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Drain spacing computation in sloping lands - an analytical approach

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Abstract
Subsurface drainage seems to be one of the feasible solutions to solve the problem of waterlogging and salinity, because by providing adequate drainage, both the excess water and harmful salts can be appropriately removed from the root zone. In the present study analytical solution of Boussinesq equation linearized by Baumann's method, incorporating constant or depth dependent evapotranspiration has been obtained to describe spatial and temporal variation of water table between two parallel drains overlying a sloping impermeable barrier. Adopting the practically feasible unsteady state drainage design criteria which stipulates that water table should be lowered by 30 cm in 2 days, once the water table reaches the soil surface, the drain spacings were computed. A computer program was written in FORTRAN 77 to compute the position of maximum water table heights, transient falling water tables for a given spacing, and drain spacing from the implicit analytical solution. Effect of slope of impermeable barrier, various rates of evapotranspiration and values of reduction factor on falling water tables and drain spacing has also been studied and discussed with the help of a numerical example.

INTRODUCTION

Land and water are the two important finite natural resources, which due to unplanned and indiscriminate exploitation, are diminishing both in qualitative and quantitative terms. According to Paroda (2000), immense pressure on our land resource can be gauged from the fact that India shares only 2 per cent geographical area of the world but supports 18 per cent of the world's population and over 15 per cent of the world's livestock. The land surface of our country is estimated as 329 Mha. Nearly 57 per cent of this is facing land degradation due to water erosion, wind erosion, loss of productivity and chemical and physical degradation. It is assessed that 8.6 M ha of agricultural land is affected by the twin problems of water logging and soil salinity, about 65 % of which is the most productive irrigated land resource. The allocation of water for agriculture, which is presently about 85 % of the developed water resources, is likely to be reduced by 10-15 % to meet the growing vital demand for drinking water and industrial use. So, in order to meet the food requirements of an ever increasing population with the available land and water resources in the developing countries, concerted efforts need be made on scientific land use planning and water management for judicious utilization of these resources.

Subsurface drainage seems to be one of the feasible solutions to solve the twin problem of water logging and soil salinity. Most of the available drainage theories attempt to describe water table behaviour in horizontal aquifers only. The problem of drainage of sloping lands none the less is also found to occur in several areas of different countries. Schmid and Luthin (1964) reported such problem areas in pre-Alps in Switzerland and adjacent countries. In India, tea gardens of hilly lands in Assam and other parts of north-eastern region, the problem of steep hill side drainage has been quite commonly experienced.

Many investigators have obtained either analytical or numerical or experimental solutions of linearized or nonlinear form of continuity equation mainly given by Boussinesq (1904) to describe spatial and temporal variation of water table between two drains located on a sloping impermeable barrier. A brief review of these studies has been presented by Upadhyaya (1999), and Upadhyaya and Chauhan (2000). Most of these studies attempt to describe water table fluctuation in a sloping unconfined aquifer on account of recharge from land surface or withdrawal from aquifer but so far, no appropriate drainage design criteria seem to have been established for sloping lands.

In the present study unsteady state analytical solution of linearized Boussinesq equation was obtained to describe spatial and temporal variation of water table between two drains lying in a sloping and horizontal unconfined aquifers subjected to constant or depth dependent evapotranspiration

THEORY

Problem Definition

The definition sketch of falling water table condition between two conventional level drains lying on a sloping impermeable barrier is given in Figure 1.

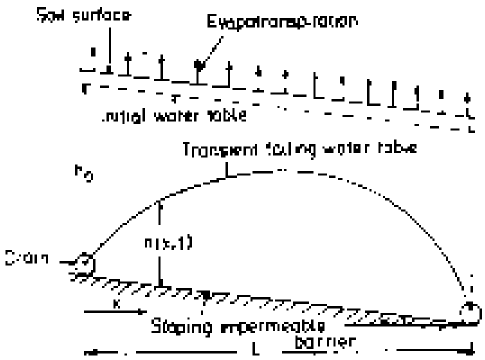

Figure 1. Definition sketch for falling water table between two drains in a sloping aquifer subjected to evapotranspiration.

In this situation, it is assumed that due to instantaneous recharge water table reaches near the land surface and it falls due to drainage or evapotranspiration from an unconfined, homogeneous, isotropic, gently sloping aquifer. It is also assumed that ground water flow in a sloping aquifer is characterized by one dimensional linearized Boussinesq equation and Dupuit-Forchheimer assumptions hold good. The flat initial shape of water table and zero water table at both the drains (neglecting the effect of seepage surface) have been considered.

The linearized form of Boussinesq equation incorporating the effect of depth dependent evapotranspiration along with appropriate initial and boundary conditions can be expressed as below:

$$\frac{\partial^2 h}{\partial x^2} - 2s\left(\frac{\partial h}{\partial t}\right) - \left[\frac{E_0 - b\ (h_0 - h)}{K D}\right] = \frac{1}{a}\frac{\partial h}{\partial t} \qquad (1)$$

$$h\ (x\ ,0) = h_0;\ \ t = 0,\ \ 0 < x < L \qquad (2a)$$

$$h\ (0\ ,t) = h\ (L\ ,t) = 0;\ \ t > 0,\ \ x = 0\ ,L \qquad (2b)$$

Here 'h' is height of water table above the sloping impermeable barrier [L] at a distance 'x' and time 't'; s = α/2D, in which, 'α' is slope of the impermeable barrier and 'D' is average depth of flow; 'E_0' is potential evapotranspiration rate at the land surface [LT^{-1}]; 'b' is reduction factor due to which evapotranspiration decreases linearly as the depth to water table increases upto a specified value [T^{-1}]; 'h_0' is initial water table height [L]; K is hydraulic conductivity of soil [LT^{-1}]; a = KD/f, in which f is drainable porosity.

ANALYTICAL SOLUTION

Analytical solution of linearized Boussinesq equation (1) considering depth dependent evapotranspiration, constant evapotranspiration and no evapotranspiration with initial and boundary conditions (2a and 2b) was obtained by devising a transformation, which converts eq (1) into a simple heat flow equation. The transformation is given as:

$$h = v\, e^{s x - s^2 a t - \frac{b t}{f}} - \frac{E_0}{b} + h_0 \qquad (3)$$

With this transformation the governing equation (1) becomes:

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{a}\frac{\partial v}{\partial t} \qquad (4)$$

and the initial and boundary conditions as:

$$v\ (x,0) = \frac{E_0}{b}\, e^{-s x} = f\ (x);\ \ t = 0,\ \ 0 < x < L \qquad (5a)$$

$$v\ (0,t) = (\frac{E_0}{b} - h_0)\, e^{s^2 a t + \frac{b t}{f}} = f_1(t);\ \ t > 0,\ \ x = 0 \qquad (5b)$$

$$v\ (L,t) = (\frac{E_0}{b} - h_0)\, e^{s^2 a t + \frac{b t}{f} - s L} = f_2(t);\ \ t > 0,\ \ x = L \qquad (5c)$$

Solution of eq (4) with general initial condition and time dependent boundary conditions is given in Ozisik (1980) as below.

$$\begin{aligned} v(x,t) = & \frac{2}{L}\sum_{m=1}^{\infty} e^{-a\beta_m^2 t}\sin\beta_m x \int_0^L f(x')\sin\beta_m x'\,d'x + (1-\frac{x}{L}) f_1(t) + \frac{x}{L} f_2(t) \\ & -\frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m}\left[f_1(0)e^{-a\beta_m^2 t} + \int_0^t e^{-a\beta_m^2(t-\tau)}\,df_1(\tau)\right] \\ & +\frac{2}{L}\sum_{m=1}^{\infty}(-1)^m\frac{\sin\beta_m x}{\beta_m}\left[f_2(0)e^{-a\beta_m^2 t} + \int_0^t e^{-a\beta_m^2(t-\tau)}\,df_2(\tau)\right] \end{aligned} \tag{6}$$

Appropriate substitution of initial and boundary conditions (5a-5c) in eq (6) and after integration and some simplifications yield:

$$\begin{aligned} v(x,t) = & \frac{2E_0}{bL}\sum_{m=1}^{\infty} e^{-a\beta_m^2 t}\sin\beta_m x\left\{\frac{\left(1-(-1)^m e^{-sL}\right)\beta_m}{\left(s^2+\beta_m^2\right)}\right\} \\ & +\left\{\left(1-\frac{x}{L}(1-e^{-sL})\right)\right\}\left\{\left(\frac{E_0}{b}-h_0\right)e^{s^2 a t+\frac{bt}{f}}\right\} - \frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m} \\ & \left[\left(\frac{E_0}{b}-h_0\right)e^{-a\beta_m^2 t}\left\{1+\left(s^2 a+\frac{b}{f}\right)\left(\frac{e^{(s^2 a+a\beta_m^2+\frac{b}{f})t}-1}{\left(s^2 a+a\beta_m^2+\frac{b}{f}\right)}\right)\right\}\right]\left[1-(-1)^m e^{-sL}\right] \end{aligned} \tag{7}$$

Substitution of eq (7) in eq (3) yields the final solution as:

$$\begin{aligned} h(x,t) = & \frac{2E_0}{bL}e^{sx-s^2at-\frac{bt}{f}}\sum_{m=1}^{\infty} e^{-a\beta_m^2 t}\sin\beta_m x\left\{\frac{\left(1-(-1)^m e^{-sL}\right)\beta_m}{\left(s^2+\beta_m^2\right)}\right\} + \left\{\left(1-\frac{x}{L}(1-e^{-sL})\right)\right\} \\ & \left\{\left(\frac{E_0}{b}-h_0\right)e^{sx}\right\} - \frac{2}{L}\sum_{m=1}^{\infty}\frac{\sin\beta_m x}{\beta_m}\left[\left(\frac{E_0}{b}-h_0\right)e^{-a\beta_m^2 t}\left\{1+\left(s^2 a+\frac{b}{f}\right)\right.\right. \\ & \left.\left.\left(\frac{e^{(s^2 a+a\beta_m^2+\frac{b}{f})t}-1}{\left(s^2 a+a\beta_m^2+\frac{b}{f}\right)}\right)\right\}\right]\left[1-(-1)^m e^{-sL}\right]\left[e^{sx-s^2at-\frac{bt}{f}}\right] - \frac{E_0}{b} + h_0 \end{aligned} \tag{8}$$

It should be possible to obtain the solution from eq (8), for the condition when constant rate of evapotranspiration is assumed to occur throughout the soil profile. But if in eq (8) the value of 'b' is substituted as zero the solution will become indeterminate and the expression for the solution can not be obtained directly. Therefore, for such a condition the solution will have to be obtained independently by considering 'b' as zero in eq (1). The transformation devised to transform such equation in the form of heat flow equation may be written as:

$$h = v\, e^{s x - s^2 a t} - \frac{E_0 t}{f} \tag{9}$$

With this transformation the initial and boundary conditions become:

$$v(x,0) = h_0\; e^{-s x} = f(x);\;\; t = 0,\;\; 0 < x < L \tag{10a}$$

$$v(0,t) = \frac{E_0 t}{f} e^{s^2 a t} = f_1(t);\;\; t > 0,\;\; x = 0 \tag{10b}$$

$$v(L,t) = \frac{E_0 t}{f}\; e^{s^2 a t - s L} = f_2(t);\;\; t > 0,\;\; x = L \tag{10c}$$

The solution to this boundary value problem with time varying boundary conditions may be obtained from the generalized solution given by eq (6) by putting the appropriate initial and boundary conditions (10a - 10c). Applying the inverse of transformation in this solution as given by eq (9), the final solution is obtained as:

$$\begin{aligned} h(x,t) = {} & \frac{2 h_0}{L} e^{s x - s^2 a t} \sum_{m=1}^{\infty} e^{-a \beta_m^2 t} \sin \beta_m x \left\{ \frac{\left(1 - (-1)^m e^{-s L}\right)\beta_m}{\left(s^2 + \beta_m^2\right)} \right\} \\ & + \left(1 - \frac{x}{L}(1 - e^{-s L})\right) \frac{E_0 t}{f} e^{s x} - \frac{2 E_0}{f L} \sum_{m=1}^{\infty} \frac{\sin \beta_m x}{\beta_m} \\ & \left[\frac{e^{s^2 a t} - e^{-a \beta_m^2 t}}{a(\beta_m^2 + s^2)} + s^2 a \left\{ \frac{t\, e^{s^2 a t}}{a(s^2 + \beta_m^2)} - \frac{(e^{s^2 a t} - e^{-a \beta_m^2 t})}{a^2 (s^2 + \beta_m^2)^2} \right\} \right] \\ & \left[1 - (-1)^m e^{-s L}\right] \left[e^{s x - s^2 a t}\right] - \frac{E_0 t}{f} \end{aligned} \tag{11}$$

Special Cases

If effect of evapotranspiration is neglected and fall of water table is due to the discharge from the conventional level drains, the solution for such a flow condition may be obtained by putting $E_0 = 0$ in eq (11) and written as:

$$h(x,t) = \frac{2 h_0}{L} e^{s x - s^2 a t} \sum_{m=1}^{\infty} e^{-a \beta_m^2 t} \sin \beta_m x \left\{ \frac{\left(1 - (-1)^m e^{-s L}\right)\beta_m}{\left(s^2 + \beta_m^2\right)} \right\} \tag{12}$$

If aquifer is assumed as horizontal, the analytical solution for falling water tables in a horizontal aquifer subjected to depth dependent evapotranspiration can be obtained by putting s = 0 in eq (8) and is written as:

$$\begin{aligned} h(x,t) = {} & \frac{4 h_0}{L} e^{-\frac{b t}{f}} \sum_{m=1,3,5}^{\infty} e^{-a \beta_m^2 t} \frac{\sin \beta_m x}{\beta_m} \\ & - \frac{4}{L} \left(\frac{E_0 - b h_0}{f} \right) e^{-\frac{b t}{f}} \sum_{m=1,3,5}^{\infty} \frac{\sin \beta_m x}{\beta_m} \left[\left\{ \frac{e^{\frac{b t}{f}} - e^{-a \beta_m^2 t}}{a \beta_m^2 + \frac{b}{f}} \right\} \right] \end{aligned} \tag{13}$$

If it is assumed that a horizontal aquifer is subjected to a constant evapotranspiration only, the solution to such situation may be obtained by putting s = 0 in eq (11) and written as:

$$h(x,t) = \frac{4h_0}{L} \sum_{m=1,3,5,...}^{\infty} e^{-a\beta_m^2 t} \frac{\sin \beta_m x}{\beta_m} - \frac{4E_0}{fL} \sum_{m=1,3,5,...}^{\infty} \frac{\sin \beta_m x}{\beta_m} \left(\frac{1 - e^{-a\beta_m^2 t}}{a\beta_m^2} \right) \quad (14)$$

If effect of evapotranspiration in a horizontal aquifer is neglected and fall of water table is due to flow of water towards drains, the solution for such a flow condition can be obtained by putting $E_0 = 0$ in eq. (14) and written as:

$$h(x,t) = \frac{4h_0}{L} \sum_{m=1,3,5,...}^{\infty} e^{-a\beta_m^2 t} \frac{\sin \beta_m x}{\beta_m} \quad (15)$$

RESULTS AND DISCUSSION

Transient falling water tables and spacings between two conventional level drains lying on a sloping/horizontal impermeable barrier were computed for depth dependent, constant and no evapotranspiration using the proposed analytical solutions obtained above with the help of a numerical example given below.

Numerical Example

Assume that two parallel level drains separated apart by a distance of 50 m are lying on an impermeable barrier having a slope of 0 %, 10 %, and 20 %. The depth to drains or impermeable barrier from the land surface is 1.8 m. Initially water table is assumed to be near the land surface due to instantaneous recharge. The hydraulic conductivity and specific yield of the soil are 3.0 m/day and 0.14, respectively. The rate of evapotranspiration is assumed as 0.008 m/day and the value of depth dependent reduction factor,'b' is 0.00667 per day. The transient fall of water tables after 1st, 2nd, 3rd, and 4th days and spacings considering unsteady state drainage criteria have been determined.

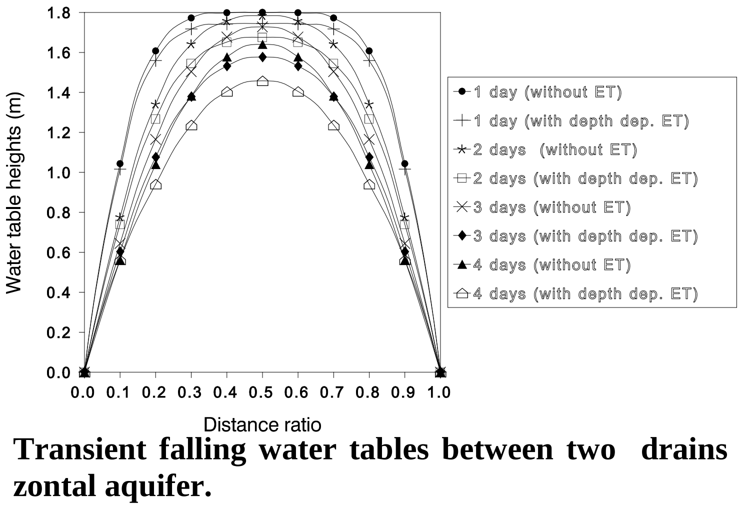

Figure 2. Transient falling water tables between two drains in a horizontal aquifer.

Falling Water Tables Between Two Drains As Computed From Analytical Solution: Analytical solution of linearized Boussinesq equation as obtained above were employed to solve the numerical example. Since depth dependent ET seems to be more realistic so transient water tables considering zero ET and depth dependent ET corresponding to 0 %, 10 %, and 20 % slopes of the impermeable barrier were determined and have been presented in Figures 1, 2 and 3, respectively.

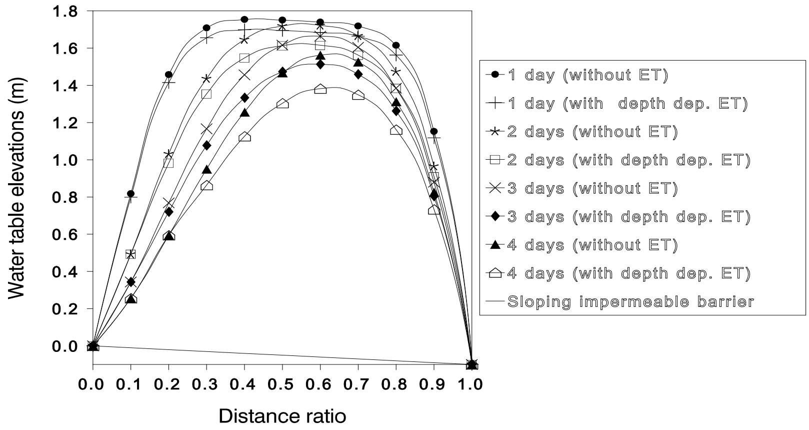

Figure 3. Falling water tables between two drains in a sloping aquifer (with 10% slope).

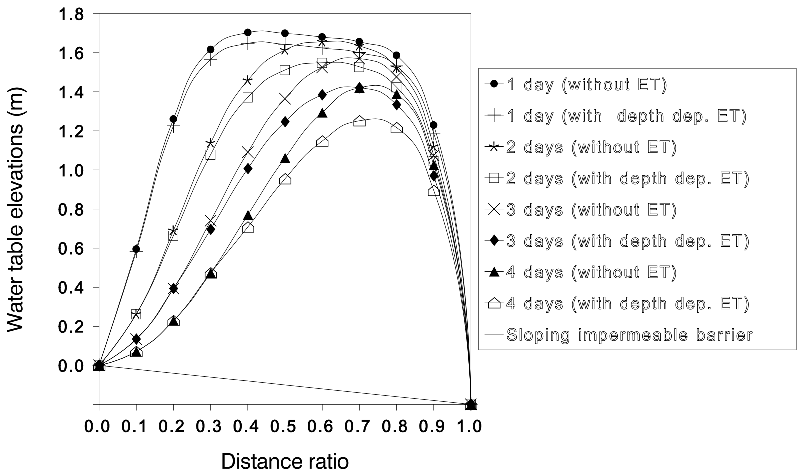

Figure 4. Falling water tables between two drains in a sloping aquifer (with 20 % slope).

It may be observed from Figures 2, 3 and 4 that with the consideration of depth dependent ET in the analytical solution of linearized Boussinesq equation, fall of water table is relatively faster as compared to the condition of zero ET. If constant ET is assumed to occur in the soil profile, the fall of water table is found to be the most rapid. Figure 2 shows that the fall of water table in a horizontal aquifer is symmetrical with maximum water table elevation occurring at the mid point. With increase in slope of impermeable barrier the position of maximum water table elevation shifts towards the downslope drain as shown in Figures 3 and 4 for 10 % and 20 % slope, respectively. The difference in maximum water table elevations due to consideration of zero ET and depth dependent ET increases with increase in time.

Spacing Between Two Drains Computed By Analytical Solution: Spacing between two drains was computed employing the assumed drainage criteria of a drop of maximum height of water table by 30 cm in 2 days from an initially flat water table near the soil surface in the implicit analytical solution for the numerical example mentioned above.

Effect of ET on spacing between two drains: The spacing between two drains was computed using analytical solution, considering zero ET, constant ET @ 0.008 m/day, and linearly decreasing ET with increase in depth to water tables for the slopes of 0 %, 10 %, 20%, 30 % and 40 %. Results are presented in Table1.

It may be observed from Table 1 that spacing between two parallel drains increases with increase in slope. The maximum increase of 17.49 % in spacing at 40% slope with respect to zero slope was observed when ET was considered as zero in the solution. Spacing between two drains also increases if effect of constant or depth dependent ET is included. The increase in spacing for the case of drains lying on a horizontal impermeable barrier as compared to zero ET was found to be 9.47 % when depth dependent ET was considered and 12.03 % when constant ET was considered whereas for 40 % slope the increase in spacing was found to be 7.11 % and 8.53 % when depth dependent ET and constant ET, respectively were taken into account.

Table 1. Computation of spacing (m) between two drains located at the horizontal/sloping impermeable barrier using transient analytical solution.

H_0 = 1.8 m, H_r = 1.5 m, K = 3.0 m/d, f = 0.14, E_0 = 0.008m/d, b = 0.00667, t = 2 days

Condition	Slope of the impermeable barrier (%)				
	0	10	20	30	40
Spacing without ET	30.42	30.76	31.74	33.89	35.74
Spacing increase with slope (%)		(1.12)	(4.34)	(11.41)	(17.49)
Spacing with constant ET Spacing increase with slope (%)	34.08	34.38	35.25	37.41	38.79
Spacing increase with constant ET compared to no ET (%)	(12.03)	(0.88) (11.77)	(3.43) (11.06)	(9.77) (10.39)	(13.82) (8.53)
Spacing with depth dependent ET	33.30	33.60	34.57	36.91	38.28
Spacing increase with slope (%)		(0.90)	(3.81)	(10.84)	(14.95)
Spacing increase with depth dependent ET compared to no ET	(9.47)	(9.23)	(8.92)	(8.91)	(7.11)

It may also be noted from Table 1 that increase in spacing becomes more with increase in slope whereas increase in spacing due to consideration of constant and depth dependent ET decreases with increase in slope.

Effect of variation of rates of ET on spacing between two drains : The effect of various rates of ET and slopes on spacing between two level drains lying on the horizontal / sloping impermeable barrier was studied and is presented in Table 2.

It may be observed from Table 2 that for the rate of ET varying from 0.004 m/day to 0.008 m/day spacing between two drains increases from 2.46 % to 9.47 % as compared to the case of zero ET for the slopes of impermeable barrier varying from 0 to 40 %. The increase in spacing for various slopes of impermeable barrier varies in the range of 1.12 % to 17.49 %.

Effect of parameter 'b' on spacing between two drains: Effect of depth dependent reduction factor 'b' on spacing between two drains lying on the horizontal / sloping impermeable barrier was also studied. The results show that with increase in the value of b from 0.0055 to 0.0088 per day the spacing between two drains decreases in the range of 1.06 % to 2.85 % as compared to the 0 value of b. The increase in spacing varies in the range of 0.88% to 15.16 % with increase in slope of impermeable barrier from 0 to 40%.

Table 2. Effect of various rates of E_0 on computed spacing (m) between two parallel drains located at the horizontal/sloping impermeable barrier using transient analytical solution.

H_0 = 1.8 m, H_r = 1.5 m, K = 3.0 m/d, f = 0.14, b = 0.00667, t = 2 days

Values of E_0 (m/d)	Slope of the impermeable barrier (%)				
	0	10	20	30	40
Spacing with E_0 = 0.0 Spacing increase with slope (%)	30.42	30.76 (1.12)	31.74 (4.34)	33.89 (11.41)	35.74 (17.49)
Spacing with E_0 = 0.004 Spacing increase with slope (%) Spacing increase with E_0 =0.004 compared to E_0 =0.00 (%)	31.45 (3.39)	31.74 (0.92) (3.19)	32.71 (4.01) (3.06)	34.91 (11.00) (3.01)	36.62 (16.44) (2.46)
Spacing with E_0 = 0.006 Spacing increase with slope (%) Spacing increase with E_0 =0.006 compared to E_0 =0.00 (%)	32.32 (6.25)	32.61 (0.90) (6.01)	33.59 (3.93) (5.83)	35.84 (10.89) (5.75)	37.40 (15.72) (4.64)
Spacing with E_0 = 0.008 Spacing increase with slope (%) Spacing increase with E_0 = 0.008 compared to E_0 =0.00 (%)	33.30 (9.47)	33.60 (0.89) (9.23)	34.57 (3.81) (8.92)	36.91 (10.84) (8.91)	38.28 (14.95) (7.11)

CONCLUSIONS

An analytical solution to the linearized Boussinesq equation for the design of subsurface drainage system in sloping lands in the presence of ET was developed. Variation in falling water tables as influenced by depth dependent ET and slope of the impermeable barrier were computed using the analytical solution. The effect of ET, reduction factor 'b', and slope of impermeable barrier on drain spacing was also studied

The following discrete conclusions may be drawn from the study.

While designing the drain spacing, if evapotranspiration is taken into account the water table falls faster leading to an increase in drain spacing by 7 to 12 % as compared to the condition with no evapotranspiration.

Consideration of depth dependent evapotranspiration depicts the real situation more closely and thus seems to be more realistic condition as compared to a constant ET one. Drain spacing increases with increase in slope of the impermeable barrier as well as with increase in the rate of evapotranspiration.

For the numerical example under study, the drain spacing decreases from 1.06 % to 2.85 % if value of reduction factor 'b' is increased from 0.0055 to 0.0088 per day.

The position of maximum water table height tends to shift towards lower drain with increase in slope of the impermeable barrier.

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