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Large-Scale Mathematical Simulation of Snow Life in Mountains

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SYNOPSIS

Theoretical ground of snow cover desription by mathematical model is defined. Non-regular scheme of numerical relief model (NRM) presenting adequate reflection of complex mountainous orography is developed. Computational scheme is given for snow depth estimation at any point of the territory. The developments can be used for snow avalanching, hydrological, recreational and other purposes.

Snow cover description for small montainous basins or separate slopes is the most actual problem of modern snow avalanche studies. Developments related to this problem may also be used for recreational, hydrological and other purposes. Purely empirical description of processes of snow cover generation and evolution requires collection of enormous amount of actual data which is difficult, first of all, due to safety considerations. Remote sensing techniques are only being progressively developed at present. That is why efforts of mathematical simulation of these processes on the base of available empirical data are of great importance.

Simulation of space distribution of snow cover parameters is to be made on the basis of large scale topographic ground (1:10000).

Effect of land surface relief on generation and evolution of hydrometeorological processes and phenomena is explicit and universally recognized. It becomes especially strong for mountainous regions, and, in particular, it appears to be the most important factor for mountainous countries glaciology.

One of the basic notions of snow avalanche studies is the notion of avalanche centre - the place of avalanche origin [1]. Formal approach to this notion definition [5] consists in determination of the ridges line by avalanche centre boundaries. Hereat the avalanche centre becomes corresponding to the watershed basin and determines relief map in contour lines as the base for avalanche centre distinguishing along with estimation of its morphologic and morphometric parameters. Taking the underlying surface effect into account is made by superimposing of characteristics of different roughness lythological and biological elements [4] on the map.

Contour lines are the main medium of relief cartographic information. It is proposed to consider each contour line as even continuous curve from the differential geometry point of view which, in principle, reflects technique of its drawing on the map. It is possible to distinguish special points on this curve: bending points or points of zero curvature and peaks - points of maximum and minimum curvature.

Set of all these points (their x_i, y_i, z_i coordinates) for given limited territory bears adequate information on geomorphological structure of its relief presented on isolines of given scale and section. Different interpolation techniques provide construction of the contour lines by their coordinates, i.e. to solve the main problem of the digital relief model (DRM). That is why it is proposed to define such set of points as one of possible DRMs. Its advantages and shortcomings are to be determined by: 1) method of its definition; 2) development of satisfactory interpolation techniques for the map drawing in contour lines; 3) possibility of different morphologic and morphometric parameters determination for the whole chosen territory, any of its parts or any point of its territory using DRM.

There are three basic techniques: manual, automatic-manual and automatic ones for derivation of the given DRM on the basis of map with contour lines. Manual method presents visual selection of characteristic points (special points) on the contour lines and their coordinates x_i, y_i definition in some conventional Decart coordinate system and z_i coordinate by corresponding contour line height.

Automatic-manual method requires preliminary preparation of cartographic information: contour lines are copied from topographic base on the tracing paper, and special points are marked out on them visually. Coordinates survey is realized by automatic-manual method using specific software on personal computer with graphic adapter. Tracing paper is superimposed on display unit screen, then cursor (marker) is moved up to characteristic point using control buttons while record of coordinates in DRM file is made automatically after a certain button pressing.

Automatic method needs further detailed technological solution but in principle it is supposed to be such as follows: 1)graphical information input using teleobjectives, scanners (with the same cartographic material preparation as for automatic-manual method); 2) tracing of each contour line; 3) selection of characteristic points.

Using DRM for the relief-mapping in contour lines both linear interpolation technique and construction technique using cubic spline are easily realized. Besides, as it is shown in [3] contour lines may be constructed by special geodetic splines of the 3-rd and 4-th order which provides maximum approximation to original relief maps.

Using special triangulation algorithm the computation of morphometric parameters becomes possible for all characteristic points and generated triangulars which provides availability of these parameters for any arbitrary point and areal objects.

Developed non-regular DRM scheme can be realized in different modes and by using numerous technical facilities. It gives the possibility both of the main DRM problem solution and realization of computations and definition of different morphologic and morphometric parameters for any site and point objects. The scheme can be used for the scientific and applied problems solution in processing of graphical representation in isolines in different research and technical fields.

Computations of snow covere parameters are possible during the whole season of snow cover deposition with 3-hour discreteness (standard meteorological period). It's quite satisfactory for the problems of avalanche hazard forecasting. It is important that model computations should be within this time interval. This puts certain demands to the computational algorithms simplicity and software facilities effectiveness. This interval can be prolonged for hydrological purposes.

The problem formulation is presented in [2]. This is the mode of water-physical snow properties developed under the guidance of Yu.M.Denisov. Differential equations of continuity, mass- and heat transfer of complex heterogenic multiphase medium serve as its base.

$$\frac{\partial}{\partial t}(\rho_i \alpha_i) + \nabla(\alpha_i \rho_i \bar{u}_i) = \sum_j \varepsilon_{ji}$$

$$\frac{\partial \bar{u}_i}{\partial t} + (\bar{u}_i \nabla)\bar{u}_i = \bar{g} - \frac{1}{\rho_i}\nabla P_i + \frac{\mu_i}{\rho_i}\Delta \bar{u}_i + \bar{F}_i \frac{1}{\rho_i} \qquad (1)$$

$$c_i \rho_i \alpha_i \left(\frac{\partial T_i}{\partial t} + \bar{u}_i \nabla T_i\right) + \sum_j L_{ij}\varepsilon_{ij} = \nabla\left(\alpha_i \lambda_i \nabla T_i + \alpha_i \bar{J}\right) + \frac{2}{K}\sum_j \frac{\lambda_i \lambda_j \beta_{ij}}{\lambda_i \frac{\alpha_j}{\beta_j} + \lambda_j \frac{\alpha_i}{\beta_i}}\left(T_i - T_j\right)$$

where

- α_i - relative volume of the i-th phase
- ρ_i - density
- $\bar{u}_i$ - mass rate
- ε_{ij} - substance amount transformed from j-th phase into the i-th one
- $\bar{g}$ - gravitational acceleration
- P_i - pressure
- μ_i - dynamical viscosity coefficient
- $\bar{F}_i$ - additional molecular forces vector
- c_i - heat capacity
- T_i - temperature
- L_{ij} - heat of the phase transfer
- $\bar{J}$ - radiant energy flow
- K - structure coefficient

λ_i - heat conductivity
β_i - relative surface

Besides, such parameters as rheological correlation for snow, boundary and initial conditions and number of additional considerations for computation of model parameters are used.

As it is mentioned by model's authors, "analytical solution of problem prescribed at the given stage is impossible due to mathematical methods absence, and numerical ones are very complicated". Indeed, numerical solution of the four-dimensional differential equations of the 2-nd order in particular derivatives with non-homogeneous coefficients is very complicated task. That is why it is necessary to stop at two-dimensional version, i.e. using one time t - coordinate and space z - coordinate. This means that we consider snow cover on elementary horizontal site. It seems that such approach gives little opportunity for avalanching processes study, as avalanching occures directly on slopes and not on horizontal sites. But, firstly, this way is only possible one for model presentations if only the model is not deteriorated in such a way that it can not also give anything, and, secondly, it is supposed to relate its output parameters with needed parameters of the snow cover on slopes using data of field observations.

Such two-dimensional version of the model appears to be operable one. Careful analysis of all its listed states made on the basis of available actual data on snow cover, atmosphere and soil observations provides specifications of some of them. The first specification concerns rheological relation for the snow cover sinking.

$$\nabla \bar{u}_1 = -\lambda^* \frac{1-\alpha_1}{\alpha_1} P_1 \qquad (2)$$

Here λ^* is some constant proportionality coefficient being $\mu_1 = 26 \cdot 10^{17}$ g cm^{-1} day^{-1} at assumed value. It is proposed to make as dimensionless non-constant coefficient.

$$\nabla \bar{u}_1 = -\frac{\lambda^*}{\mu_1} \frac{1-\alpha_1}{\alpha_1} \left(P_1 + \Delta P_a\right) \qquad (3)$$

Then it is proposed to optimize a certain quantity μ_1 and to define correlations between λ^* and other snow parameters and to take air pressure ..rfall into account.

The second specification concerns boundary condition on zero sinking rate u_1 on the soil surface.

$$u_1(0,t) = 0 \qquad (4)$$

Basing on logical considerations it does not take phase transfers on snow-soil interface (i.e. phase transfers in area but not in volume) into account. On the base on set of winter observations it is shown that in Middle Tien Shan conditions (such situation should be observed in analogous conditions of the other physical-geographical regions) melting of lower snow layer occurs on this boundary due to geothermal heating even with isotherm absence in the snow cover, and this melting amount is 10 ÷ 20% of the total winter snow reserves.

$$u_1(0,t)=\begin{cases} 0, & \text{if } \left.\dfrac{\partial T_1}{\partial z}\right|_{z\to 0} < 0 \\ \dfrac{\lambda_{rp} \left.\dfrac{\partial T_{rp}}{\partial z}\right|_{z=0}}{L_{12}\alpha_1\rho_1} & \text{if } \left.\dfrac{\partial T_1}{\partial z}\right|_{z\to 0} \geq 0 \end{cases} \qquad (5)$$

where λ_{rp} - effective ground heat conductivity,

T_{rp} - ground temperature.

Presented modifications and supplements along with the original system assigned for the case with one space coordinate and simplified by the oreder of quantities included into equation, comprise closed system of differential and analytical equations subjected to solution in relation α_i, ρ_i, β_i, u_i, ε_{ij}, T_i, H, ρ_s. The techniques for such a system solution are known and tested (differential schemes).

It is necessary to note that not all coefficients values included into the model are known. Their definition by optimization schemes, estimation of their possible correlations with other parameters requires enormous amount of the field and high-quality data. Such data are available. Unique observational and experimental data obtained by snow avalanche group of SANIGMI glaciological department on the experimental site of Dukant snow avalanche station (Western Tien Shan) comprise the base of this material. We are also faced with the problem of other snow cover paramaters estimation which are not the model output ones. Such parameters are strength and dynamical snow characteristics. Strength parameters (rigidity, shear stress and breaking stress are estimated manually using common mechanical devices which causes substantial subjectivism and errors. Nevertheless, the estimation equations were obtained for correlation between these parameters and simulated ones. Dynamical snow parameters such as Young's modulus and Poisson coefficient or the pair of Lame coefficients are not measured in the snow avalanche practice at all. It is rather unpleasantly, because the role of dynamical wave processes is likely to be very significant. Consequently, it is necessary to conduct special experiments.

For determination of the surface distribution of snow cover parameters series of pits were made on a special simulated slope. Their complete scientific processing is in prospect, but even now it is clear that there are some regularities between parameters evolution and steepness. Besides, computational scheme has been designed for the definition by DRM morphometric parameters of the snow depth at any point of locality and at any time. This scheme is capable to bear both independent and correction load in the whole complex of the problems under solution.

The analysis of data on the snow cover depth by snow stakes has shown basic capability of this significant characteristic's field computation over mountainous territory. The relation between snoe depth defined by a certain remote snow stake on meteorological site is linear one:

$$h_i = a_i h_{mp} + b_i \qquad (6)$$

where a_i and b_i are some correlation coefficients for the i-th stake. This correlation is realized with correlation coefficient r_i. It appears to be that a_i, b_i and r_i depend on morphometric parameters of underlying surface in snow stakes location. Such parameters as absolute height, exposition, azimuth angle, steepness surface curvature on plane, which are defined by DRM, are referred to as morphometric parameters of underlying surface. Commonly speaking, this relations are not linear. Their linearization and further multidimensional regression analysis provides estimation of the precise forms of correlations between a_i, b_i, and r_i coefficients and above mentioned morphometric parameters, and, consequently, it is possible to compute snow depth at any location point for which these parameters are available.

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