UM-9

MODEL PARAMETER EVALUATION USING CATCHMENT CHARACTERISTICS

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ABSTRACT

In view of growing development of water resources in many projects, it becomes necessary to estimate discharge values for ungauged watersheds or sites within a region. The simplest way is to transfer the information from catchments for which data are available to nearby ungauged catchments within a given region having similar hydrological characteristics. If loss rate parameters and unit hydrograph parameters of a rainfall-runoff model such as Nash Model are related to the catchment characteristics, then the relationships thus obtained can be used to estimate discharges for ungauged catchments.

In this User's manual the following form of non-linear equations have been adopted between Nash Model parameters and catchment characteristics.

$$NK = a\,(LLc/\sqrt{S})^{b}\ ,\ K = C\,(L)^{d},$$

where,

N & K are the average parameters of the Nash Model for each gauged catchment, L is the length of the main channel, Lc is the length from the outflow point to a point on the main stream, a,b,c and d are the unknown coefficients.

The unknown coefficients a,b,c, and d in the above equations are obtained using multiple-linear regression

programme MULTI.FOR after transforming the above equations into the linear form through logarithmic transformation. Knowing the coefficients a,b,c and d the parameters of Nash Model for ungauged catchments can be obtained after solving the above two equations. The computer programme MULTI.FOR takes the values of NK as a dependent variable and $(LLc/\sqrt{S})$ as an independent variable for first equation and the values of K as dependent variable and L as an independent variable for second equation. The programme gives the regression coefficients, the standard error of estimate, standard error of regression coefficients, correlation coefficients, t-statistics and F-statistic etc. as the output. The computer programme MULTI.FOR has been implemented and tested on VAX-11/780 system. Input and output specifications for the programme have been described. The programme could be run on other computer system,having FORTRAN compiler, after making suitable modifications in the programme.

LIST OF TABLES

1.0 INTRODUCTION

The accurate estimation of design flood hydrograph and peak discharge is a major problem to the water resources engineers and especially to the hydrologists. The unit hydrograph technique is being widely used as a practical tool by most of the water resources engineers and scientists of different countries for the design flood estimation. Majority of the hydraulic structures are being constructed on small watersheds, where the information about the discharge is either insufficient or not at all available which makes the estimation of the representative unit hydrographs for those watersheds difficult. For such situations some indirect approach must be used synthesize the values of peak flows and hydrograph of runoff. The simplest way is to develop the appropriate relationship,between the unit hydrographs of gauged watersheds for the specific region and their physical characteristics, which could be used to determine the representative unit hydrographs for ungauged watersheds of the region using information about physical characteristics of such watersheds.

If the unit hydrograph is used to develop the relationship between the catchment response and the physical characteristics of the watershed, it is necessary to restrict the UH for the catchment to a smooth curve and to describe it completely by a limited number of parameters. This implies choosing a fixed form or equation for the IUH and studying the relationship between the parameters of the model and watershed characteristics usually by some form of regression analysis.

1.1 Purpose and Capabilities

The purpose of this user's manual is to demonstrate the main steps involved in developing a relationship between the parameters of a model for gauged watersheds of a specific region and their physical characteristics. The application of the relationship for the evaluation of the model parameters for ungauged watersheds of the region using their catchment characteristics is also shown. The parameters of Nash Model for five small watersheds are estimated using the programme CONTIM.FOR which has been discussed in another user's manual. The parameters, thus obtained, could be related with the physical characteristics of the watersheds using the programme MULTI.FOR as described in this report.

1.2 Definitions and Description of Physical Parameters of the Watershed(Catchment characteristics)

Some of the important physical characteristics and storm characteristics which are generally considered in such studies are as follows:

i. Catchment Area (Ca) - It is given in km^2.

ii. (a) Length of the main stream (L) - It is a length along the longest water course from the outflow point of the watershed to the upper limit of the watershed boundary.

(B) L_C is water course length from the outflow point to point on the main stream nearest to the centroid of the watershed.

iii. Equivalent stream slope(s) - It is an equivalent stream slope estimated using the equation.

$$\frac{\sum_{i=1}^{n} l_i}{\sqrt{S}} = \sum_{i=1}^{n} \frac{l_i}{\sqrt{S}} \qquad \ldots(1)$$

where,

S is the equivalent stream slope (mm/mm)

n is the no. of segments considered along the main channel between the two contours cutting the main stream

l_i is the length of ith segment

S_i is the slope of the ith segment

iv. Form factor: Form factor, R_f is the dimensionless ratio of the basin area A to the square of basin length L i.e.

$$R_f = \frac{A}{L^2} \qquad \ldots(2)$$

v. Shape Index: It is the inverse of form factor. This ratio was used in unit hydrograph application by the U.S.Army Crops of Engineers.

vi. Circularity Ratio: Miller defined circularity ratio as the ratio of basin area A to the area of a circle A_c, having the same perimeter of the basin.

vii. Compactness coefficient: The compactness coefficient is given by the formula :

$$C_c = 0.28 \; (P_b / \sqrt{A}) \qquad \ldots(3)$$

where,

P_b = Perimeter of the basin or sub-basin

A = Area of the basin or sub-basin

viii. Elongation Ratio: Elongation ratio, R_E, as defined by Schumm is the ratio of diameter of a circle of the same area as the basin to the maximum basin length, i.e.

$$R_E = \frac{D_C}{L_m} \qquad \text{...(4)}$$

ix. Bifurcation ratio: Bifurcation ratio, R_B, is defined as the ratio of number of segments of a given order N_u to the number of segments of the higher order N_{u+1} i.e.

$$R_b = \frac{N_u}{N_{u+1}} \qquad \text{...(5)}$$

x. Watershed eccentricity: The formula for watershed eccentricity, which is a measure of shape of the watershed, is given as :

$$\tau = \frac{L_C^2 - W_L^2}{W_L} \qquad \text{...(6)}$$

where,

τ = Watershed eccentricity

L_C = length from the watershed mouth to the centre of mass of the watershed, in same unit as

W_L = the width of the watershed at the centre of mass and perpendicular to L_C

xi. Drainage density: It is an important indicator of the linear scale of land form elements in stream eroded topography. Horton defined the drainage density D as :

$$D = \frac{\Sigma L_s}{CA} \qquad \ldots(7)$$

where,

ΣL_s = Total channel segment lengths for all orders within a basin.

CA = Total area of the basin

xii. Drainage frequency: Stream frequency or channel frequency, F is the number of stream segments per unit area or

$$F = N/A \qquad \ldots(8)$$

xiii. Overland slope (L_s) : A grid of rectangular mesh are drawn on the map of the catchment such that about 100 intersections occur within the catchment boundary. At each intersection the minimum distance between adjacent contours of the same intervals are measured and the slope at each point is computed. This provides a set of slope values, of which the mean is calculated and taken as overland slope. When an intersection occurs at a point between two contours of the same value, the slope is taken as zero if the point is in a valley and as indeterminate if the point is on a hill. The latter is neglected in calculating the mean.

xiv. Coefficient of variation of the square root of the overland slope (VLS): The mean and standard deviation of the square roots of the values of the overland slope at the intersections of the grid are calculated and the coefficient of variation taken as the standard deviation devided by the mean.

xv. Mean Stream interval : The number of time the grid lines used in calculating overland slope are cut by streams on the map is devided into the total length of grid lines to give mean stream interval. This gives a measure of the average distance between streams.

xvi. Rainfall intensity and duration: The watershed response depends on the intensity and duration of the rainfall as well as the direction of the storm movement. These factors are also considered alongwith the catchment characteristics for establishing the regional relationships for model parameters.

Some of the terms used in regression analysis are as follows:

i. Multiple correlation coefficient: It provides a measure of the percent of variance in the dependent variable explained by the independent variables.

ii. Regression coefficients: It represents the change independent variable due to change in corresponding independent variable.

iii. Standard error of estimate: It is the standard deviation of the difference between the observed dependent values and the values computed from the regression equation in the units of dependent values.

iv. Standard error of regression coefficients: It represents a measure of the significance of the regression coefficients.

1.3 Description of the Model Used for the Study

Nash Model considers that the instantaneous unit hydrograph could be obtained by routing the inflow through a cascade of linear reservoirs with equal storage coefficient. The outflow from the first reservoir is considered as inflow to the second reservoir and so on. The mathematical equation developed from general differential equation for the unit hydrograph is given as :

$$U(T,t) = \frac{1}{T} \{I\ (N,t/K) - I\ (N,(t-T/K)\} \qquad \ldots(9)$$

where,

$U(T,t) = t^{th}$ ordinates for the unit hydrograph of duration T,

$I(N,t/K) =$ incomplete gamma function of order N at t/K

$I(N,(t-T)/K) =$ incomplete gamma function of order N at $(t-T)/K$

It can be seen from the above equation that the unit hydrograph of duration T for any catchment may be derived only when the values of the two parameters, N and K, are known. The parameters of Nash Model for the gauged catchments may be derived by analysing the storms of the catchments using either the method of moments or optimization method (see user's manual on unit hydrograph derivation UM-8). However, such methods can not be used to estimate the parameters N and K, of Nash Model for those catchments which are not gauged. If the relation between the model parameters for gauged catchments and catchment characteristics are established, then the parameters for ungauged catchments can be easily estimated using such relationships. Nash Model parameters, N and K, and some relevant catchment characteristics for five catchments of Godavari basin subzone 3f are given in table 1.

1.4 Scope

The programme MULTI.FOR considers ($LL_c/\sqrt{S}$) as an independent variable and NK as a dependent variable in the first regression equation (where N and K are two parameters of the Nash Model). The other regression equation is formulated taking L as an independent variable and K as a dependent variables. The form of the equation are :

$$NK = a \left(\frac{LL_c}{\sqrt{S}}\right)^b \qquad \text{...(10)}$$

$$K = c\, L^d \qquad \text{...(11)}$$

where,

a and b are the regression coefficients for equation (10) and c & d are the regression coefficients for equation (11).

The programme uses values of NK and $\frac{LL_c}{\sqrt{S}}$ for five

Table 1 - Nash Model Parameters and Catchment Characteristics

Catchment No.	Nash's Model Parameters			Length of Main stream (L)	Distance of C.G. to outlet (L_c)	Slope (s)	$LL_c/\sqrt{S}$
	N	K	NK				
			hrs.	km.	km.	m/m	x 10^4
807/1	1.96	2.63	5.15	67.2	25.75	0.00228	3.624
51	3.44	0.91	3.13	23.74	10.06	0.001299	0.663
604/2	2.63	1.38	3.63	45.95	20.44	0.00193	2.138
969/1	2.03	1.147	2.33	24.94	6.76	0.00207	0.371
566	3.40	0.73	2.48	19.55	8.37	0.004917	0.233

watersheds as input data and estimates the regression coefficients a and b. Similarly the coefficients c and d are also estimated supplying the values of K and L for five waterhseds as input data to the programme. The output of the programme includes the standard error of estimate, standard error of regression coefficients, coefficient of correlation, t-statistics and F-statistic etc. which are used as criteria for accepting the results of regression analysis.

2.0 RELATIONSHIPS BETWEEN PARAMETERS OF NASH MODEL AND CATCHMENT CHARACTERISTICS

The estimation of response from ungauged catchments is one of the major problems for the hydrologists. The easiest way of solving this problem is by transferring the informations from gauged to ungauged catchments with the help of the regional relationships developed between the parameters of a hydrological model and the catchment characteristics for the gauged catchments. Attempts have been made by various investigators in the developing such relationships for different hydrological models. Snyder (1938) related the unit hydrograph parameters with the catchment characteristics. Nash (1959) developed the relationships between the first two moments of instantaneous unit hydrograph and the catchment characteristics using multiple linear regression approach. HEC (1982) has also developed similar form of relationships' between the parameters of the Clark Model and some prominent catchment characteristics. The multiple linear regression approach could be used to develop relationship between Nash Model parameters and catchment characteristics. The procedure and main steps for developing the relationships are discussed in this user's manual.

2.1 General Description

The general form of the equation for the above relationship may be :

$$y = ax_1^{b_1} \; x_2^{b_2} \; x_3^{b_3} \; \cdots\cdots x_n^{b_n} \qquad \ldots(12a)$$

where,

y is the dependent variable, $x_1, x_2, x_3 \ldots\ldots x_n$ are the independent variables and $a, b_1, b_2 \ldots\ldots b_n$ etc. are the coefficients to be estimated.

Equation (12a) is a non linear equation which can be transformed into a linear relation by using the logarithmic transformation. After such a transformation is made , linear regression can be used to determine the coefficients that best fit the data. Equation (12b) represents the linear equation after the logarithmic transformation.

$$\log y = \log a + b_1 \log x_1 + b_2 \log x_2 \ldots\ldots b_n \log x_n \qquad \ldots(12b)$$

Since Nash Model is a two parameters model, therefore only two equations are required to develop the relationships as given in equation (10) and (11).

In the first equation, NK, which represents the lag of the catchment, is a dependent variable and the combination of catchment characteristics, $(LL_c / \sqrt{S})$ is an independent variable. The cofficients a and b are unknowns which may be estimated transforming the equation in a linear form through log transformation and using linear regr ession.

Similarly, the coefficient c and d for the second equation may also be estimated where K, storage coeffcient of a single linear reservoir, is treated as dependent variable and L, the main stream length, is an independent variable.

2.2 Data Requirements,

The data required for running the computer programme MULTI.FOR are :

i. No. of independent variables

ii. Sample size

iii. A vector containing the values of dependent variable

iv. A matrix containing the values of independent variables.

2.3 Analysis

a. Estimation of regression coefficients

The general linear model is of the form

$$y = \beta_1 x_1 + \beta_2 x_2 + \ldots\ldots + \beta_p x_p \qquad \ldots(13)$$

where

y is a dependent variable,

$x_1, x_2 \ldots\ldots x_p$ are independent variables

and $\beta_1, \beta_2 \ldots\ldots \beta_p$ are the unknown parameters

In practice n observations would be available on y with the corresponding n observations on each of the p independent variables. Thus n equations like equation(13) can be written one for each observation. Essentially we are solving n equations for the p unknown parameters ($p > n$). In practices n should be at least 3 to 4 times as large as p. Those n equations can be written in matrix notation:

$$y = x \quad \beta \qquad \ldots(14)$$

$$nx1 \quad nxp \quad px1$$

The special case of equation (14) with $x_{i,1}=1, x_{i,2}=x$ $\beta_1 = \alpha$ and $\beta_2 = \beta$ is : $y = \alpha + \beta x$

The unknown parameters 'β' can be estimated by least square approach for which the equations are to be solved are :

$$\beta = (X^T X)^{-1} X^T Y \qquad \ldots(15)$$

where,

β is the vector of regression coefficients

and X^T represents the transpose of the matrix x

b. Estimation of coefficient of correlation: The equation for the coeffcient of correlation, R, is given as

$$R = \sqrt{(\beta^T x^T y - n \bar{Y}^2) / (Y^T y - n\bar{y}^2)} \qquad \ldots(16)$$

where,

$\bar{y}$ is the mean of dependent variable observations.

c. Estimation of standard error of the regression equation: The equation for the standard error of the regression equation σ , is given as :

$$\sigma = \sqrt{(y^T y - \beta^T x^T y)/(n-p)} \qquad \ldots(17)$$

d. Estimation of standard error of the regression cofficients: The equation for estimation the standard error of the regression coefficients, S, is given as :

$$S_{\beta i} = \sqrt{C_{ii}^{-1} \sigma^2} \qquad \ldots(18)$$

where, C_{ii}^{-1} is the ith diagonal element of $(x^T x)^{-1}$

e. Estimation of t-statistic: The correctness of the linear model is judged when the quantity $(\beta i / S_{\beta_i})$ is distributed as a t-distribution with (n-p) degress of freedom. The t-statistic is computed using the relation... $t = (\beta_i - \beta_o)/S_{\beta_i}$ ...(I9)

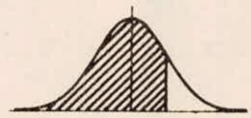
The hypothesis H_o will be rejected if $(t_1 > t_{1-\alpha/2}, (n-p)$ where α is a significance level. Otherwise accepted. The critical values of t at different confidence intervals corresponding to different degrees of freedom are given in table 2.

f. Estimation of F-statistic: When a test of the hypothesis that the entire regression equation is not explaining a significant amount of the variation in y versus at least one of these 'β's is not zero, has to be made, then the repeated application of equation(19) is not a valid way to test this hypothesis due to some correlation or interdependence between two independent variables. In that case the F-statistics which may be computed using the equation given below may be used to test the above hypothesis.

$$F = \frac{(\beta^T x^T y) - n \bar{y}^2/(p-1)}{(y^T y - \beta^T x^T y)/(n-p)} \quad \text{...(20)}$$

where hypothesis will be rejected if H_o exceeds $F_{1-\alpha}$, p-1, n-p. The critical values of F at different confidence intervals corresponding to different degrees of freedome are

Table 2 - Percentile Value ($t_{\alpha,\nu}$) for the t distribution with ν degrees of Freedom (shaded area $=\alpha$)



$t_{\alpha,\nu}$

ν	$t_{.995}$	$t_{.99}$	$t_{.975}$	$t_{.95}$	$t_{.90}$	$t_{.80}$	$t_{.75}$	$t_{.70}$	$t_{.60}$	$t_{.55}$
1	63.66	31.82	12.71	6.31	3.08	1.376	1.000	.727	.325	.158
2	9.92	6.96	4.30	2.92	1.89	1.061	.816	.617	.289	.142
3	5.84	4.54	3.18	2.35	1.64	.978	.765	.584	.277	.137
4	4.60	3.75	2.78	2.13	1.53	.941	.741	.569	.271	.134
5	4.03	3.36	2.57	2.02	1.48	.920	.727	.559	.267	.132
6	3.71	3.14	2.45	1.94	1.44	.906	.718	.553	.265	.131
7	3.50	3.00	2.36	1.90	1.42	.896	.711	.549	.263	.130
8	3.36	2.90	2.31	1.86	1.40	.889	.706	.546	.262	.130
9	3.25	2.82	2.26	1.83	1.38	.883	.703	.543	.261	.129
10	3.17	2.76	2.23	1.81	1.37	.879	.700	.542	.260	.129
11	3.11	2.72	2.20	1.80	1.36	.876	.697	.540	.260	.129
12	3.06	2.68	2.18	1.78	1.36	.873	.695	.539	.259	.128
13	3.01	-2.65	2.16	1.77	1.35	.870	.694	.538	.259	.128
14	2.98	2.62	2.14	1.76	1.34	.868	.692	.537	.258	.128
15	2.95	2.60	2.13	1.75	1.34	.866	.691	.536	.258	.128
16	2.92	2.58	2.12	1.75	1.34	.865	.690	.535	.258	.128
17	2.90	2.57	2.11	1.74	1.33	.863	.689	.534	.257	.128
18	2.88	2.55	2.10	1.73	1.33	.862	.688	.534	.257	.127
19	2.86	2.54	2.09	1.73	1.33	.861	.688	.533	.257	.127
20	2.84	2.53	2.09	1.72	1.32	.860	.687	.533	.257	.127
21	2.83	2.52	2.08	1.72	1.32	.859	.686	.532	.257	.127
22	2.82	2.51	2.07	1.72	1.32	.858	.686	.532	.256	.127
23	2.81	2.50	2.07	1.71	1.32	.858	.685	.532	.256	.127
24	2.80	2.49	2.06	1.71	1.32	.857	.685	.531	.256	.127
25	2.79	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
26	2.78	2.48	2.06	1.71	1.32	.856	.684	.531	.256	.127
27	2.77	2.47	2.05	1.70	1.31	.855	.684	.531	.256	.127
28	2.76	2.47	2.05	1.70	1.31	.855	.683	.530	.256	.127
29	2.76	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
30	2.75	2.46	2.04	1.70	1.31	.854	.683	.530	.256	.127
40	2.70	2.42	2.02	1.68	1.30	.851	.681	.529	.255	.126
60	2.66	2.39	2.00	1.67	1.30	.848	.679	.527	.254	.126
120	2.62	2.36	1.98	1.66	1.29	.845	.677	.526	.254	.126
∞	2.58	2.33	1.96	1.645	1.28	.842	.674	.524	.253	.126

given in table 3.

g. Estimation of correlation among the independent variables: If a matrix $Z_{i,j}$ is defined as $(x_{i,j} - \bar{x}_j)/\bar{S}_j$ and let $Z=(Z_{i,j})$, then $Z^T Z/(n-1)$ is a pxp correlation matrix $\underline{R} = (R_{i,j})$, where $R_{i,j}$ is the correlation coefficient between the ith and jth independent variables.

The computer programme MULTI.FOR which performs, the above analysis is discussed in Appendix-I.

2.4 Advantages and Limitations

The main advantage of developing such types of regression equations between the model parameters and catchment characteristics is to estimate the parameters for ungauged catchment of the same region. The model parameters, thus obtained, may be utilised to get the response of these ungauged catchments if the input informations are known. However, the catchments whose data is used in developing the regression equation must be similar in hydrological characteristics, geological characteristics, topography, land uses, vegetation cover and agriculture practices. The catchments should also be of the same general size, and the rainfall distribution and magnitude as well as the factors affecting loss rates should be generally be of the same order of magnitude. Great care has to be taken before including the combination of more than one variables as an indpeendent variables in the regression to avoid the building up of the

Table 3 - Cumulative F Distribution (m Numerator and n Denominator Degrees of Freedom)

α	n \ m	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	120	∞
.90	1	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.7	61.2	61.7	62.3	62.8	63.1	63.3
.95		161	200	216	225	230	234	237	239	241	242	244	246	248	250	252	253	254
.975		648	800	864	900	922	937	948	957	963	969	977	985	993	1000	1010	1010	1020
.99		4,050	5,000	5,400	5,620	5,760	5,860	5,930	5,980	6,020	6,060	6,110	6,160	6,210	6,260	6,310	6,340	6,370
.995		16,200	20,000	21,600	22,500	23,100	23,400	23,700	23,900	24,100	24,200	24,400	24,600	24,800	25,000	25,200	25,400	25,500
.90	2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.46	9.47	9.48	9.49
.95		18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
.975		38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5
.99		98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5
.995		199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
.90	3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.15	5.14	5.13
.95		10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.62	8.57	8.55	8.53
.975		17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.3	14.3	14.2	14.1	14.0	13.9	13.9
.99		34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.5	26.3	26.2	26.1
.995		55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.4	43.1	42.8	42.5	42.1	42.0	41.8
.90	4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.93	3.92	3.90	3.87	3.84	3.82	3.79	3.78	3.76
.95		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.69	5.66	5.63
.975		12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.46	8.36	8.31	8.26
.99		21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.8	13.7	13.6	13.5
.995		31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.7	20.4	20.2	19.9	19.6	19.5	19.3
.90	5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.17	3.14	3.12	3.11
.95		6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.43	4.40	4.37
.975		10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.23	6.12	6.07	6.02
.99		16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.38	9.20	9.11	9.02
.995		22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.4	13.1	12.9	12.7	12.4	12.3	12.1
.90	6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.80	2.76	2.74	2.72
.95		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.74	3.70	3.67
.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.07	4.96	4.90	4.85
.99		13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.23	7.06	6.97	6.88
.995		18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.0	9.81	9.59	9.36	9.12	9.00	8.88
.90	7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.56	2.51	2.49	2.47
.95		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.38	3.30	3.27	3.23
.975		8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.36	4.25	4.20	4.14
.99		12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	5.99	5.82	5.74	5.65
.995		16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51	8.38	8.18	7.97	7.75	7.53	7.31	7.19	7.08
.90	8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.38	2.34	2.31	2.29
.95		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	3.01	2.97	2.93
.975		7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.89	3.78	3.73	3.67
.99		11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.20	5.03	4.95	4.86
.995		14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.01	6.81	6.61	6.40	6.18	6.06	5.95

Table 3 contd.

.90	9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.25	2.21	2.18	2.16
.95		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.79	2.75	2.71
.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.56	3.45	3.39	3.33
.99		10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.65	4.48	4.40	4.31
.995		13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.23	6.03	5.83	5.62	5.41	5.30	5.19
.90	10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.15	2.11	2.08	2.06
.95		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.70	2.62	2.58	2.54
.975		6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.31	3.20	3.14	3.08
.99		10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.25	4.08	4.00	3.91
.995		12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.66	5.47	5.27	5.07	4.86	4.75	4.64
.90	12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.01	1.96	1.93	1.90
.95		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.38	2.34	2.30
.975		6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	2.96	2.85	2.79	2.72
.99		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.70	3.54	3.45	3.36
.995		11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.91	4.72	4.53	4.33	4.12	4.01	3.90
.90	15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.87	1.82	1.79	1.76
.95		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.16	2.11	2.07
.975		6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.64	2.52	2.46	2.40
.99		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.21	3.05	2.96	2.87
.995		10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.25	4.07	3.88	3.69	3.48	3.37	3.26
.90	20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.74	1.68	1.64	1.61
.95		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.95	1.90	1.84
.975		5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.35	2.22	2.16	2.09
.99		8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.78	2.61	2.52	2.42
.995		9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.68	3.50	3.32	3.12	2.92	2.81	2.69
.90	30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.61	1.54	1.50	1.46
.95		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.74	1.68	1.62
.975		5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.07	1.94	1.87	1.79
.99		7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.39	2.21	2.11	2.01
.995		9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.18	3.01	2.82	2.63	2.42	2.30	2.18
.90	60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.48	1.40	1.35	1.29
.95		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.65	1.53	1.47	1.39
.975		5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.82	1.67	1.58	1.48
.99		7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.03	1.84	1.73	1.60
.995		8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.74	2.57	2.39	2.19	1.96	1.83	1.69
.90	120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.54	1.48	1.41	1.32	1.26	1.19
.95		3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.55	1.43	1.35	1.25
.975		5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.69	1.53	1.43	1.31
.99		6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.86	1.66	1.53	1.38
.995		8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.54	2.37	2.19	1.98	1.75	1.61	1.43
.90	∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.34	1.24	1.17	1.00
.95		3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.32	1.22	1.00
.975		5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.57	1.39	1.27	1.00
.99		6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.70	1.47	1.32	1.00
.995		7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.36	2.19	2.00	1.79	1.53	1.36	1.00

larger correlations into $x^T x$ matrix. Generally one should perform regression analysis only when the number of observations for different variables involved are at least 3 or 4 times as large as the number of independent variables (Haan 1982).

3.0 RECOMMENDATIONS

The programme uses the no. of observations of dependent variables (model parameters for each catchment) and corresponding observations for independent variables(catchment characteristics) and estimates the regression coefficients,standard error of estimate, coefficient of correlation, standard error of regression coefficients, t-statistics and F-statistics etc. User's may get desired no. of regression equations at a time supplying the various combination of independent variables. The programme can accommodate maximum one hundred observations and fifty independent variables. If user wants to use the programme beyond the above maximum limit, then necessary modifications should be done by changing in DIMENSION statements of the main programme.

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APPENDIX-I

A. Computer Programme Description

The computer programme MULTI.FOR uses a set of observations for dependent variable and corresponding observations for independent variables for its running. The main variables used in the programme are described as follows:

Variable	Description
L	Maximum no. of independent variables.
N	No. of observations for dependent or independent variables
U	Matrix of observations for independent variables where the observations for each independent variables are stored columnwise.
B	A vector containing the observations for dependent variables.
N2	No. of combinations among independent variable
UNM	Unit Matrix
M	No. of independent variables in each combination
ISAVE	Position of independent variables
RR	Coefficient of determination
COR	Correlation coefficient
F	F-statistic
RC	A vector containing the regression coefficients
T	A vector containing the t-values corresponding to each regression coefficients.

SERC	A vector containing the standard error of regression coefficients
SEST	A real constant indicating the standard error of estimate
XMEAN	A vector containing the mean of independent variables
YMEAN	A real constant indicating the mean of dependent variable
ACOR	A matrix containing the values of correlation coefficient among independent variables.

The subroutines required for running the programme are described below.

i. SUBROUTINE MATR(A,RHS,N,UNM)- This subroutine calculates the inversion of a matrix. The calling arguments are :

A - A matrix for which inversion is required

RHS - A matrix which is inverse of matrix A.

N - An integer constant indicating the size of the matrix

UNM- An unit Matrix

ii. SUBROUTINE CORR(M,N,ACOR,X,XMEAN)- This subroutine estimates the coefficient of correlation among independent variables. The calling arguments of the subroutine are:

M - Maximum No. of independent variables

N - No. of observations

ACOR- A matrix containing the coefficient of correlation among independent variables.

X - A matrix containing the observations of independent variables

XMEAN- A vector containing the mean for each independent variables.

B. Input specifications

The required input for running the programme has to be supplied in two different forms : (i) some of the input information through terminal (ii) Remaining informations through files.

i. Data to be supplied through terminal:

S.No.	Matter Displayed on terminal	Variable	Description of the data to be supplied by the user
1.	INPUT FILE NAME ?	FYLE	Give input file name maximum upto six characters
2.	OUTPUT FILE NAME ?	FYLEN	Give output file name maximum upto six characters
3.	MAXIMUM NO. OF IN DEPENDENT VARIABLES ?	L	User has to supply the no. of independent variables which are being used in regression.
4.	NO. OF OBSERVATIONS ?	N	User has to supply the No. of observations being used in regression

5.	DO YOU WANT TO TAKE LOG OF THE OBSERVATIONS	IANS	If user has supplied 'YES', then the logarithmic transformed data will be used in regression.
6.	NO. OF COMBINATIONS INDEPENDENT VARIABLES?		User has to supply the no. of combinations which he wants to make among independent variables for different regression equation.

ii. Data to be supplied through a file:

Line No.	Variable	FORMAT	DESCRIPTION
1.	U	FREE	Matrix of independent variables where the observations for each independent variables are stores columnwise.
2.	B	FREE	A vector containing the observations for dependent variables.
3.	M	FREE	An integer constant indicating the no. of independent variables in each combination (one independent variable is reserved for the estimation of intercept).
4.	ISAVE	FREE	A vector containing the order of independent variables in which they have been read in data file supply

always one for the estimation of intercept.

c. OUTPUT SPECIFICATIONS

The following variables will be printed and stored with a specified format in the output file including some titles:

VARIABLE	FORMAT	DESCRIPTION
ISAVE	10I8	A vector containing the order of independent variables being used in the regression.
RR	F8.3	A real constant indicating the coefficient of determination .
F	F8.3	F-statistics.
COR	F8.3	A real constant indicating the correlation coefficient.
RC	F8.2	A vector containing the regression coefficient.
L	F8.3	A vector containing the calculated T-statistics for the regression coefficient.
SERC	F8.3	A vector containing the standard error of regression coefficients.
SEST	F8.3	A real constant indicating the standard error of estimate.
XMEAN	10F8.3	A vector containing the mean for different independent variables

		excluding the independent variable used for the independent variable used for the estimation of intercept.
YMEAN	F8.4	A real constant-indicating the mean for dependent variable.
ACOR	10F8.4	A matrix of correlation among independent variables.

D. EXAMPLE APPLICATION

Two regression equations have been developed assuming (NK) as a dependent variable and ($LL_c/\sqrt{S}$) as an independent variable in the first equation while (K) as a dependent variable and (L) as an independent variable in the second equation. The input data required for each regression are given below:

(a) Input data for first regression equation

(i) Data supplied through terminal

Matter Displayed on Terminal	Description of the test data supplied by the user
INPUT FILE NAME ?	MULT1. DAT
OUTPUT FILE NAME ?	OUT1 . DAT
MAXIMUM NO. OF INDEPE-NDENT - VARIABLES	2
NO. OF OBSERVATIONS	5
DO YOU WANT TO TAKE LOG OF THE OBSERVATIONS ?	YES
NO. OF COMBINATIONS AMONG INDEPENDENT VARIABLES	1

ii) Data supplied through file - MULT1. DAT

36240	6630	21380	3710	2330
5.15	3.13	3.63	2.33	2.48

2

1 2

(b) Input data for second regression equation:

(i) Data supplied through terminal

Matter displayed on terminal	Description of the test data supplied by the user
INPUT FILE NAME ?	MULT2 .DAT
OUTPUT FILE NAME ?	OUT2 .DAT
MAXIMUM NO. OF INDEPENDENT VARIABLES ?	2
NO. OF OBSERVATIONS	5
DO YOU WANT TO TAKE LOG OF THE OBSERVATIONS	YES
NO. OF COMBINATIONS ?	1
AMONG INDEPENDENT VARIABLES	

(ii) Data supplied through file - MULT2. DAT

67.20	23.74	45.95	24.94	19.55
2.63	0.91	1.38	1.147	0.73

2

1 2

E. TEST OUTPUT : The outputs for the input data of first regression equation and second regression equation are given in section H.

The two equation developed are of the form :

(i) $\log_e(NK) = -1.20 + 0.26 \log_e(LL_c/\sqrt{s})$

$$\text{or}\quad NK = 0.3012\,(LL_c/\sqrt{s})^{0.26} \qquad \ldots.(A)$$

(ii) $\log_e(K) = -2.94 + 0.90 \log_e(L)$

$$\text{or}\quad K = 0.0529\,(L)^{0.90} \qquad \ldots.(B)$$

Equations (A) and (B) can be solved in order to estimate the parameters for ungauged catchments using their catchment characteristics, $(LL_c/\sqrt{s})$ and L in the above equations. As an example, for a test catchment, BR No.228, of the lower Godavari subzene 3f, the parameters are estimated using its catchment characteristics as follows:

$$\frac{LL_c}{\sqrt{S}} = 12300 \qquad L = 42$$

for equation (A) and (B)

$$NK = 0.3012\,(12300)^{0.26} = 3.485$$

$$K = 0.0529\,(42)^{0.90} = 1.529$$

$$N = \frac{NK}{K} = \frac{3.485}{1.529} = 2.279$$

Since the equations (A) and (B) are developed using only five catchments, therefore, the coefficients derived may not be much reliable. If data of twenty to thirty catchments are used for the regression, then the reliability of such coefficients will be botter.

F. Test of hypothesis

(a) Equation (A) represents the form of the first equation developed having correlation coefficient equal to 0.948. Let

$$y = \log_e (NK)$$

$$x = \log_e (LL_c/\sqrt{s})$$

$$B_o = -1.20 \text{ and}$$

$$B_1 = 0.26$$

Then the equation (A) may be written as

$$y = B_o + B_1 x \qquad \ldots (C)$$

(i) t-test

The hypothesis H_o: $B_1 = 0$ has been rejected as $|t|$ is greater than $t_{0.975,3}$. Here $|t| = 5.148$ and $t_{0.975,3} = 3.18$ (from table 2). Therefore the independent variable x is contributing significantly to explain the variation in the dependent variable y.

(ii) F-test

The hypothesis H_o: $B_1 = 0$ has also been rejected on the basis of F-test as avalue of F for the regression equation exceeds the critical value of $F_{0.95,1,3}$. Here F=26.493 and critical value of $F_{0.95,1,3} = 10.1$ (from table 3). Therefore the entire regression equation is explaining a significant amount of variation in y.

(b) Equation (B) represents the form of the second equation developed having correlation coefficient equal to 0.956

Let $y' = \log_e (K)$

$$x' = \log_e (L)$$

$$B'_o = -2..94$$

$$B'_1 = 0.90$$

Then the equation (B) may he written as

$$y' = B'_o + B'_1 x' \qquad \ldots (D)$$

(i) t-test

The hypothesis H_o: $B'_1=0$ has been rejected as $|t|$ is greater than $t_{0.975,3}$. Here $|t| = 5.634$ and $t_{0.975,3} = 3.18$ (from table 2) . Therefore the independent variable x' is contributing significantly to explain the variation in the dependent variable y'.

(ii) F-test

The hypothesis H_0: $B'_1=0$ has also been rejected on the basis of F-test as value of F for the regression equation exceeds the critical value of $F_{0.95,1,3}$. Here F= 31.745 and critical value of $F_{0.95,1,3} = 10.1$ (from table 3). Therefore, the entire regression equation is explaining a significant amount of variation in y'.

G. Computer Programme

```
C       MULTIPLE LINEAR REGRESSION PROGRAMME
        DIMENSION X(200,50),B(200),H(200),Y(50,200),C(50,200),D(50)
        DIMENSION E(50,50),RC(50),VARC(50),SERC(50),V(200,200),T(50)
        DIMENSION ACOR(50,50),U(200,50),ISAVE(50),UNM(50,50),AX(200,50)
        DIMENSION AB(200),XMEAN(50),BB(200)
        DATA IYES/'YES'/
        CHARACTER*6 FYLE,FYLEN
        WRITE(5,1)
1       FORMAT(4X,'INPUT FILE NAME?'$)
        READ(5,2) FYLE
2       FORMAT(A)
        WRITE(5,3)
3       FORMAT(4X,'OUTPUT FILE NAME?'$)
        READ(5,2) FYLEN
        OPEN(UNIT=1,FILE=FYLE,STATUS='OLD')
        OPEN(UNIT=2,FILE=FYLEN,STATUS='NEW')
        WRITE(5,4)
4       FORMAT(4X,'MAXIMUM NO. OF INDEPENDENT VARIABLES?'$)
        READ(5,*) L
        WRITE(5,5)
5       FORMAT(4X,'NO. OF OBSERVATIONS?'$)
        READ(5,*)N
        DO 1133 I=1,N
        U(I,1)=1.0
1133    CONTINUE
C       READ MATRICS OF INDEPENDENT VARIABLES WITH NO. OF OBSERVATIONS
        READ(1,*)((U(I,J),I=1,N),J=2,L)
C       READ COLUMN VECTOR OF DEPENDENT VARIABLES
        READ(1,*)(B(I),I=1,N)
        WRITE(5,6)
6       FORMAT(4X,'DO YOU WANT TO TAKE LOG OF THE OBSERVATIONS?'$)
        READ(5,211) IANS
211     FORMAT(A3)
        IF(IANS.EQ.IYES) N1=1
        WRITE(5,7)
7       FORMAT(4X,'NO. OF COMBINATIONS AMONG INDEPENDENT VARIABLES?'$)
        READ(5,*) N2
        DO 8 I=1,L
        DO 8 J=1,L
        UNM(I,J)=0.0
        IF(I.EQ.J) UNM(I,J)=1.0
8       CONTINUE
        DO 703 I=1,N
703     AB(I)=B(I)
        DO 700 L1=1,N2
```

```
C     READ NO OF INDEPENDENT VARIABLES IN EACH COMBINATIONS
      READ(1,*) M
C     READ POSITION OF INDEPENDENT VARIABLES
      READ(1,*) (ISAVE(I),I=1,M)
      DO 240 I=1,N
      DO 240 J=1,M
      K=ISAVE(J)
      X(I,J)=U(I,K)
240   AX(I,J)=X(I,J)
      IF(N1.EQ.0.0) GO TO 600
      DO 500 I=1,N
      DO 500 J=2,M
      X(I,J)=ALOG(AX(I,J))
500   CONTINUE
      DO 501 I=1,N
      B(I)=ALOG(AB(I))
501   CONTINUE
      WRITE(2,706)
706   FORMAT(4X,'LOG TRANSFORMED VALUES ARE USED'//)
600   WRITE(2,80)
80    FORMAT(4X,'MULTIPLE LINEAR REGRESSION'/)
      WRITE(2,81)
81    FORMAT(4X,'----------------------------------------------'/)
      WRITE(2,701)
701   FORMAT(4X,'POSITION OF INDEPENDENT VARIABLES FOR REGRESSION'/)
      WRITE(2,702) (ISAVE(I),I=1,M)
702   FORMAT(4X,10I8)
      DO 40 I=1,N
      DO 40 J=1,M
40    Y(J,I)=X(I,J)
      DO 45 I=1,M
      DO 45 J=1,M
      C(I,J)=0.0
      DO45 K=1,N
      C(I,J)=C(I,J)+Y(I,K)*X(K,J)
45    CONTINUE
      DO 46 I=1,M
      D(I)=0.0
      DO 46 K=1,N
      D(I)=D(I)+Y(I,K)*B(K)
46    CONTINUE
      CALL MATR(C,E,M,UNM)
      DO 50 I=1,M
      RC(I)=0.0
      DO 50 J=1,M
```

```
      RC(I)=RC(I)+E(I,J)*D(J)
50    CONTINUE
      SUM=0.0
      DO 51 I=1,N
      SUM=SUM+B(I)
51    CONTINUE
      AN=N
      YMEAN=SUM/AN
      BM=0.0
      DO 52 I=1,N
      BM=BM+B(I)*B(I)
52    CONTINUE
      RRC=0.0
      DO 53 I=1,M
      RRC=RRC+RC(I)*D(I)
53    CONTINUE
      RR=(RRC-AN*YMEAN**2)/(BM-AN*YMEAN**2)
      COR=SQRT(RR)
      SIGM=BM-RRC
      AM=M
      SIGMA=SIGM/(AN-AM)
      SEST=SQRT(ABS(SIGMA))
      DO 60 I=1,M
      VARC(I)=E(I,I)*SIGMA
      SERC(I)=SQRT(ABS(VARC(I)))
60    CONTINUE
      DO 90 I=1,M
      T(I)=RC(I)/SERC(I)
90    CONTINUE
      A1=(RRC-AN*YMEAN**2)/(AM-1.0)
      A2=(BM-RRC)/(AN-AM)
      F=A1/A2
      WRITE(2,61) RR,F,COR
61    FORMAT(4X,'COEFICIENT OF DETERMINATION=',F8.3/4X,'VALUE OF F=',
      1 F8.3/4X,'CORRELATION COEFICIENT=',F8.3/)
      WRITE(2,62)
62    FORMAT(4X,'REGRESSION COEFICIENTS',6X,'T-VALUES',10X,'STAND.
      1 ERROR OF REGRESSION COEF.')
      DO 63 I=1,M
      WRITE(2,64) RC(I),T(I),SERC(I)
64    FORMAT(4X,F8.2,20X,F8.3,15X,F8.4)
63    CONTINUE
      WRITE(2,65) SEST
65    FORMAT(4X,'STAND. ERROR OF ESTIMATES=',F8.3/)
      CALL CORR(M,N,ACOR,X,XMEAN)
```

```
      WRITE(2,1111)
1111  FORMAT(4X,'MEAN OF INDEPENDENT VARIABLES')
      WRITE(2,1112)(XMEAN(I),I=2,M)
1112  FORMAT(4X,10F8.3)
      WRITE(2,1113) YMEAN
1113  FORMAT(4X,'MEAN OF DEPENDENT VARIABLE=',F8.4)
      WRITE(2,69)
69    FORMAT(4X,'MATRICS OF CORRELATION BETWEEN INDEPENDENT VARIABLES')
      K=2
      DO 70 I=2,M
      WRITE(2,71) (ACOR(I,J),J=2,K)
71    FORMAT(4X,10F8.4////)
      K=K+1
70    CONTINUE
      WRITE(2,81)
700   CONTINUE
      CLOSE(UNIT=1)
      CLOSE(UNIT=2)
      STOP
      END
C     ******************************************************************
C     THIS SUBROUTINE CALCULATES MATRICS INVERSION
C     ******************************************************************
      SUBROUTINE MATR(A,RHS,N,UNM)
      DIMENSION A(50,50),RHS(50,50),B(50,200),BRHS(50,50),CRHS(50,50)
      DIMENSION UNM(50,50)
      DO 14 I=1,N
      DO 14 J=1,N
      RHS(I,J)=UNM(I,J)
14    CONTINUE
      DO 2 I=1,N
      DIV=A(I,I)
      M=0
      IF(ABS(DIV).GT.10.E-10) GO TO 7
      K1=I
      K11=K1+1
      DO 4 J=K11,N
      IF(ABS(A(J,I)).LE.10.E-6) GO TO 4
      K2=J
      GO TO 3
4     CONTINUE
      WRITE(5,1001)
1001  FORMAT(4X,'SINGULAR MATRICS INVERSION NOT POSSIBLE')
      GO TO 1002
3     DO 103 J=1,N
```

```
      B(K2,J)=A(K2,J)
      B(K1,J)=A(K1,J)
      A(K1,J)=B(K2,J)
      A(K2,J)=B(K1,J)
      BRHS(K2,J)=RHS(K2,J)
      BRHS(K1,J)=RHS(K1,J)
      RHS(K1,J)=BRHS(K2,J)
      RHS(K2,J)=BRHS(K1,J)
103   CONTINUE
      DIV=A(K1,I)
      M=M+1
7     DO 5 J=1,N
      A(I,J)=A(I,J)/DIV
5     RHS(I,J)=RHS(I,J)/DIV
      DO 99 J=1,N
      IF(I.EQ.J) GO TO 99
      FAC=A(J,I)
      DO 98 K=1,N
      A(J,K)=A(J,K)-A(I,K)*FAC
98    RHS(J,K)=RHS(J,K)-RHS(I,K)*FAC
99    CONTINUE
2     CONTINUE
      IF(M.EQ.0) GO TO 9
      DO 11 K=1,N
      CRHS(K1,K)=RHS(K1,K)
      CRHS(K2,K)=RHS(K2,K)
      RHS(K1,K)=CRHS(K2,K)
11    RHS(K2,K)=CRHS(K1,K)
9     CONTINUE
1002  RETURN
      END
C     ************************************************************
C     THIS SUBROUTINE CALCULATES CORRELATION COEFICIENTS BETWEEN
C     INDEPENDENT VARIABLES
C     ************************************************************
      SUBROUTINE CORR(M,N,ACOR,X,XMEAN)
      DIMENSION X(200,50),ACOR(50,50),Z(200,200),V(200,200),XMEAN(50)
      DIMENSION S(50),SUM(50),SUM2(50),R(50),RR(50,50),SUM3(50,50)
      AN=N
      DO 10 I=2,M
      SUM(I)=0.0
      DO 20 J=1,N
20    SUM(I)=SUM(I)+X(J,I)
      XMEAN(I)=SUM(I)/AN
10    CONTINUE
```

```
      DO 30 I=2,M
      SUM2(I)=0.0
      DO 40 J=1,N
40    SUM2(I)=SUM2(I)+(X(J,I)-XMEAN(I))**2
      R(I)=SUM2(I)/(AN-1.0)
      S(I)=SQRT(ABS(R(I)))
30    CONTINUE
      DO 50 I=2,M
      DO 50 J=1,N
50    Z(J,I)=(X(J,I)-XMEAN(I))/S(I)
      DO 60 I=2,M
      DO 60 J=1,N
60    V(I,J)=Z(J,I)
      DO 70 I=2,M
      DO 70 J=2,M
      SUM3(I,J)=0.0
      DO 70 K=1,N
      SUM3(I,J)=SUM3(I,J)+(V(I,K)*Z(K,J))
70    CONTINUE
      DO 71 I=2,M
      DO 71 J=2,M
      ACOR(I,J)=SUM3(I,J)/(AN-1.)
71    CONTINUE
      RETURN
      END
```

H. Output

a. for First Relationship

LOG TRANSFORMED VALUES ARE USED

MULTIPLE LINEAR REGRESSION

POSITION OF INDEPENDENT VARIABLES FOR REGRESSION

1 2

COEFICIENT OF DETERMINATION= 0.898
VALUE OF F= 26.493
CORRELATION COEFICIENT= 0.948

REGRESSION COEFICIENTS	T-VALUES	STAND. ERROR OF REGRESSION COEF.
-1.20	-2.594	0.4622
0.26	5.148	0.0507

STAND. ERROR OF ESTIMATES= 0.118

MEAN OF INDEPENDENT VARIABLES
9.048
MEAN OF DEPENDENT VARIABLE= 1.1647
MATRICS OF CORRELATION BETWEEN INDEPENDENT VARIABLES
1.0000

---

b. for Second Relationship

LOG TRANSFORMED VALUES ARE USED

MULTIPLE LINEAR REGRESSION

POSITION OF INDEPENDENT VARIABLES FOR REGRESSION

1 2

COEFICIENT OF DETERMINATION= 0.914

VALUE OF F= 31.745

CORRELATION COEFICIENT= 0.956

REGRESSION COEFICIENTS	T-VALUES	STAND. ERROR OF REGRESSION COEF.
-2.94	-5.223	0.5623
0.90	5.634	0.1602

STAND. ERROR OF ESTIMATES= 0.166

MEAN OF INDEPENDENT VARIABLES

3.478

MEAN OF DEPENDENT VARIABLE= 0.2034

MATRICS OF CORRELATION BETWEEN INDEPENDENT VARIABLES

1.0000
