UM-10

MUSKINGUM CUNGE ROUTING PROCEDURE

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ABSTRACT

Experience and observations of flood flows in river show that the flood peak attenuates as it travels downstream. Although lateral flow and flow from tributaries may add up to the flow and lead to some increase in the peak. The reduction in peak is caused by irregularities in the width of the cross section of the flow and storage effect of the reach. Methods quantifying this attenuation have been tried in the past. Cunge evolved a method for estimation of these. He started with the well known Muskingum method which works on wedge and prism storage concept. The Muskingum method uses linear storage equation which is another form of stage discharge relationship. Cunge observed that the above equation will not allow the attenuation, (reduction in peak), but Muskingum method produces attenuation. Cunge explained that finite difference formulation of continuity equation, in the case of Muskingum method, produces truncation errors when compared with Taylor series expansion.

Cunge derived equations to find the routing parameter 'x' of Muskingum method in terms of average slope, and width. R.K.Price, improved upon this work. He incorporated variations in the width and slope instead of using average, since these are the features that produce attenuation.

The user manual describes the stepwise computations involved and the computer programme

The input specifications and output descriptions are explained with example. An example application to a flood observed in 1978, at Mortakka on river Narmada, is also included.

1.0 INTRODUCTION

Experience and observation of flood flows in a river indicated the natural ability to attenuate the flood peak. Hayami, (1951) observed that it is actually the irregularities in the width of the river at the water surface, that produce the attenuation. These irregularities of the sub-reaches regulate the flow of flood, acting as a series of reservoirs. Since then researchers have made many attempts in quantifying the attenuation. They either went on to a trial and error procedure of accounting the variation of width of the river by some means. Cunge started his anslysis from the unexpected attenuation caused by use of Muskingum method. He used diffusion equation to account the attenuation and an average slope and width, Price, (NERC,1975) introduced further modifications. He included the variation of widths and slope by an appropriate technique rather than just using an average width and slope. The computer model described in this user's manual adopts Price's work and describes stepwise computations involved with examples and applications.

1.1 Purpose and Capabilities

The manual introduces a simple computer programme capable of routing a flood. The programme is capable of plotting the results. The methodology used and the computations are explained stepwise. Muskingum Cunge method predicts the attenuation accurately and is physically based.

1.2 Terminology

Attenuation	:	Reduction in flood peak, as it moves downstream.
Muskingum	:	A method of flood routing, proposed by McCarthy (1938).
Numerical-Diffusion	:	This is an error, caused by the truncation of higher order terms in finite difference approximation.
Diffusion of the wave	:	This is the quantity expressed by the term $(\partial^2 Q / \partial x^2)$ in a diffusion equation.
Finite Difference	:	This is an approximation of a differential coefficient with discrete values.

1.3 Scope

When the flood water level just rises above the bank, the water starts spreading in the adjoining flood plains. Since the widths of the river at this level are irregular the flood waters are retained temporarily in the sub-reaches and the river reaches act as series of reservoirs. Muskingum-Cunge method takes these into account. In order to quantify the effect of these irregularities, the method uses the information regarding plan form of the river with markings of the extent of inundation of flood plains for various floods. These markings are used to divide the river into sub-reaches, where the width can be assumed as uniform. Each one of these sub-reaches is considered to act as a reservoir. Hence for accurate results with application of this method to route different (i.e. larger or smaller) floods, the inundation details of larger and smaller floods are needed. For floods higher than those used in the above analysis, extrapolations are necessary. However, the methodology requires only somewhat approximate markings of the extent of inundation. When these details could not be obtained, this method could not be applied.

1.4 Hardware and Software Requirements

The programme has been used in DEC 2050 and VAX-11/780 But this programme can be used on other computers also where FORTRAN compilers are available.

The memory and the computer time requirements depends upon data being used in specific problems.

2.0 SPECIFIC METHOD

2.1 General Description

Continuity equation in its lumped form and simplified momentum equation (rating curve/storage equation) are the basis of development of many hydrological flood routing methods. The Muskingum method is also one such method. Commenting on the use of rating curve in Muskingum method Cunge showed that Muskingum method should not produce attenuation, if exactly solved. However, the actual observation is on the contrary and this attenuation was explained by Cunge to be due to the numerical approximation due to truncation of higher order terms. This is termed as 'diffusion'. Instead of approximating the continuity equation of the following form:

$$\frac{\partial Q}{\partial t} + C \frac{\partial Q}{\partial s} = 0 \qquad \ldots(1)$$

the Muskingum method actually approximates

$$\frac{\partial Q}{\partial t} + C \frac{\partial Q}{\partial s} = K \frac{\partial^2 Q}{\partial s^2} \qquad \ldots(2)$$

where, Q is the discharge m^3/sec, C is the celerity($\frac{\partial Q}{\partial A}$) (m/sec.), s is the space coordinate, K is diffusion coefficient (m^2/sec.), $K = \frac{\alpha Q}{L}$, L is length of the river reach and α is attenuation parameters.

It is worth noting that Hayami (1951) produced a flood routing method based on the equation(2). He used a trial and error procedure to evaluate K. But the essential contribution of Hayami in flood routing technology is his observation that flood in natural river is attenuated by the irregularities in the channel width as stated in section 1.1. After certain mathematical reduction the following relationships were arrived at by R.K.Price. (NERC,1975):

$$\alpha = \frac{1}{2} \left[\frac{1}{L} \sum_{m=1}^{M} \frac{P_m}{S_m^{1/3}} \right]^{-3} \sum_{m=1}^{M} \left(\frac{P_m}{L_m S_m^2} \right) \quad \ldots(3)$$

where, P_m is plan area of inundated river reach of length L_m and S_m is bottom slope. The subscript refer to the reach number and L is the total length. Practical and most convenient way of estimating α is given in section 2.3.

The routing of flow is accomplished by the following recurrence formula :

$$Q_o^{n+1} = C_1 Q_1^n + C_2 Q_1^{n+1} + C_3 Q_o^n \quad \ldots(4)$$

where,

$C_1 = (kx + Dt/2)/ (k(1-x)+Dt/2)$

$C_2 = (Dt/2)- kx)/(k(1-x) + Dt/2)$

$C_3 = (k (1-x)-Dt/2)/(k(1-x) +Dt/2)$

k,x are routing parameters

The equation(4) is very similar to Muskingum method. The difference in between the method discussed and the Muskingum methos is the procedure of obtaining k, and x.

The parameter x is related to the α in the following manner:

$$x = 0.5 - (\alpha Q_p)/L^2C$$

where, Q_p - the peak of the flood (m^3/sec.), C wave speed (m/sec) and L the reach length (m)

The parameter, $K = L/C$ ...(6)

In case of long river reach, for accurate results, the reaches are to be divided into sub-reaches. If the sub-divided reaches are used with sub reach length Δs the following equations are to be used in place of (5) and (6).

$$x = 0.5 - (\alpha Q_p)/(\Delta s)^2 C \qquad \ldots(7)$$

$$k = \Delta s/C \qquad \ldots(8)$$

2.2 Data Requirement

1. Topographic map covering the entire river reach.
2. Information regarding few high flood records, as follows:
   a. An inundation map prepared for observed floods would be most preferable data. If this is not available peak discharges and corresponding water levels are required.
   b. Travel time observations or rating curves at gauging sites situated within the reach.
   c. Upstream hydrographs.
   d. Downstream hydrographs for calibration and testing.

2.3 Analysis

Practical computation starts with topographic maps. The following steps may be followed:

1. The contours occupied by the floods are marked on to the topographic maps, using flood peak elevations. (This amounts to sketching of wetted area). This produces the required inundation map.
2. The river reach is sub-divided into M sub-reaches of uniform width.
3. The length L_m and width W_m of the sub-reaches are measured from the toposheets. ($m = 1,2,......M$).
4. The plan area P_m is calculated as

$$P_m = L_m * W_m \qquad ...(9)$$

5. The attenuation parameter α for different peak discharges are calculated using equation 3 for the full reach or for the divided reaches as per section 2.1.
6. A plot of α Vs peak discharges is made. This can be used to find α for any intermediate value of discharge.
7. a) If travel time (T_p) observation are available the celerity of the flood wave C can be found by

$$C = L/T_p \qquad ...(10)$$

(Hayami (1951) introduced a refinement to the above equation, but this has not been included in this manual).

b) In case if rating curves are available the following procedure can be adopted. The slope ($\partial Q/ \partial h$) of the rating curve of the upsteam gauging site is found for various discharges.

The celerity C is calculated as per the following relationship.

$$C = \frac{1}{W} \frac{\partial Q}{\partial h} \qquad \ldots(11)$$

where,

h is the stage (m),

Q is the discharge (m^3/sec) and

W is the top width of the cross-section of flow of discharge, Q.

8. The time step Dt may be taken equal to one fifth of the time peak. This is rounded to the nearest hour and should not be greater than 3 hours.

9. The parameters K, and x are calculated as per equations (5) - (8).

10. The parameters C,Dt,Ds or L are the essential input to the programme.

2.4 Advantages and Limitations

Although theoretical basis for arriving at the expression for attenuation parameter α is complicated, the practical application is simple. The river geometry is meaningfully included in the derivation of the parameter x. The attenuation parameter α could be calculated for sub-reaches which in turn can be used to find out the parameter x of each sub reach. This possibility is of more advantageous in routing a flood in long rivers. The programme given in this manual does not include effects of flows from tributaries in routing of flood.

2.5 Programme Details

The programme capable of routing using Muskingum Method is given in Appendix-I. A programme can also be found in (NERC,1975). For the mere application of Muskingum cunge method a much simpler programme was developed in the National Institute of Hydrology. Certain salient features of these programmes are given in the following table:

2.5.1

Table 1

COMPARISON OF PROGRAM GIVEN BY (NERC,1975) AND THAT DEVELOPED IN NIH

No.	NERC 1975 FLOOD STUDIES REPORT VOL.III	THE PRESENT PROGRAM
1.	Routes the flow using $Q_{j+1}^{n+1} = C_1Q_j^n+C_2Q_j^{n+1} + C_3Q_{j+1}^n + C_4$	The same
2.	Alpha is an essential input	optional, if opted computes from the additional data provided.
3.	Takes lateral flow as a function of time and tributary flow as input hydrograph	Finds, the difference between observed inflow and outflow hydrograph; Distributes the difference according to the respective ordinates optionally either to(1) inflow (2) routed (3) or both.
4.	The routed and observed flows are printed at desired locations of the river reach and at desired time interval	Only inflow and the routed and observed flows are printed along with plots
5.	No plots are made	Plots inflow and routed flow; Plots observed and routed flow
6.	Routing intervals can be anything. For interval other than of input hydrograph linear interpolation is done	Routing interval should be the same as input hydrograph.
7.	The input specification are FORMATED	FREE FORMAT is used.

2.5.2 Input specification

Card 1

TITLE	- The heading to be written (60 characters)

Card 2

Jx	- Number of nodes greater than or equal to 2
LEND	- Number of time steps
IQH	- Number of ordinates of the upstream hydrographs
IQDNS	- Number of ordinates of the hydrographs downstream

Card 3

DxLR	- Length of the reach (m)
DT	- Time step (sec)
DTHYD	- Upstream hydrograph time interval(Sec.)
QINIT	- Initial discharge (m^3/sec.)
WSP	- Wave Speed (m/sec)
ALPHA	- Attenuation parameter
QCON	- discharge at a base line (m^3/sec)

Card 4 to M

QHYDRO	- Upstream hydrograph in Free format

Card M+1 to N

QDNS	- downstream hydrograph

Card N+1

PART	- Ratio (0 to 1) accounting lateral flow.

If ALPHA is 0 give additional data as follows:

Card N+2

NRE	- Number of sub reaches for which widths are given.

Card N+3 to N+NRE+3

DL,PM	- Length of sub reach and plan area of the same.
SLO	- The slope of respective reaches.
T1	- Name of upstream site (20A1)
T2	- Name of downstream site (20A1)

2.5.3 Output description

The programme prints

i) the length of the river reach the speed of the flood wave.

ii) the attenuation parameter the flood discharge,

iii) the number of nodes (number of sub-reaches and

iv) the valves of parameters C_1,C_2 and C_3(as per equation 4).

Two plots are made as follow:

1. Upstream, routed hydrographs
2. Observed and computed hydrographs

The discrete values of the discharges were also written by the side of the plots.

2.7 Output

The programme prints the following in sequence:

The length of the river reach	....	295.50 KM
The Speed of the flood wave	....	5.50 m/s
The attenuation parameter	....	018872.94
The lateral flow accounted upstream	....	0.0000
The number of nodes	....	5
The parameters C_1, C_2, C_3	....	0.5942, 0.1163, 0.2895

MUSKINGUM CUNGE METHOD OF FLOOD ROUTING. PLOT SHOWS OBSERVED HYDROGRAPH (o) AT MORTAKKA AND ROUTED HYDROGRAPH (*)AT GARUDESHWAR.

ROUTED	OBSERVED
3612.51	2739.45
12362.71	9691.14
23762.16	18956.43
13958.60	12283.66
30377.32	31174.96
21086.55	23287.07
11882.75	16781.90
7515.44	7443.85
5375.78	6127.56

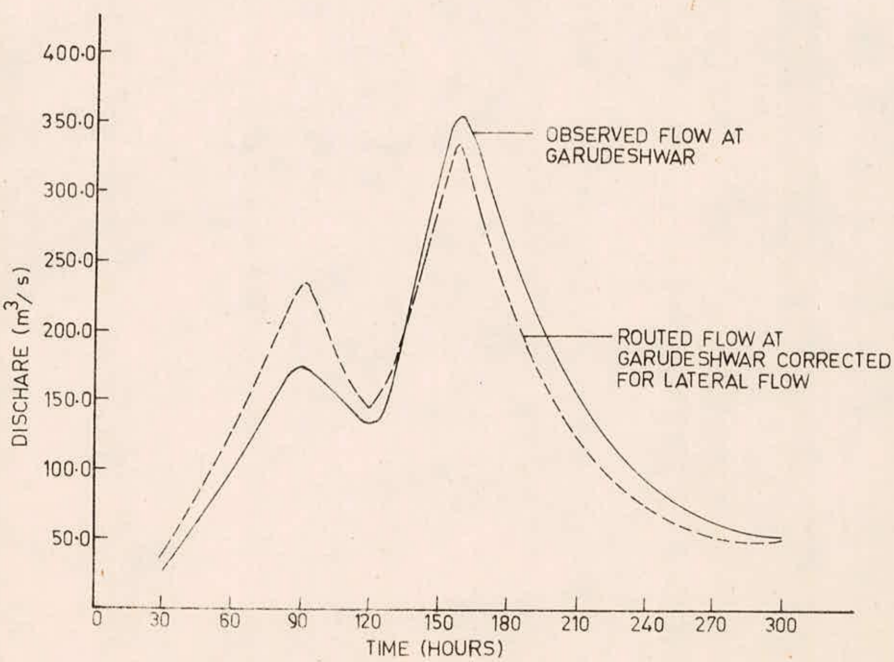

FIGURE 1 - OBSERVED AND CALCULATED HYDROGRAPH AT GARUDESHWAR

ROUTED FLOW AT GARUDESHWAR	FLOW AT MARTAKKA
3043.96	3515.17
10351.40	14397.52
19836.03	17306.58
11401.24	9835.52
23907.23	29368.80
16253.58	12429.67
8399.81	7171.36
5555.44	4953.98
4104.06	3607.14

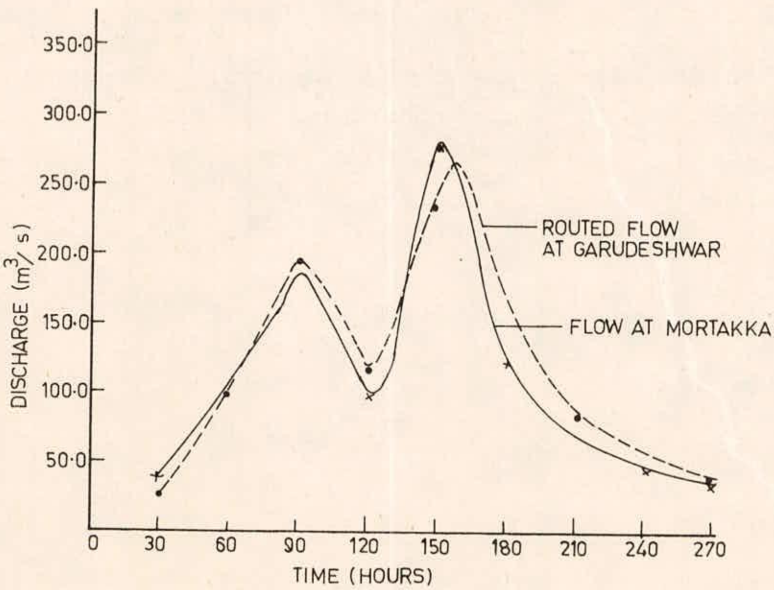

FIGURE 2 -- UPSTREAM AND ROUTED DOWNSTREAM HYDROGRAPHS

3.0 RECOMMENDATIONS

It has been well established that Muskingum-Cunge method of flood routing predicts attenuation accurately. The necessary data include high flood marks on the plan form of a river. The following are recommended for better use of this method:

i) Attenuation parameter α are to be evaluated for different flood peak records observed at the upstream site.

ii) A plot α Vs peak discharges is to be evaluated and used for finding α for any peak discharge to be routed.

iii) Similarly a plot of wave speed Vs discharge could be prepared from flood records and could be used.

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```
C   Programme  for routing  a flood  using Muskingum-Cunge method
    REAL Q1(500),Q2(500),X(500)
    DIMENSION TIME(500),QEST(500),QUP(500),QDNP(500),T1(20),T2(20)
    COMMON DX,DT,JX,DXLR,LEND,TH,J1,L1,L2,TITLE(60),JT1,JT2
    COMMON QTRIB1(500),QTRIB2(500),QHYDRO(500),DTHYD,QINIT
    COMMON QINB,QINA,TQIN,TSCQIN,ALPHA,WSP,QCON,QDNS(500)
    COMMON IQH,IQDNS
    OPEN(UNIT=3,FILE='MU1.DAT',STATUS='OLD')
    OPEN(UNIT=32,FILE='MMC.DAT',STATUS='NEW')
    OPEN(UNIT=33,FILE='MUSRT.DAT',STATUS='NEW')
    CALL DATIN
    READ(3,*) PART
    DX=DXLR/FLOAT(JX-1)
    DK=DX/WSP
    IF(ALPHA.EQ.0.0) CALL ALFHA(ALPHA)
    EPSILO=.5*(1.-QCON*ALPHA*2./(DX*WSP*DXLR))
    CC=DK*(1.-EPSILO)+DT*.5
    C1=(DK*EPSILO+DT*.5)/CC
    C2=(DT*.5-DK*EPSILO)/CC
    C3=(DK*(1.-EPSILO)-DT*.5)/CC
    C4=DT*DX/CC
    DLE=DXLR/1000.
    WRITE(32,305) DLE,WSP,ALPHA,QCON,PART,JX,C1,C2,C3
    JXM1=JX-1
    T=0.0
    TOTB=0.0
    QP1=QINIT
    QP2=QINIT
    QDNP2=QINIT
    QDNP1=QINIT
    QDN=QINIT
    QTOT=0.0
    QQQ=0.0
    QQ1=0.0
    DO 145 I=1,LEND
    QQQ=QQQ+QDNS(I)
145 QQ1=QQ1+QHYDRO(I)
    DIFF=QQQ-QQ1
    DO 127 I=1,LEND
127 QHYDRO(I)=QHYDRO(I)*(1+DIFF*PART/QQ1)
    DO 2 J=1,JX
    X(J)=DX*FLOAT(J-1)
    Q1(J) =QINIT
    Q2(J)=QINIT
2   CONTINUE
    DO 14 L=1,LEND
    TIME(L)=L
    T=T+DT
    TH=T/3600.
    QP2=QP1
```

```
      QP1=Q1(JX)
      QDNP2=QDNP1
      QDNP1=QDN
      QDN=QDNS(L)
      Q1(1)=QHYDRO(L)
      DO 4 J=1,JX
      Q2(J)=Q1(J)
 4    CONTINUE
      QTOT=0.0
      DO 6 J=2,JX
      Q1(J)=C1*Q2(J-1)+C2*Q1(J-1)+C3*Q2(J)+C4*QTOT
 6    CONTINUE
      QEST(L)=Q1(JX)
      QDNP(L)=QDN
      QUP(L)=Q1(1)
 14   CONTINUE
 305  FORMAT (12X,'The length of the river reach........'F10.2,'KM'/
     112X,       'The speed of the flood wave..........'F10.2,'m/s'/
     212x,       'The attenuation parameter............'F15.2,/
     412X,       'The average discharge................'F8.2/
     512X,       'The lateral flow accounted upstrem...'F8.4/
     612X,       'The number of nodes..................'I8/
     312X,       'The parameters C1,C2,C3..............'3F7.4/)
      READ(3,308)(T1(I),I=1,20)
 308  FORMAT(1H,(20A1))
      READ(3,308)(T2(I),I=1,20)
      WRITE(32,186) T1,T2
 186  FORMAT(10X,'MUSKINGUM CUNGE METHOD OF FLOOD ROUTING'/
     110X,       'PLOT SHOWS OBSERVED HYDROGRPH (0) 'AT',1X,20A1,/
     210X,       'AND    ROUTED   HYDROGRAPH (*) AT     ',1X,20A1)
      CALL PLOT (QEST,TIME,LEND,QUP,3,2)
      DO 128 I=1,LEND
 128  QEST(I)=QEST(I)+ DIFF*(1-PART)*QDNS(I)/QQQ
      CALL PLOT (QEST,TIME,LEND,QDNP,3,2)
      CLOSE(UNIT=3)
      CLOSE(UNIT=32)
      CLOSE(UNIT=33)
      STOP
      END
      SUBROUTINE DATIN
      COMMON DX,DT,JX,DXLR,LEND,TH,J1,L1,L2,TITLE(60),
     1JT1,JT2,QTRIB1(500),QTRIB2(500),QHYDRO(500),DTHYD,
     2QINIT,QINB,QINA,TQIN,TSCQIN,ALPHA,WSP,QCON,
     3QDNS(200)
      COMMON IQH,IQDNS
      COMMON PART
      READ(3,300)( TITLE(I),I=1,60)
      READ(3,*) JX,LEND,IQH,IQDNS
      READ(3,*)DXLR,DT,DTHYD,QINIT,WSP,ALPHA,QCON
      WRITE(33,355) (TITLE(I),I=1,60)
```

```
355 FORMAT(20X,60A1)
300 FORMAT(1H,(60A1))
    WRITE(33,301) JX,LEND,IQH,IQDNS
301 FORMAT(20X,11I8)
    WRITE(33,*  ) DXLR,DT,DTHYD,QINIT,WSP,ALPHA,QCON,DT
302 FORMAT(20X,F9.2,F7.1,F4.1,F6.2,F8.6,E9.3,2F6.1,F5.2,E9.3,F7.2,F6.2)
    READ(3,*) (QHYDRO(I),I=1,IQH)
    WRITE(22,446) (QHYDRO(I),I=1,IQH)
446 FORMAT(1X,8F10.2)
    READ(3,*) (QDNS(I),I=1,IQDNS)
    WRITE(33,306)
306 FORMAT(1H,18X,4HTIME,10X,8HDISTANCE,6X,10HCALCULATED,6X,
    19HPROTOTYPE/1H ,17X,5H(HRS),13X,3H(M),9X,9HDISCHARGE,6X,
    29HDISCHARGE/)
    RETURN
    END
C
    SUBROUTINE PLOT (CUSEC,TIME,N,QEST,L1,L2)
    DIMENSION TIME(500),CUSEC(500),A(101),QEST(500)
    DATA CHAR,GRID,BLANK,ZERO/1H*,1H+,1H ,1H0/
    QMAX=QEST(1)
    CUMAX=CUSEC(1)
    DO 10 I=1,N
    IF(CUMAX-CUSEC(I))11,9,9
11  CUMAX=CUSEC(I)
9   IF(QMAX-QEST(I))8,10,10
8   QMAX=QEST(I)
10  CONTINUE
    CUMAX=AMAX1(CUMAX,QMAX)
    SF=CUMAX/80.0
    DO 12 I=1,81
12  A(I)=GRID
    WRITE(32,13)(J,J=1,81,10)
13  FORMAT(/2X,'TIME',1X,'LINE',I2,8(6X,I4),4X,' *  ',' 0'/)
    NA=CUSEC(1)/SF+1.5
    NB=QEST(1)/SF+1.5
    I=1
    A(NA)=CHAR
    A(NB)=ZERO
    WRITE(32,14)TIME(I),(A(J),J=1,81),CUSEC(I),QEST(I)
14  FORMAT(1X,F6.1,5X,81A1,F9.3,F9.2)
    DO 15 I=1,81
15  A(I)=BLANK
    K=1
    KC=7
    DO 20 I=L1,N,L2
    KN=TIME(I)-TIME(I-1)
    AA=CUSEC(I)/SF+1.5
    NB=QEST(I)/SF+1.5
    NA=AA
```

```
22  DO 21 I1=1,81,10
21  A(I1)=GRID
    IF(KC-KN-K)24,25,26
24  KP=KC-K
    IF(KP-1)30,30,34
34  DO 31 J=2,KP
31  WRITE(32,91)(A(JJ),JJ=1,81)
91  FORMAT(12X,81A1)
30  DO 32 J=1,81
32  A(J)=GRID
    WRITE(32,92)KC,(A(JJ),JJ=1,81)
92  FORMAT(8X,I4,81A1)
    DO 33 J=1,81
33  A(J)=BLANK
    KN=KN-KC+K
    K=KC
    KC=KC+6
    GO TO 22
25  IF(KN-1) 40,40,41
41  DO 42 J=2,KN
42  WRITE(32,91)(A(J1),J1=1,81)
40  DO,43 J=1,81
43  A(J)=GRID
    A(NA)=CHAR
    A(NB)=ZERO
    WRITE(32,14)TIME(I),(A(J),J=1,81),CUSEC(I),QEST(I)
    K=KC
    KC=KC+6
    GO TO 27
26  IF(KN-1)50,50,51
51  DO 52 J=2,KN
52  WRITE(32,91)(A(JJ),JJ=1,81)
50  A(NA)=CHAR
    A(NB)=ZERO
    K=K+KN
    WRITE(32,14)TIME(I),(A(J),J=1,81),CUSEC(I),QEST(I)
27  DO 28 J=1,81
28  A(J)=BLANK
20  CONTINUE
    KP=KC-K
    IF(KP-1) 60,60,61
61  DO 62 J=1,81,10
62  A(J)=GRID
    DO 63 J=2,KP
63  WRITE(32,91)(A(JJ),JJ=1,81)
60  DO 64 J=1,81
64  A(J)=GRID
    WRITE(32,91)(A(JJ),JJ=1,81)
    WRITE(32,13)(J,J=1,81,10)
    WRITE(32,95)SF
```

```
95  FORMAT(1X,'THIS GRAPH HAS A SENSITIVITY OF',E10.3,'UNITS/SPAC
    1ING IN THE HORIZONTAL AXIS')
    WRITE(32,96)KC
96  FORMAT(1X,'SENSITIVITY IN THE VERTICAL AXIS IS 1.00 UNITS/LINE
    1'//1X,'TOTAL NUMBER OF LINES IS ',I4,'IN THIS PLOT')
    RETURN
    END
    SUBROUTINE ALFHA(ALPHA)
    DIMENSION DL(200),PM(200),SLO(200)
    READ(3,*) NRE
    READ(3,*) (DL(I),PM(I),I=1,NRE)
    READ(3,*) (SLO(I),I=1,NRE)
    PSM=0.0
    PLS=0.0
    DLE=0.0
    DO 100 I=1,NRE
    PSM=PSM + PM(I)/(SLO(I)**.3333)
    DLE=DLE+DL(I)
100 PLS=PLS+PM(I)*PM(I)/(DL(I)*SLO(I)*SLO(I))
    ALPHA=(0.5*(DLE/PSM)**3)*PLS
    RETURN
    END
```