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Unsteady seepage from a canal

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Abstract
An analytical solution for unsteady seepage from a canal is derived using Duhamel's principle, the basic solution for water level rise due to steady recharge from a strip source given by Poluborinova - Kochina and the non-linear relationship between influent seepage and hydraulic head difference proposed by Rushton and Redshaw. The parameters appearing in Rushton and Redshaw's proposed non-linear equation for influent seepage have been derived. Variation of seepage with time has been presented in non-dimensional form. Linear relationship for shallow water table condition over estimates seepage from a canal. When water table is at large depth, the linear relationship is not applicable. Seepage losses per unit area is higher for canal with smaller width.

INTRODUCTION

It has been often assumed for a stream, which is hydraulically connected with an aquifer, that the exchange flow rate between the stream and the aquifer is linearly dependent on the potential difference causing flow (Ernst, 1962; Aravin and Numerov, 1965; Herbert, 1970; Morel-Seytoux and Daly, 1975) ; Besbes et al, 1978; and Flug et al, 1980). Bouwer (1969) has reported that the recharge from a canal to an aquifer is directly proportional to the difference in the water levels in the canal and in the aquifer in the vicinity of the canal. The coefficient of proportionality, which has been designated as reach transmissivity, depends on the hydraulic conductivity and canal cross section (Morel-Seytoux, 1964, Bouwer, 1969). Only in case of confined flow, the relation between seepage and potential difference causing flow can be linear. There have been evidences that the process of stream aquifer interaction can be non-linear (Rushton and Redshaw, 1979; Dillon, 1983, 1984). Considering the fact that influent seepage from a canal is zero for zero potential difference and a finite quantity for infinite potential difference, the relationship between influent seepage and potential difference has to be non-linear in case of unconfined flow. Only for cases when the water table is very close to the canal bed, the linear relationship is applicable. In this paper, using the non-linear relationship between the influent seepage and the potential difference proposed by Rushton and Redshaw , unsteady seepage from a canal has been analysed.

STATEMENT OF THE PROBLEM

A canal having hydraulic connection with the underlying aquifer is shown in Fig. 1. The recharge from unit length of a canal to an aquifer is assumed to have the following non-linear relationship with the potential difference between the canal and the aquifer (Rushton and Redshaw, 1979):

$$Q(t) = C_2 [1 - \exp\{-C_3(h_r - h(0,t))\}] \quad (1)$$

C_2 and C_3 are constants; h_r is the hydraulic head at the canal perimeter; and h(0, t) is the hydraulic head in the aquifer under the canal axis at time t. h_r and h(0, t) are measured upwards from the impervious bed of the aquifer which has been selected as the low datum. The hydraulic head, h(0, t), is governed by the recharge from the canal which occurs up to time t. The time should be measured from the instant the seepage water from a canal joins the ground water. For convenience, it is reckoned since water is conveyed in the canal. It is aimed to find the two parameters C_2 and C_3 and the influent seepage, Q(t), at various time after the onset of running of a canal.

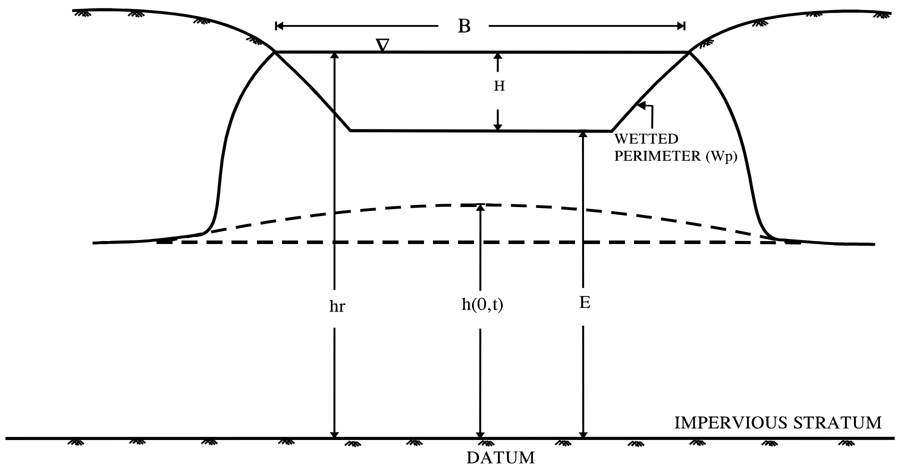

Figure 1. A canal hydraulically connected with an aquifer.

Evaluation of Constants C_2 and C_3

The seepage from unit length of a canal, when the water table is at very large depth, is given by (Kozeny, 1931; Vedernikov, 1934 , vide Harr, 1962):

$$Q = K (B+AH) \quad (2)$$

where B=width of the canal at the water surface, and H= the maximum depth of water in the canal. Using inversion of hodograph and conformal mapping , Vedernikov has derived the parameter A for canals having trapezoidal cross section. For a ditch with a curved perimeter, the parameter A is equal to 2 (Kozeny, 1931, vide Harr , 1962).

Applying the condition that the exponential term in equation (1) tends to zero for very large value of $[h_r - h(0, t)]$, i.e., when the water table lies at large depth below the canal bed, equations (1) and(2) yield

$$C_2 = K(B+AH) \quad (3)$$

For small difference between h_r and h(0, t), the higher order terms of the polynomial expansion of the exponential term appearing in equation (1) can be neglected and the seepage for small potential difference can be approximated to be:

$$Q(t) = K(B + AH)\, C_3\, [h_r - h(0,t)] \qquad (4)$$

For small potential difference between the canal and aquifer, the exchange flow rate can be assumed to have a linear relationship with the potential difference. The linear relationship proposed by several investigators is of the form:

$$Q(t) = \Gamma_r\, [\, h_r - h(0,t)] \qquad (5)$$

in which Γ_r is the constant of proportionality known as reach transmissivity. Following Herbert (1970), an expression for reach transmissivity for unit length of a canal is given by:

$$\Gamma_r = \pi K \,/\, \log_e \left[\frac{0.5(E + H)}{R} \right] \qquad (6)$$

where, E = saturated thickness of the aquifer below the bed of the canal; H= maximum depth of water in the canal; R = radius of the equivalent semi-circular section of the canal equal to W_p/π; W_p= wetted perimeter of the canal. Herbert's formula is applicable for 0.5(E+H) > R. Equating equations (4) and (5), the other constant, C_3 is found to be :

$$C_3 = \Gamma_r / \{K(B+AH)\} \qquad (7)$$

Estimation of Unsteady Seepage from a Canal

Let the time span be discretised into time-steps of equal size Δt. Let during a time-step γ the recharge rate from unit length of the canal, $Q(\gamma)$, be constant. $Q(\gamma)$ varies from one time-step to next. The hydraulic head at the end of nth unit time-step, $h(0, n\,\Delta t)$, is given by(Morel-Seytoux and Daly 1975):

$$h(0,n\Delta t) = h(0,0) + \sum_{\gamma=1}^{n} Q(\gamma)\, \delta(0,\Delta t, n - \gamma + 1) \qquad (8)$$

in which δ(.,.,.) are discrete kernel coefficients for piezometric rise under the canal. The first term within bracket of the discrete kernel coefficient is the distance from the canal axis, the second term is the uniform time-step size and the third term is convolution time index. h(0,0) is the initial water table height before the onset of recharge. The discrete kernel coefficients δ (.,.,.) are computed as follows:

The width of the strip source from which the recharge is taking place is B. If the recharge takes place at unit rate from unit length of the strip, the average rate of recharge per unit area would be 1/B. Let the rise in piezometric surface due to continuous uniform recharge , at a rate 1/B per unit area from the recharging strip of infinite length, be designated as U(x, t). Following Polubarinova-Kochina (1962), the response function U(x,t) is given by:

U(x,t) = F(x,B,t) - 0.5$\sqrt{x^2/T}$ for x ≤ - B/2 and, x ≥ B/2
= F(x,B,t) - 0.5 $(x^2+0.25B^2)/(BT)$ for -B/2 ≤ x ≤ B/2, (9)

where,

$$F(x,B,t) = \frac{\alpha\, t}{2BT}\left[\, erf\left\{ \frac{x+0.5B}{\sqrt{4\alpha\, t}} \right\} - erf\left\{ \frac{x-0.5B}{\sqrt{4\alpha\, t}} \right\} \right]$$

$$+ \frac{1}{4BT}\left[\, (x+0.5B)^2\, erf\left\{ \frac{x+0.5B}{\sqrt{4\alpha\, t}} \right\} - (x-0.5B)^2\, erf\left\{ \frac{x-0.5B}{\sqrt{4\alpha\, t}} \right\} \right]$$

$$+ \frac{\sqrt{\alpha\, t}}{2BT\sqrt{\pi}}\left[\, (x+0.5B)\exp\left\{ -\frac{(x+0.5B)^2}{4\alpha\, t} \right\} - (x-0.5B)\exp\left\{ -\frac{(x-0.5B)^2}{4\alpha t} \right\} \right]$$

in which x is the distance from the centre of the recharging strip, T is the transmissivity of the aquifer, α is the hydraulic diffusivity equal to T/ φ and φ is the specific yield.

If unit recharge per unit length of the recharging strip takes place during the first unit time period Δt, and no recharge takes place there after, the piezometric rise in the aquifer δ(.,.,.), at the end of nth time step is given by:

$$\delta(x,\Delta t,n) = [U(x,n\Delta t) - U(x,(n-1)\Delta t)]/\Delta t, \quad \text{for } n>1$$
$$= U(x,\Delta t)/\Delta t \quad \text{for } n=1 \qquad (10)$$

On substituting h(0,nΔt) from equation (8) in equation (1), and replacing h_r by [D_i - σ_r(n)], in which D_i is depth to impervious stratum measured from a high datum and σ_r(n) is depth to water level in the canal during nth time step measured from the same high datum and simplifying,

$$1 - Q(n)/C_2 = \exp\left[-C_3\left\{ D_i - \sigma_r(n) - h(0,0) - \sum_{\gamma=1}^{n} Q(\gamma)\,\delta(0,\Delta t, n-\gamma+1)\right\}\right] \qquad (11)$$

Taking logarithm of terms on either side of eq.(11),

$$\log_e[1 - Q(n)/C_2] = -C_3\left\{ D_i - \sigma_r(n) - h(0,0) - \sum_{\gamma=1}^{n} Q(\gamma)\,\delta(0,\Delta t, n-\gamma+1)\right\} \qquad (12)$$

Splitting the temporal summation into parts and rearranging,

$$\log_e[1 - Q(n)/C_2] - C_3 Q(n)\,\delta(0,\Delta t,1)$$
$$= \; = -C_3\left\{ D_i - \sigma_r(n) - h(0,0) - \sum_{\gamma=1}^{n-1} Q(\gamma)\,\delta(0,\Delta t, n-\gamma+1)\right\} \qquad (13)$$

Q(n) can be solved in succession starting from time step 1 by an iteration procedure. In particular for the first time step, the temporal summation term in equation (13) is not to be considered.

The following simplification can be adopted without much loss of accuracy. K(B+AH) being the maximum recharge rate per unit length of the canal that can occur when water table is at large depth, the ratio Q(n)/[K(B+AH)] is less than 1. Expanding the logarithmic term in equation(13) and neglecting the third and higher order terms and simplifying

$$0.5\,[1/C_2]^2 Q^2(n) + [1/C_2 + C_3\,\delta(0,\Delta t,1)]\,Q(n) - C_3\,[D_i - \sigma_r(n) - h(0,0) - \sum_{\gamma=1}^{n-1}\{Q(\gamma)\,\delta(0,\Delta t, n-\gamma+1)\}] = 0 \qquad (14)$$

Solving the above quadratic equation

$$Q(n) = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a} \qquad (15)$$

where

$a = \quad 0.5[1/C_2]^2$

$b = \quad [1/C_2 + C_3\,\delta(0, \Delta t, 1)$

$$c = -C_3\,[D_i - \sigma_r(n) - h(0,0) - \sum_{\gamma=1}^{n-1} Q(\gamma)\,\delta(0,\Delta t, n-\gamma+1)]$$

RESULTS AND DISCUSSION

The data required for the computation are : canal cross section, maximum depth of water in the canal, depth to water table, thickness of aquifer, hydraulic conductivity, and storage coefficient. Assuming a time step size Δt, the discrete kernel coefficients, $\delta(0,\Delta t,\gamma)$, γ = 1, 2……. n , were generated for known width of the canal at the water surface, hydraulic diffusivity of the aquifer. The reach transmissivity was obtained using Hurbert formula for known wetted perimeter of the canal and saturated thickness of the aquifer below the canal bed. For known trapezoidal canal cross section, the parameter 'A' was obtained from Vedernikov's graph (vide Harr,1962). The seepage was computed in succession starting from first time step for a known initial potential difference. Since the canal seepage varies with time size, the accuracy depends on the time step size. A minimum number of 10 time steps is required to compute the seepage rate at time t.

For a canal with width at water surface B=30m, maximum depth of water in the canal H=3m, canal side slope m=1.5, hydraulic conductivity K=0.01m/day, Δt=0.001 day, initial saturated thickness h(0,0)=100m, and initial potential difference $[h_r - h(0,0)]$=6m, the seepage from the canal at various time are presented in table1. Seepage rates are computed using the exact equation following an iteration procedure and by the approximate method. The error in computation by the approximate method is of the order of 2%. Therefore, approximate method which avoids iteration can be conveniently used.

For shallow water table position, if the relation between influent seepage and potential difference is approximated by a linear relation, the seepage loss is given by :

$$Q(n) = \left[\frac{D_i - \sigma_r(n) - h(0,0) - \sum_{\gamma=1}^{n-1} Q(\gamma)\,\delta(0,\Delta t, n-\gamma+1)}{\delta(0,\Delta t, 1) + 1/\Gamma_r} \right]$$

The seepage computed by a linear relationship is over estimated by about 15% . In Table-1, the seepage quantities computed by non-linear and linear relationship are compared.

Table 1. Seepage following linear and non-linear relation.

Kt/(φH)	Q(t)/(KH)		
	Non-linear (Exact)	Non-linear (Approxi-mate)	Linear
.033333	3.358783	3.429570	3.872202
.066667	3.342274	3.412055	3.847320
.100000	3.326648	3.395093	3.823281
.133333	3.311696	3.378883	3.800365
.166667	3.297804	3.363458	3.778610
.200000	3.284341	3.348776	3.757951
.233333	3.270992	3.334777	3.738298
.266667	3.258593	3.321399	3.719558
.300000	3.247157	3.308583	3.701644
.333333	3.235700	3.296276	3.684477
.366667	3.224670	3.284432	3.667989
.400000	3.214102	3.273010	3.652120
.466667	3.193938	3.251298	3.622039
.500000	3.184332	3.240948	3.607739
.566667	3.165875	3.221141	3.580443
.600000	3.157053	3.211642	3.567386
.666667	3.139717	3.193369	3.542328

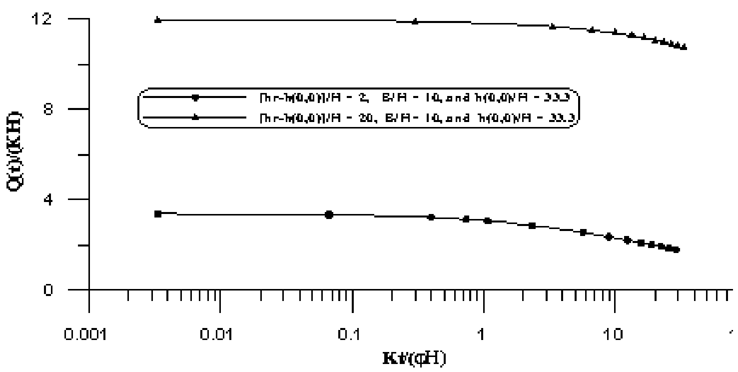

Figure 2. Variation of non-dimensional seepage with non-dimensional time parameter for shallow and deep water table position.

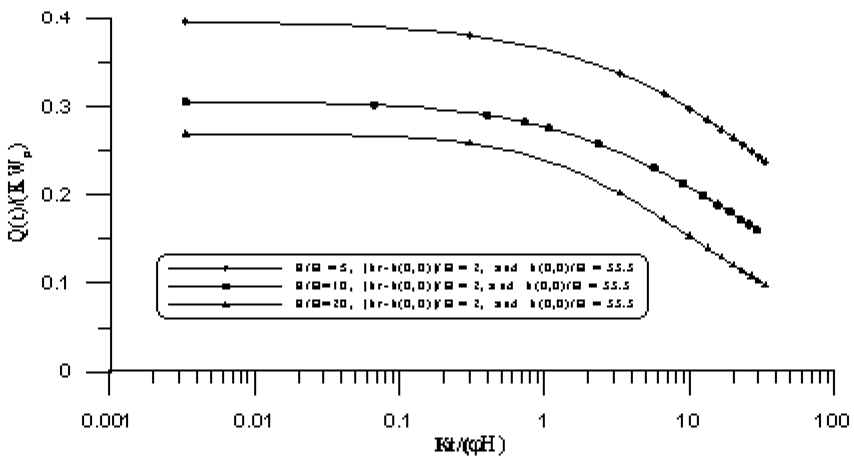

Figure 3. Variation of non-dimensional seepage for unit wetted perimeter with time for different width of the canal at the water surface.

Variations of dimensionless seepage, Q(t)/(KH) , from a canal having m=1.5 , and B/H=10, with non-dimensional time parameter, Kt/(ϕ H) , are shown in figure 2 for initial potential difference (h_r - h(0,0))/H= 20 and 2. (h_r - h(0,0))/H =20 corresponds to the case where the water table lies at a large depth below the canal for which non-linear relation between seepage and potential difference is applicable. (h_r-h(0,0))/H=2 corresponds to the case where water table lies at shallow depth for which linear law can be applied . When the water table is at shallow depth , the decrease in seepage rate is more rapid than that when the water table lies at large depth.

Variation of non-dimensional seepage for unit wetted perimeter with time for different width of the canal at the water surface is presented in figure-3. Under prolonged seepage, the seepage rate from unit surface area of a small canal [i.e., B/H=5] is 2.5 times that of from a canal of higher width [i.e, B/H= 20]. Therefore, while using canal seepage per unit wetted area, the canal width should be taken into account.

CONCLUSIONS

Unsteady seepage from a canal has been analysed using the relation given by Rushton and Redshaw. The parameters appearing in Rushton and Redshaw's equation have been derived.

Linear relationship for shallow water table condition over estimates seepage from a canal. When water table is at large depth, the linear relationship is not applicable.

Seepage losses per unit area is higher for canal with smaller width.

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