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UNSTEADY FLOW TO A MULTIAQUIFER FLOWING WELL

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1984-85

CONTENTS

LIST OF SYMBOLS

$E_i(x)$	-	exponential integral
H_i	-	initial head in the i^{th} aquifer prior to well construction
$h_i(r,t)$	-	hydraulic head in the i^{th} aquifer at distance r at time t
$h_w(I)$	-	hydraulic head at the well point during I^{th} unit time period
Δh_j	-	head change in j^{th} aquifer
Δh_w	-	water level fluctuation in a non-pumping multi-aquifer well
M	-	total number of aquifers penetrated by a multi-aquifer well
m	-	total number of time steps into which t_o is divided
n	-	total number of time steps into which t_p is divided
Q	-	constant pumping rate per unit time period
Q'	-	flow rate towards a well under steady state condition
$Q_i(I)$	-	contribution of the i^{th} aquifer during the I^{th} unit time period
r	-	radial distance from the centre of the well
r_w	-	radius of the well screen

s - drawdown

T_i - transmissivity of the i^{th} aquifer

t_o - time span between completion of well construction and commencement of pumping

t_p - duration of pumping

β_i - hydraulic diffusivity of the i^{th} aquifer

$\delta_{rwi}(I)$ discrete kernel coefficients

ϕ_i - storage coefficient of the i^{th} aquifer

LIST OF FIGURES

ABSTRACT

Using discrete kernel technique, analytical solution has been obtained for unsteady flow to a well tapping any number of aquifers which are separated by aquicludes and have different initial hydraulic heads. Results have been given for a well which is opened to three confined aquifers. Exchange of flow among the aquifers prior to and after pumping have been evaluated. The contributions of each aquifer during pumping have also been determined. The variation of composite hydraulic head at the well point with time has been obtained. It is found that when the aquifers have equal hydraulic diffusivity, the composite hydraulic head at the well point attains the near steady state value very quickly. It is also found that in a three aquifer system, when two aquifers have equal initial hydraulic head and same hydraulic diffusivity, the flows they receive from the aquifer having highest initial hydraulic head are proportional to their respective transmissivity values.

1.0 INTRODUCTION

Under field conditions the aquifer geometry rarely conforms to the concept of one aquifer system. In a borehole it is common to identify number of aquifers. Often the aquifer pumped is part of a complex aquifer system. In order to get dependable yield, wells are constructed tapping more than one aquifer. A multiple aquifer system generally consists of a series of aquifers separated from each other by confining layers. The confining layers may have negligible permeability (aquiclude) or low permeability (aquitard). When the aquifers are separated by aquicludes interaction between the aquifers is only through the well screens. However, when the aquifers are separated by aquitards, interaction between the aquifers takes place through the aquitard besides through the well screens. The mathematical solutions developed so far for determining drawdown and individual aquifer's contribution during the unsteady state flow to a multiaquifer well are intractable. Therefore, at present only a few numerical results are available for a multiaquifer well system. In the present report using discrete kernel approach, complete analytic solutions have been developed for unsteady flow to a well open to several aquifers (more than two) which are separated by aquicludes and have different potentiometric surface prior to well construction.

2.0 REVIEW

Solutions for unsteady flow to a well tapping a single aquifer of infinite areal extent have been presented by various research workers for three conditions : (1) constant discharge (Theis, 1935), (2) variable discharge (Abu - Zied and Scott, 1963; Hantush, 1964), and (3) constant head (Jacob and Lohman, 1952). These solutions could be applied to individual aquifer tapped by a multi-aquifer well if it were possible either to measure accurately the discharge from each aquifer or to control it. At present time practical means of accurate measurement or control of the discharge from each aquifer are not available. It is desirable to analyse the problem of multiaquifer well and determine the contribution of individual aquifer to total discharge of a well.

Analysis of steady flow to a well open to several aquifers has been done by Sokol. Using the Theim equation

$$Q' = \frac{2\pi T(h - h_w)}{\log_e(r/r_w)} \quad \ldots(1)$$

in which,

Q' = flow rate,

T = transmissivity of aquifer,

h = height of potentiometric surface at distance r, and

h_w = height of water level in the discharging well of radius r_w , Sokol has derived the following steady state equation relating water level fluctuation in a non-pumping multi-aquifer well, to head changes in any one aquifer penetrated by the well:

$$\Delta h_w = \frac{T_j \; \Delta h_j}{\sum_{i=1}^{M} T_i} \qquad \ldots(2)$$

In the above equation Δh_j = the head change in j^{th} aquifer, T_i = the transmissivity of the i^{th} aquifer, and M = total number of aquifers penetrated by the well.

A solution for unsteady flow to a well tapping two confined aquifers having different potentiometric surfaces has been obtained by Papadopulos. Integral transform technique has been used to obtain exact expression for head distribution but the solution is intractable for numerical calculation. Asymptotic solutions for both head and discharge distribution amenable to computation which yield results accurate enough for practical application have been derived by Papadopulos. However, no numerical results have been presented by him. The solution obtained by Papadopulos cannot be used for determining formation constants of the individual aquifers. The number of parameters involved in the solution requires that observed response curves be compared with a large number of type curves for all possible combinations of the various parameters which is not practical.

Using integral transform technique unsteady flow to a multiaquifer well open to two aquifers has also been analyzed by Khader and Verankutty (1975) who have presented numerical results for contribution of individual aquifer to the total discharge of a well.

Solution for unsteady flow to a multiaquifer well which is open to several aquifers (more than two) has been obtained by Nautiyal (1984). The time span has been discretised and the solution has been obtained by using discrete kernel theory. It has been assumed that the initial potentiometric surfaces prior to pumping in all the aquifers are same. Numerical solution for contribution of each aquifer to the total discharge of the well at the various time has been presented and the following conclusions have been derived:

(a) When all the aquifers tapped have equal diffusivity values, their contributions are proportional to the respective transmissivity values and are independent of time;

(b) That aquifer, whose hydraulic diffusivity is lowest, contributes the entire quantity of water pumped from the well in the beginning of pumping. At large time the contributions are proportional to respective transmissivity values.

In the present report unsteady flow to a well open to several aquifers, where in different potentiometric surfaces prevail prior to well construction, has been analysed.

3.0 PROBLEM DEFINITION

A schematic cross-section of a well tapping a number of confined aquifers which are separated by aquicludes is shown in Figure 1. Each of the aquifersis homogeneous, isotropic, and infinite in areal extent. The potentiometric surfaces in the aquifers are different from each other. Prior to the well construction all the aquifers are at rest condition. The well after the construction remains unpumped for a period t_o during which internal flow occurs through the well screen owing to the difference in initial heads. The multi-aquifer well is pumped at a constant rate for a period t_p. It is required to find the following:

i) the exchange of flows that takes place through the well screen among the aquifers prior to pumping owing to the difference in piezometric surfaces ,

ii) the contributions of each of the aquifers to well discharge during pumping and drawdowns in the piezometric surfaces, and

iii) the exchange of flows that takes place among the aquifers after stoppage of pumping.

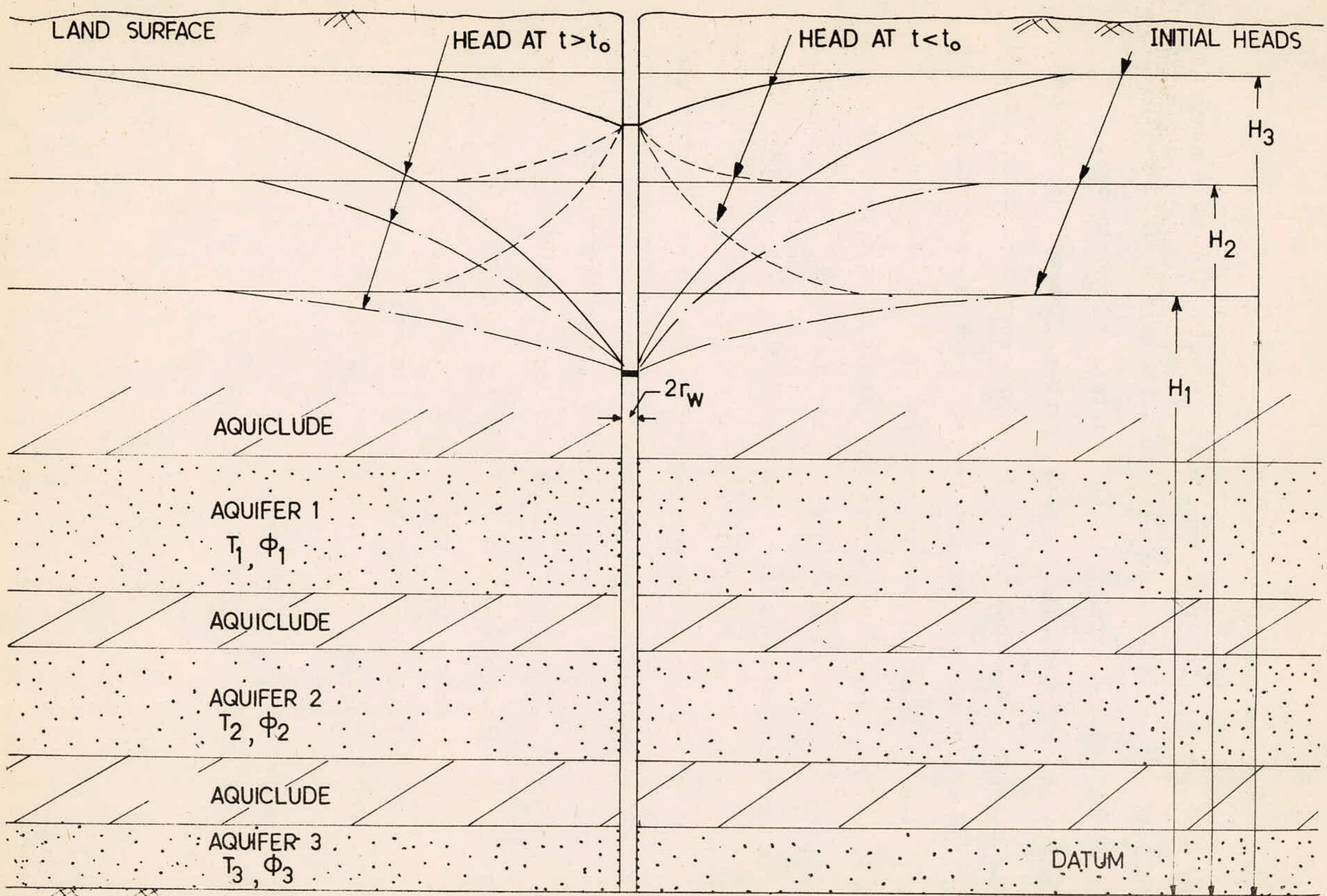

FIGURE 1 - WELL OPENED TO SEVERAL AQUIFERS WITH DIFFERENT POTENTIOMETRIC SURFACE SEPARATED BY AQUICLUDES

4.0 METHODOLOGY

The following assumptions have been made in the analysis:

(a) At any time the drawdowns in all the aquifers at the well face are same but vary with time,

(b) The time parameter is discrete. Within each time step, the abstraction rates of water derived from each of the aquifers and from well storage are separate constants.

The differential equation which describes the axially symmetric, radial, unsteady flow in each aquifer is given by

$$\frac{\partial^2 h_i}{\partial r^2} + \frac{1}{r}\frac{\partial h_i}{\partial r} = \frac{\phi_i}{T_i}\frac{\partial h_i}{\partial t}, \quad i = 1,2,......M; r > r_w \qquad ...(3)$$

where,

r_w = radius of the well screen,

M = total number of aquifers tapped by the well;

h_i = head at a distance r from the well at time t in the i^{th} aquifer;

T_i = transmissivity of the i^{th} aquifer, and

ϕ_i = storage coefficient of the i^{th} aquifer.

Solutions to the equation (3) are to be found for the initial conditions

$$h_i(r,0) = H_i, \quad i = 1,2,\ldots\ldots M, \qquad \ldots(4)$$

where, H_i is the initial head in the i^{th} aquifer prior to well construction.

The boundary conditions to be satisfied are:

$$h_i(\infty, t) = H_i; \qquad \ldots(5)$$

$$h_1(r_w,t) = h_2(r_w,t) = \ldots\ldots = h_M(r_w,t) = h_w(t) \qquad \ldots(6)$$

$$\sum_{i=1}^{M} 2\pi r_w T_i \left.\frac{\partial h_i}{\partial r}\right|_{r=r_w} - \pi r_w^2 \frac{\partial h_w(t)}{\partial t} = Q_p(t) \quad \ldots(7)$$

where $h_w(t)$ is the head in the well and $Q_p(t)$ is the pumping rate.

Let the duration t_o and t_p be discretised to m and n units of equal time step respectively. Let $Q_i(I)$ and $Q_w(I)$ are the contributions of the i^{th} aquifer and well storage respectively during the I^{th} unit time period. With the help of the discretised quantities the boundary condition prescribed at equation (7) can be rewritten as

$$Q_1(I) + Q_2(I) + Q_3(I) + \ldots\ldots + Q_M(I) + Q_w(I) = Q_p(I) \qquad \ldots(8)$$

$$\begin{aligned} Q_p(I) &= 0 \text{ for } I \leq m \\ &= Q \text{ for } m < I \leq (m+n) \\ &= 0 \text{ for } I > (m+n) \end{aligned}$$

where, Q is the pumping rate per unit time period.

Had the aquifers been tapped separately, for the initial condition $h_i(r,0) = H_i$, and the boundary condition $h_i(\infty, t) = H_i$, solution to differential equation (3), when unit impulse quantity of water is withdrawn from the i^{th} aquifer at time $t = 0$, is (Carslaw and Jaeger, 1959)

$$H_i - h_i(r,t) = \frac{1}{4\pi T_i} \frac{e^{-\frac{r^2}{4\beta_i t}}}{t}; \quad \beta_i = \frac{T_i}{\phi_i} \qquad \ldots(9)$$

Defining a unit impulse kernel

$$k_i(t) = \frac{e^{-\frac{r^2}{4\beta_i t}}}{4\pi T_i t}, \qquad \ldots(10)$$

and designating $H_i - h_i(r, t) = s_i(r,t)$ which is the drawdown in the piezometric surface in the i^{th} aquifer, drawdown for variable withdrawal from the i^{th} aquifer can be written in the form

$$s_i(r,t) = \int_0^t Q_i(c)\, k_i(t-c)\,dc, \qquad \ldots(11)$$

where $Q_i(c)$ is variable discharge rate from the i^{th} aquifer at time c. Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each timestep but varies from step to step, the drawdown at the end of time step I in the i^{th} aquifer at a distance r from the well can be written as (Morel-Seytoux, 1975)

$$s_i(r,I) = \sum_{\gamma=1}^{I} \delta_{r,i}(I-\gamma+1)\, Q_i(\gamma), \qquad \ldots(12)$$

where the discrete kernel coefficient $\delta_{r,i}(I)$ is defined as

$$\delta_{r,i}(I) = \int_0^1 k_i\,(I-c)\,dc$$

$$= \frac{1}{4\pi T_i}\left\{E_1\left(\frac{r^2}{4\beta_i I}\right) - E_1\left(\frac{r^2}{4\beta_i (I-1)}\right)\right\}, \qquad \ldots(13)$$

in which $E_1(x)$ is an exponential integral (Abramowitz and Stegun, 1970) defined as

$$E_1(x) = \int_x^\infty \frac{e^{-u}}{u}\,du.$$

When all the 'M' aquifers are tapped by a single well, during the non-pumping period water will flow from one aquifer to the other through the well screen depending on the relative values of head in the aquifers. During pumping there will be contribution from each of the aquifers through the respective well screen.

If $Q_i(\gamma)$, $\gamma = 1,2,\ldots\ldots I$, are the contributions by the i^{th} aquifer, drawdown at the well face in the i^{th} aquifer at the end of time step I is given by

$$s_i(r_w, I) = \sum_{\gamma=1}^{I} Q_i(\gamma)\,\delta_{rwi}(I-\gamma+1) \qquad \ldots(14)$$

in which the discrete kernel coefficient $\delta_{rwi}(I)$ is defined as:

$$\delta_{rwi}(I) = \frac{1}{4\pi T_i}\left\{E_1\left(\frac{r_w^2}{4\beta_i I}\right) - E_1\left(\frac{r_w^2}{4\beta_i (I-1)}\right)\right\} \qquad \ldots(15)$$

Thus the head at the well face in the i^{th} aquifer at the end of time step I can be expressed by the relation

$$h_i(I) = H_i - \sum_{\gamma=1}^{I} Q_i(\gamma)\,\delta_{rwi}(I-\gamma+1) \qquad \ldots(16)$$

Since the heads at the well face at the end of any time step in all the aquifers are equal, therefore,

$$H_1 - \sum_{\gamma=1}^{I} Q_1(\gamma)\,\delta_{rw1}(I-\gamma+1) = H_2 - \sum_{\gamma=1}^{I} Q_2(\gamma)\,\delta_{rw2}(I-\gamma+1)$$

$$= H_3 - \sum_{\gamma=1}^{I} Q_3(\gamma)\,\delta_{rw3}(I-\gamma+1) = \ldots\ldots = H_M - \sum_{\gamma=1}^{I} Q_M(\gamma)\,\delta_{rwM}(I-\gamma+1)$$

$$\ldots(17)$$

The above set of equations can be written as

$$-Q_1(I)\,\delta_{rw1}(1) + Q_2(I)\,\delta_{rw2}(1) = H_2 - H_1 + \sum_{\gamma=1}^{I-1} \delta_{rw1}(I-\gamma+1)\,Q_1(\gamma)$$

$$- \sum_{\gamma=1}^{I-1} Q_2(\gamma)\,\delta_{rw2}(I-\gamma+1)$$

$$\ldots(18)$$

$$-Q_1(I)\,\delta_{rw1}(1) + Q_3(I)\,\delta_{rw3}(1) = H_3 - H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\,\delta_{rw1}(I-\gamma+1)$$

$$- \sum_{\gamma=1}^{I-1} Q_3(\gamma)\,\delta_{rw3}(I-\gamma+1)$$

$$\ldots(19)$$

$$\vdots \qquad\qquad \vdots$$

$$-Q_1(I)\,\delta_{rw1}(1) + Q_M(I)\,\delta_{rwM}(1) = H_M - H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\,\delta_{rw1}(I-\gamma+1)$$

$$- \sum_{\gamma=1}^{I-1} Q_M(\gamma)\,\delta_{rwM}(I-\gamma+1)$$

...(20)

Let H_{max} is the maximum value of H_i. Head in the well in consequence to abstraction from well storage is given by

$$h_w(I) = H_{max} - \sum_{\gamma=1}^{I} \frac{Q_w(\gamma)}{\pi r_w^2} \qquad ...(21)$$

$h_w(I)$ is equal to $h_i(I)$ for all values of i and I at $r=r_w$. Using this relation one more quation can be written as

$$H_1 - \sum_{\gamma=1}^{I} Q_1(\gamma)\,\delta_{rw1}(I-\gamma+1) = H_{max} - \sum_{\gamma=1}^{I} \frac{Q_w(\gamma)}{\pi r_w^2} \qquad ...(22)$$

Rearranging,

$$- Q_1(I)\,\delta_{rw1}(1) + \frac{Q_w(I)}{\pi r_w^2} = H_{max} - H_1 + \sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)$$

$$- \sum_{\gamma=1}^{I-1} \frac{Q_w(\gamma)}{\pi r_w^2} \qquad ...(23)$$

In matrix notation, equations (8), (18), (19), (20) and (23) can be written as

$$
\begin{Bmatrix}
1, & 1, & 1, & \ldots, & 1, & 1 \\
-\delta_{rw_1}(1), & \delta_{rw2}(1), & 0, & \ldots, & 0, & 0 \\
-\delta_{rw1}(1), & 0, & \delta_{rw3}(1), & \ldots, & 0, & 0 \\
\cdot & & & & & \\
\cdot & & & & & \\
\cdot & & & & & \\
-\delta_{rw1}(1), & 0, & 0, & \ldots, & \delta_{rwM}(1), & 0 \\
-\delta_{rw1}(1), & 0, & 0, & \ldots, & 0, & \frac{1}{\pi r_w^2}
\end{Bmatrix}
\cdot
\begin{Bmatrix}
Q_1(I) \\ Q_2(I) \\ Q_3(I) \\ \cdot \\ \cdot \\ Q_M(I) \\ Q_w(I)
\end{Bmatrix}
=
$$

$$
\begin{Bmatrix}
Q_p(I) \\
H_2-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)-\sum_{\gamma=1}^{I-1} Q_2(\gamma)\delta_{rw2}(I-\gamma+1) \\
H_3-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)-\sum_{\gamma=1}^{I-1} Q_3(\gamma)\delta_{rw3}(I-\gamma+1) \\
\cdot \\
\cdot \\
\cdot \\
H_M-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)-\sum_{\gamma=1}^{I-1} Q_M(\gamma)\delta_{rwM}(I-\gamma+1) \\
H_{max}-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1) - \frac{1}{\pi r_w^2}\sum_{\gamma=1}^{I-1} Q_w(\gamma)
\end{Bmatrix} \quad \ldots(24)
$$

For a three aquifer system equation (24) reduces to

$$\{A\}.\{B\} = \{C\} \qquad \ldots(25)$$

in which,

$$\{A\}=\left\{\begin{bmatrix} 1 & 1 & 1 & 1 \\ -\delta_{rw1}(1), & \delta_{rw2}(1), & 0, & 0 \\ -\delta_{rw1}(1), & 0, & \delta_{rw3}(1), & 0 \\ -\delta_{rw1}(1), & 0, & 0, & \frac{1}{\pi r_w^2} \end{bmatrix}\right\}$$

$\{B\} = \{Q_1(I),\ Q_2(I),\ Q_3(I),\ Q_w(I)\}^T$, and

$$\{C\}=\left\{\begin{bmatrix} Q_p(I) \\ H_2-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)-\sum_{\gamma=1}^{I-1} Q_2(\gamma)\delta_{rw2}(I-\gamma+1) \\ H_3-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)-\sum_{\gamma=1}^{I-1} Q_3(\gamma)\delta_{rw3}(I-\gamma+1) \\ H_{max}-H_1+\sum_{\gamma=1}^{I-1} Q_1(\gamma)\delta_{rw1}(I-\gamma+1)-\frac{1}{\pi r_w^2}\sum_{\gamma=1}^{I-1} Q_w(\gamma) \end{bmatrix}\right\}$$

$Q_1(I)$, $Q_2(I)$, $Q_3(I)$, and $Q_w(I)$ can be solved in succession starting from time step 1 using the relation

$$\{B\}= \{A\}^{-1}.\ \{C\} \qquad \ldots(26)$$

Knowing $Q_i(I)$ values, the drawdown in the i^{th} aquifer can be calculated using equation (12).

5.0 RESULTS

The problem of calculating the drawdown at the well face and at different sections in the aquifers, and the quantities of flow from each aquifer, when the aquifer parameters T_i and ϕ_i and the initial heads H_i are known, has been dealt in the analysis for a well tapping any number of aquifers separated by aquicludes. Though the analysis can be used for any number of aquifers, however, results have been presented when the well is opened to three confined aquifers only. The duration starting from completion of the well to commencement of pumping and the pumping period are discretised to m and n units of equal time step. For known values of $T_1, \phi_1, T_2, \phi_2, T_3, \phi_3$, and r_w, the discrete kernel coefficients, $\delta_{rwi}(I)$, are generated making use of equation (15) for different integer values of I. For known values of m, n, H_i and $Q_p(I)$, the values of $Q_i(I)$ and $Q_w(I)$ have been found in succession, starting from time step 1.

The variations of $Q_1(t)/\{\bar{T}(H_{max} - H_1)\}$, and $Q_2(t)/\{\bar{T}(H_{max} - H_2)\}$ with $4\bar{T}t/(\bar{\phi}r_w^2)$ for $Q_p(t) = 0$ are shown in figure 2 for $(H_{max} - H_1)/(H_{max} - H_2) = 1$, T1:T2:T3 = 3:2:1 and $\phi1:\phi2:\phi3: = 3:2:1$. $\bar{T}$ and $\bar{\phi}$ are the arithmatic mean values of transmissivities and storage coefficients respectively. These results pertain to the case where i) all the aquifers have equal hydraulic diffusivity, and ii) the initial hydraulic heads in the first and in the second aquifer are the same and less than that of the third aquifer. The variation of $Q_1(t)/\{\bar{T}(H_{max}-H_1)\}$, and $Q_2(t)/\{\bar{T}(H_{max}-H_2)\}$ with $4\bar{T}t/(\bar{\phi}r_w^2)$ presented in figure 3 are for T1:T2:T3 =3:2:1, $\phi1:\phi2:\phi3:$ = 3:2:2 and $(H_{max}-H_1)/(H_{max}-H_2) = 1$. The results presented in figure 3 pertain to the case where only the first and the second aquifer have equal hydraulic diffusivity. It is found from figures 2 and 3 that if the aquifers, which receive water, have equal hydraulic diffusivity values, the flow quantities are in proportion to the respective transmissivity values. In other words if aquifer 1 and aquifer 2 have equal hydraulic diffusivity and have same initial potentiometric surface and if they are receiving water from the third aquifer, in which the potentiometric surface is at a higher level, then at all time $Q_1(t)/Q_2(t)$ values are equal to T_1/T_2.

Variations of flow from the first, second and third aquifer with time are presented in Table 1 and 2 for different levels of the initial potentiometric surface for $Q_p(t) = 0$. The -ve sign indicates that water is flowing into the aquifer. As seen from the tables water always frows from the aquifer with highest initial hydraulic head. The aquifer with lowest initial hydraulic head always receives water. The aquifer having intermediate initial hydraulic head will either gain or lose water at any time depending upon the value of its initial hydraulic head and the composite hydraulic head that would prevail at the well point. The composite hydraulic head at the well point is a function

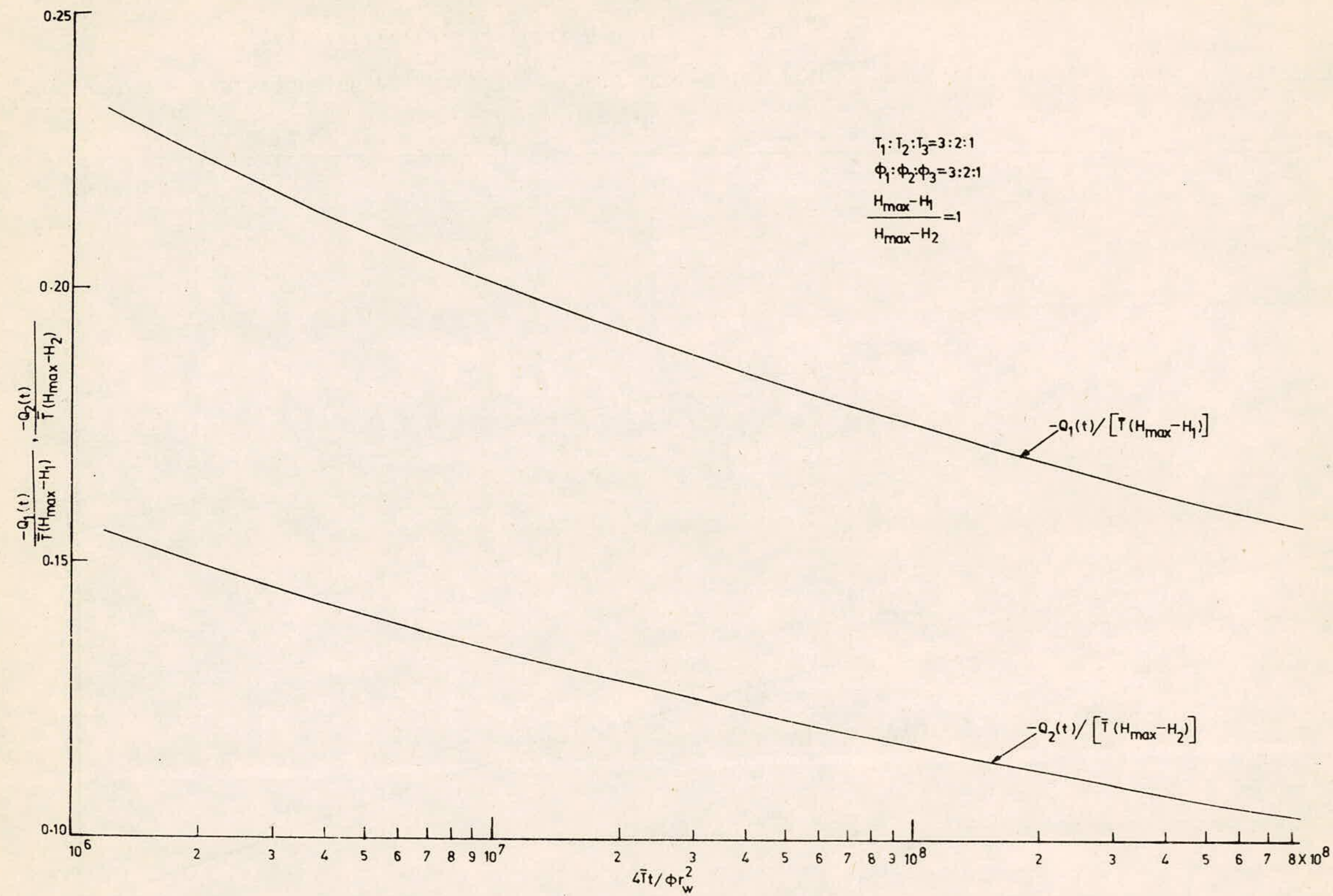

FIGURE 2 - VARIATIONS OF $-Q_1(t)/\{\bar{T}(H_{max}-H_1)\}$ AND $-Q_2(t)/\{\bar{T}(H_{max}-H_2)\}$ WITH $4\bar{T}t/(\bar{\phi}r_w^2)$ WHEN $T_1:T_2:T_3=3:2:1, \phi_1:\phi_2:\phi_3=3:2:1$ AND $(H_{max}-H_1)/(H_{max}-H_2) = 1$

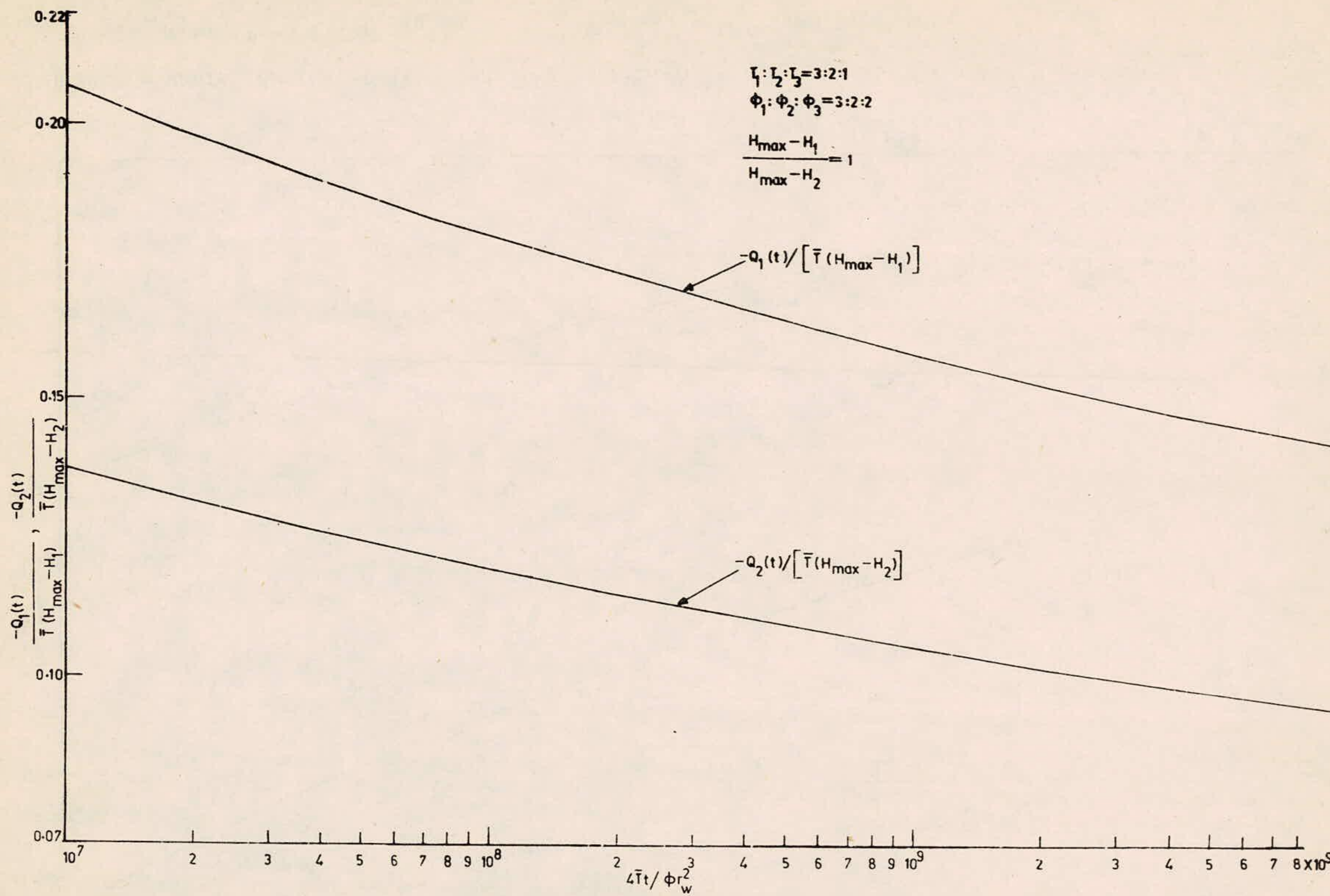

FIGURE 3 - VARIATIONS OF $-Q_1(t)/\{\bar{T}(H_{max}-H_1)\}$ AND $-Q_2(t)/\{\bar{T}(H_{max}-H_2)\}$ WITH $4\bar{T}t/(\bar{\phi}r_w^2)$ WHEN $T_1:T_2:T_3=3:2:1, \phi_1:\phi_2:\phi_3=3:2:2$ AND $(H_{max}-H_1)/(H_{max}-H_2 = 1$

Table 1 - Flow From Different Aquifers Due to Difference in potentiometric surfaces when T_1= 900 m^2/day, T_2=600 m^2/day, T_3=300 m^2/day, ϕ_1= .03, ϕ_2= .02, ϕ'_3=.01, H_1=201.0 m, H_2 = 202.0 m, H_3 = 203.0 m, r_w=.1 m and $Q_p(t)$ = 0.

Time (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)
1	- 476.2	158.7	317.5
2	- 456.6	152.2	304.4
3	- 445.7	148.6	297.1
4	- 438.2	146.1	292.1
5	- 432.6	144.2	288.4
6	- 428.1	142.7	285.4
7	- 424.4	141.5	282.9
8	- 421.2	140.4	280.8
9	- 418.5	139.5	279.0
10	- 416.1	138.7	277.4
20	- 401.0	133.7	267.4
30	- 392.5	130.8	261.7
40	- 386.7	128.9	257.8
50	- 382.4	127.5	254.9
60	- 378.9	126.3	252.6
70	- 375.9	125.3	250.6
80	- 373.5	124.5	249.0
90	- 371.3	123.8	247.5
100	- 369.4	123.1	246.3

Table 2 - Flow from Different Aquifers Due to Difference in potentiometric surfaces when $T_1=900\ m^2/day$, $T_2=600\ m^2/day$, $T_3=300\ m^2/day$, $\phi_1 = .03$, $\phi_2 = .02$, $\phi_3=.01$, $H_1 = 202.0$ m, $H_2 = 205.0$ m, $H_3 = 203.0$ m, $r_w = .1$ m $Q_p(t) = 0$.

Time (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)
1	- 834.6	874.3	- 39.7
2	- 799.2	837.2	- 38.0
3	- 780.0	817.1	- 37.1
4	- 766.9	803.4	- 36.5
5	- 757.0	793.1	- 36.1
6	- 749.2	784.9	- 35.7
7	- 742.7	778.1	- 35.4
8	- 737.2	772.3	- 35.1
9	- 732.4	767.3	- 34.9
10	- 728.1	762.8	- 34.7
20	- 701.8	735.2	- 33.4
30	- 686.9	719.6	- 32.7
40	- 676.8	709.0	- 32.2
50	- 669.1	701.0	- 31.9
60	- 663.0	694.6	- 31.6
70	- 657.9	689.2	- 31.3
80	- 653.6	684.7	- 31.1
90	- 649.8	680.7	- 30.9
100	- 646.4	677.2	- 30.8

of time and is found to attain a near steady state value very quickly when all the aquifers have equal hydraulic diffusivity. According to Sokol (1963), the composite steady state hydraulic head at the well point is given by $h_w = \sum_{i=1}^{M} (T_i H_i) / \sum_{i=1}^{M} (T_i)$. When the intermediate initial hydraulic head is lower than the composite hydraulic head at the well point, the aquifer with intermediate initial hydraulic head will receive water. In such situation the water which flows from the aquifer with highest initial hydraulic head is shared by the other two aquifers. However, when the intermediate initial head is higher than the composite hydraulic head at the well point, water also flows from the aquifer with intermediate initial head to the aquifer with lowest initial hydraulic head.

The variations of $(H_1-h_1)/(H_{max}-H_1)$ with $4\bar{T}t/(\bar{\phi}r_w^2)$ are shown in figure 4 for two sets of aquifer parameters. In one set the parameters have the value $(H_{max}-H_1)/(H_{max}-H_2)=1$, $T_1:T_2:T_3 = 3:2:1$, and $\phi_1:\phi_2:\phi_3 = 3:2:1$. In the other set $\phi_1:\phi_2:\phi_3 = 3:2:2$ all other parameters remaining unchanged. As seen from figure 4, when all the aquifers have same hydraulic diffiusivity, the near steady state drawdown value at the well point is attained quickly. On the other hand when all the aquifers do not have equal hydraulic diffusivity value the near steady state drawdown at the well point will be attained after a long time.

Variation of hydraulic head with time at the well point is presented in Table 3 for a case where the initial

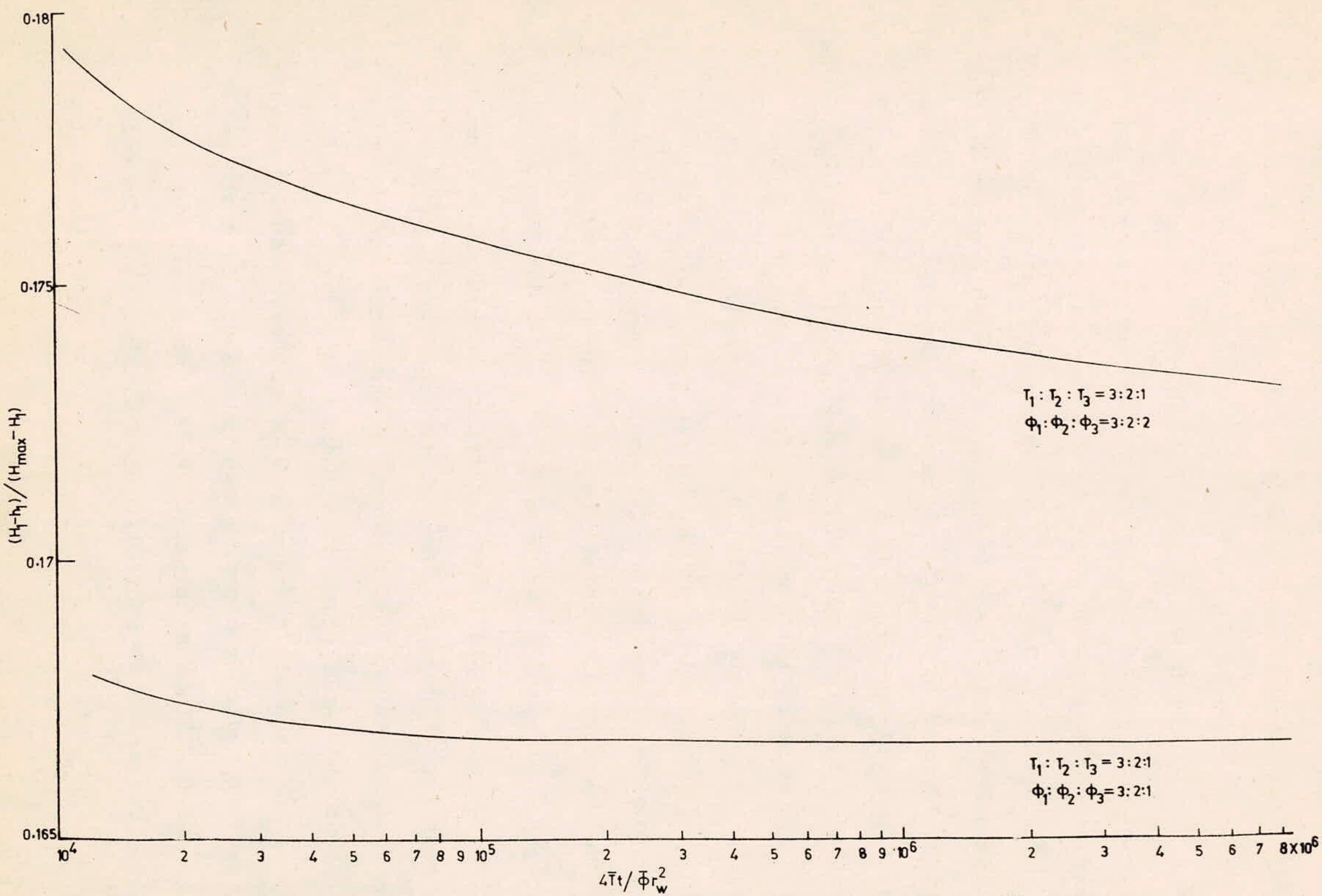

FIGURE 4 - VARIATIONS OF $(H_1-h_i)/(H_{max}-H_1)$ AT THE WELL POINT WITH $4\bar{T}t/(\bar{\phi}r_w^2)$ FOR $(H_{max}-H_1)/(H_{max}-H_2) = 1$, $T_1:T_2:T_3=3:2:1$, WHEN $\phi_1:\phi_2:\phi_3=$ (i) 3:2:1 AND (ii) 3:2:2

Table 3 - Hydraulic head at the well point at different time evaluated with aquifer parameter $T_1 = 900\ m^2/day$ $T_2 = 600\ m^2/day$, $T_3 = 300\ m^2/day$, $\phi_1 = 0.03$, $\phi_2 = 0.02$, $\phi_3 = 0.01$, $H_1 = 201.0\ m$, $H_2 = 202.0\ m$, $H_3 = 203.0\ m$, $r_w = 1m$ and $Q_p(t) = 0$

Time (day)	Hydraulic head (m)
$\frac{1}{50^{th}}$	201.6667938
$\frac{1}{20^{th}}$	201.6667022
$\frac{1}{10^{th}}$	201.6666870
1	201.6666717
10	201.6666717
100	201.6666717

potentiometric surfaces are at different levels and all the aquifers have equal hydraulic diffusivity. The case of aquifers having equal hydraulic diffusivity has been choosen as the steady state composite hydraulic head at the well point is attained immediately when the aquifers have same hydraulic diffusivity. According to Sokol, the steady state composite hydraulic head for these choosen values of H_i and T_i is 201.66666. As seen from Table 3 the near steady state composite head at the well point is equal to 201.66667.

The response of a multiaquifer well tapping several aquifers with different potentiometric surfaces to pumping can conveniently be decomposed to the following two parts:

i) Part 1 : Response due to difference in potentiometric surfaces as existing in the field but $Q_p(t) = 0$.

Part 2 : Response due to pumping i.e. :

$$Q_p(t) = \begin{cases} 0 \text{ for } 0 < t \leq t_o, \\ Q \text{ for } t_o < t \leq (t_o + t_p), \\ 0 \text{ for } t > (t_o + t_p), \end{cases}$$

when all the initial hydraulic heads are equal to the lowest initial hydraulic head.

The responses of an aquifer corresponding to part 1 and 2 when added would give its response to the pumping for the case when the potentiometric surfaces are at different levels. This can be seen from the results presented in Tables 4,5 and

Table 4 - Contributions of each aquifer and head at the well point when $T_1=1000\ m^2/day$, $T_2 = 100\ m^2/day$, $T_3 = 50\ m^2/day$, $\phi_1 = 0.2$, $\phi_2 = .01$, $\phi_3 = .01$, $r_w = 0.1$ m, $H_1 = 200.0$m, $H_2 = 200.0$ m, $H_3 = 201.00$ m, and $Q_p(t) = 0$

t (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)	$Q_w(t)$ (m^3/day)	$h_i(t)$ at well point (m)
1	-39.15	-3.732	42.89	approx.zero	200.04368
2	-37.27	-3.561	40.83	"	200.04365
3	-36.25	-3.468	39.72	"	200.04364
50	-30.56	-2.943	33.50	"	200.04362
51	-30.53	-2.939	33.47	"	200.04362
52	-30.49	-2.936	33.43	"	200.04362
80	-29.78	-2.871	32.65	"	200.043609
81	-29.76	-2.869	32.63	"	200.043609
82	-29.74	-2.867	32.60	"	200.043609
83	-29.72	-2.865	32.58	"	200.043609
90	-29.59	-2.853	32.44	"	200.043609
100	-29.42	-2.837	32.26	"	200.043609

Table 5 - Contributions of each aquifer and head at the well point when $T_1 = 1000\ m^2/day$, $T_2 = 100\ m^2/day$, $T_3 = 50\ m^2/day$, $\phi_1 = 0.2$, $\phi_2 = .01$, $\phi_3 = .01$, $r_w = 0.1\ m$, $H_1 = H_2 = H_3 = 200.0\ m$, $t_o = 50$ days, $t_p = 30$ days, $Q = 1000\ m^3/day$

t (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)	$Q_w(t)$ (m^3/day)	$h_i(t)$ at well point (m)
50	0.00	0.00	0.00	0.00	200.0
51	873.00	83.20	43.66	.0024	199.032
52	873.0	83.35	43.65	.001	198.984
80	872.47	83.91	43.62	approx zero	198.7966
81	-.6820	.7175	-.0333	-.0024	199.7623
82	-.5340	.5614	-.0260	-.0014	199.8081
83	-.4475	.4700	-.0217	-.0008	199.8340
90	-.2351	.2464	-.0110	-.0001	199.9040
100	-.1471	.1538	-.0066	approx zero	199.9366

Table 6 - Contributions of each aquifer and head at the well point when $T_1 = 1000\ m^2/day$, $T_2 = 100\ m^2/day$, $T_3 = 50\ m^2/day$, $\phi_1 = 0.2$, $\phi_2 = .01$, $\phi_3 = .01$, $r_w = 0.1\ m$, $H_1 = 200.0\ m$, $H_2 = 200.0\ m$, $H_3 = 201.0\ m$, $Q_p(t) = 1000\ m^3/day$, $t_o = 50$ days, $t_p=30$ days

t (day)	$Q_1(t)$ (m^3/day)	$Q_2(t)$ (m^3/day)	$Q_3(t)$ (m^3/day)	$Q_w(t)$ (m^3/day)	$h_i(t)$ at well point (m)
1	-39.155	-3.732	42.89	approx.zero	200.04368
2	-37.265	-3.561	40.825	"	200.04365
3	-36.25	-3.468	39.72	"	200.04364
50	-30.55	-2.943	33.50	"	200.04362
51	842.50	80.25	77.10	.00246	199.07563
52	842.50	80.45	77.05	.00114	199.0276
80	842.50	80.55	77.05	approx.zero	198.8402
81	-30.435	-2.150	32.59	-.0004	199.8059
82	-30.255	-2.316	32.57	-.0002	199.8517
83	-30.15	-2.403	32.56	-.0001	199.8776
90	-29.82	-2.605	32.43	-.0001	199.9476
100	-29.57	-2.683	32.25	approx.zero	199.9801