International Symposium on
Hydrology of Mountainous Areas
Shimla (INDIA) May 28-30, 1992

Mathematical Modelling of Mountainous River Basins A Case Study in South India

L.C. Kandasamy*, Dr. E.J.James, Dr. H. Suresh Rao*** and Dr. K.Elango*****

*Scientist 'E', Centre for Water Resources Development & Management, Kozhikode
** Scientist E1, Centre for Water Resources Development & Management, Kozhikode
***Indian Institute of Technology, Madras (INDIA)

Synopsis

The study is aimed at the application of selected rainfall- runoff models to chosen river basins of Kerala, a state in India, characterized by their mountainous features. The application of four different rainfall-runoff models is carried out on two types of basins. The models considered are (i) Linear (ii) Linear perturbation (iii) Constrained Linear System and (iv) the TANK model. The comparative performance of the models are evaluated by means of the statistical performance indices computed for volume reproduction, mean, summation and deviation characteristics of the residuals. As a result of this investigation, the Linear Perturbation model is identified as the most suitable model for the steep and relatively small basins as encountered in Kerala which is subjected to seasonal rainfall.

1.0 INTRODUCTION

Hydrological prediction is one of the primary purposes of Applied hydrology and it is useful for the estimation of runoff. To obtain satisfactory estimates, it may become necessary to simulate the physical processes involved. In the past, due to the limitations imposed by manual computation, forecasting had to be very simple. The analysis of rainfall-runoff process acquired further sophistication with the arrival of high speed computers making it possible to model several aspects of the process in great detail. The rainfall-runoff relationships have a role to play in the estimation of water yields with relative ease not only on medium and large computers but also on microcomputers.

There has been intensive research over the past five decades starting with the work of Sherman on unit hydrograph theory. Currently there exists a variety of models suitable for different situations. Highly complex rainfall-runoff models are being developed that take in consideration the physical and geomorphological conditions of the basins, in addition to their hydrometeorological characteristics. An international exercise on identification of most promising models and their domain of application was carried out under the auspices of World Meteorological Organization (WMO,1975).

1.1 National context

The water resource wealth of India as a whole is substantial but its uneven seasonal and spatial distribution leads to problems. The inherent uncertainty in the timing and quantum of rainfall has necessitated building of storage reservoirs for the controlled regulation of river flows. The average annual rainfall for the country is about 1170 mm. It is estimated that the runoff inclusive of that received from snow of Himalayas is of the order of 180 Mha m. Many water resource projects have been implemented raising the irrigation potential to about 61 M ha. and the hydropower installed capacity to almost 16 GW in addition to the provision of regulated water supply to industries and urban requirements.

1.2 Need for the Study

Kerala is a state in south western part of India, with as many as 44 rivers and copious amount of rainfall in its different physiographic regions. Many of these river systems await the establishment of rainfall-runoff models for development of the basins with their diversified hydrometeorological, physiographic and landuse characteristics. The type of the relationship and the computational facility to be employed in developing or identifying such relationships would depend on the available data and the extent of the development conceived in the basin.

At present, the assessment of the surface water potential is made empirically on an annual basis. A better scientific computation based on daily runoff is essential for the proper planning of water resource projects which are operated based on daily and weekly water requirements.

Considering these needs, an attempt has been made in this study to establish the rainfall-runoff relationship using mathematical modeling and to compare selected models by analyzing their applicability to the basins of Kerala (Kandasamy, 1991).

1.3 Objective and Scope of the Study

The specific objective of the study is to apply the following four rainfall-runoff models to selected river basins of Kerala and to carry out statistical evaluation of the performances of the models.

i. Single input-single output linear system model(LIN)

ii. Single input-single output linear perturbation model (PER)

iii.Constrained linear system model (CLS).

iv. Tank model (TANK)

Application of the above mentioned four models to the Kizu basin of Japan as a test case, since this has the distinction of being one of the test-basins considered for evaluation of rainfall-runoff models in an international study conducted by the World Meteorological Organization (WMO, 1975).

2.0 LITERATURE REVIEW

2.1 Definition of Mathematical Model

The diverse and hybrid approaches to the study of hydrologic problems, as evident from the proliferating number of books and periodicals in hydrology (Singh, 1989), can be broadly divided into physical science approach and systems approach. The former is referred to as a basic, pure, causal, dynamic or theoretical approach; and the latter as an operational, applied, empirical, black box or parametric approach.

The systems approach is utilized for the explicit purpose of establishing an input-output relationship that can be used for reconstructing past events and then for prediction of future events. According to Clarke, a mathematical model is a simplified representation of a complex system in which the behavior of the system is represented by a set of equations, including logical conditions, expressing relation between variable and parameters.

2.2 Necessity of Rainfall-Runoff Models

Rainfall may be considered as the primary and independent hydrological process. Other hydrological processes such as runoff, infiltration, evaporation, etc are from this considera-tion, dependent on, and derivable from the rainfall process. Among the dependent processes, runoff is probably the most important in the sense that this process determines the magnitude as well as the spatial and temporal distribution of all surface flows. Historical information on surface flow, more specifically on streamflow, is generally of limited availability for a number of river basins

Most of these models have been developed by agencies to apply them for specific conditions. It is quite common to find the models widely applied by users not associated with their development and only few attempts have been made for the systematic comparison of models. With large array of variety of models available, it would seem obvious that these models should be objectively and critically reviewed.

WMO initiated a project for an internationally coordinated intercomparison of the conceptual models for operational hydrological forecasting. Several graphical and statistical verification criteria were used in the evaluation and intercomparison of the simulations produced by the selected models. The models used for intercomparison purpose were: (i) CBM model of Commonwealth Bureau of Meteorology, (ii) GIRARD I model or Orstom (Paris), (iii) Serial Storage Type model, Tank I of National Research Centre for Disaster Prevention, Tokyo (iv) Serial Storage Type Model, Tank II (v) Flood Forecasting model (IMH2-SSVP) of Institute of Meteorology and Hydrology (Bucharest), (vi) SSARR model of US Corps of Engineers, (vii) NWSH model of National Weather Service, (Maryland), (viii) SRFCH model of National Weather Service, (California), (ix) HMC model of Hydrometeorological Centre of USSR, and (x) Constrained Linear Systems model of IBM Scientific Centre, Pavia.

3 SELECTION OF BASINS AND RAINFALL-RUNOFF MODELS

Out of the large number of models that nave been reported in the literature, four models are identified and selected, Keeping in view the nature of basins, climatology,data availability and the nature of purposes the model results would be expected to serve. In this regard, the study carried out by World Meteorological Organization is adopted as a guideline (WMO, 1975).

3.1 Selection of Basins

Kerala State is situated on the south-west coast of the Indian peninsula between 8°18'and 12°48'north latitudes and between 74°52' and 77°22' east longitudes. The State has a total area of 38,864 km^2. The coastline of the state is about 550 km long. The width of the state nowhere exceeds 100 km and it narrows down to just 12 km towards the south, the average width being about 70 km.

3.1.1 Meteorology

India Meteorological Department has prepared comprehensive maps showing the principal features of rainfall over India. The annual rainfall in the lowland ranges from 900 mm in the south to 3500 mm in the north of Kerala. In the midland, annual rainfall ranges from 1400 mm in the south to 4000 mm in the north. In the highlands, annual rainfall varies from 2500 mm in the south to about 5000 mm in the north. The rainy seasons in the state are South-west monsoon (June-September) and North-east monsoon (October -November). The average annual rainfall in the state is 2615 mm. The morphology of the Ghats is one of the decisive factors in the rainfall pattern.

The preliminary studies indicated that the linear regression of monthly rainfall and discharge values and the multiple linear and power regressions can give in general an approximate estimate of monthly discharges and peak discharges respectively.

In order to identify a manageable number of basins say, about five for the present study, the selected basins are to be representative covering the range of the physical characteristics encountered in Kerala. These characteristics include, the average yield, mean slope and maximum daily discharge. Their length ranges from 12 to 152 km, with the area from 58 to 1951 km^2, slope from 2 to 63 x 10^{-3} and maximum daily discharge from 3 to 737 Mm^3 day^{-1}.

Chaliyar river is one of the large rivers of Kerala, having a length of 169 km and a catchment of 2923 km^2. The Kuttiyadi river is a medium sized river having a length of 74 km and catchment area of 583 km^2. The Chaliyar river has ten basins gauged systematically and the Kuttiyadi river has one such gauging station. By considering the hydrological and geomorphological nature and data availability of all the basins, four basins of Chaliyar and the Kuttiyadi basin are chosen for this study.Chaliyar river originates from the Ilambaleri hills in Gudalur taluk of Nilgiri district in Tamilnadu, at an elevation of 2,066 m above mean sea level. It has a total drainage area of 2,923 km^2. It has a length of about 170 km and confluences with the Arabian sea at Beypore.

The Kuttiyadi is one of the medium rivers of Kerala state. It originates from the Narikota ranges situated on the western slopes of the Wynad hills at an elevation of 1,220 m above mean sea level. The river has a length of 170 km and drains an area of 583 km^2. In the initial stage, the river is very steep and at the Orakuzhi falls about 5 km from the origin it loses height, by about 680 m in 3.5 km of length. It joins the Arabian sea at Kottakal.

Kizu basin is one of the six standard basins considered by the World Meteorological Organization (WMO, 1975) for the intercomparison of ten different models for their performance. In the present study this basin is taken up as a bench mark basin for calibrating the models. The Kizu river is located in Kiuki region in Japan, having an area of about 1445 km^2.

3.2 Selection of Models

From the consideration of models, it is decided to choose four models, two of which are the hydrologic models used in the WMO study and the other two being the relatively recent models of University College Galway (Kachroo, 1989). They are as follows:

1. Linear model (LIN)
2. Linear perturbation model (PER)
3. CLS model (CLS)
4. Tank model (TANK)

3.2.1 Linear Model (LIN)

In the Linear model the input - output relationship is expressed in terms of the pulse response as: considering m number of time intervals.

$$Y_i = \sum_{j=1}^{m} x_{i-j+1} h_{lj} + e_i \qquad (1)$$

where h_{lj} refers to the pulse response,

e_i is the error term,

i is the index representing the discretized value i x t

for the continuous or historical time variable t,

j is the index of the discretized interval j x t

representing the continuous form of time difference,

t is discretization interval; and

m is the maximum number of time intervals with which a pulse of rainfall gets completely disposed off as a runoff at the gauging station.

Criteria which express model accuracy are generally linked with the objective used for optimizing its parameters. A commonly used objective function even for non-linear or conceptual models is minimizing the sum of squares of differences between the observed (y) and the estimated (y') discharge, by summing up over whole of the calibration period. The model efficiency is defined as R^2 analogous to the coefficient of determination as in linear regression the complement of the proportion of the initial variance represented by F.

3.2.2 Linear Perturbation Model (PER)

The Linear Perturbation Model confines the dependence on linearity to the perturbations and utilizes the input and output to a portion of the system of the information on observed seasonal behavior (Kachroo, 1989).

For a single input series the Linear Perturbation Model may be described by:

$$v_i = \sum_{j=1}^{m} u_{i-j+1} \, h_{pj} + e_i \qquad (2)$$

$i = 1,2, \ldots\ldots\ldots n$

where $v_i = y_i - y_d$

$u_i = x_i - x_d \quad d = 1,2,\ldots\ldots\ldots 365.$

3.2.3 C.L.S.Model

The concept of a time-invariant linear system with constraints is utilized in the CLS model. A further modification of this method was made by Todini and Wallis (1977) to account for a time-variant non-linear system. This was accomplished by introducing multiple-impulse responses for the catchment based on an antecedent precipitation index (API) and threshold level concept.

The linear system of CLS model with N inputs can be defined as:

$$y_i = \sum_{j=1}^{m} x_i * h_{cj} + e_i \qquad (3)$$

where h_{cj} is a pulse response vector

y_i is vector of length m of discrete outputs sampled with time interval of dt,

e_i is residual vector; and

x_i is precipitation matrix of order (m x Nkl)

in which m = length of input or output vector,

kl = length of each individual pulse response.

The solution of above equation used in CLS is obtained by least square fitting method, giving the objective of minimization of the sum of the squares of the components of the residual vector.

3.2.4 TANK Model

The TANK model was developed as a simple conceptual rainfall-runoff model for simulation of flood events and daily runoff of a basin (Sugawara 1967). The daily rainfall-runoff analysis model applicable to humid basins is assumed to consist of four tanks laid vertically in series.

Herein the action of a drainage basin is conceptualized to consist of flow in and out of horizontal slices. Each slice defined with its areal and vertical extent is assumed to behave as a tank or linear reservoir. The outflow rate at any instant is proportional to the storage driving that outflow. The outflow may be either lateral through a side vent being used for representing this, or vertical represented by the bottom vent in a tank. These tanks are connected serially. The lateral flows add up to direct runoff and the vertical flows move to the next lower tank.

3.2.5 Data Requirements of Models

For the linear and linear perturbation models the requirements of data include:

1) Daily average rainfall over the basin in mm.

2) Daily discharge at the outlet of the basin in Mm^3 day^{-1}

3) Total number of data and the leap year information.

4) Area of basin in km^2.

5) No. of data to be used for calibration and the rest of the values will be automatically utilized for verification.

6) No. of days of memory for pulse response function.

The CLS model computes the pulse response function of the basin utilizing the self optimization routine and the data required for this model are the same as above with the following additions.

1) Daily evaporation values computed for that basin, if the same is to be considered.

2) Kernel length in no of days, similar to memory length of linear and linear perturbation models.

The TANK Model requires the following additional data.

1) Daily mean evaporation values of the basin. If the observed evaporation values are not available for the period under consideration, monthly mean of daily evapotranspiration values may be provided.

3.2.6 Evaluation of Model Performance

In the present study the super-set made up of following fourteen numerical verification parameters out of which seven are adopted from the four models and seven are new ones. is taken up for evaluation of the models.

1) Mean of Residuals

2) Mean of Absolute Residuals

3) Standard Deviation of Residuals

4) Maximum Positive Residual

5) Maximum Negative Residual

6) Maximum Observed Discharge

7) Maximum Estimated Discharge

8) Percentage Error between the Maximum Discharges

9) Sum of Observed Discharges

10) sum of Estimated Discharges

11) Volume Difference

12) Model Efficiency

13) Coefficient of Persistence

14) Reproduction Index

Reproduction Index (RI) is a factor independent of the regime and aims at expressing the proportion of the number of estimated values that lie within a tolerance limit of observed values.

Graphical Comparison

The model performance can also be assessed by graphical plots in addition to evaluation of numerical parameters. The linear scale plot of estimated and observed hydrograph, scatter diagram and double mass plot are utilized for this purpose.

4 DISCUSSION OF RESULTS

4.1 Preliminary Studies-Choice of Memory Length

The investigations with the data of different years as well as with different basins lead to the conclusion that a memory length of 3 days for LIN and PER models, and 5 days for the CLS model are satisfactory for the Kerala basins. Similar application for the Kizu basin of Japan leads to the choice of a memory length of 5 days for all the three models.

4.2 Role of Calibration Length

In the present study for the steep and small basins of Kerala, it has been found that longer period of calibration does not appreciably improve the performance. For the Ariakode basin, for the PER model the mean and standard deviation of residuals, volume difference and model efficiency do not improve significantly with the increase in the number of years of data used for calibration.

4.3 Results of Kizu Basin

The results of WMO study with TANK and CLS models are compared with those obtained in the present study. The hydrologic data of the Kizu basin for the years 1963 to 1965 are adopted in the study. The hydrographs, scatter diagram and double-mass plot of the observed and estimated discharges obtained from verification for CLS and TANK models by WMO and the present studies compare favourably.

4.4 Results of Kerala Basins

The hydrological model studies are conducted for the four conceptual models for the five selected river basins of Kerala and typical results of Ariakode Basin are presented.

4.4.1 Graphical Comparison

Out of the hydrographic for the four models shown in fig. 1 to 6, the PER model leads to an equally good reproduction of both large flows as well as lean flows where as in the case of TANK model the peak flows are not predicted properly and this is also brought out by the scatter diagram. In general, the PER estimates are closet to the matching line than the estimates from the other three models. In verification mode, the PER model has the overall better assessment than the rest except for the peak reproduction.

4.4.2 Numerical Performance Indices

From the numerical values, the PER model gave the least volume balance, maximum model efficiency, low coefficient of persistence and high reproduction index. Thus during calibration as well as verification of Ariakode, the PER model was considered to be better than the other models.

5 SUMMARY

As a result of this investigation, the following conclusions are drawn.

1. The numerical performance indices and the graphical representations such as super imposed observed and estimated hydrographs, scatter diagram and the double mass graph adopted for the inter-comparison of rainfall-runoff models are found to be very effective.

2. Using the data of Kizu river of Japan, the CLS and TANK models performed satisfactorily. It is conformed from the study, that quite comparable results are obtained as that of the WMO study for the Kizu basin.

3. From the results of Kerala basins, it is found that in calibration the Linear perturbation model performs better than the others; however in verification, the linear perturbation model gives better overall performance for 50%, the linear model for 30% and the TANK model for 20% of the station years of data.

4. It is recommended that the linear perturbation model be preferred for application to small, mountainous catchments subjected to monsoonic rainfall.

6 REFERENCES

1. Kachroo R.K (1989), "Lecture notes on River Flow Forecasting workshop. Part I, Discussion of the Principles" University College, Galway, Ireland. I, 1-14.

2. Kandasamy L.C., (1991) "Application of Rainfall-Runoff Models to Mountainous River Basins" Ph.D. Thesis Department of Civil Engg.,Indian Institute of Technology, Madras, India.

3. Singh V.P. (1989) "Hydrological systems Watershed Modelling." II. Prentice Hall, New Jersey. 246-259.

4. Sugawara, M.(1967). "The Flood Forecasting by a Series Storage Type Model." International Symposium on Flood and Their Computation, Leningrad, August.

5. Todini, E., and Wallis J.R., (1977). "Using CLS for Daily or longer period Rainfall-runoff modelling." in Mathematical models for Surface Water Hydrology. edited by T.A.Ciriani, et al. John Wiley, 149-168.

6. WMO (1975) "Intercomparison of Conceptual Models used in operational Hydrological forecasting." Operational Hydrology Report 7. Geneva, Switzerland.

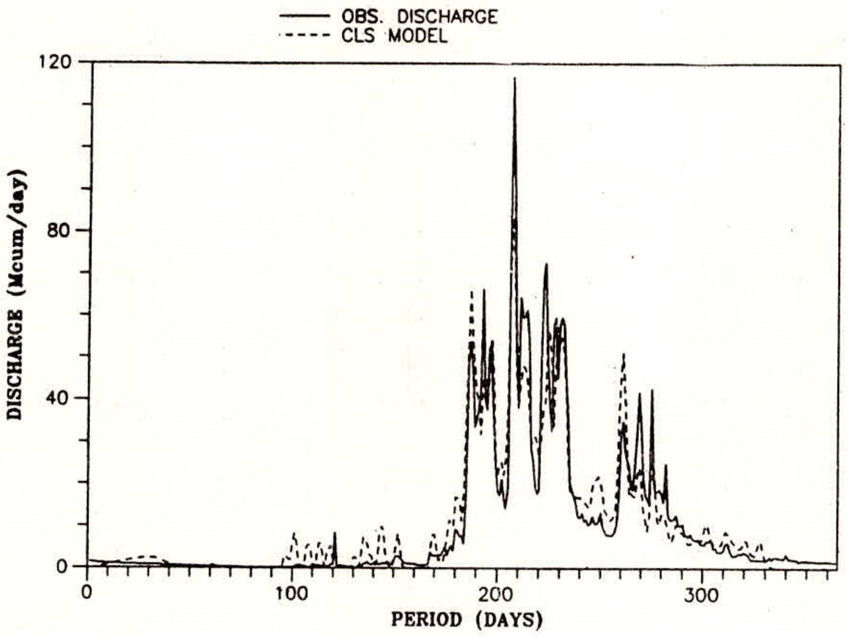

FIG 1 HYDROGRAPH ARIAKODE–CALIBRATION–1974

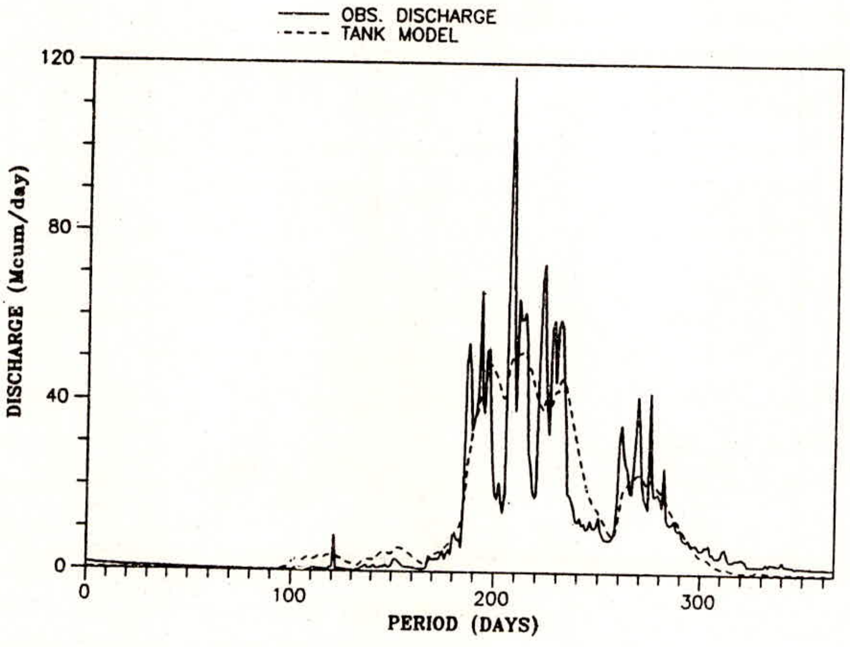

FIG 2 HYDROGRAPH ARIAKODE–CALIBRATION–1974

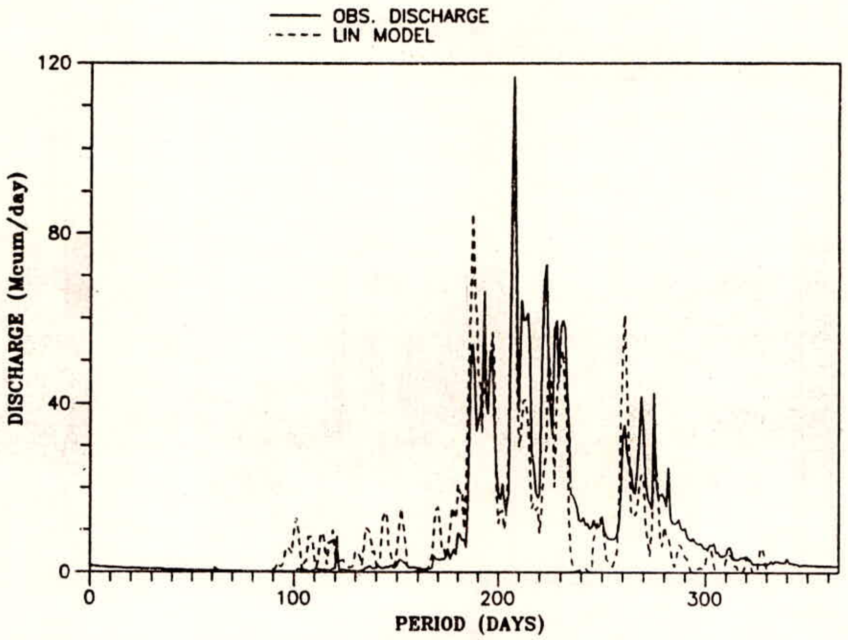

FIG 3 HYDROGRAPH ARIAKODE-CALIBRATION-1974

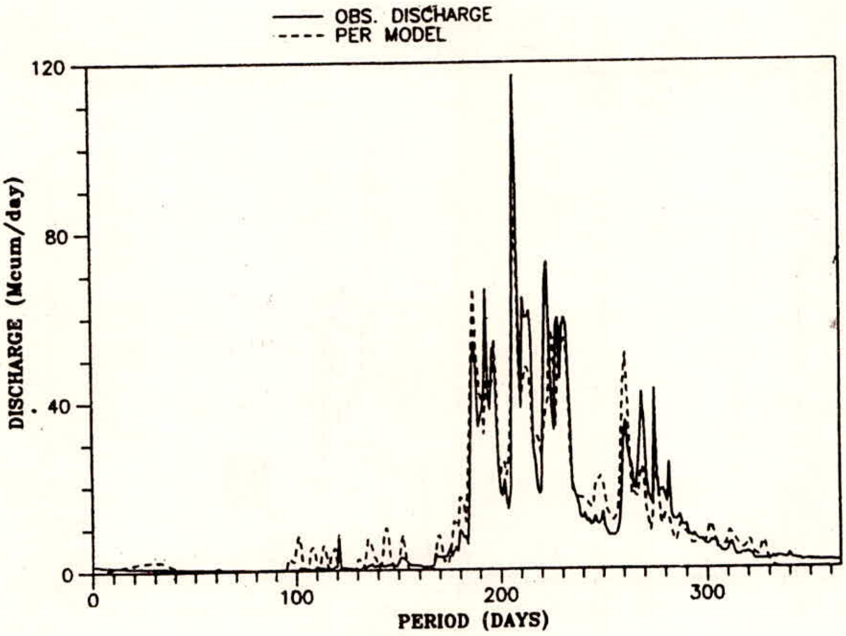

FIG 4 HYDROGRAPH ARIAKODE-CALIBRATION-1974

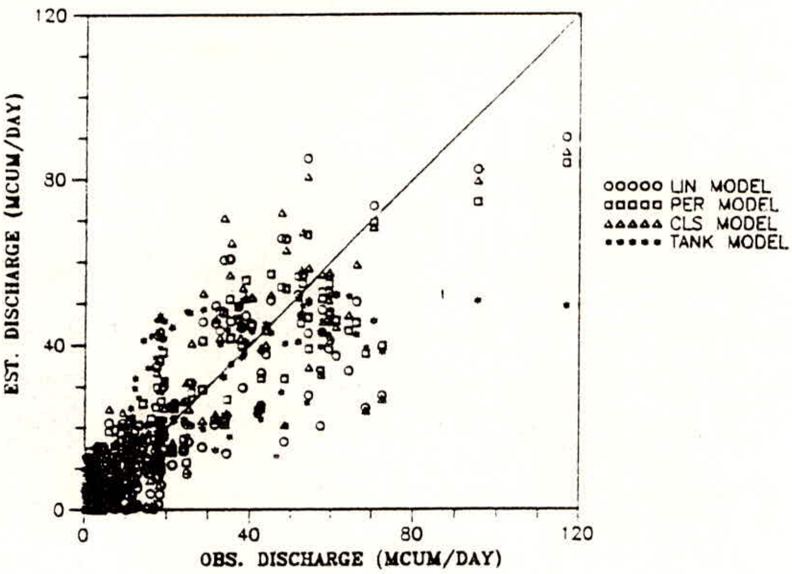

Fig5. SCATTER DIAGRAM ARIAKODE CALIBRATION -1974

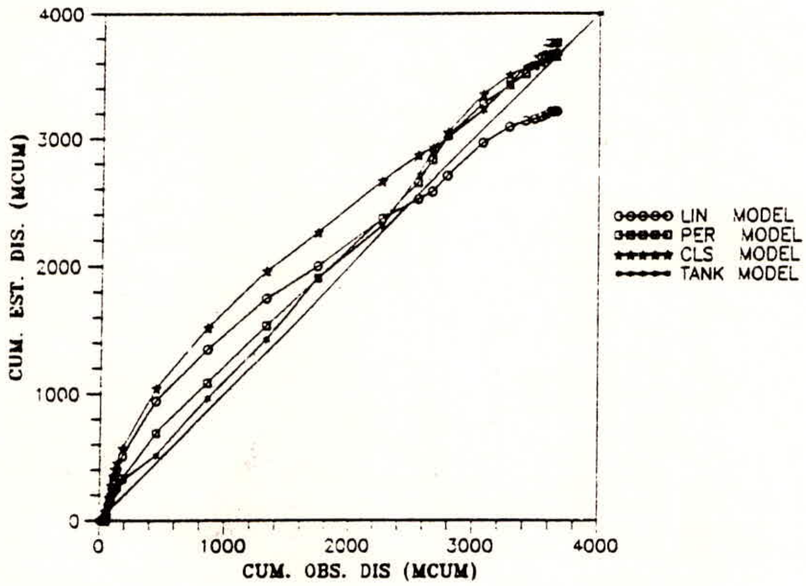

Fig6. DOUBLE MASS GRAPH ARIAKODE CALIBRATION -1974