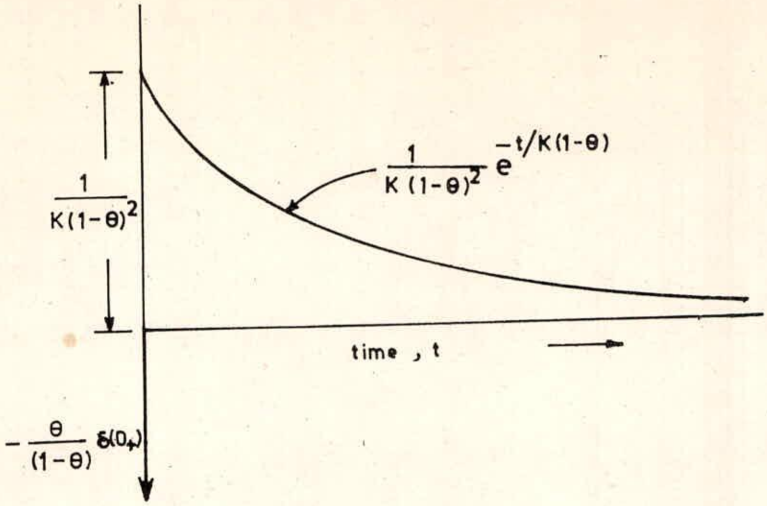

Figure 1 - THE I.U.H. of the Muskingum Channel Reach

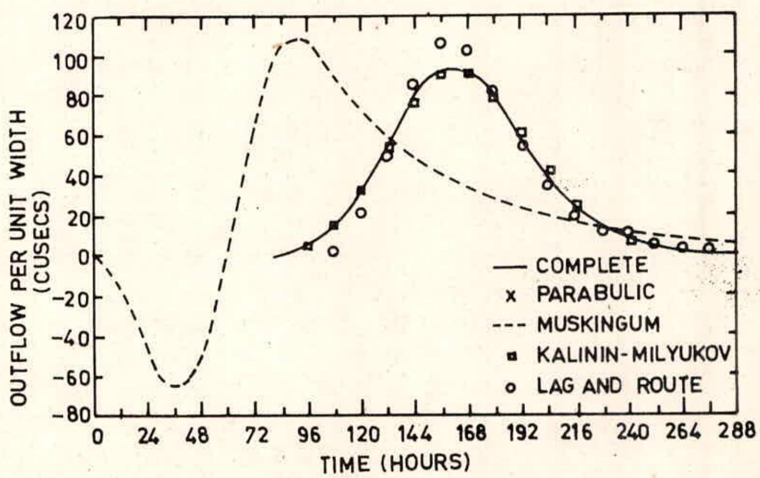

FIGURE 2 - Simulation by two parameter models
(Adapted from Dooge 1973)

$\frac{\Theta}{(1-\theta)}\delta(\Theta_+)$ in which θ is the Muskingum weighting factor and $\delta(0_+)$ is the Dirac-delta function applied at the origin. The Dirac-delta function is defined as a function whose width is nearly zero and whose height is nearly infinite in such a manner that the area beneath it is unity. In a comparative study of the performance of various two parameter hydrologic flood routing methods, Dooge(1973) also brought out this defect more explicitly, as shown in figure 2 by routing a hypothetical inflow hydrograph using these methods.

It is an undeniable fact that the problem of negative or reduced outflow exists, whether one is dealing with the conventional Muskingum method or with that of diffusion analogy based Muskingum methods. Many remedial measures have been suggested to overcome this defect. However, as pointed out by Kundzewicz(1980), and Perumal and Seth (1984) that the problem is not really solved, but it is either minimized or skipped.

Not convinced with such remedial measures some researchers (Meehan,1979; and Meehan and Wiggins,1979) have even suggested for rejecting the Muskingum method for field use. The protoganists argue infavour of the method pointing to the fact that the method is simple, useful in the field, and the solution with negative or reduced outflow which develops in the beginning of the routed hydrograph for a short duration is mathematically correct (Nash,1959; Weinmann and Laurenson,1979; and Strupczewski and Kundzewicz,1980). On the other hand, the antagonists of this method argue that since the unrealistic outflow is produced in the beginningof the routed hydrograph, the method should be suitably amended (Gill,1980) or rejected (Meehan,1979; and Meehan and Wiggins,1979). However the arguments of both these groups are not based on theoretical consideration but from intuitive notion.

It is the purpose of this note to bring out the theoretical basis for this negative or reduced outflow in the beginning of the routed hydrograph arrived using Muskingum method. This might form the basis for accepting or rejecting the views of the protagonists and antagonists of this method.

2.0 REVIEW

The Muskingum method employs the lumped continuity equation given as:

$$I - Q = \frac{dS}{dt} \qquad \text{...(1)}$$

and the storage equation, given as:

$$S = K\,[\,\theta I + (1-\theta)Q] \qquad \text{...(2)}$$

in which, I, Q and S are the inflow, outflow and storage at time t respectively; θ and K are the parameters of the method. The conventional or classical Muskingum routing equation is obtained by expressing equation(1) and (2) in the finite difference form (Miller and Cunge, 1975) and their simplification leads to:

$$Q_2 = C_o\, I_2 + C_1 I_1 + C_2 Q_1 \qquad \text{...(3)}$$

in which, Q_1, Q_2 and I_1, I_2 respectively are the outflow and inflow at the beginning and end of the routing time interval, Δt. The coefficients C_o C_1 and C_2 are expresse as:

$$C_0 = \frac{-K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad \text{...(4)}$$

$$C_1 = \frac{K\theta + \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad \text{...(5)}$$

$$C_2 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad \text{....(6)}$$

The sum cf C_0, C_1 and C_2 are equal to 1.0. Since I_1, I_2 and Q_1 are known for every time increment, routing is accomplished by solving equation (3) recursively.

It can be seen from the structure of equation(3) and the expressions for coefficients C_0, C_1 and C_2 given by equations(4),(5),(6) that the solution may yield negative or reduced outflow ordinates when

$C_0<0$. Therefore the recommendation(U.S. Army Corps of Engineers, 1960; Miller and Cunge, 1975) that $\Delta t > 2K\theta$ has been made from the consideration of avoiding negative or reduced outflow in the Muskingum solution. Recently Hjelmfelt (1985) has shown that this condition ensures non-negative Muskingum solution. Although this condition apparently avoids the negative or reduced outflow formation, there is an obvious shortcoming as a result of its application. Suppose for a given reach and for a given observed flood, the value of $\theta=0.3$ and $K=5$ hours were estimated and the use of this condition for this flood to avoid negative or reduced outflow demands $\Delta t > 3.0$ hours which may be greather than the time interval, generally one hour, at which the observations are available. Therefore, the observed information at one hour intervals Would not be considered for the sake of avoiding negative or reduced outflow. This shortcoming may be serious in flood forecasting operations.

Equations(1) and (2) can also be solved exactly(Kulandaiswamy, 1966; and Diskin, 1967). For the initial condition of $Q_0=I_0$ at $t = 0$, the exact solution is given as:

$$Q = -\frac{\theta}{(1-\theta)} I + \frac{e^{-t/K(1-\theta)}}{K(1-\theta)^2} \int_0^t I\, e^{t/K(1-\theta)} dt + \frac{I_0}{(1-\theta)} e^{-t/K(1-\theta)} \qquad \ldots(7)$$

and for the initial condition of $Q_0=0$ at $t=0$, the solutioin reduces to:

$$Q = -\frac{\theta}{(1-\theta)} I + \frac{e^{-t/K(1-\theta)}}{K(1-\theta)^2} \int_0^t I\, e^{t/K(1-\theta)} dt \quad \ldots.(8)$$

Equation (7) results in reduced outflow in the beginning due to the incorporation of initial steady flow present in the channel and equation(8) results in negative outflow in the beginning due to the consideration of zero flow in the channel as the initial condition. The formation of negative or reduced outflow solution of Muskingum method is not very obvious from the solutions given by equations(7) and (8). However, for a Dirac-delta input function, it can be explicitly shown based on equation (8) as:

$$u(0,t) = -\frac{\theta}{(1-\theta)}\ \delta\ (0_+) + \frac{1}{K(1-\theta)^2}\ e^{-t/K(1-\theta)} \quad \ldots(9)$$

The above solution depicted in figure 1 was given by **Venetis** (1969) as the Instantaneous Unit Hydrograph of the Muskingum reach. This depictioin implies that for any instantaneous input there is bound to be an instantaneous negative outflow at the same time and in addition an exponentially decaying outflow.

The remedial measures suggested by U.S.Army Corps of Engineers (1960), Weinmann and Laurenson(1979), Ponce and Theurer,(1982) and Chang et al.(1983) eigher reduces the magnitude of the negative or reduced outflow ordinates or it skips this unrealistic outflow zone. But it is proved by equations(7),(8) and (9) that this defect exists for any positive input. Nash (1959) had suggested the use of lag and route method instead of Muskingum method for routing floods in steep rivers wherein this defect may be predominant.

Realizing that this defect is built in within the method, Meehan(1979) and Meehan and Wiggins(1979) recommended that the classical Muskingum method be retired from field use and suggested the use of Muskingum-Cunge method. However, these investigators did not realize that Muskingum-Cunge method also has this defect and can avoid the formation of negative outflow by adopting the condition $\Delta t > 2K\theta$.

Some investigators (Laurenson and Weinmann,1979; and Gill,1979) attempted to find the reason behind the formation of this defect. Laurenson and Weinmann(1977) reasoned based on the use of conventional solution given by equation(3) that when $\Delta t \ll K$, the discharge at the outflow section of the reach is evaluated before any disturbance realized at the inflow section of the reach has been able to travel the reach length under consideration. However, their reasoning is not correct as the solution in the continuous time domain given by equations (7),(8) and (9) also show the presence of this defect.

Gill(1979) attempted to solve the puzzle of Muskingum method by modifying the initial conditions required for the solution of Muskingum equations(1) and (2). Instead of considering the initial conditions.

$$I(0) = Q(0) \quad \text{at } t = 0 \qquad ...(10)$$

$$\text{and } I(0) = Q(0) = I_0 \quad \text{at } t = 0 \qquad ...(11)$$

he considered the initial condition:

$$I(0) = Q(\tau) \quad \text{for } \tau > 0 \qquad ...(12)$$

Gill argued that the initial conditions given by equations(10) and(11) assumes that the effect of inflow reaches the outlet of the reach under consideration instantaneously which contradicts with the flood movement characteristics in natural river for which the initial condition given by equation(12) is more appropriate. Therefore based on the concept that the effect of inflow reaches the downstream point of the reach after certain time, of the entry of input, Gill solved the equations (1) and (2) as:

$$Q(t) = -\frac{\theta}{(1-\theta)} I(t) + \frac{e^{-t/K(1-\theta)}}{K(1-\theta)^2} \int_{\tau}^{t} I(t)\, e^{t/K(1-\theta)} dt$$

$$+ \left[\frac{\theta}{(1-\theta)} I(\tau) + Q(\tau)\right] e^{-(t-\tau)/K(1-\theta)} \qquad ...(13)$$

for $t > \tau$

But Gill (1970) used this solution to put forward his theory on pure translatory behaviour of the Muskingum method. This resulted in many controversies. Singh and McCann(1980), and Strupczewski and Kundzewicz (1980) criticized Gill's approach on the context that the initial conditions given by equations(10) and (11) are mathematically correct and unequivocally proved there exists no translatory behaviour of the Muskingum solution. Also in a different sense, Gill's (1979) argument to solve the defect of Muskingum method may not be acceptable as other conceptual model such as n-linear reservoirs in series (Dooge, 1973) applied for flood routing in channels assumes that the effect of inflow is felt at the outlet of the reach instantaneously without creating any unrealistic solution as in the case of Muskingum method.

It seems that the cause for the negative or reduced outflow in the beginning of the Muskingum solution May be attributed to the form of storage equation employed. But the way in which it is responsible is brought out in this note.

3.0 PROBLEM DEFINITION

It is required to find the reason behind the formation of negative or reduced outflow which arises in the beginning of the solution of Muskingum method.

4.0 METHODOLOGY

A physically based flood routing methodology is presented herein with assumptions involved. Incidentally it is seen that the mathematical analysis of the method results in the physical justification of the Muskingum method for routing floods in river channel The relationships between the parameters of the Muskingum method, and the channel and flow characteristics are derived.

It should be pointed out herein that Cunge(1969) and Dooge, et al.(1982) presented different approaches on the justification of the use of Muskingum method for routing floods in river channels, and their approaches enable to relate the parameters of the Muskingum method with channel and flow characteristics. However, these authors have not given the reason behind the formation of reduced or negative outflow. Unlike their approaches the present approach not only relates the parameters of the Muskingum method with flow and channel characteristics, but establish the reason for the formation of unrealistic outflow in the beginning of the Muskingum solution.

4.1 Physical Basis for the Model Development

During steady flow in a river reach there exists a unique relationship between stage and discharge at any cross-section. This situation is altered during unsteady flow, with the discharge appearing first in a cross-section and at the same time the stage which corresponds to that discharge during steady flow appears at a section upstream of it . This concept has been used by Kalinin and Milyukov (as quoted by Miller and Cunge 1975) to determine the 'unit length of reach' required for flood routing in long river reaches. However, the Kalinin -Milyukov method appears to be less flexible since the 'unit reach length' of the channel is fixed and it is possible that rating curves may not be available at the end section of the unit reach length. Conversely, interpolation of hydrographs at intermediate cross sections increases the level of uncertainty in the results. Thus there is a potential problem in the determination of stage hydrographs from the calculated discharge hydrographs of the Kalinin-Milyukov method.

In this report, it is shown that the extension of the Kalinin-Milyukov concept leads to a flood routing model which is devoid of the restriction imposed by the Kalinin-Milyukov method as mentioned above. During unsteady flow in a prismatic channel with linearly varying water stage along the river reach, the channel storage S in the river reach of length Δx is uniquely related to the mean water stage of the reach which corresponds to the steady flow stage of the discharge, Q which is observed at the outlet of the reach. This is the concept of the Kalinin Milyukov method with the reach length, Δx corresponding to the unit reach length. This concept is extended in the proposed method with the assumption that the mean water stage is uniquely related to the discharge at a section located 'l' distance upstream of the outlet section, where the discharge Q is being recorded, and the discharge, Q itself is uniquely related to the stage observed at 'l' distance upstream of the outlet section.

The mathematical description of this method incorporating this concept is given in the following pages with the assumptions involved.

4.2 Assumptions

The following assumptions are made in this method:

1. Section is wide rectangular

2. The inflow is known

3. There is no lateral inflow into the reach or lateral outflow from the reach

4. The water stage is linearly varying

5. During unsteady flow while the discharge Q arrives at the outflow section of the reach, the stage which corresponds to the flow, Q during steady flow condition arrives at a distance 'l' upstream of the outflow section.

6. The discharge at the section'l' distance upstream of the outflow section has its steady flow stage located at the middle of the reach

7. The flow at any section of the routing reach is proportional to the water stage. This implies that the flow velocity remains constant at a section.

4.3 Development of the Model

Figure 3 depicts a river reach with uniform rectangular cross-section and the upstream and downstream sections, where the inflow and outflow hydrographs are measured resepctively have been denoted as section(1) and (2). Let the distance between these sections be Δx.

Based on assumption(5), the discharge Q at section(2) is related uniquely with the stage at section(3) which is located'l' distance upstream of section(2). Therefore while the discharge, Q appears at section(2), the corresponding normal depth, y appears at section(3). Based on assumption(6), the discharge at section(3) is uniquely related with the stage at the midele of the reach between

SECTION ①-① : CORRESPONDS TO THE INFLOW POINT

SECTION ②-② : CORRESPONDS TO THE OUTFLOW POINT

SECTION ③-③ : CORRESPONDS TO THE POINT WHERE THE DISCHARGE IS UNIQUELY RELATED WITH THE STAGE AT MIDDLE OF THE REACH

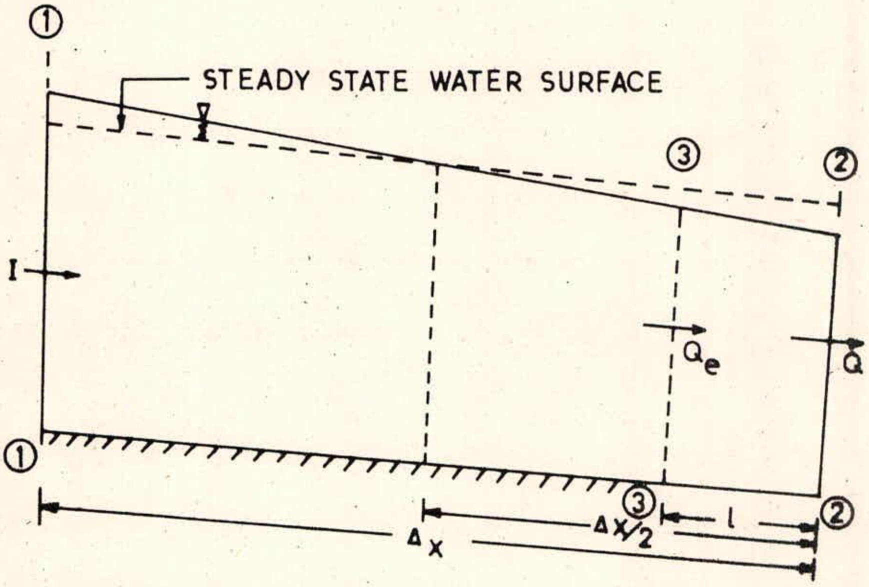

FIGURE 3 - Definition sketch of the reach under consideration

sections(1) and (2). The discharge, Q_e at section(3) is expressed as:

$$Q_e = A\ V \qquad \ldots(14)$$

in which,

A= Area at section(3)

V= Velocity at section(3)

The velocity, V can be expressed by Chezy's formula as:

$$V = cR^{1/2}\ S_f^{1/2} \qquad \ldots(15)$$

where,

c = Chezy's coefficient

R = Hydraulic radius,

S_f= Energy slope.

Substitution of equation(15) in equation(14) gives the expression for Q_e as:

$$Q_e = AcR^{1/2}S_f^{\ 1/2} \qquad \ldots(16)$$

The energy slope, S_f can be expressed in terms of bed slope, S_0 and and the water surface slope, $\frac{\partial y}{\partial x}$ after eliminating the local and convective acceleration terms from the momentum equation of St. Venant's equation (Henderson, 1966) as:

$$S_f = S_0 - \frac{\partial y}{\partial x} \qquad \ldots(17)$$

in which, y is the depth at section (3). Substituting equation(17) in equation (16) gives:

$$Q_e = AcR^{1/2}\ S_0^{1/2}\ \left(1 - \frac{1}{S_0}.\ \frac{\partial y}{\partial x}\right)^{1/2} \qquad \ldots(18)$$

But

$$AcR^{1/2}\ S_0^{1/2} = Q \qquad \ldots(19)$$

Substituting equation(19) in equation(18) and expanding the term $(1-\frac{1}{S_0}\ \frac{\partial y}{\partial x})^{1/2}$ is Binomial series leads to :

$$Q_e = Q\ \left(1 - \frac{1}{2S_0}\ \frac{\partial y}{\partial x} + \frac{1/2\ (1/2-1)}{\underline{|2}}\ \left(-\ \frac{1}{S_0}\ \frac{\partial y}{\partial x}\right)^2 + \ldots\ldots\right) \qquad \ldots(20)$$

when the bed slope is not very flat, $S_0 >> \frac{\partial y}{\partial x}$ (Henderson, 1966). Therefore, the terms $(\frac{1}{S_o} \frac{\partial y}{\partial x})^2$ and higher orders can be assumed very small when compared with $\frac{1}{2S_0} \frac{\partial y}{\partial x}$.

Under such condition equation (20) can be approximated as:

$$Q_e = Q \left(1 - \frac{1}{2S_0} \frac{\partial y}{\partial x} \right) \qquad ...(21)$$

Since, Q the outflow is uniquely related to the depth y,

$$\frac{\partial y}{\partial x} = \frac{(B + 2Y)y}{(B/2\ B+2y)Q} \frac{\partial Q}{\partial x} \qquad ...(22)$$

But for large rectangular channels $R \approx y$ and therefore equation (22) can be approximated as:

$$\frac{\partial y}{\partial x} = \frac{1}{BC_Q} \frac{\partial Q}{\partial x} \qquad ...(23)$$

in which, C_Q is the flood wave celerity corresponding to the discharge, Q.

In the subsequent mathematics of the problem the large rectangular concept is adopted and so the use of equation(23) is continued.

Substitution of equation(23) in equation(21) leads to:

$$Q_e = Q - \frac{Q}{2S_oBC_Q} \frac{\partial Q}{\partial x} \qquad ...(24)$$

Since the discharge at section(3) is uniquely related to the depth at the middle of the reach between sections(1) and (2):

$$\frac{\partial y_e}{\partial t} = \frac{1}{BC_{Q_e}} \frac{\partial Q_e}{\partial t} \qquad ...(25)$$

where C_{Q_e} is the celerity of the flood wave corresponding to the discharge Q_e. Due to assumption(7), the wave celerity does not change with the magnitude of discharge and therefore $Q_{Q_e} = C_Q = C$.

Based on assumption (4), one can simulate that for every unsteady flow situation observed in the channel reach there exists a corresponding steady flow situation having the discharge equal to that observed at section(3) and the stage equal to that observed in the middle of the reach. This one to one correspondence between steady flow and unsteady flow situations is made possible by simply tilting the water surface profile of the unsteady flow situation about the stage at the middle of the reach in such a way that it becomes parallel to the bed surface.

Under the corresponding steady flow situation, equation(25) can be used in the hydraulic continuity equation to solve for Q_e. The hydraulic continuity equation is written as:

$$\frac{\partial Q_e}{\partial x} + B \frac{\partial y_e}{\partial t} = 0 \qquad ...(26)$$

Substituing equation(25) in equation(26) and modifying it gives:

$$\frac{\partial Q_e}{\partial t} + C \frac{\partial Q_e}{\partial x} = 0 \qquad ...(27)$$

Because of the assumptions (4) and (7), $\frac{\partial Q_e}{\partial x}$ is written as:

$$\frac{\partial Q_e}{\partial x} = \frac{\partial Q}{\partial x} \qquad ...(28)$$

and the differentiation of Q_e with reference to time leads to:

$$\frac{\partial Q_e}{\partial t} = \frac{\partial Q}{\partial t} - \frac{Q}{2S_0BC} \frac{\partial^2 Q}{\partial t \partial x} - \frac{1}{2S_0BC} \left(\frac{\partial Q}{\partial x}\right) \left(\frac{\partial Q}{\partial t}\right) \qquad ...(29)$$

The last term of the above equation is very small when compared with other terms of the equation. It can be easily seen by comparing the order of magnitude of this term with the term $\frac{\partial Q}{\partial t}$. The order of magnitude of the last term is $\frac{1}{2S_0BC} \frac{\partial Q}{\partial x}$. Using equation(23) it can be seen that it corresponds to the order of $\frac{1}{2S_0} \frac{\partial y}{\partial x}$. Note the order of magnitude of the first term is unity. As $S_0 >> \frac{\partial y}{\partial x}$, the last term of equation(29) may be considered negligible. Therefore equation(29) is approximated as:

$$\frac{\partial Q_e}{\partial t} = \frac{\partial Q}{\partial t} - \frac{Q}{2S_oBC} \frac{\partial^2 Q}{\partial t \partial x} \qquad ...(30)$$

Substituting equation(28) and (30) in equation(27) gives:

$$\frac{\partial Q}{\partial t} + C\frac{\partial Q}{\partial x} = \frac{Q}{2S_0\ BC}\ \frac{\partial^2 Q}{\partial t\,\partial x} \qquad \ldots(31)$$

The expression for $\frac{\partial Q}{\partial x}$ can be approximated in lumped form as:

$$\frac{\partial Q}{\partial x} = \frac{Q-I}{\Delta x} \qquad \ldots(32)$$

where, I is the inflow at section(1). Substitution of equation(32) in equation(31) gives:

$$\frac{\partial Q}{\partial t} + C\frac{(Q - I)}{\Delta x} = \frac{Q}{2S_0 BC\Delta x}\ \frac{\partial}{\partial t}(Q - I) \qquad \ldots(33)$$

The variable $\frac{Q}{2S_0\ BC\Delta x}$ can be fixed by taking a reference discharge, Q_0 and it is written as:

$$\theta = \frac{Q_0}{2S_0 BC\,\Delta x} \qquad \ldots(34)$$

The average travel time of flood wave in traversing the reach from section (1) to (2) is given as:

$$K = \frac{\Delta x}{C} \qquad \ldots(35)$$

Now Q and I vary only as a function of time, t and so the partial derivative operator'∂' is replaced by full derivative operator'd'. Substituting equations(34) and (35) in equation(33) leads to:

$$I - Q = \frac{d}{dt}\left[\ K\ (\theta I + (1-\theta)\ Q)\ \right] \qquad \ldots(36)$$

It can be seen that equation(36) is the Muskingum differential equation with the storage of the reach given as:

$$S = K\left[\theta I + (1-\theta)\ Q\ \right] \qquad \ldots(37)$$

where

θ = the weighting factor, and

K = the travel time

The term within the bracket of equation(37) corresponds to discharge at section(3). Multiplying both sides of equation(36) by (1-θ) gives:

$$I-[\theta\ I + (1-\theta)\,Q] = \frac{d}{dt}\ K(1-\theta)\ [\ \theta I + (1-\theta)Q] \qquad \ldots(38)$$

i.e.

$$I - Q_e = \frac{d}{dt}[\,K\,(1-\theta)\,Q_e] \qquad \ldots(39)$$

The solution of equation(39) for Q_e gives:

$$Q_e = \frac{e^{-t/K(1-\theta)}}{K(1-\theta)} \int_0^t I\, e^{t/K(1-\theta)}\, dt \quad \text{when } I = 0, \text{ at } t=0 \qquad \ldots(40)$$

or

$$Q_e = \frac{e^{-t/K(1-\theta)}}{K(1-\theta)} \int_0^t I\, e^{t/K(1-\theta)}\, dt + I_o e^{-t/K(1-\theta)}$$

$$\text{when } I = I_o, \text{ at } t = 0 \qquad \ldots(41)$$

Equation(40) implies that the solution for Q_e is obtained by convoluting the inflow, I with the Instantaneous Unit Hydrograph of a single linear reservoir.

The solution for Q is obtained taking into consideration that Q_e is a linear variation of inflow and outflow.

Accordingly Q is given as

$$Q = -\frac{\theta}{(1-\theta)}\, I + \frac{e^{-t/K(1-\theta)}}{K(1-\theta)^2} \int_0^t I\, e^{t/K(1-\theta)}\, dt \qquad \ldots(42)$$

Equation(42) is the solution of Muskingum equation, when I=0 at t =0. Alternatively when $I = I_0$ at t =0

$$Q = -\frac{\theta}{(1-\theta)}\, I + \frac{e^{-t/K(1-\theta)}}{K(1-\theta)^2} \int_0^t I\, e^{t/K(1-\theta)}\, dt + \frac{I_0}{(1-\theta)}\, e^{-t/K(1-\theta)} \qquad \ldots.(43)$$

4.4 Range of Weighting Parameter

Before exploring the cause for negative outflow in the Muskingum solution, it is essential to interpret the limits of the Weighting parameter θ so as to confirm to the limits of the Muskingum method specified in practice. Two cases can be visualized with regard to the extreme limits of θ. The case of the weighting parameter nearing zero indicates that section(3) coincides with section(2) and such a situation results in the Kalinin-Milyukov method. The other extreme of the weighting factor nearing 0.5 indicates that section(3) apoproaches the mid-section of the reach. It can be visualized from the physical basis of the proposed method, that θ =0.5 is the limiting case where the discharge preceedes the corresponding steady stage state. The situation in which $\theta > 0.5$ implies the location of section(3) upstream of mid-section of the reach and based on the physical basis of the model (i.e., the discharge precedes the corresponding steady flow stage in unstready situation), the change of direction of flow could be realized. Accordingly, the computed hydrograph at the outflow section, i.e., at section(2), would be the amplification of inflow hydrograph. This mathematical, but so far physically unexplained behaviour of the Muskingum method solution has been noted by many researchers(Dooge,1973, and Strupczewski and Kundzewicz 1980).

Therefore, it can be inferred that the weighting paramter, θ is limited to zero on its lower limit and to 0.5 on its upper limit.

5.0 CAUSE OF NEGATIVE OUTFLOW

It can be seen that the solution for the outflow given by equation (42) or (43) and the interpretation of the weighting parameter unequivocally proves that the proposed method results, in the physical justification of the Muskingum flood routing method. It should be possible to ascertain the cause of negative or reduced outflow in the beginning of the Muskingum solution through this physically based method.

Equations(40) and (41) indicate that at section(3) $Q_e = 0$ and and $Q_e = I_0$ respectively when t=0, Invoking the assumption of linear variation of discharge along the reach implied by the assumptions(4) and (7) from t=0 cnwards necessarily leads to a discharge less than the initial flow, either zero or I_0 at section(2) during the beginning of the solution depending on whether equation (40) or (41) is used. Therefore the formation of negative or reduced flow in the solution of Muskingum method may be avoided.if the assumption of linear variation of discharge is not imposed in the beginning of the routing process. The description of the details of such remedial measure is beyond the scope of this study.

6.0 CONCLUSIONS

It is shown that the method described herein gives physical justification for the Muskingum method for routing floods in river channels. It is inferred that invoking the assumption of linear variation of discharge from t = 0 onwards leads to the formation of negative or reduced outflow at section (2), when section(3) which is located upstream of section(2) itself has solution for outflow nearer to zero in the beginning.

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