TN-8

STUDY OF REACH TRANSMISSIVITY FOR STREAM AQUIFER INTERACTION

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1984-85

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LIST OF SYMBOLS

b	Excess river width at the water surface
b'	$\sinh^2 \dfrac{L\pi}{2D_i}$
c	Constant
D_i	Thickness of pervious stratum below the river bed
D_w	Depth to water level in the aquifer measured from the water surface in the river
d	Location of point D of Z plane in t plane
E	Saturated thickness of an aquifer
e	Location of point E of Z plane in t plane
$F(\frac{\pi}{2},\cdot)$	Complete elliptic integral of the 1st kind
H	Depth of water in the river
h_1	Height of water in the aquifer measured from river bed level at a distance L from a vertical river bank
i	$\sqrt{-1}$
K	Coefficient of permeability
L	Distance between the river and observation point
L'	Length of a river reach
M	Complex constant
p	Pressure
Q_r	Return flow from an aquifer to a river
q	Total flow from the river to the aquifer per unit time per unit length of the river
q'	Flow occurring through the river bed to the aquifer per unit time per unit length of the river
s_r	Drawdown in the aquifer in the vicinity of a river reach
T	Aquifer transmissivity

w	Complex potential = $\phi + i\Psi$
W_p	Wetted perimeter of a canal
x, y	Cartesian coordinates
Z	Complex variable = $x + iy$
α	A parameter
Γ_r	Reach transmissivity
σ_r	Drawdown to the water level in the stream
θ	Zhukovsky's plane = $Z + \frac{iw}{K}$
ϕ	Velocity potential function
Ψ	Stream function

LIST OF FIGURES

ABSTRACT

A river comprising boundary of flow domain is often encountered in regional groundwater flow modelling. When a river fully penetrates an aquifer and has considerable stream discharge, the river is to be treated as a boundary of prescribed head. In such a case, the region on each side of the river behaves independently. However, a situation is rarely seen where a river completely penetrates an aquifer. In case of partially penetrating river with considerable stream discharge in comparison to the seepage losses besides treating the river as specific head boundary, the exchange of flow between the river and the aquifer has to be introduced through boundary while modelling the groundwater flow. The recharge from a river to an aquifer is proportional to a difference in the level of water in the river and in the aquifer in the vicinity of river. The coefficient of proportionality is recognised as reach transmissivity, which depends on the stream-bed characteristics and shape of stream cross section besides the aquifer parameters.

In this report the reach transmissivity has been determined for a river with large width. The recharge from a river with large width to an aquifer, where water table exists at shallow depth, has been quantified using conformal

mapping technique. The recharges occuring through bed and side of the river have been estimated separately. The suitable position of observation well to monitor ground water table, which can be used in assessing river recharge to the aquifer, has been identified. The influence of river stage and position of water table on reach transmissivity has been analyzed. It is found that, when the observation well is located at a distance more than 0.5 D_i, where D_i is the depth to impervious stratum below the river bed, the corresponding reach transmissivity is independent of the water table position at the observation well. The reach transmissivity depends on the river stage.

1.0 INTRODUCTION

A river, comprising a boundary of flow is often encountered in regional ground water flow modelling. When a river reach cuts right through to the base of an aquifer, it conforms to the Dirilchelt type of boundary. A fully penetrating river reach with large stream discharge as compared to the seepage losses, can be conveniently treated as a boundary of prescribed head. In such a case the region on each side of the river would behave independently. However, a situation is rarely seen where a river completely penetrates an aquifer. In the case of a partially penetrating river with considerable stream discharge, besides treating the river as a prescribed head boundary, the exchange of flow between the river and aquifer has to be introduced through the boundary nodes while modelling the groundwater flow (Rushton and Redshaw, 1978). The recharge from a river to an aquifer is proportional to a difference in the level of water in the river and in the aquifer in the vicinity of the river (Bouwer, 1969). The coefficient of proportionality recognised as reach transmissivity depends on the stream bed characteristics and shape of the stream cross-section (Morel Seytoux, 1964; Bouwer, 1969). The water level in the aquifer depends on all the abstractions and recharges including recharge from the river. Such an implicit and complex stream-aquifer interaction problem has been analyzed by Morel Seytoux and Daly (1975) who have used reach transmissivity and discrete kernel theory for finding an expression for recharge. The expression of reach transmissivity developed by Morel Seytoux et al (1979) is valid for a river or a canal with small width. In the present analysis, the recharge from a river of large width when water table is at shallow depth has been found for a steady state condition using conformal mapping. By knowing the recharge for a given position of the water table at a point near the river, the corresponding reach transmissivity can be obtained.

2.0 REVIEW

The return flow from an aquifer to a river reach could be expressed in the form (Morel Seytoux et al., 1973; Morel Seytoux, 1975).

$$Q_r = \Gamma_r \, (\sigma_r - S_r) \qquad \ldots(1)$$

where

Γ_r = the reach transmissivity,

σ_r = the drawdown to the water level in the stream,

S_r = the drawdown in the aquifer in the vicinity of the reach both measured from the same datum.

A positive value of Q_r means the flow is taking place from the aquifer to the river. Studies of the literature on seepage from canals (Bouwer, 1969) indicate that the reach transmissivity could be expressed in terms of stream and aquifer parameters.

Under a steady-state regime a simple application of potential theory for saturated flow leads to the formula (Morel Seytoux et al, 1979)

$$\Gamma_r = \frac{TL'}{E} \; \frac{0.5\, W_p + E}{5\, W_p + 0.5\, E} \qquad \ldots(2)$$

where T is the aquifer transmissivity in the vicinity of the river, E its saturated thickness, L' the length of the reach, and W_p is wetted perimeter. Equation (1) has been applied to transient condition on the basis that a transient system could be well represented as a continuous succession of steady states.

Mishra (vide Hydrology Report, 1980) has obtained an approximate expression for reach transmissivity for a river having large width. The flow from the river occurring through the bank has been estimated using Dupuit's theory for known river stage and position of water level in the aquifer at an observation point. The flow occurring through the bed has been obtained using Schwarz Christoffel conformal mapping. The expression for total seepage has been found to be

$$q = \frac{K\ (H + h_1)\ (H - h_1)}{2L} + K(H - h_1)\ \frac{F\{\frac{\pi}{2}, \sqrt{(\frac{1}{1 + b'})}\}}{F\{\frac{\pi}{2}, \sqrt{(\frac{b'}{1 + b'})}\}} \qquad \ldots(3)$$

in which

$b' = \text{Sinh}^2\ \frac{L\pi}{2D_i}$

h_1 = height of water in the aquifer measured from river bed level at a distance L from the vertical river bank

D_i = thickness of pervious stratum below the river bed

H = depth of water in the river.

The expression for reach transmissivity has been derived from equation (3) and is given by

$$\Gamma_r = \frac{T_2}{L} + \frac{T_1}{D} \frac{F\{\frac{\pi}{2}, \sqrt{(\frac{1}{1 + b'})}\}}{F\{\frac{\pi}{2}, \sqrt{(\frac{b'}{1 + b'})}\}} \quad \ldots(4)$$

in which $T_1 = \dfrac{K(H + h_1)}{2}$ and

$$T_2 = K\, D_i .$$

The above expression for reach transmissivity is only valid for situation where the water level in the aquifer is above the river bed level. A general and more rigorous expression of reach transmissivity has been obtained in the present study.

3.0 PROBLEM DEFINITION

Figure 1(a) shows a schematic cross-section of a river in Z plane. The river is partially penetrating and has large width. An impervious stratum is underlying at a depth D_i below the river bed. When the width of a river bed is more than $4D_i$, the river can be regarded to have large width. The depth of water in the river is H. The river bank has a slope equal to $\tan^{-1}(b/H)$, where b is the excess river width at the water surface. At a distance L from the river the water table in the aquifer is at a depth D_w below the level of water in the river. It is required to find the quantity of water recharged by the river to the aquifer.

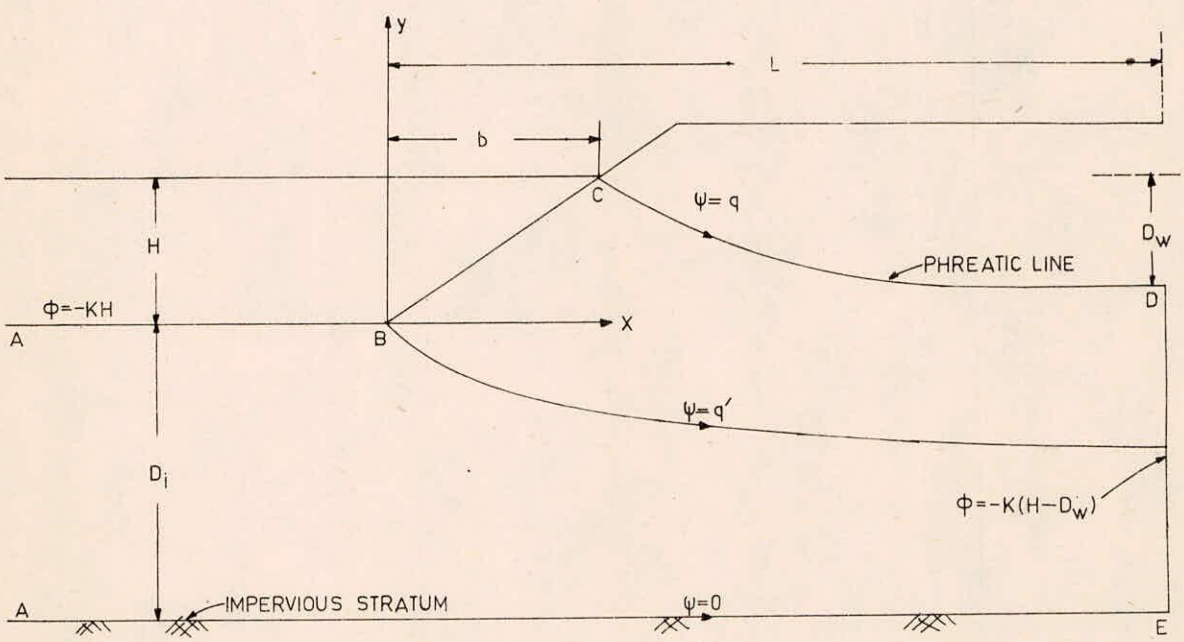

FIGURE 1(a) - PHYSICAL FLOW DOMAIN (Z-PLANE)

4.0 METHODOLOGY

The pertinent complex potential plane w, where $w = \phi + i\psi$ is shown in Figure 1(b), in which ψ = stream function and ϕ is the velocity potential function defined as

$$\phi = -K\left(\frac{p}{\gamma_w} + y\right) + c \qquad \ldots(5)$$

where,

K = coefficient of permeability,

p = pressure,

γ_w = unit weight of water,

y = elevation head, and

c = constant which has been taken as zero

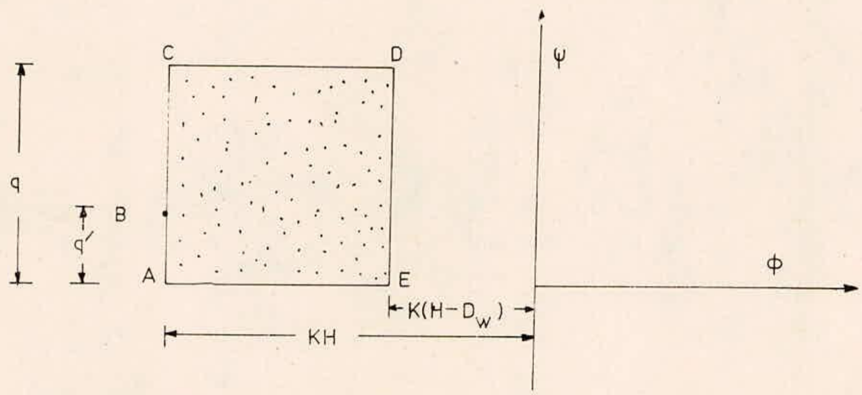

FIGURE 1(b) - COMPLEX POTENTIAL PLANE (w-PLANE)

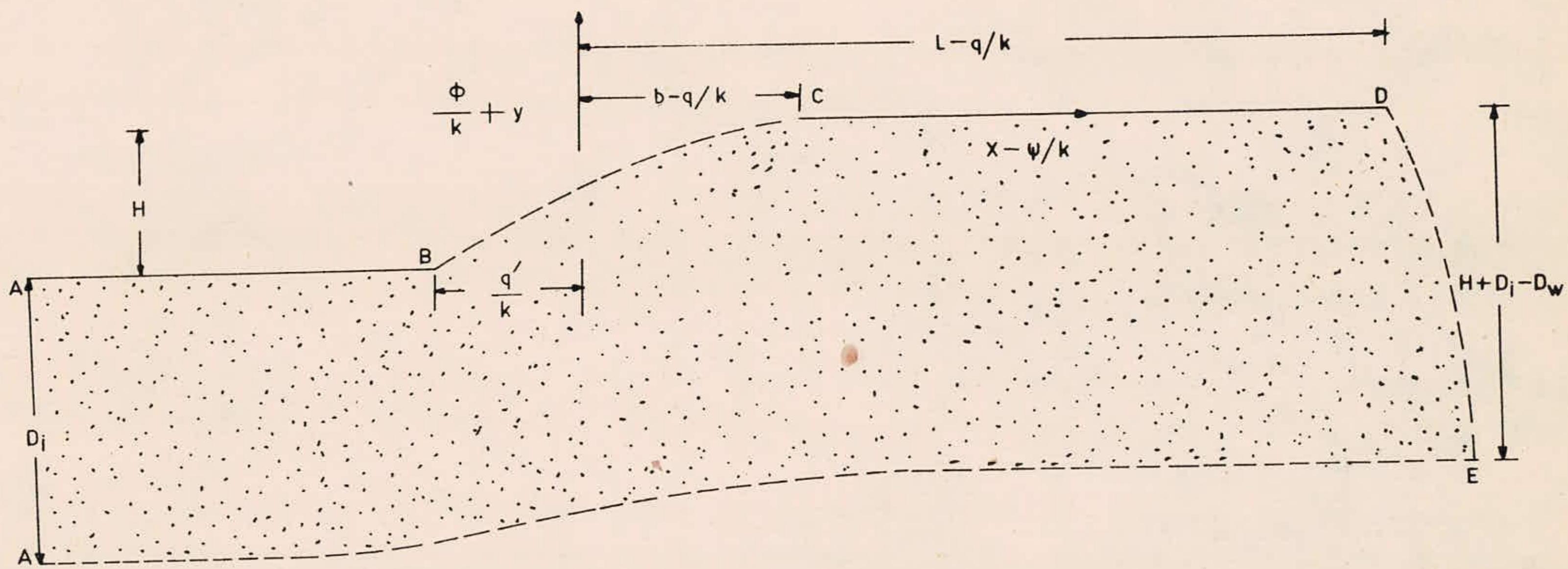

FIGURE 1(c) - ZHUKOVSKY'S θ PLANE

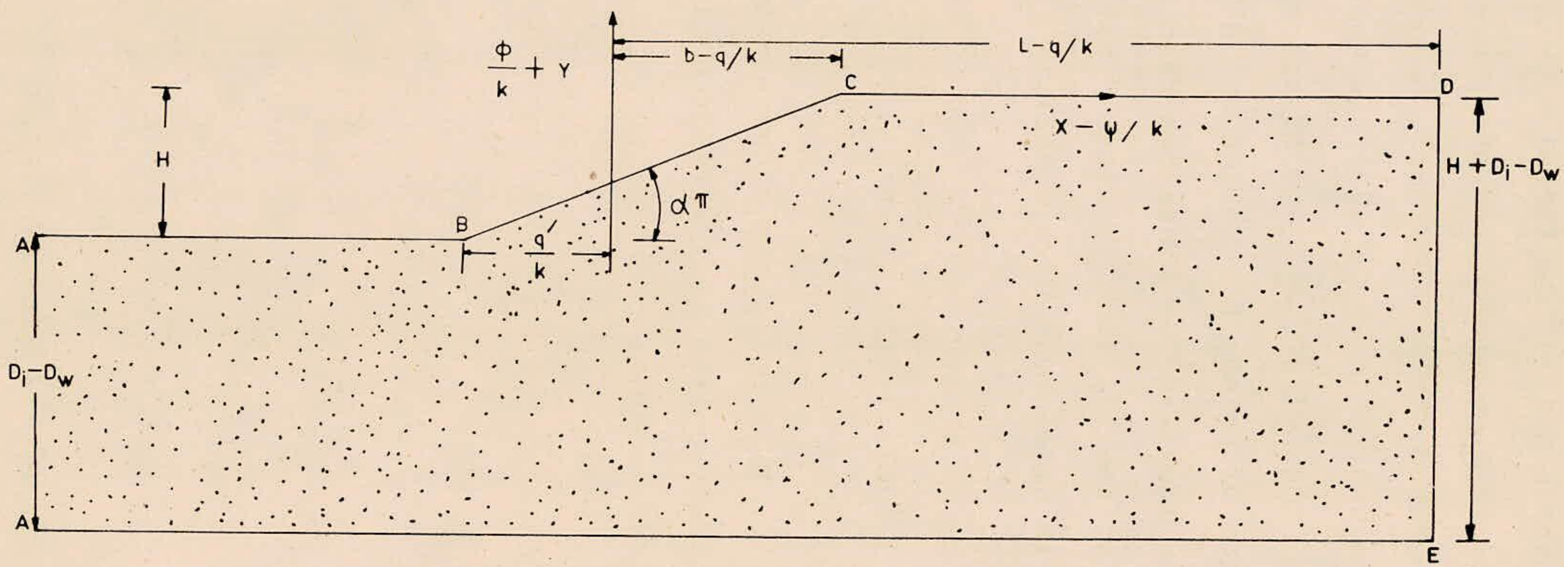

FIGURE 1(d) - IDEALIZED θ PLANE

The flow domain consists of a phreatic line which is curvilinear and unknown priori. Conformal mapping can be applied to analyze the unconfined flow after transforming the flow domain to Zhukovsky's θ plane. The pertinent Zhukovsky's θ plane where $\theta = Z + \frac{iw}{K}$ is shown in figure 1 (c). The loci of BC, DE, and AE are not known though the locations of points B, C, D, E and A are known. In figure 1(d), the idealized θ plane has been shown. The conformal mapping of the θ plane to lower half of the auxiliary t plane is given by (Harr, 1962).

$$\theta = M \int_{o}^{t'} \frac{t^{\alpha}\, dt}{(1-t)^{\alpha}\, (d-t)^{\frac{1}{2}}\, (e-t)^{\frac{1}{2}}} - iH - \frac{q'}{K} \qquad \ldots(6)$$

the vertices A, B, C, D, E being mapped onto points $-\infty$, O, 1, d, e respectively on the real axis of t plane as shown in figure 1(e). M is a complex constant to be evaluated.

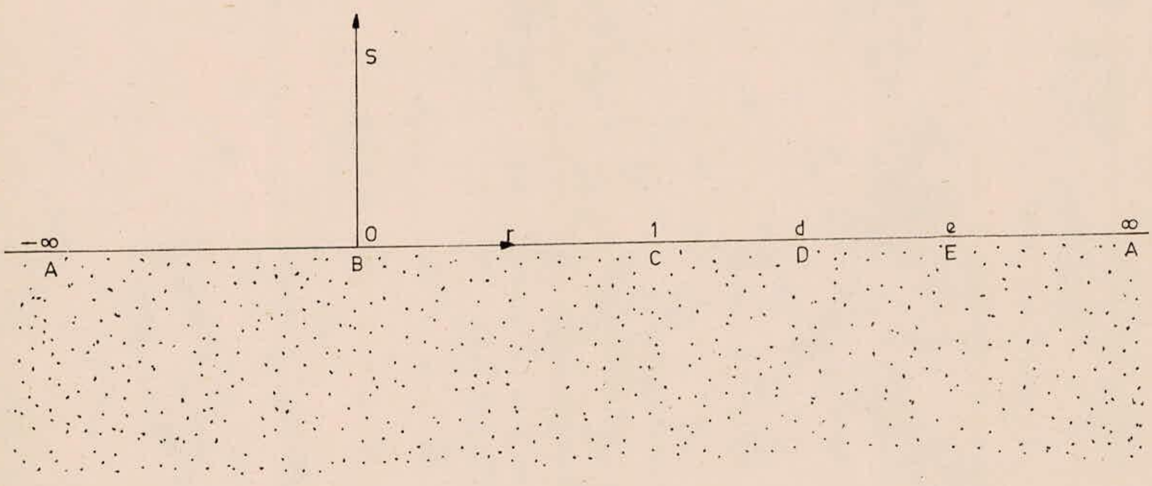

FIGURE 1(e) - AUXILIARY t PLANE

Making use of the relations between θ and t planes at points C,D,E the following equations are obtained:

$$\sqrt{\{(b - \frac{q}{K} + \frac{q'}{K})^2 + H^2\}} = |M| \int_o^1 \frac{t^\alpha dt}{(1-t)^\alpha (d-t)^{\frac{1}{2}} (e-t)^{\frac{1}{2}}} \qquad \ldots(7)$$

$$(L-b) = |M| \int_1^d \frac{t^\alpha dt}{(1-t)^\alpha \ (d-t)^{\frac{1}{2}} \ (e-t)^{\frac{1}{2}}} \qquad \ldots(8)$$

$$H + D_i - D_w = |M| \int_d^e \frac{t^\alpha \ dt}{(t-1)^\alpha \ (t-d)^{\frac{1}{2}} \ (e-t)^{\frac{1}{2}}} \qquad \ldots(9)$$

The conformal mapping of the complex potential plane to lower half of the t plane is given by

$$w = M' \int_1^{t'} \frac{dt}{(1-t)^{\frac{1}{2}} \ (d-t)^{\frac{1}{2}} \ (e-t)^{\frac{1}{2}}} - KH + iq \qquad \ldots(10)$$

in which, M' is a constant.

Making use of the relations between w and t planes at points D and E the following expressions are obtained:

$$M' = \frac{\sqrt{-1} \ KD_w \ \sqrt{(e-1)}}{2F\{\frac{\pi}{2}, \ \sqrt{(\frac{d-1}{e-1})}\}} \qquad \ldots(11)$$

$$\frac{q}{KD_w} = \frac{F\{\frac{\pi}{2}, \ \sqrt{(\frac{e-d}{e-1})}\}}{F\{\frac{\pi}{2}, \ \sqrt{(\frac{d-1}{e-1})}\}} \qquad \ldots(12)$$

in which $F\{\frac{\pi}{2}, \surd(\frac{d-1}{e-1})$ is complete elliptic integral of 1st kind with argument equal to $\surd(\frac{d-1}{e-1})$. The expression for q' which specifies the recharge taking place through the river bed is found as follows:

for $-\infty \leq t \leq 1$,

$$w = M' \int_{-\infty}^{t'} \frac{dt}{(1-t)^{\frac{1}{2}} \quad (d-t)^{\frac{1}{2}} \quad (e-t)^{\frac{1}{2}}} - KH \qquad \ldots(13)$$

Integrating and applying the condition at point B for which $t = 0$ and $w = - KH + iq'$, the following expression for q' is obtained:

$$\frac{q'}{KD_w} = \frac{F\{\mathrm{Sin}^{-1} \surd(\frac{e-1}{e}), \ \surd(\frac{e-d}{e-1})\}}{F\{\frac{\pi}{2}, \ \surd(\frac{d-1}{e-1})\}} \qquad \ldots(14)$$

From geometry of figure 1(d), the following equation can be derived:

$$\alpha\pi = \tan^{-1} \left(\frac{H}{b + \frac{q'}{K} - \frac{q}{K}} \right) \qquad \ldots(15)$$

5.0 RESULTS

The six unknowns M,d,e,q, q' and α can be found from equations 7,8,9,12,14 and 15. However, it is found convenient to assume numerical values for the parameters d and e and evaluate corresponding values of D_i and L. The numerical results have been obtained in the following manner:

Assuming numerical values for H, b, D_w, d, and e; q, q' and α are obtained from equations 12, 14 and 15 respectively. The constant M is then obtained from equation 7. The values of D_i and L corresponding to the assumed values of d and e are obtained from equations 8 and 9 respectively. The integral appearing in equation 7, 8 and 9 have been evaluated numerically after converting the improper integral to proper intengral by method of substitution. For assumed values of d, H, b, and D_w, the value of e is varied and the result for a particular value of D_i is obtained by an iteration procedure.

The variations of q/KD_w and q'/KD_w with L/D_i are shown in Figures 2(a) and 2(b) for $D_w/D_i = 0.01$ and 0.1 respectively. L can be envisaged as the distance of an observation well and q/D_w can be regarded as the reach transmissivity for unit length of river.

Variation of q/KD_w with D_w/D_i is shown in Figure 3 for $b/D_i = 0.1$, $H/D_i = 0.1$ and $L/D_i = 0.5$ and 1. As seen from the figure the nondimensional values of q/KD_w are independent of D_w for $L/D_i \geq 0.5$. Thus, if the observation well is located beyond 0.5 D_i the corresponding reach transmissivity is independent of the drawdown D_w. However, the reach transmissivity is dependent on the river stage. As seen from Figure 4, q/KD_w increases as H/D_i increases. For a river with a given geometry, the reach transmissivity for various stages at a known observation point could be determined. This can be used to assess river recharge under varying river stage.

The Locus of the phreatic line has been shown in Figure 5 for a particular river cross-section.

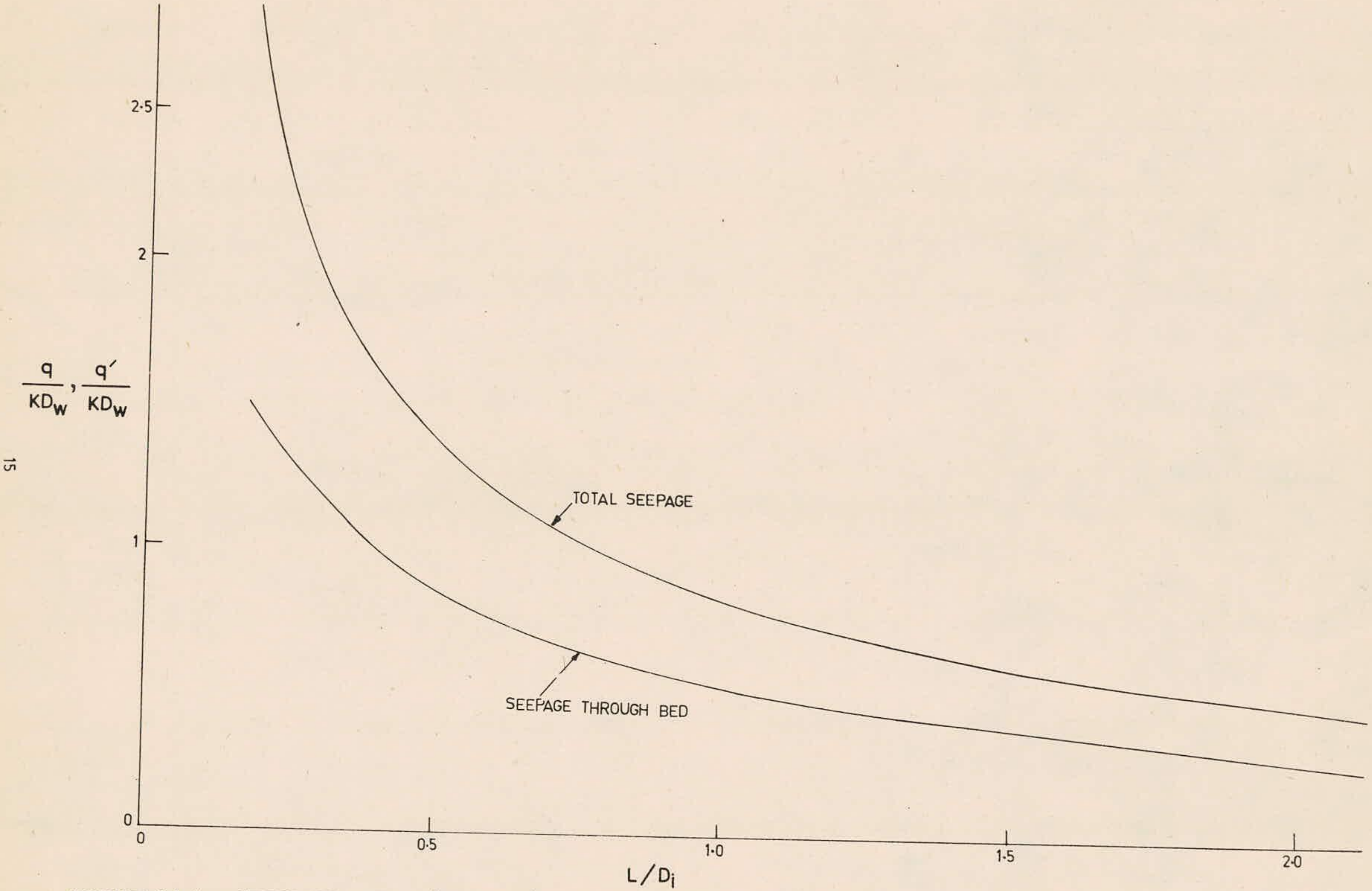

FIGURE 2(a) - VARIATION OF q/KD_w AND q'/KD_w WITH L/D_i FOR $H/D_i = 0.1$, $b/D_i=0.1$ AND $D_w/D_i=0.01$.

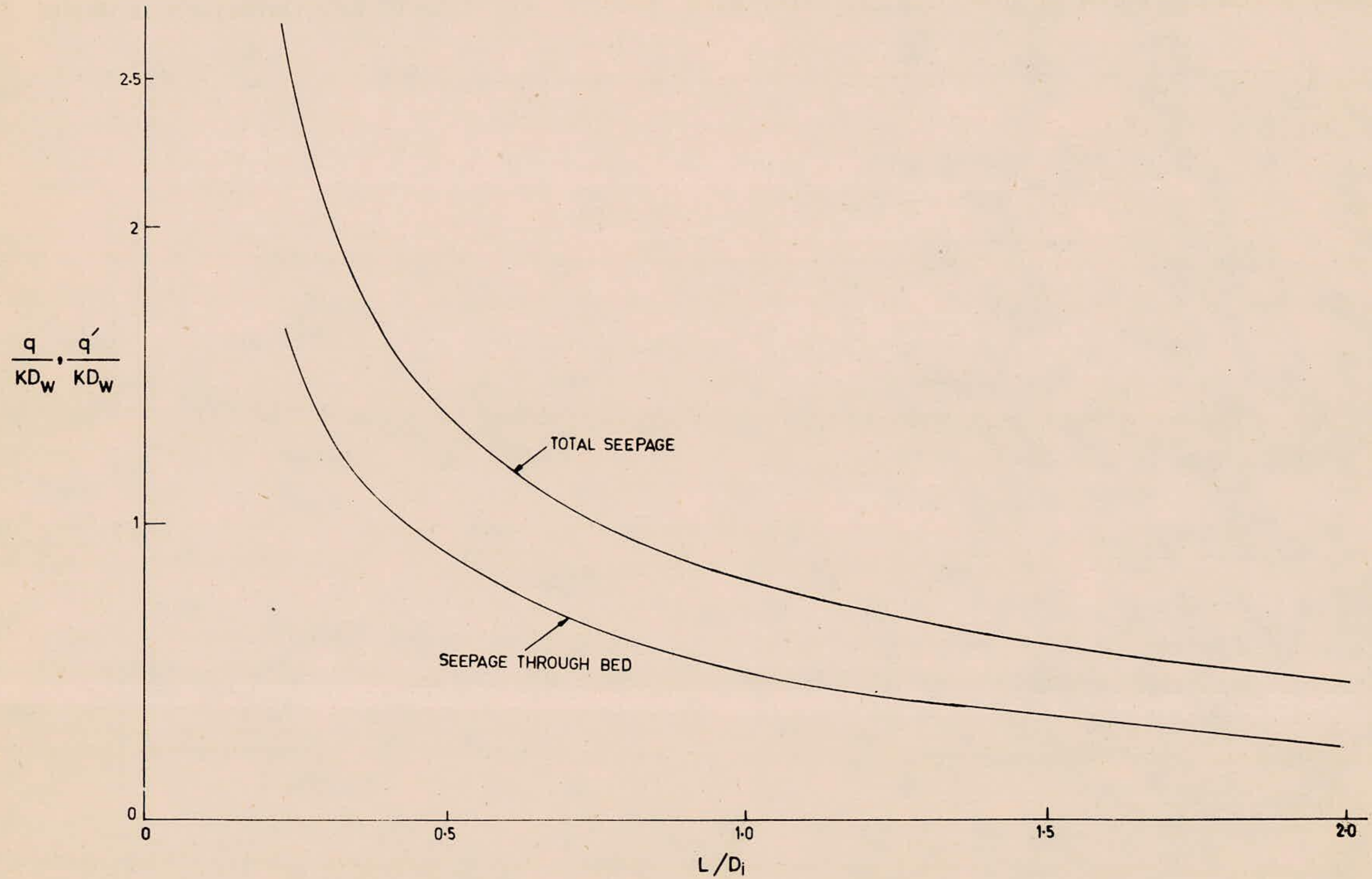

FIGURE 2(b) - VARIATION OF q/KD_w AND q'/KD_w WITH L/D_i FOR $H/D_i=0.1$, $b/D_i=0.1$, AND $D_w/D_i=0.1$

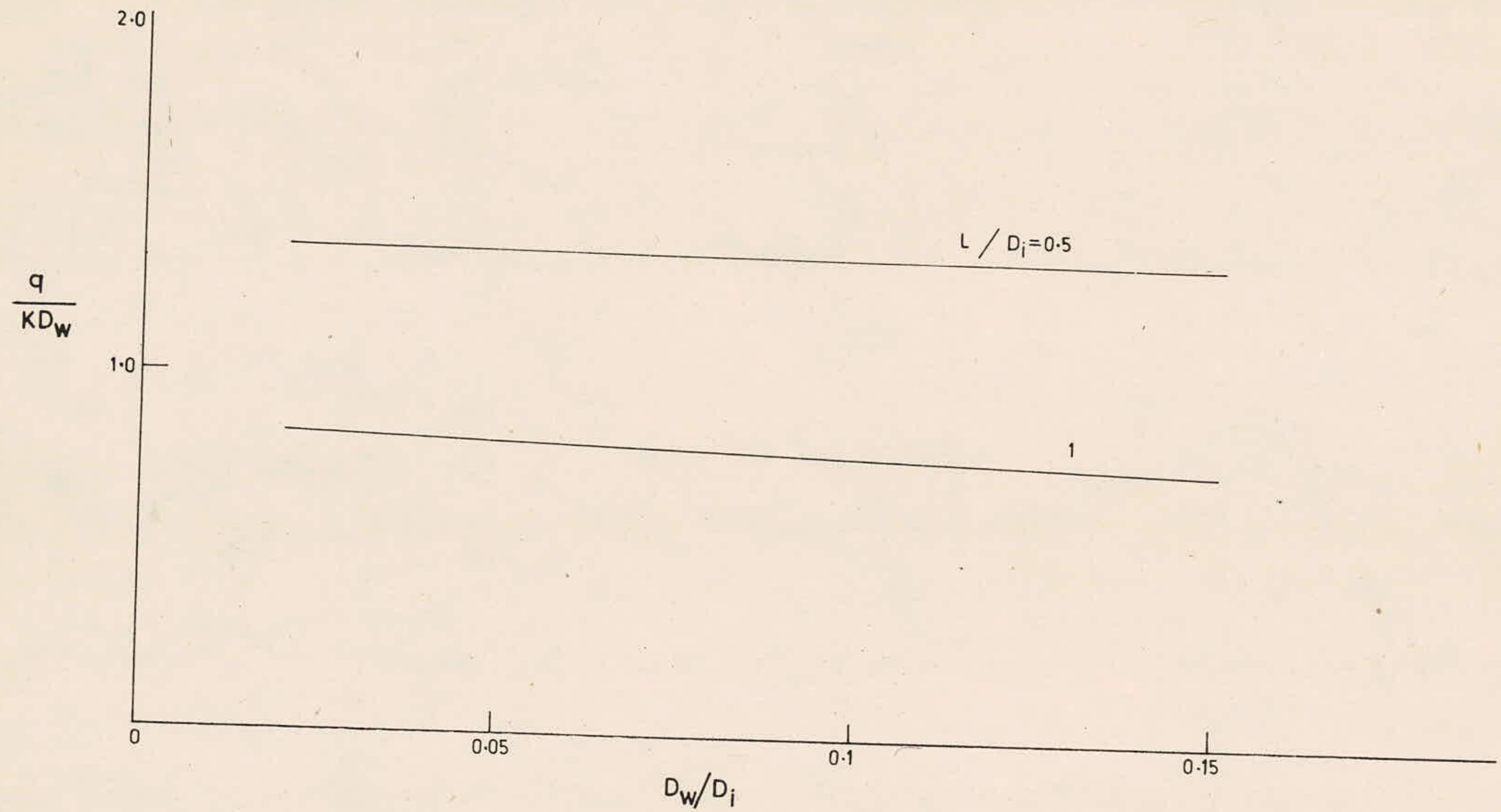

FIGURE 3 - VARIATION OF q/KD_w WITH D_w/D_i FOR $b/D_i=0.1$ AND $H/D_i=0.1$

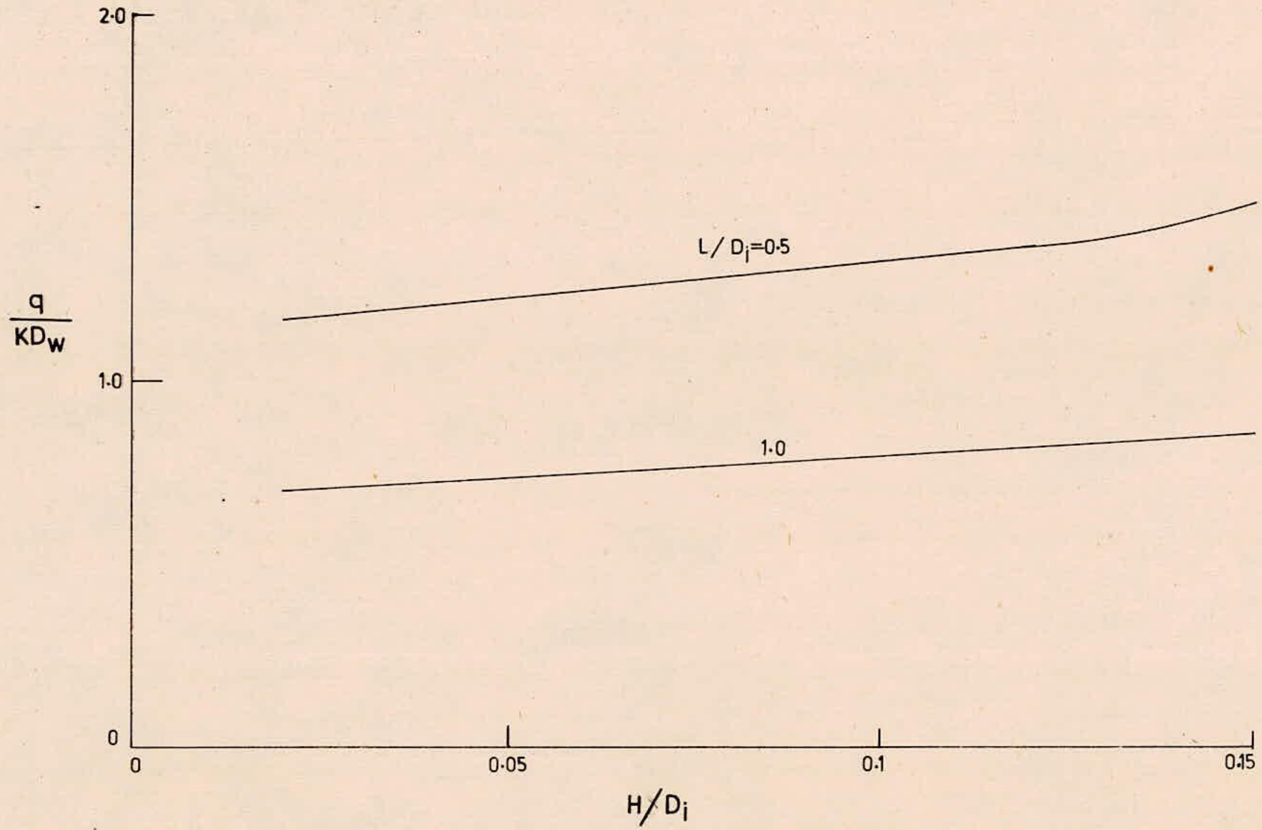

FIGURE 4 - VARIATION OF q/KD_w WITH H/D_i FOR $b/D_i = 0.1$ AND $D_w/D_i = 0.1$

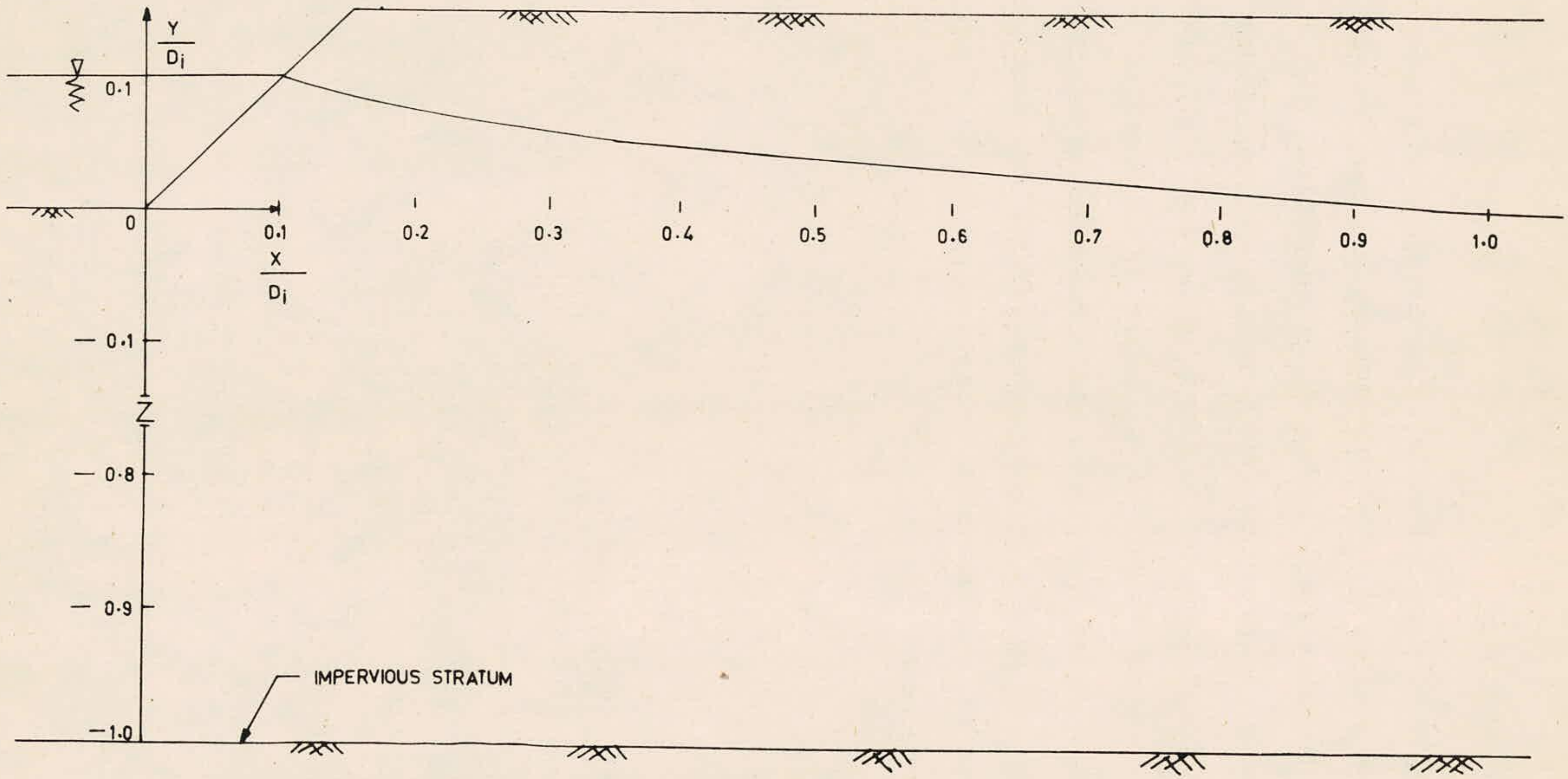

FIGURE 5 - LOCUS OF THE PHREATIC LINE

6.0 CONCLUSIONS

Recharge from a partially penetrating river of large width to a shallow water table aquifer has been found using conformal mapping for a steady state condition. The quantities of seepage occurring through the bed of the river have been identified. It is found that when the observation point is located beyond 0.5 D_i, where D_i is depth to impervious stratum under the river, the corresponding reach transmissivity is almost independent of the drawdown at the observation point. The reach transmissivity values depend on the river stage.

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