

# Mountainous Convective Storm Rainfall Model: Hydrological Applications

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## INTRODUCTION

The purposes of this paper are:

1. To provide a stochastic system framework for analyzing convective storm rainfall events on mountainous areas.
2. To present examples of utilizing an event-based precipitation model to drive hydrologic models in evaluating watershed response to land use activities and studying the occurrence and severity of droughts.

## PRECIPITATION MODELING

Stochastic precipitation models are characterized by at least two and sometimes three classes of random variables and their probability distribution functions. First, there is the random number of events or alternatively, the interarrival time between events. In this case, an event is defined as one in which a storm center (point of maximum rainfall) occurs within a watershed. Second, there is a description of the event characteristic such as storm depth, duration or maximum intensity for a specified period of time, or a combination of these variables. A third class is the location of the storm center on a particular sub-area of a watershed. In this paper, the event-based precipitation model presented by Fogel and Duckstein (1969) and Fogel and Hyun (1990) is adopted to (1) describe an stochastic modeling framework for analyzing occurrence and magnitude of convective storm rainfall events on mountainous areas, and (2) to illustrate with application the linkage between precipitation models and hydrologic models.

The modeling approach presented here is based on statistical analysis of summer thunderstorm events in southwestern United States (Fogel et al. 1971). In this model, the simulation is conducted on two different interactive scales. First, the overall characteristics of the monsoon season, which include the number of storms per season ( $N$ ), the starting day

of the season ( $D_1$ ), and the length of the season ( $L$ ). Second, the individual storm characteristics, which include the maximum rainfall depth at the storm center ( $R_{\max}$ ), and the interarrival time between two consecutive storms. The number of storms per season is drawn from a Poisson distribution with a mean that is related to the mean elevation of the watershed (Fogel and Hyun 1990). Monsoon season starting date ( $D_1$ ), and length ( $L$ ) are drawn from normal distributions with means and standard deviations that vary regionally (Fogel and Hyun 1990). They are considered to be independent of elevation. Once the seasonal variables are defined for the  $i$ -th year ( $N$ ,  $D$ , and  $L$ ), storms are distributed throughout the season by drawing  $N_i$  times from a uniform distribution. This assumes that storms are equally likely to occur anytime during the season, an assumption that does not always hold. Each storm is located on a particular sub-area of the watershed based on the probabilities of storm center occurrence determined from the size and average elevation of each sub-area. Storm duration and intensity variations are not accounted for in this model. The next step is to generate maximum storm depth at the storm center. One possible probability distribution of  $R_{\max}$  for southwestern United States, is the gamma distribution (Fogel 1979).

## HYDROLOGIC MODELING

Land use activities on mountainous areas have increased worldwide. These activities have allegedly contributed to flooding, soil erosion, desertification, and water quality degradation. Assessment of the environmental impact of these activities is of great concern both before and after they take place. Since land use changes are often contemplated where little or no data are available, it is becoming widely accepted that hydrologic modeling is the only available means of making predictions of the environmental impacts of these activities. Several models have been used for predicting the hydrologic effects of land modification on watersheds. Examples include statistical models, usually based on multiple regression analysis (Bock et al. 1972), regional analysis (Thomas and Benson 1970), simple hydrologic models based on the unit hydrograph method (Sherman 1932), and more complex simulation modeling techniques using prescriptive models such as the Stanford Watershed model (Crawford 1966) and the Sacramento model (Burnash 1973) or descriptive models as described by Beyen (1985). All these models have some limitations when used in conjunction with limited data.

An alternative modeling approach for predicting the hydrologic effects of land modification is to link an event-based precipitation model to a deterministic rainfall-runoff relationship that transforms precipitation inputs into probabilistic distributions of hydrologic variables such as water yield, peak discharge, and sediment yield (Fogel et al. 1974). The development and application of event-based precipitation-driven models start with the definition of an event which is dependent on the intended use of the model and available data. For example, if the intent is to simulate the short-duration thunderstorm type of rainfall and only daily data are available, then an event may be defined as any day during the season when thunderstorms are likely to occur in which measurable rainfall was

recorded. If the concern is for runoff from such storms, the event could be defined as one in which daily rainfall exceeded a certain threshold value in excess of which runoff occurred. Similar definitions may be used for other storm types.

For this paper, we linked the previously described precipitation model to the Soil Conservation Service (SCS) Method (McCuen 1982). Although the SCS method has its limitations it can be used as an initial approximation to rainfall-runoff analysis in ungauged watersheds. The distribution function for rainfall depth,  $R$ , was transformed into a distribution function for storm runoff volume,  $V$ , by using the SCS method (Fogel et al. 1976). The distribution function of  $V$  was combined with the distribution for the number of runoff events per seasons to produce a distribution for the annual maximum flow volumes from which the return period was calculated. The maximal distribution of runoff generated by the model was compared to time series data from an experimental watershed located near Tucson, Arizona (Fig. 1), whose record length is 20 years. The results show a problem often encountered with analyzing short-term records. One extreme runoff value for the 20-year period more than doubled the next highest value. A question arises as to whether this is a sample from the same population as the other values, and if so, what influence should one value have on the predictive capability of a distribution. Note that the model exhibits a greater variance than a type 1 Gumbel distribution fitted to the data, which if nothing else, predicts values that are on the safe side when compared to the time series data. Carrying the above procedure one step further, distribution functions can also be obtained for forecasting snowmelt runoff (Hanes et al. 1977), modeling groundwater discharge along stream beds and sediment yield from watersheds (Fogel et al. 1974).

## DROUGHT ANALYSIS

A necessary component of a drought analysis is specification of the method by which drought events will be abstracted from a time series of precipitation events. The method employed here uses an event-based stochastic model to generate twenty 100-year sequences of summer thunderstorm events for determining probabilities of summer seasons with below-normal precipitation. The effect of elevation, which is of particular importance for the occurrence of thunderstorm rainfall events, was also considered in the analysis. A statistical analysis of the periods with below-normal precipitation was performed for the 20 sequences of synthetic precipitation data. A drought- severity index suitable to the environmental conditions of southwestern United States was defined.

### Drought Index

Recognizing that both the total amounts of seasonal rainfall and the number of events that result in this total are significant indicators of a dry season, we propose the following statistical index as a descriptor of drought occurrence and severity in southwestern United States:

$$S_i = \begin{cases} (1 - R_i/R_0)(1 - N_i/N_0) & \text{if } N_i < N_0 \text{ and } R_i < R_0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where  $S_i$  is the statistical drought-severity index for the  $i$ -th season,  $R_i$  is the total rainfall for the  $i$ -th season,  $R_0$  is the long-term average seasonal rainfall,  $N_i$  is the number of rainfall events for the  $i$ -th season,  $N_0$  is the long-term average number of events per season, and  $I$  is the number of seasons. Notice that by including the number of events per season in Eq. 1, we account for the effect of interarrival time between events in determining drought severity. A drought-severity index equal to 1 defines the most severe seasonal drought in terms of total absence of precipitation ( $R_i = 0$  and  $N_i = 0$ ). The severity of this phenomenon would increase drastically with its prolonged duration over a number of seasons. For this study, the mean and variance of  $R_{\max}$  were assumed to be independent of elevation as available information indicated that this dependency was not very significant. The distribution of storm rainfall was based on storm center depth and circular isohyets determined by a bell-shaped function (Fogel and Duckstein 1969). The values of the simulation statistics used for the different random variables described in the model are shown in Table 1 and are based on a hypothetical watershed of 100 Km<sup>2</sup> in size.

Twenty sequences of one hundred summer seasons were generated following the above criteria for seven different elevations, ranging from 1,000 m to 2,500 m. For each elevation, the total seasonal precipitation was calculated by adding the precipitation for each storm within the season. The average number of storms and the average total seasonal precipitation were computed from the 2000 seasons for each elevation. The drought index for each season was calculated using Equation 1. At each elevation, the probability of drought occurrence was determined as the number of occurrences of  $S_i > 0$  divided by the total number of seasons. The conditional probabilities of drought severity exceeding a specified value (if a drought occurs) were developed by ranking values of  $S_i$  for each elevation and performing a maximum value analysis on the ranked sequences. The result of this analysis is shown in Fig. 2. Multiplying the latter value by the proper value from Table 2 yields the absolute probability of occurrence of a drought with a given severity index. It can be seen from Table 2 that summer droughts, independent of their severity, have the same likelihood of occurrence for all elevations which makes sense as the overall meteorologic conditions should be the same for all elevations. However, Table 2 shows that elevation is an important parameter in determining the severity of summer droughts. The fact that the mean number of storm events/area/season increases with elevation contributes significantly to the observation of decreasing drought severity as elevation increases.

## CONCLUSIONS

The study of various aspects of watershed management requires stochastic inputs to determine the economic and environmentally-sound alternatives under natural uncertainty conditions. The application of an event-based precipitation-driven model is beneficial for this purpose. This modeling approach allows consideration of both extreme wet and dry periods.

A drought analysis performed in this study for the southwestern United States using the described precipitation model revealed the following points:

1. The occurrence and severity of a dry season can be described in terms of a statistical index related to the number of storms and the total precipitation during the season.

2. Periods of below-long-term-average precipitation and number of precipitation events per season have the same likelihood of occurrence for all elevations. However, elevation is an important parameter in determining the severity of summer droughts in southwestern United States.

3. Further research is needed to determine the influence of elevation on the maximum rainfall depth at the storm center ( $R_{\max}$ ), the starting day of the season ( $D_1$ ), and the length of the season ( $L$ ).

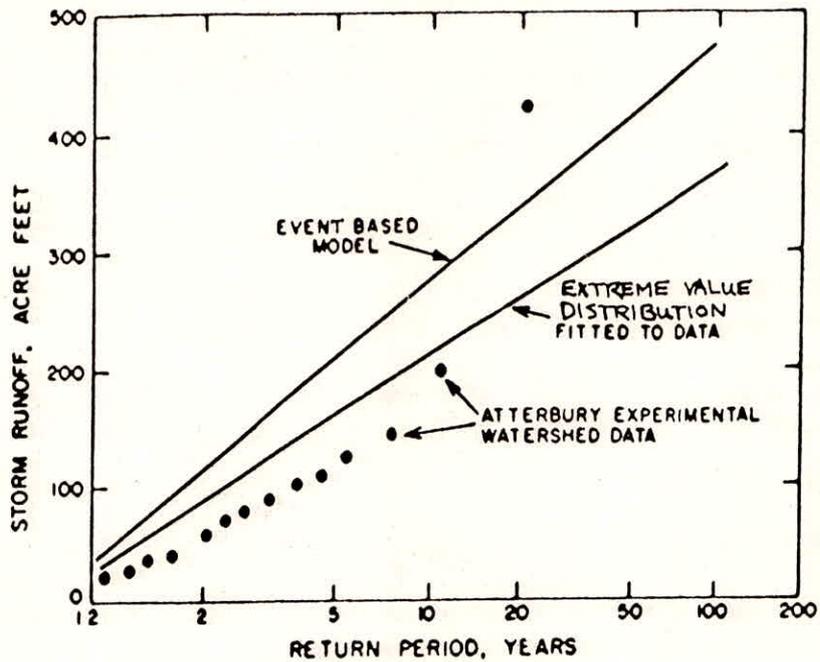


Fig. 1. Distribution of storm runoff: A comparison between event-based model and annual maximum series.

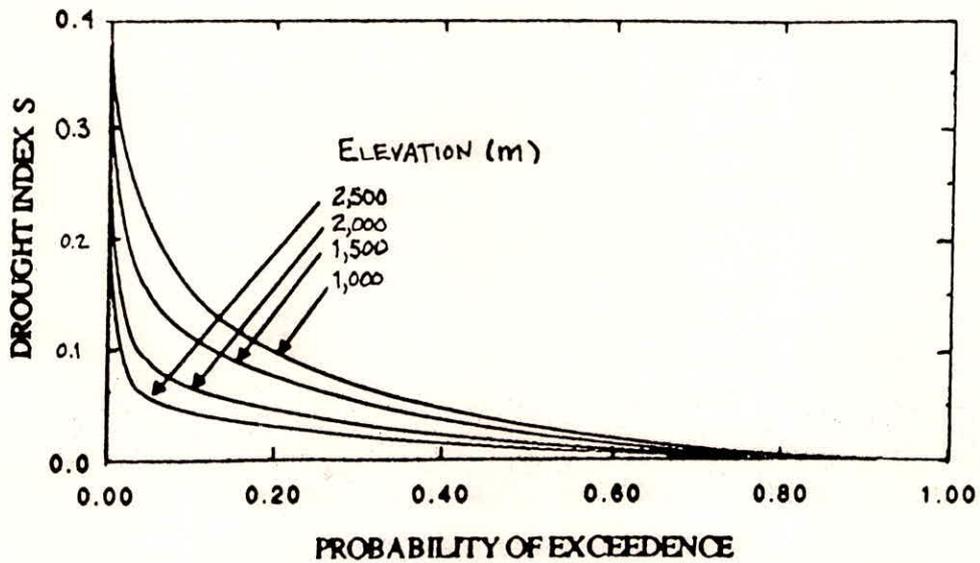


Fig. 2.- Conditional Probability of exceedence for drought severity index given the mean elevation of the watershed.

Table 1. Values of simulation statistics for random variables of event-based precipitation for a 100 Km<sup>2</sup> watershed

Variable	Distribution	Statistics	
		mean	variance
Mean no. events/season	Poisson		
1,000 m elev.		19	-
1,500 m elev.		30	-
2,000 m elev.		41	-
2,500 m elev.		52	-
Max. storm depth	Gamma	39.4 m	299 m <sup>2</sup>
Season length	Normal	85 jd	225 jd <sup>2</sup>
Starting day	Normal	189 jd	144 jd <sup>2</sup>

jd = Julian date

Table 2. Probabilities of drought occurrence and drought Index statistics at various elevations

Elevation (m)	Probability of drought occurrence	Drought Index S		
		mean	std.dev.	max
1,000	0.26	0.021	0.024	0.203
1,500	0.25	0.015	0.018	0.110
2,000	0.25	0.013	0.015	0.084
2,500	0.26	0.012	0.013	0.079

## REFERENCES

- Beven, K. 1985. Distributed models. In: Anderson, M. G. and Burt, T. P. (eds.). 1985. Hydrological Forecasting. John Wiley & Sons, New York, pp. 405-435.
- Bock, P., Enger, I., Malhotra, G. P. and Chisholm, D. A. et al. 1972. Estimating peak runoff rates from ungaged small rural watersheds. Report No. 136, National Cooperative Highway Research Program, Highway Research Board, National Academy of Sciences, Washington, D.C.
- Burnash, J. 1973. A generalized streamflow simulation system. Joint Federal-State River Forecast Center.
- Crawford, N. H. and Linsley, R. K. 1966. Digital simulation in hydrology: Stanford Watershed Model IV. Tech. Report No. 39, Civil Engineering Dept., Stanford Univ.
- Fogel, M. M. 1979. Precipitation in the desert. In: D. D. Evans and J. L. Thames (eds.). Water in desert ecosystems. Dowden, Hutchinson and Ross, Inc., Stroudsburg, PA, pp. 365-391.
- Fogel, M. M. and Duckstein, L. 1969. Point rainfall frequencies in convective storms. *Water Resources Research*, 5(6):1129-1137.
- Fogel, M. M. and Hyun K. K. 1990. Simulating spatially varied thunderstorm rainfall. In: Proceedings, ASCE Intl. Symp., Hydraulics/Hydrology of Arid Lands, San Diego, CA, pp. 513-518.
- Fogel, M. M., Duckstein, L. and Kisiel, C. 1971. Space-time validation of a thunderstorm rainfall model. *Water Resources Research*, 7(2):309-316.
- Fogel, M. M., Duckstein, L. and Kisiel, C. 1974. Modeling the hydrologic effects of land modifications. *Transactions ASAE*, 17(6):1006-1010.
- Fogel, M. M., Duckstein, L. and Musy, A. 1976. Event-based formulation of watershed management. Proc. of ASCE Conference on Environmental Impact of Irrigation and Drainage, Ottawa, Ontario, Canada, pp. 349-373.
- Hanes, W. T., Fogel, M. M. and Duckstein, L. 1977. Forecasting snowmelt runoff: probabilistic model. *Journal of the Irrigation and Drainage Division, ASCE*, 103(IR3):343-355.
- McCuen, R. H. 1982. A guide to hydrologic analysis using SCS methods. Prentice-Hall, Englewood Cliff, New Jersey.



Sherman, L. K. 1932. Streamflow from rainfall by unit-graph method. Eng. News Record  
No. 108:501-505.

Thomas, D. M. and Benson, M. A. 1970. Generalization of streamflow characteristics from  
drainage basin characteristics. USGS Water Supply Paper, Washington, D.C.