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Kinematic Wave Modelling Where Do We Go From Here?

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The kinematic wave model of surface water hydrology is reviewed with a look towards the future. The paper focuses on three areas of concern: (1) the applicability of kinematic waves, (2) the role of numerical diffusion in kinematic wave modelling, and (3) the nature of kinematic shock. A brief discussion on the role of the Vedernikov number in flood wave diffusion is included. The paper closes with an attempt to answer the question: Where do we go from here in kinematic wave modelling?

INTRODUCTION

The hydrology of surface waters in mountainous areas is characterized by steep slopes. Flow in steep channels is governed primarily by the gravitational and frictional forces (i.e., those associated with the channel bed slope S_o and friction slope S_f, respectively), and to a much lesser extent by the force originating in the flow depth gradient (i.e., the water surface slope minus the channel bed slope), or by the inertial force. The kinematic wave is a shallow water wave model that considers only the gravitational and frictional forces.

The theory of kinematic waves dates back to the middle 1950's (Lighthill and Whitham, 1955). In the last three decades, its application to overland flow and streamflow has gained considerable momentum (Wooding, 1965; Woolhiser and Liggett, 1967; Ponce and Simons, 1977; Hydrologic Engineering Center, 1990). There is a substantial body of knowledge on the kinematic wave, and papers continue to appear in the literature describing what the model can and cannot do (Hromadka and DeVries, 1988; Ponce, 1990a). There is, however, still some misunderstanding about the precise role that kinematic waves play in surface water hydrology.

Current areas of concern focus on the following issues: (1) Are kinematic waves applicable to mountain streams and well as to alluvial rivers? (2) Can the kinematic wave describe physical diffusion? If so, under what circumstances? (3) Under what conditions will the kinematic wave steepen to the point where it becomes a kinematic shock? This paper answers these as well as the following question: Given the current status of kinematic wave modelling, where do we go from here?

KINEMATIC WAVES

The kinematic wave can be defined in more than one way. First and foremost, a kinematic wave is a wave that transports mass, in contrast to the inertial or "gravity" wave of classical mechanics, which transports energy. In flood hydrology, a kinematic wave is characterized by the existence of a *one-to-one* relationship between discharge and stage. In surface water hydrology, a kinematic wave is a shallow water wave that considers only the gravitational and frictional forces and neglects the forces arising from the flow depth gradient and inertia.

These definitions are all related. To put them in the proper perspective, we turn to Lighthill and Whitham (1955), who, in introducing the concept of kinematic wave, saw fit to subtitle their paper *Flood Movement in Long Rivers*. Flood waves transport mass; kinematic waves also transport mass. However, while flood waves are kinematic in nature, not all kinematic waves are flood waves. To clearly distinguish between flood waves and kinematic waves, we explore Lighthill and Whitham's subtitle a little further. What is a long river? Surely, they did not mean to imply that the kinematic wave could be used only for long rivers. If that is the case, the kinematic wave could not be applied to mountain streams, which are short compared to most alluvial rivers. We now know that kinematic waves apply to both "short" mountain streams as well as to "long" alluvial rivers.

The resolution of this conflict was made possible by the work of Ponce and Simons (1977), who identified the length parameter describing the applicability of kinematic waves. In fact, the latter is controlled, not by the "length" of the river, or by the length L of the shallow wave, by rather, by the dimensionless ratio L_o/L, where L_o is a reference channel length (a function of channel bed slope and frictional characteristics), defined as the length of channel in which the reference flow drops a head equal to its depth: $L_o = d_o/S_o$, with d_o= reference flow depth, and S_o= channel bed slope. According to Ponce and Simons (1977), a wave is kinematic if the dimensionless wave number $\sigma = (2\pi/L)L_o$ is sufficiently small. For a given Froude number F_o [$F_o = u_o/(gd_o)^{1/2}$, in which u_o= mean flow velocity and g= gravitational acceleration], the smaller the value of σ, the more kinematic the wave is. Therefore, it is neither L_o nor L that determines whether a wave is kinematic, but rather their ratio L_o/L.

The concept of reference channel length L_o is clear, provided a value of reference flow depth can be established, which is usually the case. The same cannot be said for the wavelength L, which needs to be converted to the temporal domain if it is going to be of any practical use in hydrology. (Understandably, hydrologists are reluctant to relate to the concept of flood wavelength, and prefer instead to describe flood waves in terms of flood and/or stage hydrographs). Since $L = cT$, where c is the wave celerity and T is the wave period, the ratio L_o/L can be expressed as: $L_o/L = d_o/(S_o cT)$. We use Seddon's law (Seddon, 1990; Chow, 1959) to express the kinematic wave celerity c in terms of the mean flow velocity: $c = (3/2)u_o$, applicable to Chezy friction in wide channels. Therefore, the ratio L_o/L can be expressed as: $L_o/L = (2/3)d_o/(TS_o u_o)$. Furthermore, assuming that the flood wave period T (a concept foreign to flood hydrologists) can be expressed in terms of the more familiar flood wave time-of-rise t_r, say $T = 2t_r$, then: $L_o/L = (1/3)d_o/(t_r S_o u_o)$.

While t_r and S_o tend to vary within a broad range in nature, d_o and u_o are usually restricted within a narrow range. In fact, the flood wave time-of-rise t_r can be as short as 5 to 15 minutes in small steep catchments, and as long as 3 to 6 months in large mild-relief catchments. (To give an extreme example, the time-of-rise of the Upper Paraguay river at Porto Murtinho, Brazil, is approximately 6 months). The channel bed slope S_o typically varies between S_o= 0.1 (or steeper) in mountain stream situations, and S_o= 0.00001 in some deltaic and tidal settings. Thus, in general, the ratio L_o/L is inversely related to the product $t_r S_o$. It is this latter product that determines whether a kinematic wave is applicable: For a given Froude number, the larger the product $t_r S_o$ (and therefore, the smaller the ratio L_o/L), the more kinematic the wave is.

In light of the preceding considerations, the meaning of Lighthill and Whitham's subtitle *Flood Movement in Long Rivers* is now fully elucidated: The adjective "long" should be construed as referring to a large $t_r S_o$ product. This implies that either t_r or S_o, or both, should be large. Experience reveals that Mother Nature has contrived not to make these two parameters large simultaneously. Either the time-of-rise t_r is long (as in the case of a large mild-relief catchment), or the channel bed slope S_o is steep (as in the case of a mountain stream, or alternatively, steep overland flow), but usually not at the same time. This behavior confirms the wide range of field situations in which the kinematic wave is applicable: for *both* steep and mild catchments, and *both* fast-rising and slow-rising hydrographs, provided the product $t_r S_o$ is sufficiently large.

Ponce et al (1978) developed a criterion for the applicability of kinematic waves, and subsequently modified it for practical applications (Ponce, 1989). The criterion states that for a shallow water wave (whether flood wave or overland flow wave) to be kinematic, it should satisfy the following dimensionless inequality: $N = t_r S_o (u_o / d_o) > 85$. The larger the value of N, the more kinematic the wave is. For instance, if $t_r = 6$ h, $S_o = 0.01$, mean velocity $u_o = 2$ ms^{-1}, and flow depth $d_o = 1$ m, it follows that $N = 432 > 85$, confirming that this wave is kinematic. According to our definitions, this wave will have the following properties: (a) it will transport mass, (b) it will not diffuse appreciably, (c) it will describe a one-to-one relationship between discharge and stage at any cross-section, and (d) the forces arising from the flow depth gradient and inertia will be so small so as to be negligible compared to the gravitational and frictional forces.

DIFFUSION WAVES

The specification of a one-to-one relationship between discharge and stage, a key trait of the kinematic wave, imposes a significant physical and mathematical constraint: The wave cannot diffuse; i.e., it can travel downstream and transport mass in the process, but it cannot dissipate (i.e., spread in space and time) its discharge or stage. This limitation of the kinematic wave is grounded in the mathematics: The neglect of the flow depth gradient and inertia terms results in a first-order partial differential equation governing the motion. This equation cannot describe diffusion, since diffusion is a second-order process. From a physical perspective, the one-to-one stage-discharge relationship implies that wave diffusion is clearly out of the picture, since wave diffusion is caused by the presence of a loop (however small!) in the stage-discharge rating.

Since in nature there exist shallow water waves which do diffuse--although in small amounts--the theory of kinematic waves is incomplete without a means of incorporating this important diffusion mechanism. Lighthill and Whitham (1955) clearly saw this when they suggested the extension of kinematic waves to the realm of *diffusion waves*, i.e. of kinematic waves that incorporate a small amount of diffusion. To accomplish this, the mathematics of kinematic waves is modified to include the flow-depth gradient term, while still excluding the inertia terms. This significant extension allows the description of looped stage-discharge ratings, and consequently, of the diffusion of kinematic waves, properly now, *diffusion waves*. To put it in a nutshell: diffusion waves are still kinematic in nature; they still transport mass; however, unlike kinematic waves, diffusion waves have the capability to undergo small amounts of *physical* diffusion.

This physical diffusion is confirmed by theory and experience: As long as the flow depth gradient is not negligible, it will produce a looped stage-discharge rating for every shallow wave, which will in turn cause the wave in question to dissipate as it travels downstream. In practice, as the channel bed slope S_o decreases (as the flow moves from mountain streams to alluvial rivers), the friction slope S_f decreases accordingly (as channel roughness typically decreases in a downstream direction), and the flow depth gradient becomes increasingly too important to be disregarded. Intuitively, while kinematic waves are seen to apply to mountain streams, diffusion waves are seen to apply to valley streams and alluvial rivers. A rule of thumb, validated by experience, says that if the channel bed slope is greater than 1 percent ($S_o > 0.01$), the wave is most likely to be kinematic (and to feature a one-to-one relationship between discharge and stage, and not diffuse appreciably). If the channel bed slope is less than 1 percent, the wave may not be kinematic; it may be a diffusion wave. If so, it will feature a looped stage-discharge rating and show a small but appreciable amount of diffusion.

Ponce (1989) has presented a practical criterion for the applicability of diffusion waves. The criterion states that for a shallow wave (whether flood wave or overland flow wave) to be a diffusion wave, the following dimensionless inequality should be satisfied: $M = t_r S_o (g/d_o)^{1/2} > 15$. For instance, if $t_r = 6$ h, $S_o = 0.001$, and flow depth $d_o = 1$ m, it follows that $M = 67.6 > 15$, confirming that this wave is a diffusion wave. According to our definitions, this wave will have the following properties: (a) it will transport mass, like the kinematic wave; (b) it will diffuse appreciably, unlike the kinematic wave; (c) it will describe a looped stage-discharge relationship at any cross-section, and (d) the force arising from the flow depth gradient can no longer be neglected.

It should be noted that in the example of the previous section, had the channel bed slope been $S_o = 0.001$, then $N = 43.2$, and the wave would not have qualified as a kinematic wave. However, in the example of this section, if the slope is $S_o = 0.01$ instead, then $M = 676$, and the wave would still qualify as a diffusion wave. It is concluded that while the kinematic wave model *does not apply* to diffusion waves, the diffusion wave model *does apply* to kinematic waves. In other words, the theory of diffusion waves (represented by the diffusion wave equation) can properly describe *both* kinematic and diffusion waves. The converse does not hold true: The theory of kinematic waves (represented by the kinematic wave equation) is limited only to kinematic waves and cannot describe diffusion waves.

DYNAMIC WAVES

At this point, we leave Lighthill and Whitham and their concept of kinematic/diffusion waves and approach the problem of shallow water wave propagation in its most general form, i.e., by considering the "dynamic" wave, that which, in addition to gravitational, frictional and flow-depth gradient forces, also includes the inertial force. This leads us to a set of two partial differential equations of continuity and motion, also referred to as the "Saint Venant equations."

Before we give up on theory and resort to our computer models, branding the often-repeated dictum "There is no known analytical solution of the Saint Venant equations," it is worthwhile to reckon the existence of a number of incomplete yet illuminating analytical solutions which are scattered throughout the literature (Lighthill and Whitham, 1955; Dooge, 1973; Ponce and Simons, 1977). In particular, the linear solution of Ponce and Simons is significant because it gives us great insight into the behavior of shallow water waves, including kinematic, difusion, dynamic, and inertial waves.

The work of Ponce and Simons (1977) can be summarized in the following statements:

1. The dynamic wave lies towards the middle of the dimensionless wave number spectrum ($10^0 < \sigma < 10^2$), while kinematic/diffusion waves lie to the left ($10^{-2} < \sigma < 10^0$) and inertial waves to the right ($10^1 < \sigma < 10^4$).

2. In the stable flow regime (Vedernikov number $V < 1$, Chow, 1959; Ponce, 1990), the dynamic wave shows very strong diffusive tendencies.

3. At the threshold of flow instability ($V = 1$), the Seddon and Lagrange speeds (Chow, 1959) are the same, and kinematic, dynamic, and inertial waves have the same celerity and lack diffusion.

4. In the unstable flow regime ($V > 1$), kinematic, dynamic, and inertial waves have a tendency to amplify during propagation.

The findings of Ponce and Simons (1977) pose an interesting theoretical question which helps place the nature of shallow water waves in the proper perspective. Granted that kinematic waves (and by extension, diffusion waves), lying to the left of the σ spectrum, transport mass. This is an intuitive conclusion which cannot be challenged. On the other hand, the inertial wave, the so-called "gravity" wave of classical mechanics, lying to the right of the σ spectrum, transports energy. What, then, do dynamic waves transport, since they lie towards the middle of the wave number spectrum? Mass, or energy? It stands to logic that the answer is: both. Therein lies the reason for the markedly strong dissipative tendencies of the dynamic wave: Shallow water waves can transport mass and energy simultaneously *only* at the expense of wave diffusion. In the stable flow regime ($V < 1$), the more dynamic a wave is, the more strongly dissipative it is (Ponce et al, 1978). At the threshold of flow instability ($V = 1$), dynamic waves lose their ability to dissipate, and their properties coalesce with those of kinematic and inertial waves.

The preceding discussion raises a practical question which has been in the minds of many researchers and practitioners who have dealt with the dynamic wave. If the dynamic wave is so strongly dissipative in most cases of practical interest, is it worth attempting to compute it? Would it not dissipate shortly after it is generated, with its mass going to join the underlying larger, kinematic/diffusion, wave? Or, can it be tracked downstream as it propagates? If so, what is its characteristic speed? A more practical question is: If the dynamic wave is so strongly dissipative, could it be properly construed as a flood wave? These questions continue to trouble those who use the dynamic wave. Lighthill and Whitham (1955) put it in a nutshell when they stated (op. cit., p. 293): "Under the conditions appropriate for flood waves... the dynamic waves rapidly become negligible, and it is the kinematic waves, following at lower speed, which assume the dominant role."

In summary, dynamic waves *do not apply* to floods in mountain streams. Attempts to do this will be futile, given the accumulated body of theoretical and practical experience pointing otherwise. There is still the unresolved question of whether dynamic waves apply to the routing of flood waves in any physically realistic setting. Dam-break flood waves notwithstanding, perhaps the only clear statement that can be made today is that the dynamic wave applies to tidal flow and similar such situations where there is a significant downstream control of the flow.

PHYSICAL VS NUMERICAL DIFFUSION

If kinematic waves cannot diffuse, why is it that numerical models of kinematic waves are able to show some wave diffusion? The resolution of this paradox lies in the conversion of a partial differential equation into a finite difference equation. This conversion can only be done at the expense of introducing an error. This error is a function of the grid size (Δx and Δt) and tends to disappear as the grid size is progressively refined. In flood routing, the error that creeps into a typical computation using finite differences manifests itself as *numerical diffusion* and *numerical dispersion* effects.

These effects are the direct result of specifying a discrete space-time domain, and are not necessarily related to the physical diffusion and dispersion which are inherent in the nature of flood waves.

Numerical diffusion arises because the calculated wave amplitude is smaller than the physical wave amplitude. Numerical dispersion arises when the calculated wave celerity is different from the physical wave celerity. In conventional finite difference (F.D.) shallow water wave models, the aim is to minimize numerical diffusion and dispersion by choosing a grid size sufficiently small to drive these errors to inconsequential amounts. Then, the convection and diffusion of the shallow water wave can be properly described by the numerical model.

Unfortunately, not all F.D. kinematic wave models have sought to minimize numerical diffusion and dispersion. Often, a F.D. kinematic wave model has inadvertedly used the numerical diffusion as a way of showing a certain amount of "physically realistic" diffusion in the calculated results (Li et al, 1975; Curtis et al, 1978). A detailed treatment of this subject is out of the scope of this paper. The interested reader is referred to the paper by Ponce et al (1979), which treats the various numerical schemes of the convection-diffusion equation (of which the kinematic wave equation is a special type), and their numerical diffusion/dispersion effects (amplitude and phase portraits). For our present purpose, we quote Cunge (1969) in stating that finite difference schemes of the kinematic wave equation introduce varying amounts of numerical diffusion and dispersion. The latter interfere with the physical effects, modifying them (Abbott, 1976; Ponce, 1990a). Thus, a finite difference kinematic wave model may be able to show some diffusion, the actual amount being a function of the grid size and weighting factors (used in discretizing the terms of the kinematic wave equation). The fact that this diffusion is artificial and intrinsically related to the grid size can be readily demonstrated by solving the same problem several times, each time halving the spatial increment Δx and temporal increment Δt. Carried to the practical limit, this test leads to the eventual disappearance of the numerical diffusion in question, with the result approaching the analytical solution of the kinematic wave, which is nondiffusive.

We are now in a quandary! If we solve the kinematic wave *properly*, achieving the complete elimination of numerical diffusion and dispersion, we can only hope to describe kinematic waves, but not diffusion waves; if the problem does have some physical diffusion, the latter would be entirely missing from this approach. Conversely, if we solve the kinematic wave *improperly*, introducing numerical diffusion and dispersion by our choice of grid size, there is no guarantee that these will be related to the diffusion and dispersion, if any, of the physical problem. Any arbitrary choice of grid size will cause some numerical diffusion and/or dispersion, and since the latter are unrelated to the physical problem, the solution degrades accordingly, from deterministic to *conceptual*. It would be hit and miss, as far as the accurate reproduction of wave properties is concerned.

Fortunately, there is a way out of this difficulty. As shown by Cunge (1969), and subsequently by others (Natural Environment Research Council, 1975; Ponce and Yevjevich, 1979; Ponce, 1989), the numerical diffusion and dispersion of F.D. kinematic wave models *can* be managed! There is a way to optimize the numerical diffusion while minimizing the numerical dispersion, to make the method--and its inherent errors--work for us instead of against us! By a careful match of the numerical diffusion with the physical diffusion, the F.D. kinematic wave model can reproduce *both* kinematic and diffusion waves, in a methodology that has been referred to as the Muskingum-Cunge (M-C) method.

In a nutshell, the M-C method is a variant of the Muskingum method of flood routing in which the parameters K and X are calculated directly, based on hydraulic data (channel friction, bed slope, and cross-sectional characteristics), instead of indirectly, based on the conventional hydrologic data (storage-weighted flow relations). The M-C method was first applied to open channel flow, and later to overland flow (Ponce, 1986). Extensive tests have shown that the method holds promise for overland flow, since unlike conventional finite difference kinematic wave models, the M-C model is essentially grid independent. In other words, the solution does not depend on the choice of grid size (Ponce, 1986).

In summary, a conventional F. D. kinematic wave model will diffuse numerically, with the diffusion being dependent on the choice of grid size. If the grid size is refined to eliminate the numerical diffusion, no physical diffusion (if present) can be simulated. If the grid size is not refined, the amount of numerical diffusion is arbitrary and not related to the physical diffusion (if any), and the model degrades into a conceptual status. If the M-C method is used, the numerical diffusion is matched with the physical diffusion; consequently, the result is independent of the grid size, and thus, the deterministic character of the method is preserved.

KINEMATIC SHOCK

Kinematic waves lack physical diffusion. However, kinematic waves are nonlinear (or rather, quasilinear), a property which gives them the inherent tendency to change their shape upon propagation: either steepen or flatten, depending on the stage relative to the channel cross-section (flow inbank or out-of-bank). Under the right set of circumstances, a kinematic flood wave can steepen to the point where it becomes for all practical purposes a "wall of water." (In overland flow situations, the "wall of water" would be a small discontinuity in the water surface profile). This is the *kinematic shock,* i.e., a kinematic wave that has steepened upon propagation to the point of being nearly discontinuous.

Contrary to conventional wisdom (Kibler and Woolhiser, 1970; Cunge, 1969), there is no physical unreality about the kinematic shock. If the steepening tendency is allowed to continue unchecked, the kinematic shock *will form* in due time. Diffusion, however, acts to counteract the steepening tendency. Therefore, in cases where diffusion, either physical or numerical, is present, the development of the kinematic shock is likely to be arrested. This explains the pervasive presence of kinematic shocks in analytical solutions of the kinematic wave, which have no diffusion, numerical or otherwise. On the other hand, kinematic shocks are shown to be conspicuously absent from finite difference kinematic wave models, particularly from those that have appreciable amounts of built-in numerical diffusion.

Ponce and Windingland (1985) have clarified the conditions under which the kinematic shock is likely to develop. Based on theoretical considerations supported by extensive numerical experiments, they established the following conditions for kinematic shock development:

1. The wave must be kinematic, i.e., it must have negligible physical diffusion. Diffusion tends to counteract the development of the shock.
2. The ratio of base-to-peak flow Q_b/Q_p must be small, with zero as the lower limit, such as in the case of an ephemeral stream (recall the flash floods occurring on dry beds).
3. The channel is (a) hydraulically wide, i.e., of nearly constant wetted perimeter, to allow the wave steepening to progress unchecked by the cross-sectional shape; and (b) sufficiently long to allow enough time for the shock to develop.

4. The flow is at high Froude number, within the stable flow regime ($V < 1$). The higher the Froude number within the stable flow regime, the smaller the physical diffusion, and the more likely the shock can continue to develop unchecked. In the limit, as the Vedernikov number approaches 1 (and the Froude number approaches 2, for hydraulically wide channels with Chezy friction), diffusion vanishes as the flow reaches the threshold of instability.

In practice, all four conditions may prevail at the same time in a given situation. Whether a kinematic shock will form will depend on the strength of any one condition, or, if more than one is present, on their combined strength. For instance, an analytical solution of the kinematic wave in an overland flow plane satisfies conditions 1 and 3 (a), and maybe even 3 (b) if the plane is long enough. The case of a flash flood in an ephemeral stream in an arid or semiarid region satisfies condition 2, and probably even 3 (a), 3 (b), and 4. The fact that kinematic shocks are not a common sight in nature points to the practical difficulty of satisfying all of these conditions.

Condition 1 is satisfied in channels where the product $t_r S_o$ is large. Condition 2 is satisfied in ephemeral streams. Condition 3 (a) is satisfied in inbank flow in wide rectangular channels, but not if the flow goes overbank, since in this case the wetted perimeter would cease to be nearly constant. Condition 3 (b) is dependent of the catchment's physiography, geology, and drainage density. The longer a stream, uninterrupted by lateral inflow at tributary confluences, the better the chances for the shock to develop. Condition 4 is dependent on the channel aspect ratio, boundary friction, and presence or absence of riparian vegetation. In this regard, we echo Jarrett (1984) in reminding the reader that Mother Nature does not like high-Froude-number flows! So condition 4 is more likely to be the exception rather than the rule.

In closing, it should be noted that kinematic shocks, particularly those associated with flash floods, are very difficult to document precisely, given the obvious likelihood of bodily harm and possibly even death for those daring enough to attempt it. For the conditions prevalent in mountainous areas, kinematic shocks (and flash floods) would be associated with one or more of the following: (1) intense cloud bursts, (2) an arid or semiarid region, (3) a steep ephemeral stream, (4) a low-friction channel (in both bed and banks), and (5) a catchment of low drainage density.

ROLE OF THE VEDERNIKOV NUMBER IN FLOOD WAVE DIFFUSION

As pointed out by Hayami (1951) in his classical paper on diffusion waves, the hydraulic diffusivity is the physical parameter controlling the diffusion of diffusion waves. The hydraulic diffusivity is: $\nu = q_o/(2S_o)$, in which q_o = reference unit-width discharge, and S_o = channel bed slope. Therefore, the amount of diffusion that a flood wave undergoes during propagation is directly proportional to the unit-width discharge and inversely proportional to the channel bed slope. In other words, the steeper the channel bed slope, the lesser the amount of flood wave diffusion. In the limit, as the channel bed slope increases, the diffusion disappears and the flood wave becomes a kinematic wave.

Hayami's hydraulic diffusivity is properly a *kinematic* hydraulic diffusivity [$\nu_k = q_o/(2S_o)$], because it lacks inertia altogether. It is strictly applicable to flow well within the stable regime, i.e., for small Vedernikov numbers, in the range $0 < V < 0.25$ ($0 < F_o < 0.5$, for hydraulically wide channels with Chezy friction). By including inertia in the formulation, Dooge (1973) and Dooge et al (1982) have extended the concept of hydraulic diffusivity to the realm of dynamic waves. This leads to the concept of *dynamic* hydraulic diffusivity: $\nu_d = (1 - V^2)\, q_o/(2S_o)$ (Ponce, 1990a; 1990b).

It is seen that unlike its kinematic counterpart, the dynamic hydraulic diffusivity is also a function of the Vedernikov number. As the Vedernikov number approaches 0 (in the case of low-Froude-number flows), n_d reduces to n_k. Conversely, as the Vedernikov number approaches 1 ($V = 1$ is the threshold of flow instability), n_d reduces to 0, and wave diffusion vanishes (obviously, a process which could not be simulated with n_k). It is seen that n_d applies through a wider range of flow conditions than n_k (in the range $0 < V < 1$) . Since n_d does not significantly complicate the expression for hydraulic diffusivity, it should be the preferred way of modelling flood wave diffusion. In practice, since wave diffusion is usually small (most flood waves are diffusion waves!), the dynamic contribution to wave diffusion turns out to be also small.

The inclusion of the Vedernikov number in the expression for hydraulic diffusivity has the advantage that it can also account for channels of arbitrary cross-sectional shape, i.e., those other than hydraulically wide. Taken to the limit, i.e., for the inherently stable channel (Ponce, 1990b), $V = 0$, regardless of Froude number, and the kinematic and dynamic hydraulic diffusivities are one and the same. It is seen that in this case, the flood wave attenuation is governed by the kinematic hydraulic diffusivity, for all values of discharge or stage.

WHERE DO WE GO FROM HERE?

Having reviewed the status of kinematic waves, it is only fitting that we now take a stab at the question: Where do we go from here? We know very well that kinematic waves are useful tools in applied hydrology. They describe the flow in steep streams (recall the theme of this symposium: *The Hydrology of Mountainous Areas*), and they do it very well. With the applicability issue now clearly settled, there is no doubt that kinematic waves will continue to be used in the future. In fact, when the extension is made to diffusion waves, the applicability issue is no longer a serious roadblock. In this the last decade of the century, the burden of proof is seen to be slowly shifting to the dynamic wave. The dynamic wave has yet to show, beyond reasonable doubt, that in most cases of practical interest, *it is there* for us to calculate it.

Caution should be exercised when applying the kinematic wave to overland flow and streamflow in the context of a numerical computer model, now that we know that numerical diffusion is likely to creep in and degrade the accuracy of the computation. In this regard, the method of matched diffusivities (M-C method) holds particular promise, given its demonstrated grid independent for a wide range of grid sizes. The M-C method is an analog of the diffusion wave model, and therefore, can be used to solve for *both* kinematic and diffusion waves. Furthermore, when the dynamic hydraulic diffusivity is used in lieu of its kinematic counterpart, the method can account for most of the wave dynamics, including the Vedernikov number and its effect on cross-sectional shape and boundary friction.

More research is needed into the nature of kinematic shock and its relevance to the modelling of flash floods. Given that the conditions under which these shocks develop have now been clearly identified, the following question is posed: Can a hazard rating be established for flash floods, in terms of regional climate, catchment geology, physiography, and drainage density, and channel slope, friction, and cross-sectional shape? This question is in need of immediate attention if we are going to apply the theory of kinematic waves to guarantee the safety of the populations that are currently at risk all over the world.

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