STORAGE-YIELD ANALYSIS

by

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INTRODUCTION

A reservoir is constructed to change the temporal and spatial availability of water of a stream. Since the natural flow in a stream varies in quantity with time and it seldom follows the demand pattern, it is essential to store the water when the availability is more than the requirements and release it from storage when the situation reverse. The storage-yield analysis is an important aspect of reservoir design.

THE RESERVOIR SYSTEM

The commonly used terms which are relevant to storage-yield analysis of a reservoir are defined here.

a) Dead Storage Zone

This is the bottom most zone in a reservoir and the corresponding storage is also termed as inactive storage. Generally it is mean to cater for the sediment entering in the reservoir or to provide minimum pool for recreation facilities. Usually all the outlets are located above this zone. The withdrawals from this zone if any at all, are made only in extreme dry conditions. The entire reservoir storage which lies above the inactive storage is called live or active storage.

b) Conservation Zone

Water is stored in this zone to cater for various conservation requirements like irrigation, water supply and hydropower generation etc. This zone, normally accounts for most of the storage space available in a reservoir.

c) Flood Control Zone or Surcharge Zone

This storage zone is used to moderate floods impinging the reservoir. Depending upon the flood volume and downstream release constraints, water is stored in this zone to attenuate a flood peak. After the flood peak has passed, this zone is emptied as soon as possible to prepare for subsequent flood events.

d) Full Reservoir Level (FRL)

This is the highest level of the reservoir at which water is intended to be held for various conservation uses, including part or total of the flood storage without allowing any passage of water through the spillway.

e) Maximum Water Level (MWL)

It is the highest level to which the reservoir water will rise while passing the design flood with the spillway facilities in full operation. This level refers to the top of spill zone.

f) Release

Release or draft is the amount of controlled outflow from a reservoir during a given time interval to satisfy the various demands.

g) Yield

For the reservoirs serving for conservation purposes, the amount of water released for these purpose is called the reservoir yield. For the reservoirs where the stored water is used to generate hydroelectric power, yield is defined as the amount of power delivered during a time interval.

h) Firm Yield

Firm water yield from a reservoir is defined as the maximum quantity of water that can be guaranteed to be delivered with 100% reliability. The firm power yield of a reservoir can also be described in a similar manner.

i) Reliability

Reliability of a system is described by the probability α that the system is in the satisfactory state :

$$\alpha = \text{Prob}[X_t \in S]$$

where X_t is the state of the system at time t and S is the domain of admissible states. Further, risk, which is the probability of failure is $(1-\alpha)$.

TECHNIQUES FOR STORAGE-YIELD ANALYSIS

Depending upon the type of data and the computational technique used, the popularly used reservoir capacity computation procedures are classified into following categories :

i) Critical period techniques,
ii) Optimization techniques, and
iii) Simulation techniques.

Among these techniques, those based upon critical period concepts are the earliest techniques. One such method, known as 'Mass Curve Technique' was the first rational method proposed to compute the required storage capacity of a reservoir.

With the advent of computer, the techniques, which beneficially use its computational capabilities are increasingly being used. Among the category optimization techniques, those based on Linear Programming (LP) have been found to be particularly suitable.

THE MASS CURVE METHOD

The mass curve technique is essentially a graphical method based on critical period concept. The second method which uses a popular optimization technique linear programming, is a computer based method. The third method is simulation which can also be used to further modify and test the results of first two methods.

The critical period of defined as the duration in which an initially full reservoir depletes and passing through various states (without spilling), empties. In the methods based on critical period concept a sequence of streamflows containing a critical period is routed through an initially full reservoir in presence of specified demands. The reservoir capacity is obtained by finding the maximum difference between cumulative inflows and cumulative releases.

If we define a function X(t) as

$$X(t) = \int_{t_0}^{t_f} x(t)\, dt$$

then the graph of X(t) versus time is known as the mass curve. The mass curve technique, proposed by Ripple in 1883 to determine storage capacity of a reservoir, is a graphical integration technique. A similar method was a introduced in Europe and was known as 'The Stretched - Thread Rule'. Let x be the series of inflows to the reservoir and q be the outflow or draft series and Z be defined as

$$Z_t = \int_0^t (x-q)\, d\tau$$

or $$Z_t = \int_0^t x\, d\tau - \int_0^t q\, d\tau = X_t - Q_t$$

The plot of Z_t with respect to time represents storage fluctuations in an unconstrained reservoir subject to inflow x and outflow q. This graph can be used to find the smallest size of the reservoir required to supply draft series throughout the critical period without failure. Here it is assumed that the reservoir does not spill during the critical period or it is a topless reservoir. Many times, it is more convenient to express release as a ratio of mean inflow and this ratio is called the degree of regulation. Similarly, the storage capacity can also be expressed as a ratio of mean annual inflow and is called storage ratio or storage coefficient.

The mass curve technique, although very simple and straight forward, has a few shortcomings. One drawback pointed out many times is the implicit assumption that the storage which would have been adequate in past will also be adequate in future. Although this is not clearly true, the error caused is not really serious particularly if sufficiently long flow series has been considered. Secondly this problem will arise in many other method since future is not known. Some methods try to address this problem by explicitly considering the stochasticity of the inflows.

Another criticism which is in general true for all critical period techniques is the circularity of the definition of critical period. By definition, the critical periods depends upon the operating policy of the reservoir up to that period, its capacity and demand level. These are essentially the factors which our analysis aims to find out.

One more drawback of the mass curve is that explicit economic analysis cannot be done in this technique. The storage size can not be related to the economic life of the project. Further, it can not be computed for a particular level of reliability.

OPTIMIZATION TECHNIQUES FOR RESERVOIR CAPACITY COMPUTATION

The advent of computer and the development of optimization techniques has led to the use of both of these to storage-yield analysis. Among the various available optimization techniques, Linear Programming (LP) and Dynamic Programming (DP) are two techniques which have been used extensively. Here, only a LP based formulation is being discussed. Before this, the theory of linear programming is being discussed very briefly. The problem formulation is essentially same in case of DP.

Let us consider a situation in which a reservoir is to be constructed at a particular site. Monthly inflow data for past n months is available. The projected demand of water during a critical year is known along with its distributions among each month. The losses from the reservoir are neglected for the time being. The problem is to find out the minimum capacity of reservoir which will supply the required quantity of water without failure. Let X be the annual water demand from the reservoir and α_i = 1,2..... 12 be its fractions for different months. Hence the demand in a particular month will be $\alpha_i X$. Let I_i be the inflow to the reservoir during the i^{th} month and R_i be the water actually released from the reservoir.

Representing by S_i the storage content of the reservoir at the beginning of month i, the continuity equation is :

$$S_i + I_i - R_i = S_{i+1}$$

This equation has to be satisfied for each of the n months and hence we shall have n such equations which will be constraints in the formulation. The value of S_i is given as input.

It is also required that the amount of water actually released from the reservoir must be more than or equal to the amount demanded. This can be mathematically expressed as

$$R_i \geq \alpha_i X \qquad i = 1,2......n$$

Since this condition also must hold for each month, there will be n such constraints.

If the capacity of the required reservoir is C then in any month, from physical point of view, the storage content of the reservoir must be equal to or less than this value. Hence

$$C \geq S_i \qquad i = 1,2....n$$

Moreover, the storage S_i capacity C and release R can take only positive values. This completes the problem formulation. The problem is quite easy to solve particularly due to availability of standard package programs.

THE METHOD OF SIMULATION

Simulation is essentially a search procedure. It is one of the most widely used techniques to solve a large variety of problems associated with the design and operation of a water resources systems. The reason is that this approach can be realistically and conveniently used to examine and evaluate the performance of a set of alternative options available.

Assume that a site has been identified for the construction of a dam. The reservoir has to cater for irrigation for a nearby area and the target demand of water for different months is given. The elevation-area-capacity table for the site is available. A sufficiently long series of streamflows at the site is available. Further, it is required that the reliability of the reservoir should be least 75%. An efficient procedure of binary search can be used to determine the required storage capacity. In this method, first the upper and lower bounds on the capacity of the reservoir are determined. The lower bound can be taken to be zero or the dead storage and the upper bound can be determined from physical factors such as water availability etc. A trial value for the reservoir capacity is selected which is the mean of upper bound and lower bound.

Now, starting with a suitable value of initial storage content, the reservoir is operated using the streamflow data. The effect of this initial storage value will not be very significant if the inflow series for a sufficiently long period, say 30-40 years is being used. During any time period, the release is made equal to the demand if that much water is available in the storage. Otherwise whatever can be made available is released and the reservoir is said to have failed in that period. The evaporation losses can be easily considered if the information about the depth of evaporation is available. In this way, the reservoir is operated for the entire period of record. Now the the reliability of the reservoir is computed. If this reliability is less than the desired value, it means that the capacity of the reservoir must be increased. In this case the present capacity is adopted as the lower bound for next iteration. The feasible region below this lower bound is discarded and the trial value for the next iteration is chosen midway the upper bound and new lower bound. If, however, the reliability comes out to be higher than the required limit, the size of the reservoir is bigger than what it should have been and hence the region between the current value and upper bound is discarded for further examination. The present capacity value becomes the new upper bound. Again the trial value for the next iteration is chosen as mean of new upper bound and old lower bound.

The computations are repeatedly performed in this manner and they are terminated when the required convergence is achieved. This method converges quite rapidly as the feasible region is halved every time. It may be seen that in this method, generation of hydroelectric power can also be easily considered.

STORAGE-YIELD ANALYSIS USING OPTIMIZATION & SIMULATION

In the following a methodology which combines simulation and optimization for storage yield analysis will be described. It is assumed that the data like elevation- area- capacity table, inflow, normal evaporation depth are available. The monthly distribution of yield is also known. If storage required is to be calculated then yield is known otherwise the storage capacity is known. The Fibonacci search technique for the computation of dependent variable, reservoir capacity or annual yield, till desired reliability is achieved with permissible tolerances, supplied by the user.

At the beginning of iteration, the upper bound of the variable the average inflow volume in a year given by

$$Y_{upl} = \left(\sum_{t=1}^{NMONTH} I_t \;/\; NMONTH \right) * 12$$

where Y_{upl} = upper bound of variable Y,

I_t = reservoir inflow during time period t, and

NMONTH = total number of months.

The lower bound of storage is taken as dead storage S_{min}, whereas for annual yield lower bound is taken as zero. With the desired accuracy, specified lower bound and calculated upper bound, one dimensional search is carried out to reach the optimum value of variable.

In Fibonacci search technique, objective function is required, which is computed as described here. the reliability achieved is computed after complete reservoir operation computations, based on mass balance equation, given by

$$S_{t+1} = S_t + I_t - R_t - E_t$$

subjected to

$$S_{min} \leq S_t \leq C$$

where E_t = evaporation loss during t^{th} interval, and

D_t = demand during interval t.

The evaporation loss E_t is function of both S_t and S_{t+1}. Hence an iterative method is applied using elevation-area-capacity table till absolute difference between two successive relative evaporation losses (E_t/S_t) are less than DIFMAX, which is user supplied. At each time interval, attempt is made to satisfy the demand to the extent possible. if the available water in reservoir is less than S_{min}, no release is made and the storage is depleted by evaporation only and the reservoir is assumed to have failed during that particular month. If during any period, $S_t + I_t \geq C$, the extra water over the storage capacity after meeting the demand is spilled. If there is not enough water in the reservoir to meet the demand any period, the demand is met to the extent possible and the month is treated as failure month.

The reliability achieved (REL) is computed by

$$REL = 1.0 - FAIL/NMONTH$$

Where FALL = number of failures (number of periods when $R_t < D_t$).

The objective function used in Fibonacci search is

$$OF = | REL - RELI |$$

where RELI is the reliability desired.

The detail of Fibonacci search method, which is a unidirectional search method for nonlinear optimization problems, can be found in texts such as Rao (1979). The choice of this method over other univariate nonlinear programming techniques is somewhat subjective.

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