THEME 7

STATISTICAL ANALYSIS OF DROUGHT AND LOW FLOWS

International Seminar on Hydrology of Extremes

1-3 December, 1988
Roorkee (India)

ON DROUGHT ANALYSIS OF PERIODIC—STOCHASTIC PROCESSES

Pedro Guerrero Salazar
Professor of Agricultural Engineering
National Agrarian University, La Molina, Lima, Peru

SYNOPSIS

Periodic stochastic series are studied by defining drought and its parameters for this particular type of hydrologic processes. New approaches and techniques are presented with a case study illustrating the power of these new approaches.

1.0 INTRODUCTION

Pressure for a higher standard of living and the increase of world population continuously requires more food, energy, raw materials, industrial production and various services. The inevitable result is the increase (in pressure) with time on alltypes of world wide available water resources.

Because these renewable resources on continental areas are constant, in their averages, regardless of their space and time variations, sooner or later the increase in water demand faces space and time shortages because of the stochastic variations in water supply and demand. Experience and investigation show that risks of water shortage increases rapidly with an increase of utilization of the available water resources in an area. Particularly sensitive in this regard is the food production as the most important commodity on the margins of balance between food supply and food demand. Usually water shortages of drought proportions have the largest impact on the agricultural production.

Water supplies and water demands usually are periodic-stochastic space-time processes, which must be used either individually or in combination (and according to the problem on hand) for the analysis of shortages, deficits and droughts, through their differences.

2.0 DROUGHTS, DEFICITS, SHORTAGES AND LOW FLOWS

Water deficiencies can be analyzed using three criteria. Droughts, which are a creeping - type disaster phenomena, and are associated with water deficiencies of long duration, high intensity and large area coverage, usually involving all water resources variables and users, having significant economic and social consequences. Deficits which can be related to the lack of water at a given place for a given time interval, with relatively moderate consequences. Shortages which are a small negative difference between water demand and water supply, with readily acceptable consequences.

In studying their physical aspects, the following properties are of practical significance: duration, total water deficiency over this duration, area coverage by this total deficiency, intensity of largest deficiency and similar random variables. These variables are best described by joint or marginal probability distributions of individual variables. The properties of these random variables are related either to populations or to samples of various sizes. Assuming a multivariate or a univariate series of water supply variable(s) as the process, and a multivariate or a univariate series of water demand variable (s) as the output process of agricultural and water resources systems, the crossing of these two time processes provides the necessary information for computing or estimating the probabilities of drought properties.

An objective definition of droughts, based on the theory of runs, has been used for stationary time series, such as annual series. For the univariate case and discrete time series of water supply, a selected arbitrary variable value or truncation level Xo may give two new truncated series of positive and negative difference. A sequence of consecutive negative deviations preceded and followed by positive deviations is called a negative run- length and it may be associated with a duration of a drought.

The sum of all negative deviations over such a run -length is called the negative run-sum, and the ratio of the negative run - sum and the negative run-length is called the negative run - intensity. In this context, the definition was used by Llamas and Siddiqui, 1969, Saldarriaga and Yevjevich, 1970; Millan and Yevjevich, 1971; Millan, 1972 and Guerrero-Salazar; and Yevjevich; 1975.

An extension for bivariate processes, mainly concerned with droughts of two representative variables were considered by Guerrero-Salazar, 1973.

3.0 DROUGHT ANALYSIS OF PERIODIC-STOCHASTIC PROCESSES

3.1 The Problem

The drought analysis of periodic-stochastic processes is more complex than the drought analysis of stationary stochastic processes. The use of the theory of runs for periodic-stochastic processes may no longer be the best approach unless some decomposition of series is performed for these processes. However, any such transformation may affect objectives and results of analysis.

Monthly series will exemplify the periodic-stochastic processes in the following analysis. Series with a shorter time interval than a month may also serve the same purpose. A review of presently available techniques, and of some potential techniques for drought analysis of these processes, are presented here. A case study is given in which the drought parameters used are specific drought magnitude criteria.

3.2 A Review of Presently Available Techniques

Drought analysis depends whether the water flow is regulated or not. Generally the instantaneous extremes are used in case of no regulation, while deficits during the critical periods are used when flow regulation is involved.

In the theory of extremes, droughts also called low flows are defined as instantaneous or interval smallest annual values, with every year giving one lowest value or the drought(Gumbel, 1963). The problem was to find probabilities of these lowest values, called the minimum drought values, either positive or zero. Using the symbol X for random variables defining droughts, the return period T(X), as the expected number of years between the exceedences of a deficit, is:

$$T(X) = \frac{1}{P(X \leq x)} = \frac{1}{F(x)} \quad (1)$$

Since the exact distribution of drought variables as defined is not available, the asymptotic bonded exponential distribution of the smallest value of a positive variable was used by Gumbel. This approach to droughts of periodic stochastic processes may be well used in pollution control problems, attaching probabilities to levels of critical concentrations of pollutants. Similarly, probabilities of minimum consecutive n-days values, with n often 7, 15 and 30 days (Gannon, 1963) are determined. This particular definition of drought as a couple-of-days lowest values in a year may be acceptable for perennial but not for intermittent streams.

For studying droughts in case of flow regulations, Askew et.al(1969) defined the critical period as the time duration during which the hydrologic record would give most critical deficit with respect to demand. The maximum permissible water extraction rate is used as a variable of this critical period. This permissible rate is based on the active storage available in a hypothetical system of reservoirs. The demand can be smaller, equal or greater than the maximum permissible extraction rate. Generally, the extraction is assumed to be constant during the critical period. Whenever the rate of demand is greater than the maximum permissible extraction rate, the deficit may be conceived as a drought.

Another parameter used for definition of droughts in case of flow regulation is the yield criterion (Beard, 1963).

The number of shortage periods per year, and the amount of annual firm yield, are defined as drought parameters. Firm yield should be well defined for a reservoir system, with the characteristics of this system specified how it produces the firm yield in terms of monthly and total annual use of water. A single index of the economic effect of shortages was suggested by Beard (1963), in form of the sum squares of annual shortages in a 100 year period, beginning with an initial or representative amount of water in all storage capacities. The yield needed to be met by the system is the total water requirements of all water users and all losses. Beard and Kubik (1972), in studying the operation rules of a reservoir system, stated that many theoretical studies of potential yield are based on providing a uniform yield throughout the year, whereas virtually all water uses vary seasonally. As a consequence of it, and in order to consider a more realistic situation, they suggested a detailed sequential analysis of the process of runoff storage use, both for making a reliable estimate of required storage and for deriving operation rules of the system.

The water supply in form of runoff time series have been studied extensively. Their description by mathematical models of periodic-stochastic nature of monthly, weekly or daily series has been extensively investigated. The water use time series have not been studied in such details as the water supply time series. Salas and Yevjevich (1972), in studying the actual water demand or water use time series, concluded that the demand series are basically trend-periodic-stochastic series. A need exists for a development of methodology of estimating these parameters and producing the realistic realizations of future samples of water use time series. The lack of these sequences is a likely reason for considering only trend and periodic components in water demand time series. Only the periodic water demand series are used here.

3.3 Potential Techniques for Drought Analysis of Periodic-Stochastic Processes

The first alternative in treating the drought of trend-periodic stochastic series is to remove trends and periodicities in parameters, using either the parametric or nonparametric method of their removal. The procedure in this approach is relatively simple, namely it is assumed that water demand series have both trends and periodicities in basic parameters, with these periodicities being in phase with periodicities in parameters of water supply series. An additional simplification is that they all have the same amplitudes. Llamas and Siddiqui (1969) used this approach for the analysis of a univariate monthly precipitation periodic-stochastic series. The nonparametric method of removing periodicities in parameters was used, and the theory of runs was applied in the drought analysis in case of a dependent stationary time series. It can be shown that the stochastic component of monthly precipitation could be approximated by an independent series for all practical purposes. This fact simplifies the study of droughts for the stochastic component of monthly precipitation in a univariate case. For the bivariate case and removed periodicities in parameters, in this approach the exact expressions for distributions of different runs can be used. However, the run-sums may not have a clear meaning if the general but different standard deviations of the two series are not retained while removing periodicities. Run-lengths can be investigated on the standarized stochastic components without too many problems. For dependent second-order stationary univariate or bivariate series either the exact or approximate expressions of the theory of runs, will produce the properties of droughts. The analysis of droughts for trend-periodic-stochastic processes, by removing trends and periodicities in parameters, depends on the characteristics of demand series.

The second alternative is to use the supply-less-demand series. Since the supply series is periodic-stochastic and the demand series is assumed to be only periodic, in phase with and of the same amplitude as the periodicity in supply series, differences between supply and demand represent a first order stationary process of deficits and excesses. In case of high variability between the low and high flows, the excess-deficit series still can be periodic, in which case the theory of runs of stationary processes may not be meaningfully applied. This approach has the disadvantage of not being adequate when periodicties in demand are out phase with and of different amplitude than the periodicity in supply.

The third alternative, used this paper, is the "drought-magnitude and drought-duration criteria". The magnitude of a drought depends on the demand imposed on the water system. During the planning stage of a water resource development scheme, for example, the choice of droughts for analysis is related to contemplated demand series. As shown by Texas Water Development Board (1971), the severity of the most critical drought affects the selection of the ultimate plan, by influencing decisions on the size and the number of facilities required for optimal performance of a system. The more severe this most critical drought, the larger or more numerous are facilities that are needed to insure an adequate performance of the entire system. Of great interest in the planning process are droughts which require new storage capacity to insure uninterrupted deliveries, or which require importation of water from other sources.

When severity of a drought is studied, special consideration must be given to relation between the drought duration and all the physical storage and other capacities of the system, which are required to meet the demand during the drought period. A drought of a given duration, equal to or longer than the time required to use the storage system from a full to an empty state, will have quite a different effect on the system than a short drought not requiring more than the total water storage.

The magnitude of a drought can be defined as the maximum absolute value of monthly differences between supply and demand over the drought duration. In mathematical terms, this magnitude is:

$$M = \min_t \left(\min \sum_{i=k+1}^{k+t} \frac{X_i - D_i}{t} \right) \quad (2)$$

with X_i the monthly supply (in the case of a system of reservoirs, it is the sum of the monthly inflows to all the reservoirs), D_i is the monthly demand (in case of complex systems, it is the sum of monthly demands at all system demand points), k any starting time point for studying droughts, and t the duration of the critical drought period in samples used for analysis. This concept is analogous to studying the negative run-intensity for univariates or the joint negative-negative run-intensity for bivariates.

Another parameter, proposed by the Texas Water Development Board (1971), is the drought time position, defined by the time of the drought mid-point. For a drought with duration t and the absolute starting time k, the position is:

$$L_t = K + \frac{t}{2} \quad (3)$$

There may be individual months during drought periods when supply exceeds demand. However, effects of these months may not be sufficiently large to overcome the general drought consequences, since all the other months would have significant deficits. Because some sort of flow regulation may always be involved, the storage easily takes care of individual months with small surplus and distributes it over the months of significant deficit. If no regulation is involved, the surplus of these couple of months is simply lost.

This alternative for drought measuring parameters has the basic disadvantage that the theory of runs cannot be easily applied, since the periodicities are involved. It is a somewhat different approach to drought definition. The main advantage over the other approaches to drought definitions is that it can easily treat the cases of demand being out of phase and of different amplitude in comparison with those of water supply.

A fourth alternative is based on a simultaneous generation of annual and monthly series, by jointly preserving their parameters such as the variance, serial correlation coefficients, among the others. Harms and Campbell (1967) used this type of generation, claiming to preserve the normal distribution of annual flows, the log normal distribution of monthly flows, and the serial correlation of annual an monthly flows. The technique is based on the assumption that a first-order linear autoregressive model is adequate to represent the dependence of annual flow series, and that the Thomas-Fiering model is adequate to represent the structure of monthly flow series, with an adjustment being sufficient to take care of the linkage between the annual monthly flow series. Another technique available for this type of generation is the disaggregtion process, outlined by Valencia and Schaake (1973). For the cases considered in this paper, namely the first-order linear autoregressive model for both the annual and monthly series, the technique which considers a sequential generation of annual events with a disaggregation model for generating seasonal, monthly, weekly, or daily events within the year can be adjusted and used. Due to computer storage requirements, these authors suggest first to generate seasonal values and then to repeat the process on a season by season basis to generate monthly values in a second disaggregation step.

For this alternative of simultaneous generation of annual and monthly time series, once samples are generated, the theoretical analysis or approximations in case of dependent processes can be applied to annual series to determine the probabilities of drought runs. For example, if the annual process in inferred to be stationary process having the first serial correlation coefficient ρ_1, then the probabilities of a long drought or probabilities of the longest run-length, say a project of economic life of 50 years, can be determined. For simultaneous generation, a k-year or the longest drought in annual series may be singled out, and the monthly series of this period can be investigated. The annual series permit the identification of critical drought periods to design the system, with the sequential patterns of monthly series studied for these periods. The main advantage of this alternative is the use of a more reliable estimation procedure for probabilities of droughts rather than obtaining these probabilities from less reliable frequencies of historical records.

Further advantage of the fourth approach relates to the use of optimization techniques in design and operation of water resources systems, because, after the critical droughts of given probabilities are determined, the optimization procedures can be applied to parts of monthly series during these critical periods instead of optimization extended throughout the total generated monthly series.

The approaoch of drought magnitude and drought duration criteria, as outlined herein, has the potential to be developed in a technique of drought analysis of periodic-stochastic processes. To demonstrate this potential, a case study has been worked out and presented in the next section.

4.0 A CASE STUDY

The drought analysis of periodic-stochastic processes is complex not only due to periodicities, but also because the number of parameters for both the supply and demand series is much greater than in the case of stationary stochastic processes. The large number of parameters requires a large number of cases to be studied in the general form, and this number can be excessive. As a consequence, no attempt is made here to generalize all cases or to cover some or most of them in this paper. A study study is given only in order to show the use of drought parameters, and therefore, the case study covers a small number of parameters. In spite of simplifications it is thought that the case has a practical significance.

The monthly supply series is assumed to be a periodic-stochastic process, with periodic mean and periodic standard deviation composed only of the 12-month harmonic. The resulting stationary stochastic component follows the first-order linear autoregressive model.

Then $\bar{x}$: the overall mean (2.885), $C_1(\mu)$: the amplitude of the 12-month harmonic in the mean (1.889), $\theta_1(\mu)$: the phase of the first harmonic in the means (θ), $\bar{s}_\tau$: the overall mean standard deviation (1.848), $C_1(\sigma)$: the amplitude of the first harmonic in the standard deviation (0.946), $\theta_1(\sigma)$: the phase of the first harmonic in the standard deviation (θ), and ρ_1: the first serial correlation coefficient (0.5). The independent stochastic component is assumed to follow the three-parameter lognormal distribution.

$$f(x) = \frac{1}{(X-\beta)S_n\sqrt{2\pi}}\exp\left\{-\frac{(\ln(x-\beta)-\mu_n)^2}{2S^2}\right\} \tag{4}$$

With the lower bound $\beta=1.5$, and $\mu_n = 0.2216$ and $S_n^2 = 0.3677$
The monthly demand series is assumed to be a periodic process, with periodicity composed only of a 12-month harmonic. This is a simplification in comparison to reality. Nevertheless, it is a common practice in water resources planning to simplify the complex nature, because quite often the lack of data may not justify the more complex models. The demand model then is

$$D_i = \bar{D} + (1 * \cos(\frac{\pi}{6}\tau + \theta_1^*) \tag{5}$$

with $\bar{D}$ the overall mean (2.50), C_1^* the amplitude of the first harmonic (1.00), and θ_1^* the phase angle of the first harmonic (θ). For supply and demand series in phase, $\theta_1(\mu) = \theta_1^* = 0.0$, otherwise they are different. In this case study, three alternatives are used for phase differences $(\theta_1(\mu)-\theta_1^*)=0.0;\ \pi/2;$ and π.

Since the monthly demand has been shown by Salas and Yevjevich (1971) to be periodic-stochastic processes, the demand could have been modeled as

$$D_i = \bar{D} + C_1^*(\mu)\cos\left(\frac{\pi}{6}\tau+\theta_1^*(\mu)\right) \tag{6}$$

$$+\ \bar{S}_D + C_1^*(\sigma)\cos\left(\frac{\pi}{6}\tau + \theta_1^*(\sigma)\right)\left(\rho_1^*\ \varepsilon_{i-1} + \xi_i\right)$$

with parameters $\bar{D}$, $C_1^*(\mu)$, $\theta_1^*(u)$, $C_1^*(\sigma)$, $\theta_1(\sigma)$, and ρ_1 the independent stochastic component of demand series ξ_1 as the counterpart of that of the supply series. Since results are expected to be similar to those when only the periodic demand is considered, as far as computing the drought characteristics, the case study treats only the periodic demand.

A program was prepared to compute drought characteristics as defined before, namely the drought magnitude (Eq. 2), the drought durations for its given magnitude and the corresponding deficit.

Figures 1, 2, and 3 show supply and demand series for the three cases of phase differences.

Also, figures 4, 5 and 6 show the drought magnitude computed for a set of samples each of 30 years or monthly flows, as well as the cumulative deficit during the drought of a given duration for the three phase differences between the supply and the demand.

Values of drought characteristics as shown in Figs.4, 5 and 6, are presented in Table 1. Figures 4,5 and 6, give an idea of the range of values of the drought magnitude criteria and its duration. The selected values of the drought magnitude criteria and its duration. The selected value corresponds to the maximum deficit. It should be noted that generally the criteria decrease with an increase in duration. The analysis of the values obtained show that phase differences (up to half a cycle) do not influence very much the drought duration or the volume of deficit for the case study. However, it should be recognized that these results apply only to the selected values of the case study and no generalization could be made.

Table 1 : Values for Drought Characteristics of the Case Study with Three Phase Differences between Supply and Demand Series.

Phase difference	Drought magnitude	Duration in months	Volume deficit
0	.6376	43	27.4169
$\pi/2$	.6969	42	29.2717
π	.6734	42	28.2812

5.0 CONCLUSIONS

The present theory of runs is not adequate to treat droughts in periodic-stochastic processes, therefore an alternative type of drought analysis has to be used.

Drought magnitude and drought duration criteria have been used and a case study based on a particular monthly series has been performed. The parameters are not affected significantly by the differences in phases up to a half cycle between supply and demand.

6.0 REFERENCES

1. Askew, A.J., Yeh, W.W.G., and Hall, W.A., 1970, Stream flow generating techniques: a comparison of their abilities to simulate critical periods of drought, University of California, Water Resources Center, Contribution 131, January, 1970.

2. Beard, L.R., 1963, Estimating long term storage requirements and firm yield of rivers, IASH Berkeley Symposium, August, 1963, Pub.63, pp.151-166.

3. Beard, L.R. and Kubik, H.E., 1972, Drought severity and water supply dependability, Journal of the Irrigation and Drainage Division, IR-3, September, 1972, pp.433-442.

4. Cannon, J.J., 1963, Definition of river drought flow characteristics, IASH Berkeley Symposium, August, 1963, Pub. 63, pp.137-150.

5. Gumbel, E.J., 1963, Statistical forecast of droughts, Bulletin IASH, Vol.VIII, No.1, pp.5-23.

6. Harms, A.A. and Campbell, T.H., 1967, An extension of the Thomas-Fiering model for the sequential generation

of streamflow, Water Resources Research, Vol.3.

7. Guerrero-Salazar P, and Yevjevich 1975. Analysis of Drought Characteristics by the theory of Runs, Hydrology Paper No.80, Colorado State University.

8. Llamas, J. and Siddiqqi, M.M., 1969. Runs of precipitation series, Hydrology Paper No.33, Colorado State University.

9. Salas-La Cruz, J.D. and Yevjevich, V., 1972, Stochastic structure of water use time series, Hydrology Paper No.52, Colorado State University, June, 1972.

10. Saldarriaga, J. and Yevjevich, V., 1970, Application of run lengths to hydrologic series, Hydrology Paper No.40, Colorado State University.

11. Texas Water Development Board, 1971, Stochastic optimization and simulations techniques for management of regional water resource systems, Report 131, July, 1971, 129 p.

12. Valencia, D.R. and Schaake, J.C.Jr., 1973, Disaggregation processes in stochastic hydrology, Water Resources Research, Vol.9, No.3, June, 1973, pp.580-585.

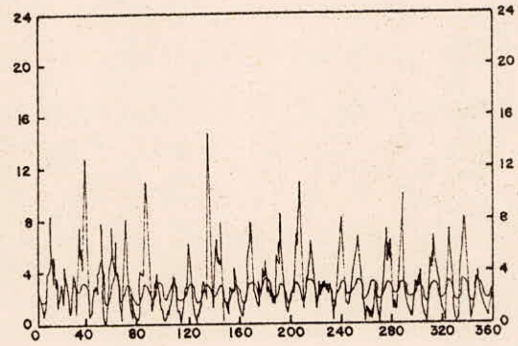

Fig.1.
Supply Series (Periodic-Stochastic) and Demand Series (Periodic) for the Case Study with No Phase Difference$[\theta_1(\mu)-\theta_1^*=0.0]$

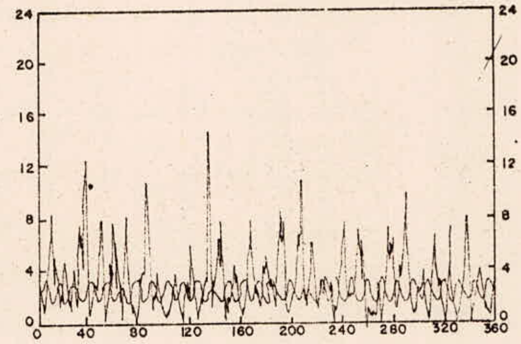

Fig.3.
Supply Series (Periodic-Stochastic) and Demand Series (Periodic) for the Case Study with Phase Difference of $\pi,[\theta_1(\mu).\theta_1^*\pi]$

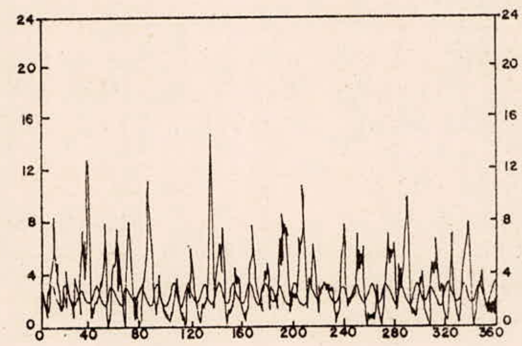

Fig.2.
Supply Series (Periodic-Stochastic) and Demand Series (Periodic) for the Case Study with Phase Difference of$[\theta_1(\mu)-\theta_1^*]=\pi/2$

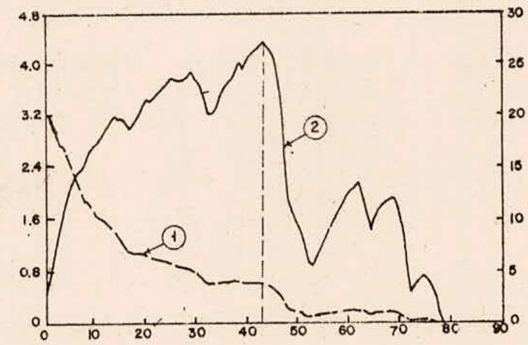

Fig.4.
Drought Magnitude (1) and Corresponding Volume Deficit (2) for Given Drought Duration, Which Correspond to the Case Study of No Phase Difference

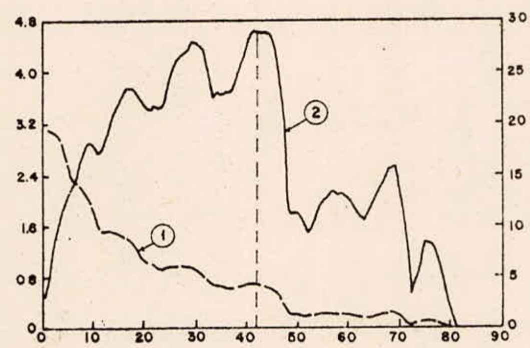

Fig.5.
Drought Magnitude (1) and Corresponding Volumen Deficit (2) for Given Drought Duration, Which Correspond to the Case Study of Phase Difference Equal to $\pi/2$

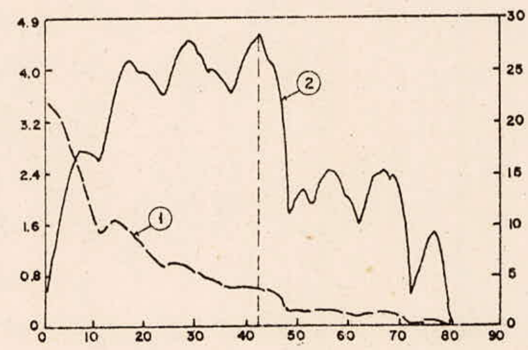

Fig.6.
Drought Magnitude (1) and Corresponding Volume Deficit (2) for Given Drought Duration, Which Correspond to the Case Study of Phase Difference Equal to π