LECTURE-6

UNIT HYDROGRAPH ANALYSIS

6.0 OBJECTIVES

After attending this lecture, the participants would be able to know about the following aspects :

(i) Unit hydrograph (UH) theory, its assumptions and limitations,

(ii) Various factors affecting unit hydrograph shape,

(iii) Derivation of unit hydrographs,

(iv) Instantaneous unit hydrograph (IUH) and S curve hydrograph,

and (v) Application of the unit hydragraph.

6.1 INTRODUCTION

Design of structures to control river flows must consider both extremes of runoff (that is, droughts and floods). Analyses are required to size the capacity of outlet works (spillways, bypasses etc.) to cater for floods. It is often necessary in hydrologic design to have details of both the peak flows and the distribution of flow with time, in other words, the hydrograph, so that the runoff volume can be estimated.

As streamflow records are somewhat limited for most locations, it is neceesary to relate runoff to rainfall. Knowing rainfall rates, a function to convert rainfall to runoff is required. Unit Hydrograph (UH) is such a tool.

Unit hydrographs can be derived from analysis of rainfall and runoff records in those catchments where such data are available. The procedures used to derive a unit hydrograph are dependent upon whether the storm from which a unit hydrograph is to be calculated is :

* a simple or single period storm, or
* a multi period storm

Thunder-storms are usually intense and of short duration, and more likely to be treated as single period storms. Frontal storms are usually of longer duration and therefore, are generally suitable for single period analysis. A multiperiod approach should be followed for these.

Generally, the unit hydrographs derived from various events are not the same. The estimation of design flood requires a representative unit hydrograph for the watershed. There are two possible approaches for the derivation of the representative unit hydrograph for the watershed.

In first approach, the unit hydrographs derived from various events are averaged by the conventional averaging method. However, the second approach considers the joint event, obtained from clubbing the various events together for the analysis and provides a single representative unit hydrograph for the watershed.

As mentioned above, the unit hydrograph is a conversion factor which converts the rainfall (excess rainfall) to runoff (direct surface runoff). For this conversion, the duration of unit hydrograph should be the same as the duration of each of the excess rainfall blocks. If the durations of unit hydrograph and excess rainfall blocks differ, then the duration of unit hydrograph should be changed to the duration of excess rainfall, and the unit hydrograph with changed duration should be used for converting the excess rainfall to the direct surface runoff. Two approaches based on S-curve and superimposition methods are envisaged for changing the duration of unit hydrograph. The former approach of S-curve is more general than the later one which is used for changing the duration of unit hydrograph only when the new duration is the integer multiple of the original duration.

In this lecture most of the above aspects are discussed with the help of some illustrative examples. If runoff data is not available (ungauged catchment case), technique have been developed to derive unit hydrographs using purely catchment characteristics. These are termed as 'Regional unit hydrographs' or 'Synthetic unit hydrographs'. The regional unit hydrographs developed for some specific regions in India are discussed in lecture no. 10 of this note.

6.2 THE PURPOSE OF A UNIT HYDROGRAPH

Basically, the unit hydrograph is a multiplier, converting rainfall to runoff. A unit hydrograph is a hydrograph and thus its multiplying effect varies with time, producing from rainfall a time distribution of surface runoff. A unit hydrograph only measures direct surface runoff. Therefore, the base flow must be separated from the stream flow hydrograph in the derivation of a unit hydrograph and must be added to surface runoff derived using unit hydrograph techniques to obtain total runoff. The base flow separation techniques have already been described in lecture No. 5.

When variations in rainfall with time, the unit hydrograph can be applied to each rainfall unit and from an addition of the resultant individual runoff hydrographs, the surface runoff hydrograph can be determined.

6.3 WHAT IS A UNIT HYDROGRAPH ?

(a) A unit hydrograph is a hydrograph of surface runoff that would result at a given point in a stream from unit rainfall excess occurring in unit time uniformly over the catchment area above that given point.

It deals only with rainfall excess and thus loss rates must be deducted from total rainfall.

(b) Earlier the normel practice was to take unit rainfall excess as one inch, With metrication, one millimetre is now commonly used, but it may be stressed that the unit rainfall excess can be any selected value (such as 5, 10. 20, 50, 100 mm etc,) depending on the size of the catchment and the rainfall magnitude.

(c) The unit time is arbitrarily selected taking into account the duration of the storm and the size of the catchment area being examined. For small catchments use short periods, say 1, 2 hours and for larger catchments 3, 4, 6 or even 12 hours.

(d) Basically it can therefore be said :

(i) A unit hydrograph is a flow hydrograph;

(ii) A unit hydrograph is a hydrograph of surface runoff not total runoff;

(iii) The hydrograph of surface runoff results from the rainfall excess;

(iv) The rainfall excess represents total rainfall minus losses (abstractions);

(v) For any unit time period, the rainfall excess is assumed to occur uniformly over the catchment;

(vi) Typical unit times used in unit hydrograph analysis are 1, 2, 3, 6, 8 and 12 hour periods. Time periods which are not multiples of 24 hours, should not be used.

6.4 UNIT HYDROGRAPH THEORY AND ASSUMPTIONS

(a) Constant Base Length Proposition : The duration of surface runoff (t_r) for rainfall of a known duration (t_p) for one catchment is constant and independent of the total volume of runoff. See Fig. 6.1.

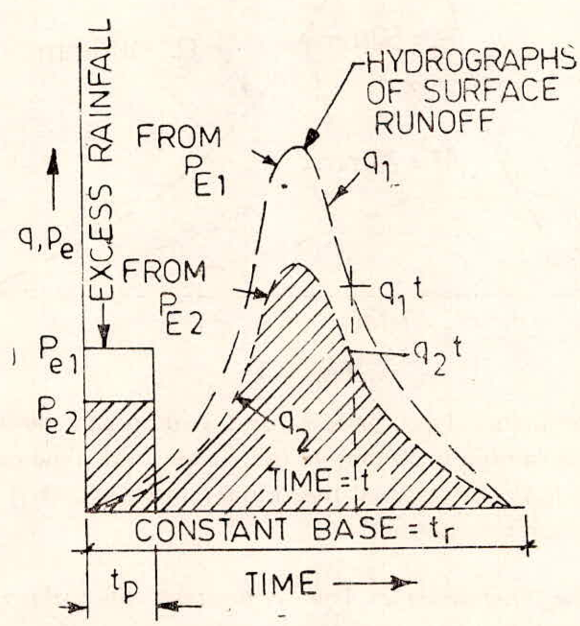

Fig. 6,1

If t_p = Duration of excess rainfall, then for all rainfall events of this duration, the base length of the surface rudoff is constant and equal to t_r.

(b) Proportional ordinate Proposition : For storms of equal duration (unit periods) on one catchment, at time t, the rate of surface runoff is proportional to the total volume of rainfall excess. See Fig. 6.1.

For storm 1 — rainfall excess = P_{E1} over time period $t_p = P_{e1} \times t_p$

For storm 2 — rainfall excess = P_{E2} over time period $t_p = P_{e2} \times t_p$

At Time t :

$$\frac{q_1(t)}{q_2(t)} = \frac{P_{e1} \times t_p}{P_{e2} \times t_p} = \frac{P_{E1}}{P_{E2}}$$

Example 6.1

Fig. 6.2 shows the surface runoff hydrographs for three different excess rainfall volumes of 100 mm, 50 mm, 25 mm of same unit period. The ratios of q 100 : q 50 : q 25 = 100 : 50 : 25 = 4 : 2 : 1

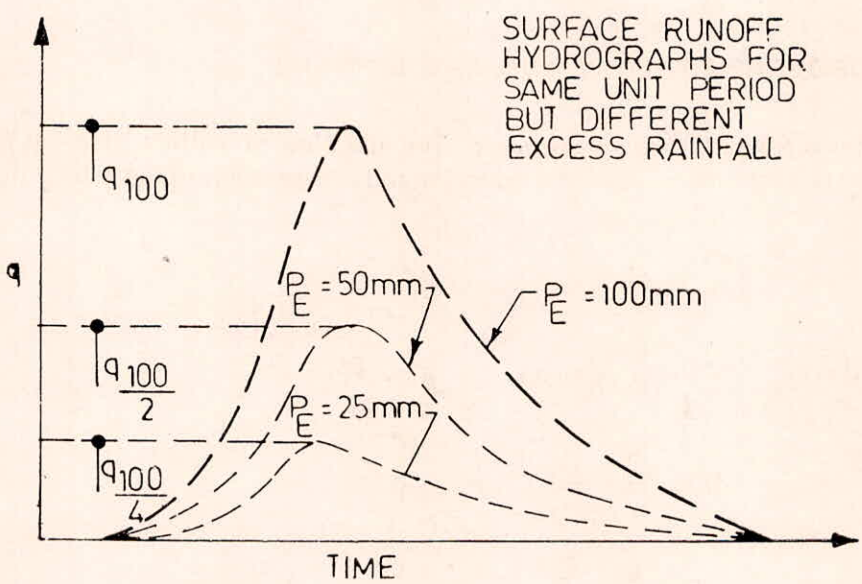

Fig. 6.2

Therefore, in unit hydrograph theory if you have a unit hydrograph derived for 100 mm of excess rainfall and the actual excess rainfall is 60 mm in the same unit time period, the runoff resulting is 0.6 times the unit hydrograph ordinates. If the actual rainfall excess is 220 mm, then the runoff is 2.2 times the unit hydrograph and so on.

(c) Concurrent flow Proposition : The hydrograph of surface runoff resulting from a particular portion of storm rainfall is not affected by concurrent runoff resulting from other portions of the storm.

In other words, the total hydropraph of surface runoff is the sum of the surface runoff produced by the individual portions of the excess rainfall. See Fig. 6.3 where two hydrographs of surface runoff are shown one resulting from excess rainfall P_{E1} and other from excess rainfall P_{E2}.

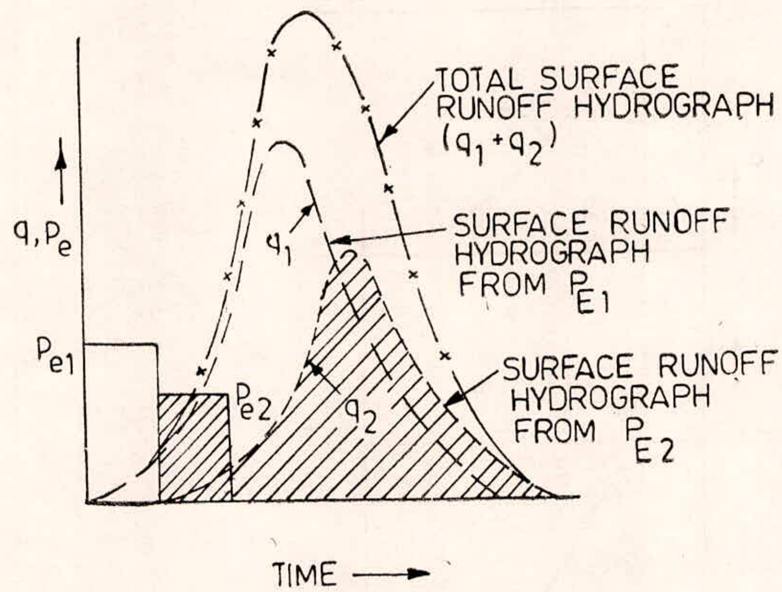

Fig. 6.3

Under this principle, (concurrenf flow proposition), the surface runoff (q_1) from the first rainfall excess (P_{E1}) is not affected by the surface runoff (q_2) from the second rainfall excess (P_{E2}). The total surface runoff is therefore the sum of $q_1 + q_2$.

In unit hydrograph derivation, it is necessary to derive a unit hydrograph which will produce q_1 when multiplied by P_{E1} and q_2 when multiplied by P_{E2}.

Example 6.2

Fig. 6.4 shows the example for concurrent flow proposition where A is a given hydrograph for a depth of net rainfall of 5 cm and a duration $\triangle t$. The hydrograph for a rain consisting of two successive periods, each with the same duration Δt as the given one but with depth of excess rainfall of 6 and 3 cm respectively, can be found by adding the ordinates of the hydrographs B and C. Here B and C are the hydrographs due to the excess rainfall of 6 and 3 cms respectively.

(d) Uniform rainfall excess in time : The assumption requires that :

(i) the storm selected should be of short duration and intense.
(ii) the resulting hydrograph should be of single peaked and short time base.

(e) Uniform rainfall excess in space :

(i) the catchment considered should be small.

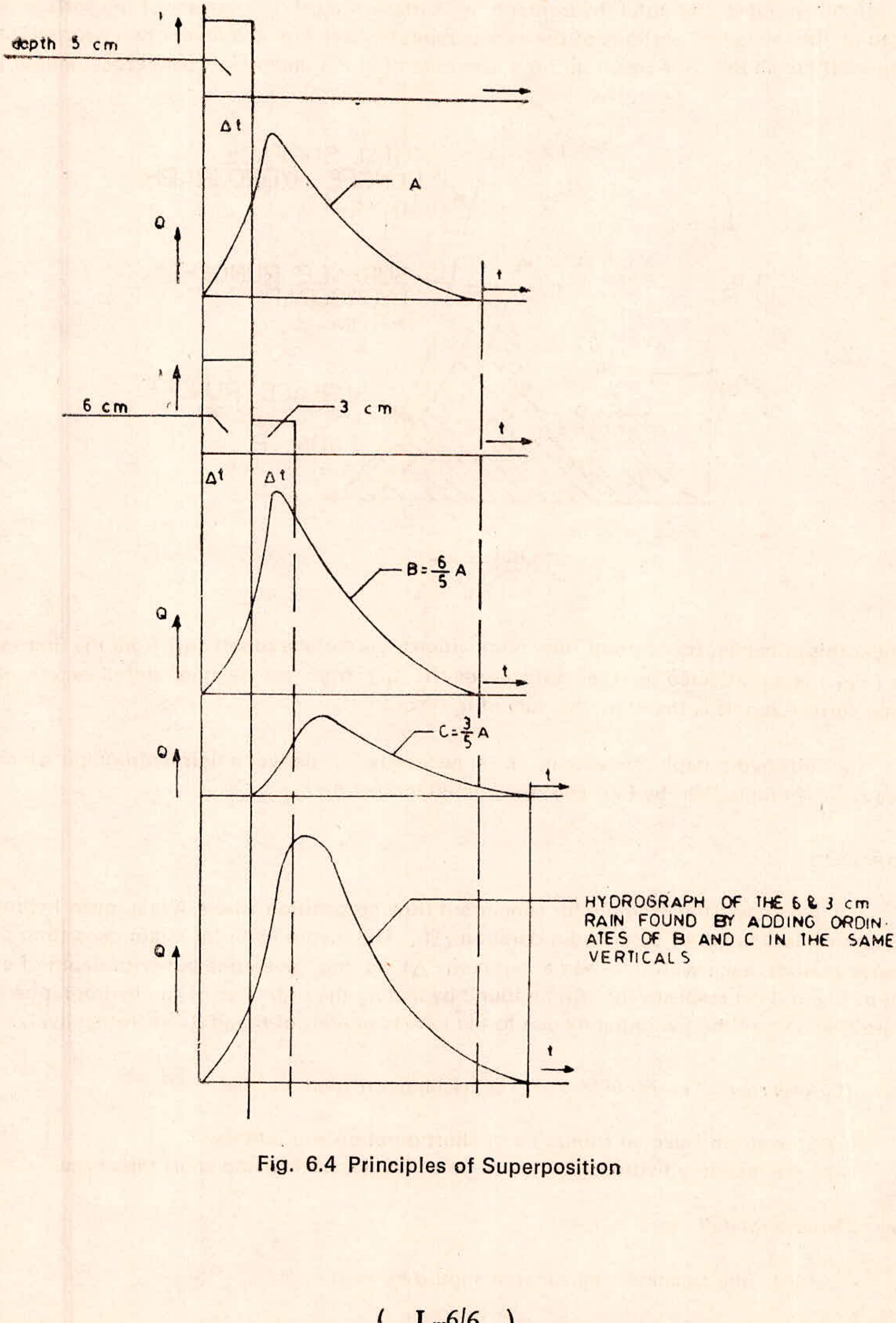

Fig. 6.4 Principles of Superposition

(ii) the unit hydrograph theory becomes inapplicable for lhe large watershed (more than 5000 sq. km.)

(iii) the unit hydrograph theory can be applied by sub dividing the large watersheds.

(*f*) *Time invariance propositions* : This principle is valid only when :

(i) the physical characteristics of the watershed do not change will time due to man made adjustments and land use effects.

(ii) storm pattern and its movement do not change with time.

6.5 SUMMARY

(i) the principles of proportionality and superposition along with the assupmption of uniform rainfall in time and space constitute the unit hydrograph theory.

(ii) the principle of proportionality states that for the same unit period of excess rainfall the surface runoff hydrograph is directly proportional to the excess rainfall volume.

(iii) When correctly sequenced with respect to the timing of the rainfall, surface runoff hydrograph can be superimposed and added to arrive at total surface runoff. The term used to correctly sequence the surface runoff hydrograph is lagging.

(iv) An isolated, intense, short duration storm of nearly uniform distribution in space and time is the most desirable for the unit hydrograph derivation. However, such storms are rare in practice. Therefore, one has to use the complex storms for deriving the unit hydrograph.

(v) The shapes of the unit hydrograph are strongly dependent on the effective storm pattern and the physical condition of the watershed prior to the storm. As a result unit hydrographs derived from differant storms of the same watershed may vary.

6.6 FACTORS AFFECTING UNIT HYDROGRAPH SHAPE

The two basic factors that affect the shape of the unit hydrograph are :

(a) Rainfall distribution over the catchment

(b) The physiography of the catchment-its shape, slope, vegetation cover, soil type etc.

(*a*) *Rainfall distribuiion* : The primary variations in unit hydrographs, derived from different storms, are due to variations in areal pattern of rainfall, rainfall duration and time intensity pattern.

A hydrograph resulting from precipitation concentrated in the lower part of a basin will have a rapid rise, a sharp peak and a rapid recession. Precipitation concentrated in the upper part of the same basin will have a slower rise and recession and a lower, broader peak. The unit hydrographs developed from different areal distributions of runoff would have very different shapes. The differences in spatial distribution of rainfall may be considerably reduced if the application of the unit hydrograph technique is restricted to small basins.

The limitation of small basins does not ordinarily apply rainfall variations caused by topographic effects, since the effects can be considered as relatively fixed characteristics of the basin.

Variations from the normal areal pattern of precipitation cause the differences in unit hydrographs. The time base of the unit hydrograph will be lengthened and the peak will be lowered as the duration of rainfall is increased for the same amount of runoff. A separate unit hydrograph is theoretically necessary for each possible duration of rainfall. Time intensity pattern of rainfall can have a significant effect on unit hydrographs and the effect is directly related to basin size on large basins, where changes in storm intensity must last for several hours to cause distinguishable effects on the hydrograph. The short bursts of rainfall lasting only for few minutes may cause the clearly defined peaks in the hydrographs of very small basins.

For large basins, valley storage tends to eliminate the effects of short time intensities and only major changes in the time intensity pattern are reflected in the hydrograph.

The effects of changes in the time intensity pattern can usually be lessened by selecting the computational interval to be used in developing a unit hynrograph short enough so that the changes are not large from one computational interval to the next.

(*b*) *Physiography of the Catchment* : Changes in the physical characteristic do occur from natural and man mode causes. As a result the shape of the unit hydrographs also changes.

Catchments of the same size but with different physiographic factors produce differently shaped hydrographs. Steep catchment slopes result in faster runoff and thus tend to have more peaked unit hydrographs with the peaks occuring earlier than catchments with flatter slopes.

Urbanization of a catchment causes drastic changes in the shape of hydrographs and unit hydrograph developed. Due to urbanization, the natural valley storage of the basin and the average loss rates would be reduced. As a result the unit hydrographs will tend to have higher peaks and shorter times of concentration.

Under natural conditions, changes in physical characteristics can occur due to seasonal and long term changes in vegetation or to other causes, such as fires etc.

Some regional relationship, between the unit hydrograph parameters and existing basin characteristics, can be utilised to derive the unit hydrographs for the basin due to change in the catchment characteristics. The development of regional relationships will be covered in lecture No. 10.

6.7 CHARACTERISTICS OF THE UNIT HYDROGRAPH AND THE DISTRIBUTION GRAPH

The main characteristics of the unit hydrograph and the distribution graph are :

(i) It gives the time distribution of the discharge hydrograph of a watershed produced by a uniform net rain of given depth precipitated on the area.

(i i) It shows how this rain is transformed into discharge at the outlet. This transformation is assumed to be a linear process.

(iii) It is a characteristic for a given watershed, it shows the integrated effect of the surface features on the routing of the rain through the catchment.

(iv) To derive a unit hydrograph it is necessary to dispose of a number of records of, at least, moderate floods.

(v) The principles of the unit hydrograph can be applied for estimating the design flood, supplementing missing flood records and short term flood forecasting based on recorded rainfall.

(vi) The distribution graph shows the percentage of total unit hydrograph occurring during successive time periods. The sum of all the percentaga must be equal to 100.

6.8 DERIVATION OF UNIT HYDROGRAPH

6.8.1 Conventional Method

The unit hydrograph for a gauging station is derived as follows from analysis of recorded floods for the single period storm :

(i) Derive the total hydrographs for the major floods from stream flow records using gauge height flow ratings (rating curves).

(i i) The base flow should be separated from the total Hydrograph to produce the direct surface runoff hydrograph and thus the surface runoff volume Q_s in millimeters. In practise it is usual to analyse floods which produce a result for Q_s greater than 20 mm. Often this is not possible and lower values can be used, but with some caution.

(iii) Rainfall records for the stations in the catchment must be analysed and the average rainfall for the storm which produced that flood determined using one of the procedures stated in the earlier lecture no. 3.

(iv) The hydrograph of the total average rainfall is derived for time periods equivalent to the required unit period of the unit hydrograph.

(v) Loss rates are evaluated to apply to the total rainfall to estimate the excess rainfall in each of the unit periods.

(vi) Having determined the hydrographs of excess rainfall and confirming that the excess rainfall occurs in only one single unit period, the derivation of the unit hydrograph is quite simple.

(vii) Remembering that the unit hydrograph is a multiplying function to convert excess rainfall to surface runoff, the following procedure is followed :

- Plot the surface runoff hydrograph
- Determine the volume of excess rainfall, P_E, in the single unit period. (P_E also equals the volume of the surface runoff hydrograg Q_s).
- Relate the value of P_E to the unit volume required in the unit hydrograph, Q_{UH}

For example, if $P_E = 22.5$ mm and you want to derive a unit hydrograph for 10 mm ($Q_{UH} = 10$ mm), then the proportionality factor (F) used to convert the surface runoff hydrograph ordinates to unit hydrograph ordinates is :

$$F = \frac{P_E}{Q_{UH}} = \frac{22.5}{10} = 2.25$$

If the unit hydrograph is to represent 50 mm then

$$F = \frac{22.5}{50} = 0.450$$

Divide the surface runoff hydrographs by F and this gives the unit hydrograph ordinates.

Example 6.3

The thunder storm of six hours durations with the excess rainfall of 154 mm produces the following surface runoff hydrograph :

Date and Time (hrs.)		Surface Runoff (m^3/s)
June 15,	0600	10
	1200	500
	1800	1600
	M. N.	3500
June 16,	0600	5200
	1200	3100
	1800	1500
	M. N.	650
June 17,	0600	250
	1200	0
	1800	0

Find out the ordinates of 6 hour unit hydrograph having unit volume equal to 100 mm.

Solution :

A unit hydrograph for 100 mm is required

Hence,

$P_E = 154, Q_{UH} = 100$

$\therefore F = \frac{154}{100} = 1.54$

The required unit hydrograph ordinates are found dividing the surface runoff hydrograph ordinates by the factor F = 1.54. The calculation of 6 hour unit hydrograph for the above storm in shown in Table 6.1.

TABLE—6.1 Computation of 6-hour Unit Hydrograph

Date	Time (Hrs)	Surface Runoff (m^3/s)	P_E (mm)	6—Hour 100 mm unit hydrograph (m^3/s)
		(Q)		(Q/F)
June 15	0600	10	154	7
	1200	500		325
	1800	1600		1039
	2400	3500		2272
June 16	0600	5200		3377
	1200	3100		2013
	1800	1500		974
	2400	650		422
June 17	0600	250		162
	1200	0		0
	1800	0		0

6.8.2 Derivation of unit hydrographs from Multi Period Storms

When a storm is of short duration and of fairly uniform intensity the unit hydrograph can be derived simply by applying the single period storm technique.

For a storm of long duration or a shorter one with variable intensity, the storm must be treated as consisting of a series of storms and the derivation of the unit hydrograph is slightly more complex.

There are number of methods to derive unit hydrographs from multiperiod storms but all are based on unit hydrograph theory. Let :

P_{ej} = Excess Rainfall Intensity (mm/hour)

P_{Ei} = Excess Rainfall volume (mm)

U_l = Ordinates of Unit hydrograph (m^3/sec)

q_i = Ordinates of surface runoff hydrograph (m^3/sec).

Before illustrating how to derive unit hydrographs for multi period storm, the following example shows how the unit hydrograph theory is used to produce the surface runoff hydrograph from a multi period storm.

If P_{el} = Rainfall rate in hours in period T_l (mm/hrs)

then P_{Ei} = Volume of rainfall

$= P_{el} \times T_l$

e.g. If $P_{e1} = 2$ mm/hour

$T_1 = 2$ hours

$\therefore \quad P_{E1} = 2 \times 2 = 4$ mm

To determine the surface runoff hydrograph from 4 mm of rainfall in 2 hours, we use the unit hydrograph of 2 hour duration which has 7 unit period ($7 \times 2 = 14$ hours). Then in unit hydrograph theory :

(i) The base length of the surface runoff for rainfall of 2 hours duration is 14 hours.

(i i) For a 10 mm unit hydrograph, under the proportionality principle, the average runoff in the various periods for the first rainfall excess of 4 mm, is :

$q_1 = 0.4 \times U_1$ $\qquad q_5 = 0.4 \times U_5$

$q_2 = 0.4 \times U_2$ $\qquad q_6 = 0.4 \times U_6$

$q_3 = 0.4 \times U_3$ $\qquad q_7 = 0.4 \times U_7$

$q_4 = 0.4 \times U_4$

(iii) If $P_{e2} = 4$ mm/hour (2nd period of rainfall)

$P_{E2} = 4 \times 2 = 8$ mm

Then the ordinates for the second period of rainfall are :

$q_1 = 0.8 \times U_1$

$q_2 = 0.8 \times U_2$ etc.

However, the rainfall in the second period started (lagged) T_1 (2 hours) later and thus q_1 for the second period of rainfall must be added to the second period of runoff for first hydrograph.

This principle continues for the third and subsequent rainfalls.

The equations for the combined surface runoff can therefore be generalised as follows (Fig. 6.5).

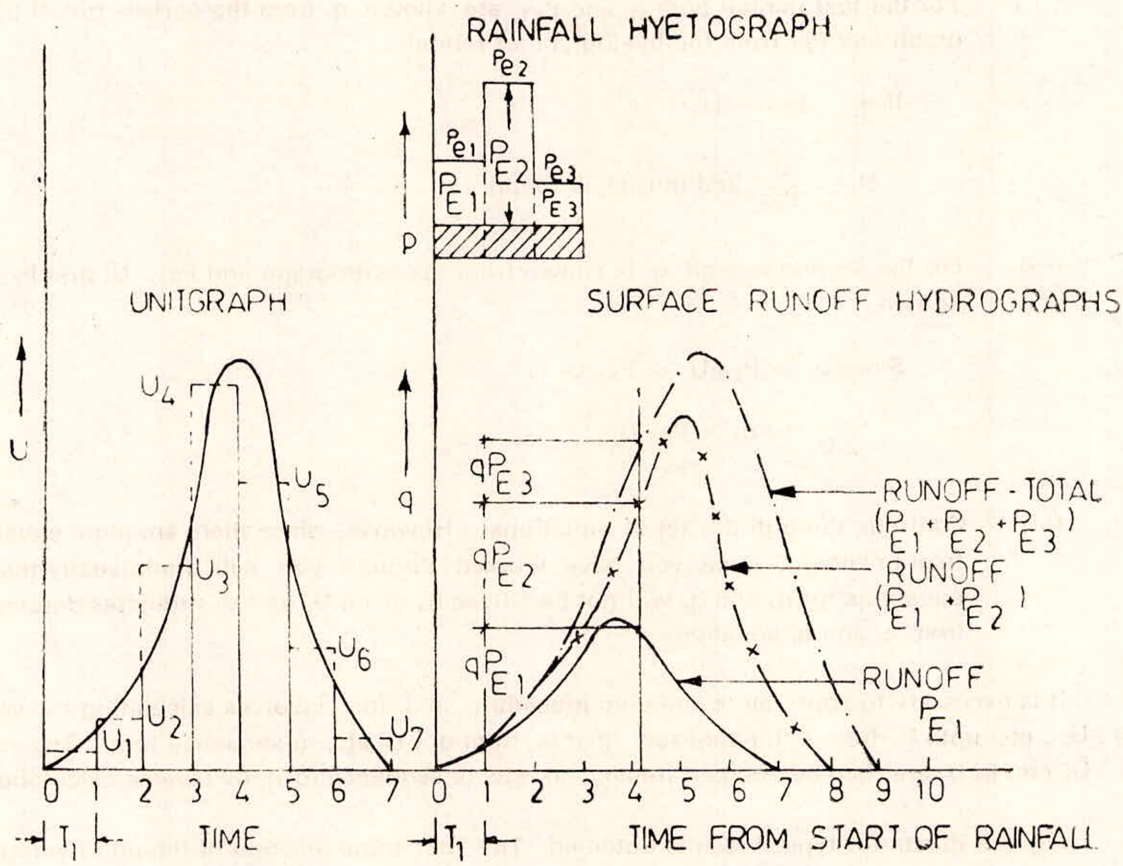

Fig. 6.5

$$q_1 = P_{E1} U_1 \qquad (6.1)$$

$$q_2 = P_{E1} U_2 + P_{E2} U_1 \qquad (6.2)$$

$$q_3 = P_{E1} U_3 + P_{E2} U_2 + P_{E3} U_1 \qquad (6.3)$$

$$q_4 = P_{E1} U_4 + P_{E2} U_3 + P_{E3} U_2 \qquad (6.4)$$

$$q_5 = P_{E1} U_5 + P_{E2} U_4 + P_{E3} U_3 \qquad (6.5)$$

Ordinates of Surface Runoff Hydrograph

$$q_6 = P_{E1} U_6 + P_{E2} U_5 + P_{E3} U_4 \qquad (6.6)$$

$$q_7 = P_{E1} U_7 + P_{E2} U_6 + P_{E3} U_5 \qquad (6.7)$$

$$q_8 = \qquad + P_{E2} U_7 + P_{E3} U_6 \qquad (6.8)$$

$$q_9 = 0 \quad + 0 \quad + P_{E3} U_7 \qquad (6.9)$$

All the procedures used for derivation of unit hydrographs from multi-period storms use the principles of proportionality and superposition which are evidenced in the above equations.

a The Method of Single Division : This procedure is basically a trial and error approach. The procedure is to solve each of the quations relating to the unit hydrograph ordinates. (Refer to Fig. 6.5)

(i) For the first rainfall both q_1 and P_{E1} are known, q_1 from the surface runoff hydrograph and P_{E1} from the hyetograph of rainfall.

If $q_1 = P_{E1} \times U_1$

$$U_1 = \frac{q_1}{P_{E1}} \text{ and thus } U_1 \text{ is found}$$

(i i) For the second rainfall, q_2 is known from the hydrograph and P_{E1}, U_1 and P_{E2} are known.

Since $q_2 = P_{E2}\, U_2 + P_{E2}\, U_1$

$$U_2 = \frac{q_2 - P_{E2}\, U_1}{P_{E1}}$$

(iii) Continue through the set of equations. However, since there are more equations than unknown, once you have worked through you will find usually that the equations for q_8 and q_9 will not be solved by using U_6 and U_7 ordinates determined from q_6 and q_7 equations.

It is necessary to apply an 'averaging' procedure and this involves calculating the values of U_1 , U_2 , etc. upto U_7 from a 'forward run', that is, from q_1 , q_2 etc, in sequence to q_7. The values of U_7 , U_8 etc. to U_1 are then determined from q_9 , q_8 etc, in sequence to q_1 by reverse calculations.

Fig. 6.6 illustrates typical results obtained. The first approximation of the unit hydrograph is obtained by 'average' the values from the two runs.

Test the unit hydrograph by checking how the derived hydrograph compared with the recorded hydrograph—THIS TEST SHOULD BE DONE IN ALL UNIT HYDROGRAPH DERIVATIONS.

When testing the unit hydrograph obtained by averaging, it is some times found that the reproduction is not good. This can be due to many things such as a small error in U_1 and in U_2 and in U_6 and U_7, which are the leading and trailing ordinates, can magnify errors in later calculations. For averaging place more reliance on U_1 and U_2 from forward runs and U_6 and U_7 for reverse runs.

The records of rainfall and runoff are inadequate and do not represent the actual situation to any acceptable degree. Unit hydrograph theory itself may not apply to the storm. Storms

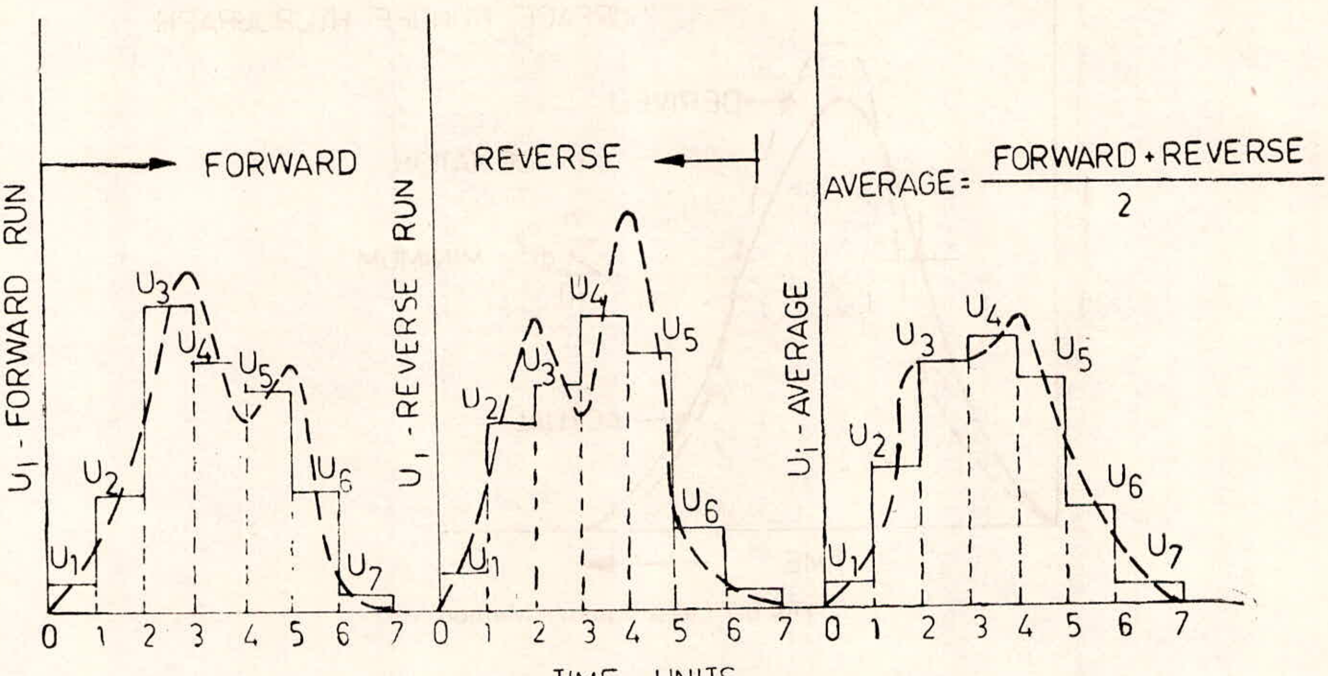

Fig. 6.6 Method of Single Division

which are concentrated rather than uniformly distributed over the catchment tend to make unit hydrograph derivation in this way difficult.

(*b*) *The Least Squares Method :* As evidenced in the single Division Method, there are more equations than unknown and the solution adopted depends on which set of equations are used.

The Least Squares method is a statistical curve fitting procedure which provides a 'line of best fit'. The equations which provide the estimates of unit hydrograph ordinates using this method are described in APPENDIX-I in matrix notations.

The basic principle of the Least Squares method is the fact that the line of best fit produces the best reproduction of the hydrograph when the unit hydrograph is re-applied to the rainfall excess. (ref. to Fig. 6.7). The unit hydrograph adopted is the one that satisfied the statistical objective of minimising the sum of the squares of the deviations of the ordinates of the reproduced hydrograph from the ordinates of the actual hydrograph.

As the method involves some tedious computations to satisfy the statistical objective, it is not a practical proposition to carry out least squares derivations by hand. Computers are used

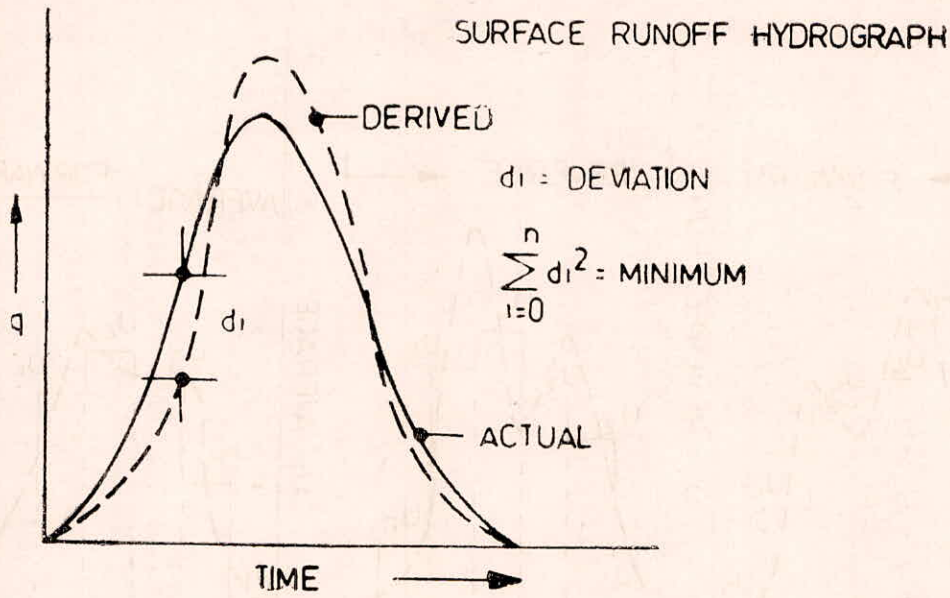

Fig. 6.7 Least Squares Method

for this, and numerous package programmes are readily available. It is probably one of the better methods, provided that :

— The hydrograph is representative of the basin,

— The rainfall and hydrograph ordinates are accurate,

— The unit hydrograph theory applies to the catchment.

If these criteria are not met, the shape of the 'best fit' unit hydrograph can be strange and ordinates can even be negative. It must be stressed that the Unit hydrograph so derived is the mathematical multiplying function and does not necessarily represent the physical constraints of runoff peculiar to the particular catchment.

Kutchment (1967) stated a procedure to derive smooth unit hydrograph. It involves a regularization factor, α, which is to be determined by trial and error procedure.

(c) Collin's Method : This is also a trial and error solution. Assuming the unit hydrograph has four units of time T_2 (U_1, U_2, U_3 and U_4), and there are three periods of time T_1 of excess rainfall (P_{E1} , P_{E2} and P_{E3}) with P_{E2} the maximum rainfall volume, then the surface runoff ordinate are determined as described earlier in section 6.3.2 of this lecture. In the previous example the unit hydrograph had 7 ordinates whereas in this example there are only 4. It is not necessary when converting rainfall to runoff as shown in the example for the selected unit hydrograph ordinates

to be over the same time interval as the rainfall excess. All that is important is that the unit hydrograph represents runoff for excess rainfall of the same interval as that of the excess rainfall which is being examined.

Ordinates of Surface Runoff Hydrograph

$$q_1 = P_{E1} U_1 \quad (6.10)$$

$$q_2 = P_{E1} U_2 + P_{E2} U_1 \quad (6.11)$$

$$q_3 = P_{E1} U_3 + P_{E2} U_2 + P_{E3} U_1 \quad (6.12)$$

$$q_4 = P_{E1} U_4 + P_{E2} U_3 + P_{E3} U_2 \quad (6.13)$$

$$q_5 = 0 + P_{E2} U_4 + P_{E3} U_3 \quad (6.14)$$

$$q_6 = 0 + 0 + P_{E3} U_4 \quad (6.15)$$

The surface runoff hydrograph is shown in Fig. 6.8

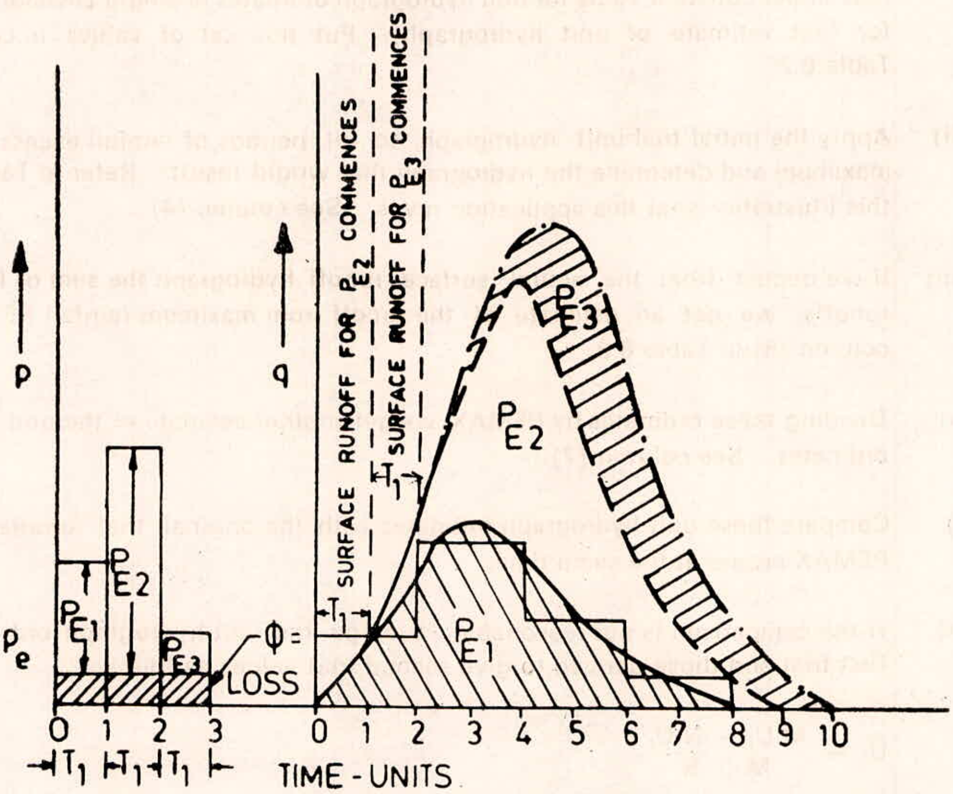

Fig. 6.8

Procedure :

The following steps may be adopted for the unit hydrograph derivation by Collin's Method for a gauged catchment :

(i) Derive the total hydrographs for the major floods from stream flow records using rating curves.

(ii) Estimate the surface runoff separating the base flow using one of the procedures described in earlier lecture, no. 5.

(iii) Estimate the average rainfall hyetograph using the procedure described in earlier lecture no. 3.

(iv) Compute the effective rainfall hydrograph separating the losses from total rainfall hyetograph.

(v) Ignore the terms in the equation that contain the maximum rainfall (in this example P_{E2}, termed PEMAX).

(vi) Assume a first trial unit hydrograph, that is, a set of U_1 values that look reasonable. (use either constant value for unit hydrograph ordinates or single Division procedure for first estimate of unit hydrograph). Put this set of values in column 3 of Table 6.2

(vii) Apply the initial trial unit hydrograph to all periods of rainfall excess except the maximum and determine the hydrograph that would result. Refer to Table 6.2 and this illustrates what this application gives See column (4).

(viii) If we deduct from the actual surface runoff hydrograph the sum of P_{E1} and P_{E3} runoffs, we get an estimate of the runoff from maximum rainfall PEMAX. See column (6) in Table 6.2

(ix) Dividing these ordinates by PEMAX, we get another estimate of the unit hydrograph ordinates. See column (7).

(x) Compare these unit hydrograph ordinates with the original trial ordinates U_i^1 for PEMAX occurs at the same time.

(xi) If the comparison is not reasonable, average the unit hydrograph ordinates of the first trial and those derived to give second trial values as follows :

$$\overline{U_i} = \frac{M\,U_i + N\,U_i^{i}}{M + N} \qquad (6.16)$$

Where M = total rainfall excess except the largest one

N = largest block of rainfall excess.

It is important with trial ordinates of the unit hydrograph to ensure the unit runoff volume corresponds to that required (this can be any multiple of 1 mm)

(xii) Repeat step (vii) to (xi) until the calculated unit hydrograph agrees with the trial unit hydrograph.

In Table 6.2 the various terms used are described as follows :

U_i = Average trial unit hydrograph ordinate in period i.

q_i = Average surface runoff ordinate in period i

P_{Ei} = Excess rainfall volume in period i

U_i' = Adjusted trial unit hydrograph ordinate in period i.

$\overline{U_i} = (M\ U_i + N\ U_i') \ / \ (M + N)$

$M = P_{E1} + P_{E3},\ N = PEMAX$

$F_E = \Sigma \frac{\overline{U_i}}{U_{UH}}$, where U_{UH} = Unit hydrograph unit volume.

and $\Sigma\ U_i$ = sum of column (8) between limits of PEMAX influence.

Example 6.4

The excess rainfall and surface runoff ordinates for a storm of a typical catchment are given below :

Time Periods (6 hrs)	P_E (mm)	q average ($m^3/s/6$ hours)
1	40	250
2	100	1050
3	60	2050
4		4350
5		4150
6		2300
7		1070
8		450
9		120

The area of the catchment is 170,000 hectares. Find out the 6-hours unit hydrograph using Collin's Method.

Solution :

Table 6.3 and 6.4 presents two trial runs on Collin's method. The objective is to find a 6 hour unit hydrograph for 100 mm of surface runoff.

Briefly the table consists of :

(i) Indication of time period-either in hours or number of periods.

(ii) Rainfall excess determined from analysis of recorded rainfall and runoff.

(iii) Trial unit hydrograph ordinates-The number of periods in the unit hydrograph is determined from a combination of the number of periods of excess rainfall and the number of periods of surface runoff (Principle of superposition). For example, if there are 7 units on the surface runoff hydrograph and only 1 period of rainfall, the unit hydrograph has 7 units. If 2 periods of rainfall produce the surface runoff then there are 6 units in the unit hydrograph.

The general equation for the base length relationships is :

Periods in surface runoff base =

Periods in Unit hydrograph Base + Number of Rainfall Periods — 1

or conversely

Periods in Unit hydrograph base =

Periods in Surface Runoff Base — Number of Rainfall Periods + 1

e.g. (1) U.G. Base = 7 — 1 + 1 = 7 (for 1 period of rainfall)

(2) U.G. Base = 7 – 2 + 1 = 6 (for 2 periods of rainfall)

In Table 6.3

Surface Runoff Base = 9 (six — hour periods)

Rainfall periods = 3 (six — hour periods)

∴ U.G. Base = 9 — 3 + 1 = 7 six hour periods (42 hours)

In table 6.3 the first trial assumes a uniform unit hydrograph and this is determined as follows :

Calculate total surface runoff (from Volume under hydrograph) or by planimetering area = 340×10^6 m^3

Catchment area = 170,000 hectares

= 1700×10^6 square metres.

For unit runoff of 1 mm unit hydrolograph volume = 1.7×10^6 m^3

For unit runoff of 100 mm unit hydrograph volume = 170×10^6 m^3

Excess Runoff in hydrograph $= \dfrac{340 \times 10^6}{1.7 \times 10^6}$

$= 200$ mm

$P_{E1} = 40$ mm $\therefore \dfrac{P_{E1}}{100} = 0.40$

$P_{E2} = 100$ mm $\therefore \dfrac{P_{E2}}{100} = 1.0$

$P_{E3} = 60$ mm $\therefore \dfrac{P_{E3}}{100} = 0.6$

Total = 200 mm.

First trial assume constant flow in Unit Hydrograph.

First trial Average

$$U_i = \frac{170 \times 10^6}{42} = 4.05 \times 10^6 \text{ m}^3/\text{hour}$$

$$= \frac{4.05 \times 10^6}{3600} = 1125 \text{ m}^3/\text{sec}$$

(iv) Repeat the stated procedure until $\overline{U}$ (adjusted) are almost equivalent to trial units then adopt as unit hydrograph.

6.9 CHANGE OF UNIT VOLUME IN A UNIT HYDROGRAPH

To convert unit hydrographs from X mm (say $QU_G = 100$ mm) to Y mm (say $QU_G = 50$ mm) simply divide the ordinates of the unit hydrograph (X mm) by the ratio of the volumes.

i.e. $\dfrac{X}{Y} = \dfrac{100}{50} = 2$

Note that the duration of the unit hydrograph will be the same.

6.10 SELECTION OF UNIT PERIOD FOR UNIT HYDROGRAPH

The time period should be determined using the following criteria :

(i) It should be short enough to define the hyetograph and the hydrograph with reasonable precision.

(ii) It should be a simple fraction of 24-hours do not use 5-hours or 7 hours etc. or periods that are not whole fraction of 24 hours.

(iii) A convention adopted in practice is to use a unit period approximately 1/3 to 1/4 the period of rise of the hydrograph. (This is not always practical).

Table 6.2 Collin's Method

(1) Time Unit Periods	(2) Rainfall excess mm	(3) Trial Unit Hydrograph (m³/s) U_1	U_2	U_3	U_4	(4) $\Sigma P_{Ei}\ U_i$ Excludes PEMAX	(5) Actual Hydrograph of surface runoff (m³/s)	(6) = (5) − (4) $q_i - \Sigma P_{Ei}\ U_i$	(7) (6) ÷ PEMAX	(8) $\overline{U_i}$ (m³/s)	(9) Adjusted for volume correctness U_i (m³/s)
0							q_0				
1	P_{E1}	$P_{E1}\ U_1$				$P_{E1}U_1$	q_1	$q_1 - P_{E1}U_1$ Limit of PEMAX Influence			
2	PEMAX	—	$P_{E1}U_2$			$P_{E1}U_2$	q_2	$q_2 - P_{E1}U_2$	U'_1	$\overline{U_1}$	$\overline{U_1} \div F_c$
3	P_{E3}	$P_{E3}U_1$	—	$P_{E1}U_3$	—	$P_{E3}U_1 + P_{E1}U_3$	q_3	$q_3 - (P_{E3}U_1 + P_{E1}U_3)$	U'_2	$\overline{U_2}$	$\overline{U_2} \div F_c$
4	—	—	$P_{E3}U_2$	—	$P_{E1}U_4$	$P_{E3}U_2 + P_{E1}U_4$	q_4	$q_4 - (P_{E3}U_2 + P_{E1}U_4)$	U'_3	$\overline{U_3}$	$\overline{U_3} \div F_c$
5				$P_{E3}U_3$	—	$P_{E3}U_3$	q_5	$q_5 - P_{E3}$ Limit of PEMAX Influence	U'_4	$\overline{U_4}$	$\overline{U_4} \div F_c$
6					$P_{E3}U_4$	$P_{E3}U_4$	q_6	$q_6 - P_{E3}U_4$			

Table 6.3 Collin's Method (example)

Time Period (6 hrs)	P_E (mm)	For 100mm Unit Hydro-graph excess rainfall (mm) P_E / 100	TRIAL (1) Unit hydrograph (m^3/s) U_1 1125	U_2 1125	U_3 1125	U_4 1125	U_5 1125	U_6 1125	U_7 1125	$\Sigma P_{Ei} XU_i$ (m^3/s)	q (m^3/s/ 6 hrs)	$U' = \frac{q - P_{Ei} U_i}{PEMAX}$	$\overline{U}$ (m^3/s)	$\overline{U}$ (adjusted for 100 mm volume) (m^3/s)
1	40	0.40	450							450	250	—	—	—
2	100	1.00	—	450						450	1050	600	863	821
3	60	0.60	675	—	450					1125	2050	925	1025	975
4				675	—	450				1125	4350	3225	2175	2069
5					675	—	450			1125	4150	3025	2075	1974
6						675	—	450		1125	2300	1175	1150	1094
7							675	—	450	1125	1070	—55	535	509
8								675	—	675	450	—225	450	425
9									675	675	120	—	—	—
Total	200										15790		8273	7870
												100	7870	Factor = 1.05

Table 6.4 Collin's Method (example)

Time Period 6 hrs	P_E (mm)	For 100mm Unit hydro graph excess rain (mm) P_E / 100	TRIAL (2) Unit Hydrograph (m^3/s) U_1 821	U_2 975	U_3 2069	U_4 1974	U_5 1094	U_6 509	U_7 428	$\Sigma P_{Ei} XU_i$ (m^3/s)	q (m^3/s/ 6-hrs)	$\overline{U'} = \frac{q - P_{Ei} U_i}{PEMAX}$	$\overline{U}$ (m^3/s)	$\overline{U}$ adjusted for 100 mm value (m^3/s)
1	40	0.40	328.4							328.4	250	—	—	—
2	100	1.00	—	390						390.0	1050	664	741	729
3	60	0.60	492.60	—	827.6					1320.2	2050	729.8	852	838
4				585	—	789.6				1374.6	4350	2975.4	2522	2480
5					1241.4	—	487.6			1679.0	4150	2471.0	2223	2186
6						1184.4	—	203.6		1388.0	2300	912.0	1003	986
7							656.4	—	171.2	827.6	1070	242.4	376	370
8								305.4	—	305.4	450	144 6	286	281
9									256.80	126.80	120	—	—	—
Total											15790		8003	7870
												100	7870	Factor =1.02

6.11 INSTANTANEOUS UNIT HYDROGRAPH (IUH)

If the period of a unit hydrograph is shortened but the depth is kept constant, the peak flows will increase and the base lengfh will decrease. If the period becomes zero and depth remain the same, the intensity of the rain becomes infinite. This corresponds with the instantaneous application of a sheet of water over entire catchment area. This water drains off by gravity and the resulting unit hydrograph is the instantaneous unit hydrograph (IUH) Fig. 6.9.

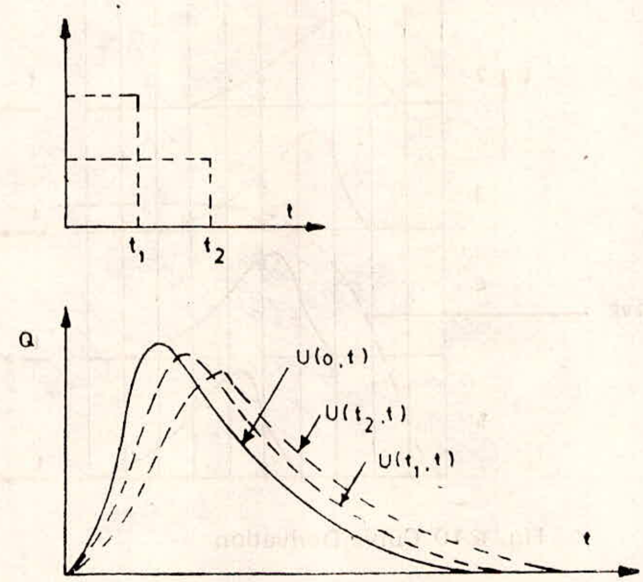

Fig. 6.2 Definition Sketch of Instantaneous Unit Hydrograph

Thus, the instaneous unit hydrograph is a purely theoretical concept and represents the unit hydrograph obtained when the unit excess rainfall volume occurs instantaneously. IUH represents a more characteristics curve of a catchment area than a T-hours unit hydrogradh as it is not affected by the duration. It is a useful tool in regional unit hydrograph studies.

6.12 S–CURVE HYDROGRAPH

An S—curve is the hydrograph of direct surface runoff that would result from excess rainfall of unit volume occuring per unit period continuously (see Fig. 6.10) S—curve may be derived using the following steps ;

(i) Plot the unit hydrograph of unit volume U_1 at time $t = 0$

(ii) Plot the unit hydrograph successively lagged time T_1. which equals the unit period of unit hydrograph.

(iii) At tima t_b which is the base length the first unit hydrograph the S–curve becomes constant.

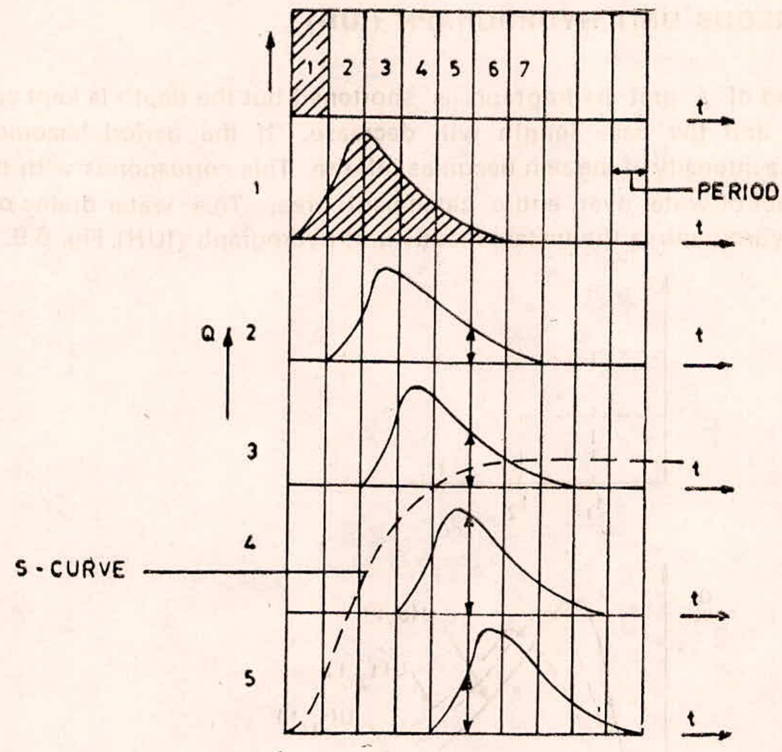

Fig. 6.10 Curve Derivation

(iv) The S—curve for the unit period (T_1 hours) has been determined. It is generally noticed that the S—curve, some times, fluctuates in the upper range. These fluctuations are commonly referred to as a 'hunting effect', and this is characteristics of S-curve derivation. The procedure to be followed is to plot a curve of best fit through the points and use the values from the curve ordinates.

(v) The Maximum ordinate of the **S**-curve corresponds with the equilibrium discharge

$$Q_{max} = \frac{0.2778\ AU_1}{T_1}$$

Where A is the catchment area (sq. km.), U_1 is the unit volume (mm) and T_1 is the duration of the unit hydrograph (hours)

The derivation of S—curve from a given unit hydrograph is explained in the following example :

Example 6.5

For a typical catchment, the ordinates of 6-hour unit hydrograph with volume 10 mm are given below :

Dated	Time (hours)	Interval (hours)	6-hour unit hydrograph volume 10 mm (m/s)
June 15	0600	0	0
	0900	3	200
	1200	6	500
	1500	9	1000
	1800	12	1600
	2100	15	2400
	2400	18	3500
June 16	0300	21	4200
	0600	24	5200
	0900	27	4400
	1200	30	3100
	1500	33	2300
	1800	36	1500
	2100	39	1000
	2400	42	650
June 17	0300	45	400
	0600	48	250
	0900	51	150
	1200	54	0

if the equilibrium discharge is 16250 cumec, compute the S—curve ordinates.

Solution :

The steps are : (Ref. Table 6.5)

(i) Enter the ordinates of 6-hour unit hydrograph in column (4).

(ii) Lag the 6-hour unit hydrograph by 6-hour, 12 hour, etc. upto 54 hours, (the time base of the unit hydrograph) and enter these values in column (5) to (13).

(iii) Compute the S—curve addition summing the values of column (5) to (13) at each interval. These values are shown in column (14).

(iv) Plot the S—curve obtained at step (iii).

(v) The plotted S—curve shows some hunting effect in the upper range. Plot a best fit curve through these points in order to get required S-curve. Fig. 6.11 shows the smooth S-curve.

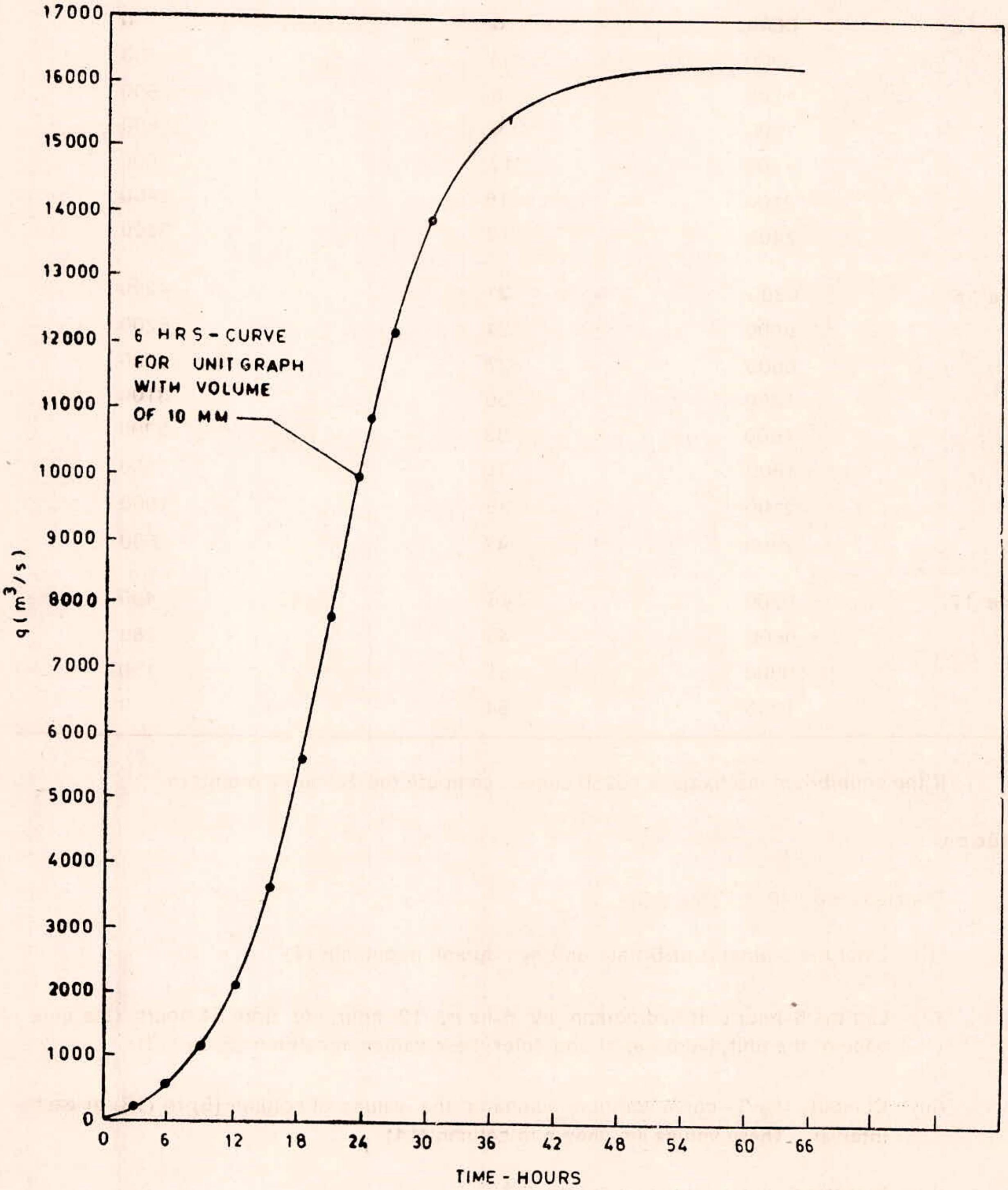

Fig. 6.11 S-Curve

(vi) Read the values of the S—curve, obtained at step (v), and enter in the table as column (15). This is the S—curve of intensity 10/6 mm/hour.

(vii) Multiply the ordinates of column (15) by 6 to get the S—curve of intensity 10 mm/hour (column 16).

6.13 RELATIONSHIP BETWEEN THE S-CURVE AND UH

A simple relation exists between the S-curve of a watershed and its UH. In Fig. 6.12 S_t is the S-curve for a continuous rain of i cm/hour and S_{t-T} is the same curve shifted over a distance of T—hours to the right. This means that the differences in the ordinates of the two (S_t and S_{-T}) represent the ordinates of a hydrograph resulting due to excess rainfall depth of T_i cm occuring over the catchment for T hours. Therefore :

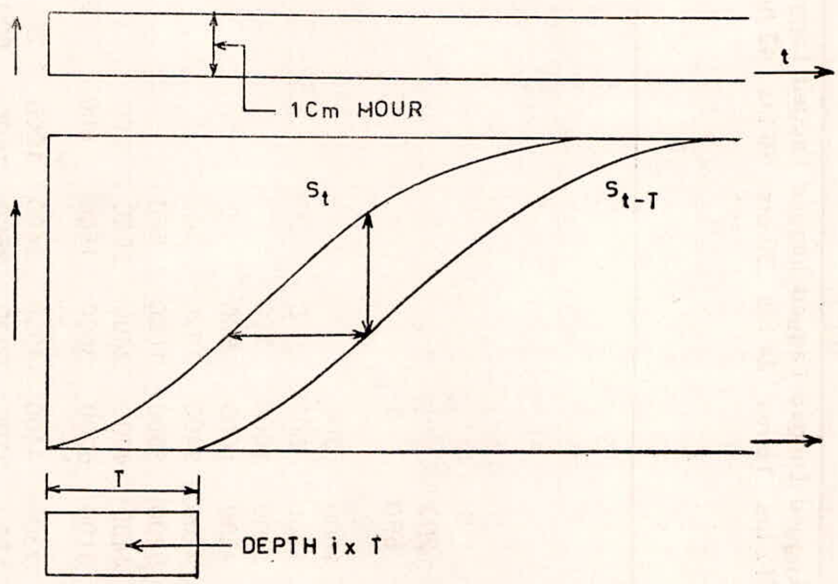

Fig. 6.12 Relationship Between the S—Curve and UH

$$U_{\text{depth } i\,T}(T, t) = S_t - S_{t-T} \tag{6.17}$$

$$\text{or } U_{\text{depth } d}(T, t) = \frac{S - S_{t-T}}{i\,T} d \tag{6.18a}$$

$$= \frac{d}{i} \; \frac{S_t - S_{t-T}}{T} \tag{6.18b}$$

here d = depth of the T-hour unit hydrograph

i = net rainfall intensity belonging to the S-curve.

if d = 1, i = 1

Table 6.5 S—Curve Derivation

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
				(m^3/s)											
Day	Time	Interval (hours)	6 hour U.H. Volume 10 mm	Lagged 6 hrs	Lagged 12 hrs	Lagged 18 hrs	Lagged 24 hrs	Lagged 30 hrs	Lagged 36 hrs	Lagged 42 hrs	Lagged 48 hrs	Lagged 54 hrs	S-curve by addition	S-curve from graph	S-curve of intensity 10 mm/hour
June 15	0600	0	0										0	0	0
	0900	3	200										200	200	1200
	1200	6	500	0									500	500	3000
	1500	9	1000	200									1200	1200	7200
	1800	12	1600	500	0								2100	2100	12600
	2100	15	2400	1000	200								3600	3600	21600
	2400	18	3500	1600	500	0							5600	5600	33600
June 16	0300	21	4200	2400	1000	200							7800	7900	47400
	0600	24	5200	3500	1600	500	0						10,800	10,400	62400
	0900	27	4400	4200	2400	1000	200						12,200	12,400	74400
	1200	30	3100	5200	3500	1600	500	0					13,900	13,800	82800
	1500	33	2300	4400	4200	2400	1000	200					14,500	14,700	88200
	1800	36	1500	3100	5200	3500	1600	500	0				15,400	15,300	91800
	2100	39	1000	2300	4400	4200	2400	1000	200				15,500	15,700	94200
	2400	42	650	1500	3100	5200	3500	1600	500	0			16,050	15,900	95400
June 17	0300	45	400	1000	2300	4400	4200	2400	1000	200			15,900	16.100	96600
	0600	48	250	650	1500	3100	5200	3500	1600	500	0		16,300	16,200	97200
	0900	51	150	400	1000	2300	4400	4200	2400	1000	200		16,050	16,230	97380
	1200	54	0	250	650	1500	3100	5200	3500	1600	500	0	16,300	16,250	97500

$$\therefore U(T, t) = \frac{S_t - S_{t-T}}{T} \quad (6.19)$$

where S_t is the S-curve of unit intensity (1 cm/hour).

6.14 RELATIONSHIP BETWEEN THE S-CURVE AND IUH

Since $U_{\text{depth } d}(T, t) = \frac{S_t - S_{t-T}}{i\,T} d \quad (6.20)$

As T approaches zero, U (T, t) approaches to U (O, t) and U depth d (T, t) approaches to $\frac{d\,S_t}{dt} \frac{d}{i}$

$$\therefore U(O, t) = u(t) = \frac{d\,S_t}{dt} \frac{d}{i} \quad (6.21)$$

$$\therefore S_t = \frac{i}{d} \int_o^t u(t)\, dt \quad (6.22)$$

If i = 1 and d = 1, the above equation becomes

$$S_t = \int_O^t u(t)\, dt \quad (6.23)$$

Fig. 6.13 shows the IUH and corresponding S-curve.

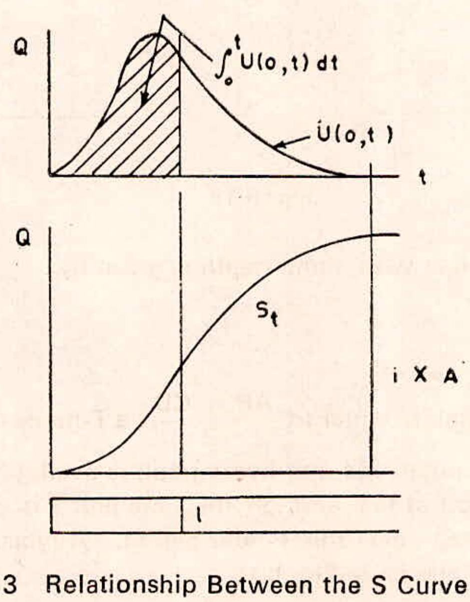

Fig. 6.13 Relationship Between the S Curve and IUH

6.15 RELATIONSHIP BETWEEN UH AND IUH

The relationship between the IUH (u (t), and the T-hours unit hydrograph (U (T,t)), both with the same unit depth can be found by again using the two S-curve (Fig. 6.14)

$$S_t = \int_O^t u\ (t)\ dt \tag{6.24}$$

$$T.\ U(T,t) = S_t - S_{t-T} \tag{6.25}$$

$$= \int_O^t u\ (t)\ dt - \int_O^{t-T} u\ (t)\ dt$$

$$= \int_{t-T}^t u\ (t)\ dt \tag{6.26}$$

$$= A' \text{ (Fig. 6.14)}$$

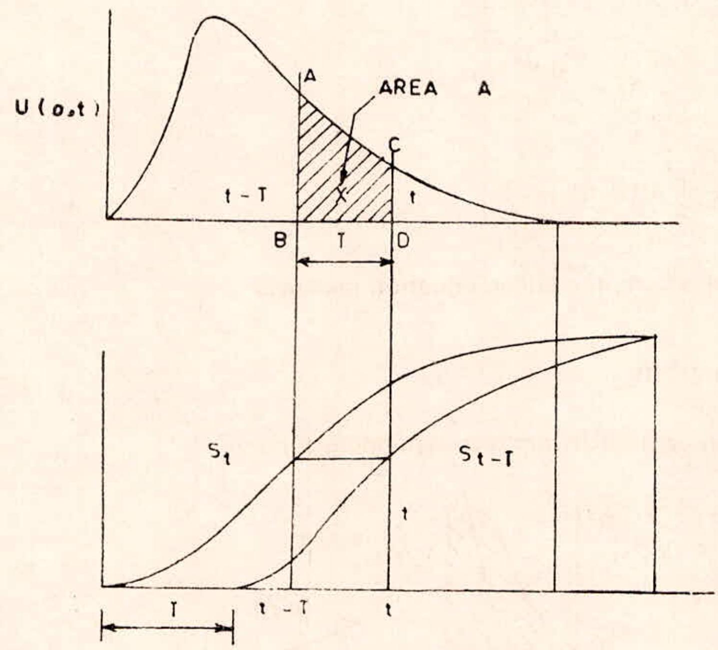

Fig. 6.14

The T-hour unit hydrograph with a unit depth is given by :

$$U\ (T,\ t) = \frac{A'}{T} \tag{6.27}$$

Since $\frac{A'}{T}$ is approximately equal to $\frac{AB + CD}{2}$, a T-hours unit hydrograph can be easily derived from an IUH. If the instantaneous unit hydrograph is available, the ordinate of the T-hour unit hydrograph (1, 2, 3 hours etc.) at the end of the time unit T is equal to the average ordinate of the instantaneous unit hydrograph over the T-hour period. A typical unit hydrograph of T-hour duration, obtained in this way, is shown in Fig. 6.15.

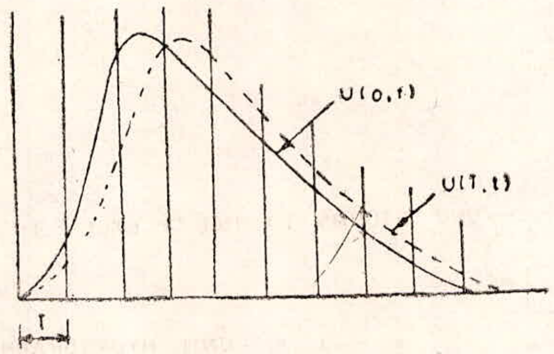

Fig. 6.15 Relationship Between IUH and UH

6.16 CHANGE OF UNIT PERIOD OF A UNIT HYDROGRAPH

Having derived a unit hydrograph for a particular unit period (say 6 hours) and you want to change the period the following procedures can be used.

- Superimposition method :—Only suitable when the new duration of unit hydrograph is integer multiple of the original duration (say original duration 6-hours and new duration 12 hours).
- S-curve method :—More general method

6.16.1 Superimposition method

The Unit hydrograph of 2 t_r duration can be derived from a unit hydrograph of t_r duration in the following steps :

(i) Add the ordinates of t_r — hour unit hydrograph to the ordinates of an identical unit hydrograph lagged by t_r — hour.

(ii) Divide the ordinates of the resulting hydrograph of step (i) by 2 to obtain a unit hydrograph for a unit duration of 2 t_r. Fig. 6.16 shows conversion of a unit hydrograph of t_r duration to duration 2 t_r.

Note that the unit hydrograph of n t_r duration can be derived by n time successive lagging of the t_r duration unit hydrograph and then dividing the resulting hydrograph by n, where n is an integer (n = 1,2...etc.)

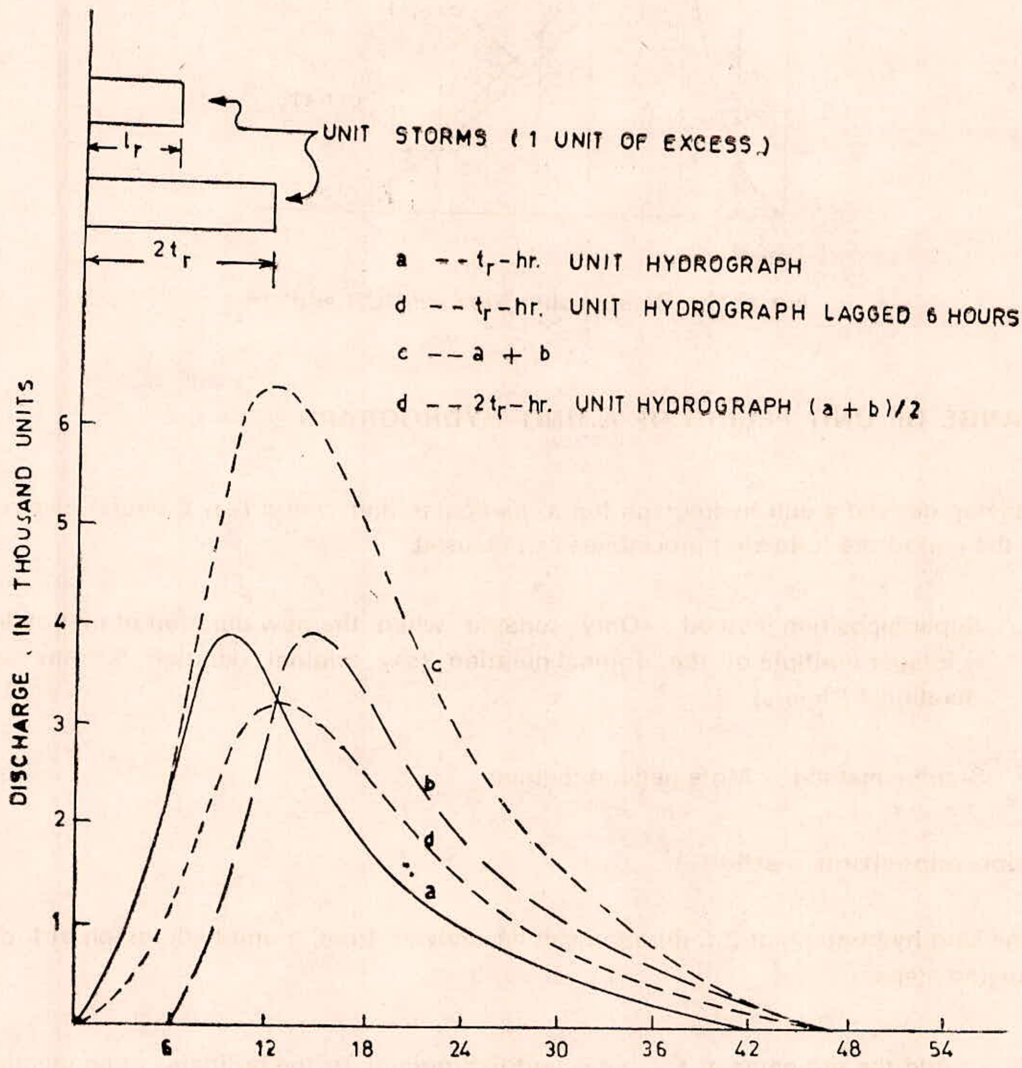

Fig. 6.16 Conversion of a unit Hydrograph to Duration $2t_r$

Example 6.6

The ordinates of 6-hour unit hydrograph is given below :

Time (hrs)	6-hour unit hydrograph ordinates (m^3/s)
0	0
3	200
6	500
9	1000
12	1600
15	2400
18	3500
21	4200
24	5200
27	4400
30	3100
33	2300
36	1500
39	1000
42	650
45	400
48	250
51	150
54	0

Derive 12-hour unit hydrograph of the same unit volume as of the 6-hour unit hydrograph.

Solution :

The computional steps are : (Ref. Table 6.6)

(i) Enter the ordinates of 6-hour unit hydrograph in column (2).

(i i) Lag the 6-hour unit hydrograph by 6-hour and enter these values in column (3).

(iii) Add the respective ordinates of column (2) and column (3). The resulting hydrograph ordinates are shown in column (4).

(iv) Divide the ordinates of the hydrograph, obtained from step (iii), by 2 to get required unit hydrograph of 12-hour duration. The ordinates of this unit hydrograph is given in column (5).

Table 6.6 Derivation of 12-hour Unit Hydrograph from 6-hour Unit Hydrograph

Time (hrs)	6-hour unit hydrograph ordinates (m^3/s)	6-hour unit hydrograph ordinates lagged by 6 hour (m^3/s)	Superimposed hydrograph (m^3/s)	12-hour unit (m^3/s)
(1)	(2)	(3)	(4)=(2)+(3)	(5)=(4)÷2
0	0	—	0	0
3	200	—	200	100
6	500	0	500	250
9	1000	200	1200	600
12	1600	500	2100	1050
15	2400	1000	3400	1700
18	3500	1600	5100	2550
21	4200	2400	6600	3300
24	5200	3500	8700	4350
27	4400	4200	8600	4300
30	3100	5200	8300	4150
33	2300	4400	6700	3350
36	1500	3100	4600	2300
39	1000	2300	3300	1650
42	650	1500	2150	1075
45	400	1000	1400	700
48	250	650	900	450
51	150	400	550	275
54	0	250	250	125
57	—	150	150	75
60	—	0	0	0

6.16.2 S-Curve Method :

After deriving the S-curve of unit intensity (or some other intensity), from t_1 duration unit hydrograph the unit hydrograph of t_2 duration can be obtained as follows :

(i) Shift the S-curve by t_2 hours to get the curve S_2. (Fig. 6.17)

(i i) Subtract the S_2 curve from S_1 curve giving another curve.

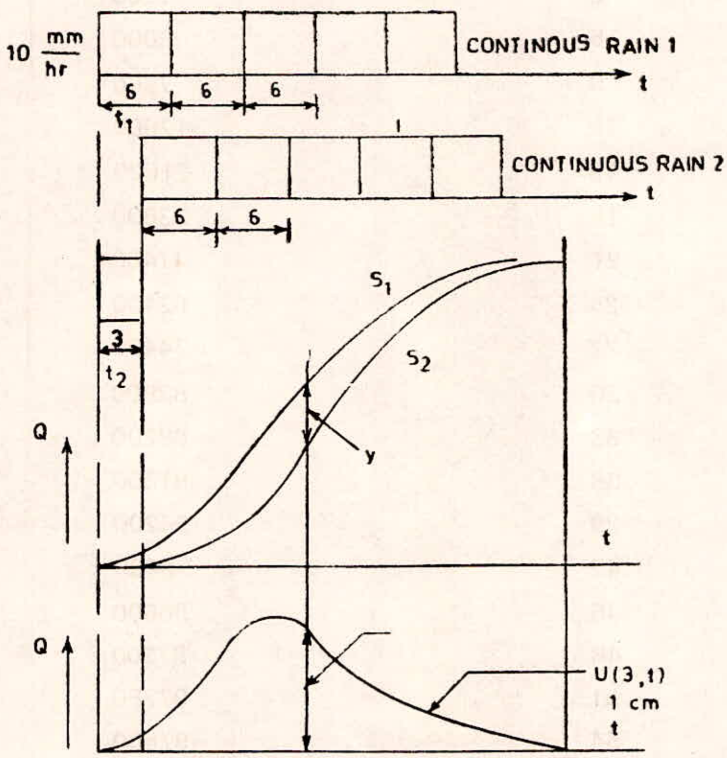

Fig. 6.17 Conversion of a 6 Hour Duration unit Hydrograph to 3 hours Duration unit Hydrograph

(iii) The difference between the two S curves represents the unit hydrograph for time t_2 with a unit volume equal to t_2/t_1 of U_1, where U_1 is the unit volume of the original duration unit hydrograph.

(iv) To produce a unit hydrograph for the period t_2 with a unit volume U_1 multiply the difference between the S-curves by t_1/t_2.

Example 6.7 :

The S-curve ordinates of intensity 1 cm/hour, derived from 6 hour 1 cm unit hydrograph, are given below :

Time (hrs)	S-curve hydrograph ordinates (m^3/s)
0	0
3	1200
6	3000
9	7200
12	12600
15	21600
18	33600
21	47400
24	62400
27	74400
30	82800
33	88200
36	91800
39	94200
42	95400
45	96600
48	97200
51	97380
54	97500
57	97500

Derive a unit hydrograph of 3-hour duration with 1 cm volume.

Solution :

The computational steps are : (Ref. Table 6.7)

(i) Enter the S-curve ordinates in column (2).
(S-curve is derived from 6-hour UH in the example 6.5 of S-curve derivation)

(i i) Shift the S-curve by 3 hours and enter these values in column (3).

(iii) Subtract the two S-curves (column (2) and column (3)) and enter these values in column (4). This represents the unit hydrograph of duration 3 hours with a unit volume equal to 3 cm.

(iv) Compute the unit hydrograph of duration 3 hour with a unit volume equal to 1 cm after dividing the hydrograph obtained from step (iii) by 3, The ordinates of the resulting unit hydrograph is given in column (5).

Table 6.7 Derivation of 3-Hour Unit Hydrograph from 6-Hour Unit Hydrograph-S-Curve Method

Time (hours)	S-curve ordinates of intensity 1 cm/hour (m^3/s)	S-curve lagged 3 hours (m^3/s)	3-hour unit hydrograph with volume' 3 cm (m^3/s)	3-hour unit hydrograph with volume 3 cm (m^3/s)
(1)	(2)	(3)	(4)=(2)−(3)	(5)=(4) / 3
0	0	—	0	0
3	1200	0	1200	400
6	3000	1200	1800	600
9	7200	3000	4200	1400
12	12600	7200	5400	1800
15	21600	12600	9000	3000
18	33600	21600	12000	4000
21	47400	33600	13800	4600
24	62400	47400	15000	5000
27	74400	62400	12000	4000
30	82800	74400	8400	2800
33	88200	82800	5400	1800
36	91800	88200	3600	1200
39	94200	91800	2400	800
42	95400	94200	1200	400
45	96600	95400	1200	400
48	97200	96600	600	200
51	97380	97200	180	60
54	97500	97380	120	40
57	97500	97500	0	0

6.17 AVERAGING OF UNIT HYDROGRAPHS

In practice, when unit hydrographs are derived for a particular catchment, the unit hydrographs that result will not be the same. The unit hydrographs are thus usually averaged.

6.17.1 Text Book method of averaging

The text book method (e.g Wilson (1974), Linsley etal (1975)) of averaging is performed in the following steps :

(i) Plot the unit hydrographs, derived from a no. of individual events, on a single diagram.

(i i) Mark a point which corresponds to the average peak ordinate and average time to peak (Fig. 6.18).

(iii) Sketch an average unit hydrograph by eye so that it passes through the average peak point, has unit volume, and generally conforms to the characteristics shape of the individual unit hydrographs.

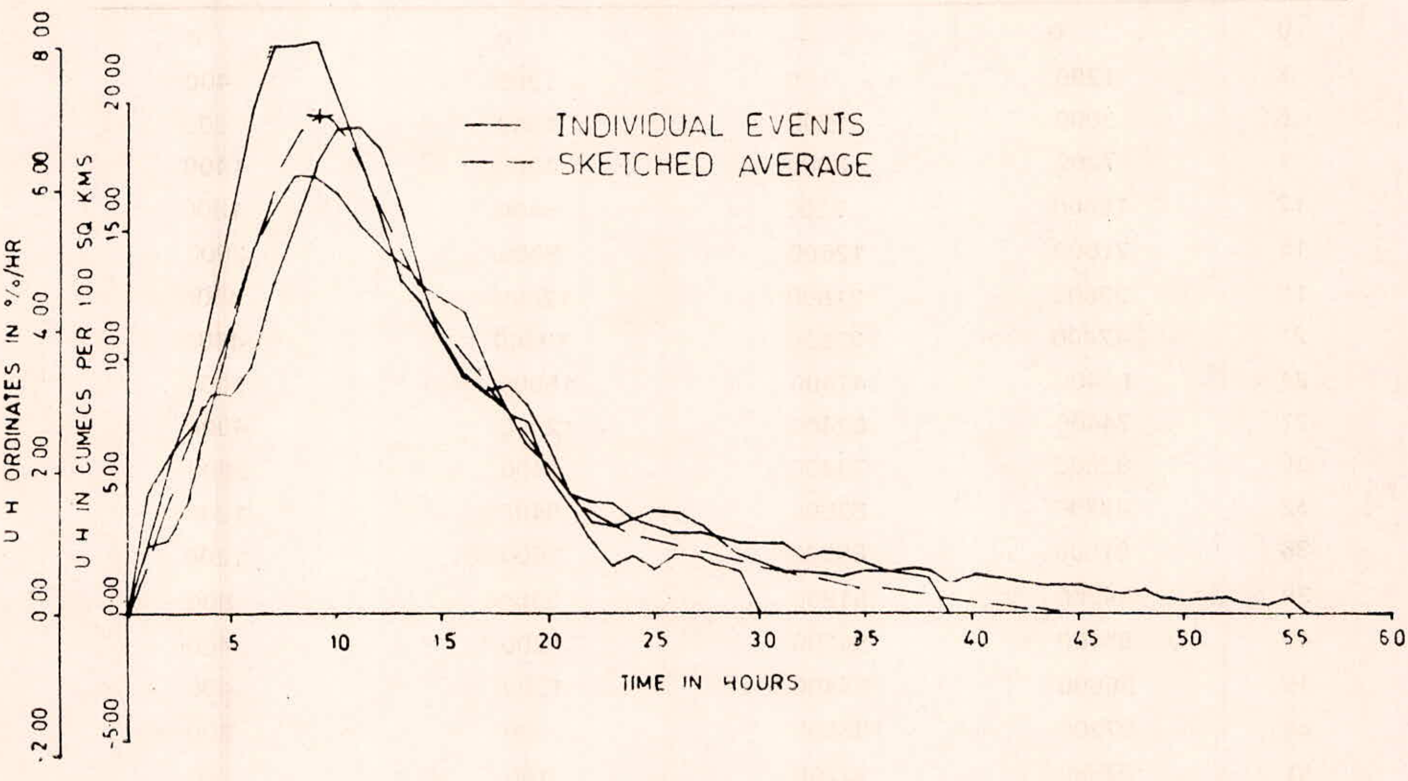

Fig. 6.18 Textbook Method ot Averaging

(iv) The procedure relies on a limited amount of trial and error (to preserve volume) and on a good deal of subjective judgement.

6.17.2 Ordinate by ordinate averaging

Various ordinate by ordinate averaging methods are possible and four such schemes are :

(a) Mean method

(a) Median method

(c) Mean peaks aligned method

(d) Median peaks aligned method

(*a*) *Mean method :* In this method the average unit hydrograph ordinates are computed by taking the mean of the corresponding ordinates of individual unit hydrographs.

Fig. 6.19 shows the average unit hydrograph computed by this method. It can be seen from the Fig. 6.19 that the average unit hydrograph so derived tends to have a low peak and could produce a lower hydrograph. However. as far as possible you should apply the unit hydrograph to the excess rainfall to reproduce the surface runoff hydrographs for the recorded storm used for unit hydrograph derivation.

The average unit hydrograph should also be reapplied to see how it reproduce the actual hydrographs.

(*b*) *Median method* : This method computes the average unit hydrograph by taking the median of the individual unit hydrograph ordinates. This method reduce the effect of outlier ordinates.

(*c*) *Mean peaks aligned method* : It is a refinement of simple ordinate by ordinate averaging. This gives a better representation of unit hydrograph shape, but if time to peak is important it is essential that the rising limb is adjusted so that the commencement of runoff becomes the critical characteristics.

Plot unit hydrographs so that peak coincides, and then average the rising and falling limbs Check the volume to maintain unit volume. Fig. 6.20 illustrates this.

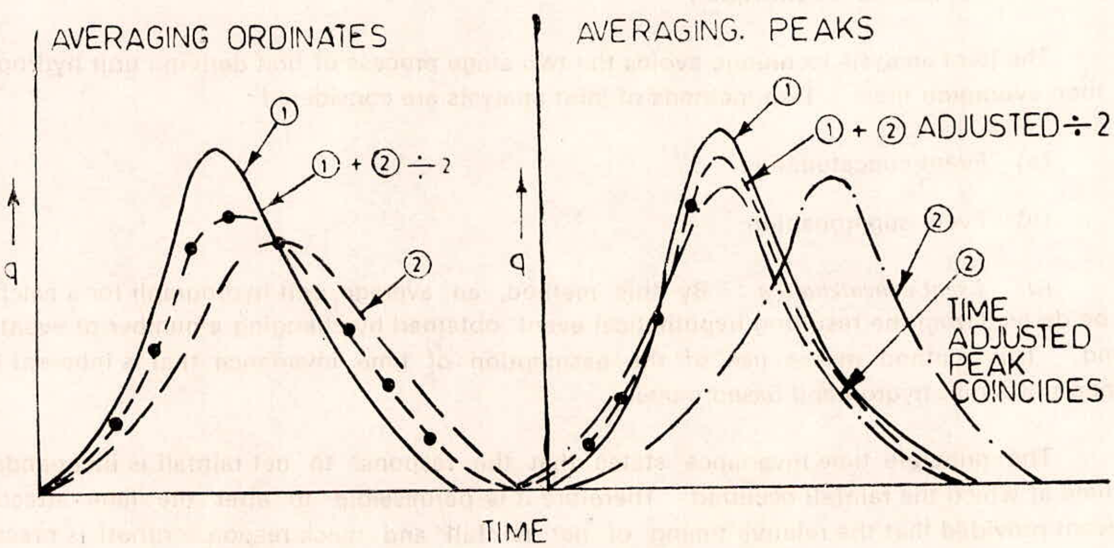

Fig. 6.19 Mean Method Fig. 6.20 Mean Peaks Aligned Method

(d) Median peaks aligned method : In this method the steps are :

— Plot unit hydrographs so that peaks coincide and then compute the median of the rising and falling limbs.

— Check the volume to maintain unit volume.

6.17.3 Shape Factor Averaging

The basis of the approach is the characterization of unit hydrographs by statistics namely the volume, the mean, the co-efficient of variation, skewness and peakedness. Strictly speaking, the volume and mean characterize the scale and location of the distribution (namely the unit hydrograph) rather than its shape, but the five statistics collectively referred as shape factors for convenient only.

The method of using shape factors to determine an average unit hydrograph is as follows :

(i) Calculate the shape factors values for individual derived unit hydrograph and average the values. (A median form of averaging again has the merits of minimizing the effect of outliers).

(i) The shape factor values of the individual unit hydrographs are then scanned until the unit hydrograph is noted that has shape factors approximately equal to the average values.

(iii) The unit hydrograph obtained from step (ii) would represent the average unit hydrograph for the catchment.

6.17.4 Joint Analysis Technique :

The joint analysis technique avoids the two stage process of first deriving unit hydrographs and then averaging them. Two methods of joint analysis are considered :

(a) Event concatenation

(b) Event superposition.

(a) Event concatenation : By this method, an average unit hydrograph for a catchment can be derived from the resulting hypothetical event obtained by changing a number of events end to end. This method makes use of the assumption of time invariance that is inherent in the adoption of a unit hydrograpd based model.

The principle time invariance states that the response to net rainfall is independent of the time at which the rainfall occurred. Therefore it is permissible to alter the time attached to an event provided that the relative timing of net rainfall and quick response runotf is preserved. A simple example of event concatenation is shown in Fig. 6.21.

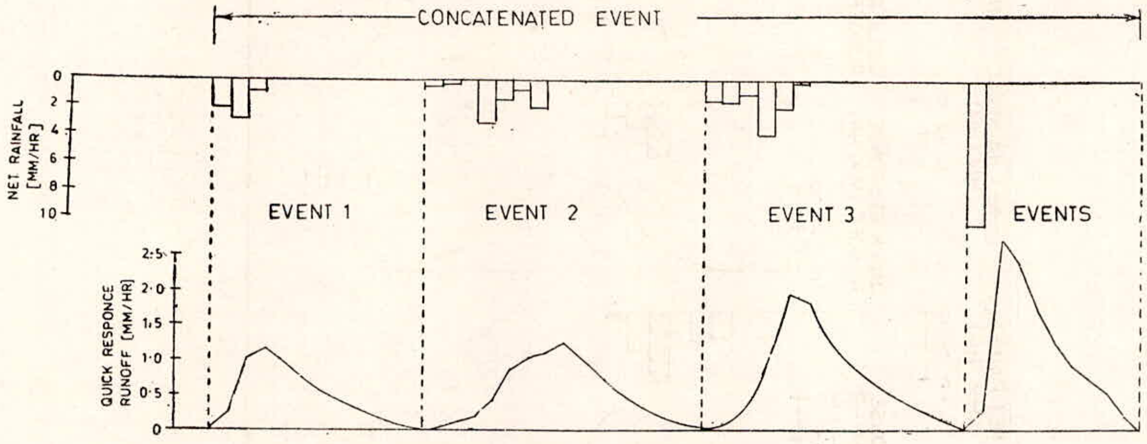

Fig. 6.21 Event Concatenation

(*b*) *Event superposition :* This technique relies on the unit hydrograph assumptions of linearity and time-invariance. The steps to be carried out, for deriving the average unit hydrograph by this method, are :

— Preserve the relative timing of net rainfall and quick response runoff for each event.

— Align the peak elements of net rainfall of each event.

— Superpose the event excess rainfall data by adding the corresponding excess rainfall blocks of each avent, obtained from the alignment of the peak rainfall of each event.

— Similarly superpose the surface runoff hydrographs of each event.

— Derive the unit hydrograph from the resulting superposed event. This gives the average unit hydrograph for the catchment.

Fig. 6.22 illustrate the superposition for the same example as that used to illustrate concentration.

6.18 CATCHMENTS SIZE LIMITATION FOR UNIT HYDROGRAPH

Areal uniformity is a principle of the unit hydrograph theory, and for larger catchments it is difficult to obtain. The upper limit for the application of this principle is usually adopted as 2500 to 7500 square kilometres. If the catchmont is greater then 7500 square kilometers, one unit hydrograph should not be used, but the catchment should be divided into sub-areas, usually major tributaries.

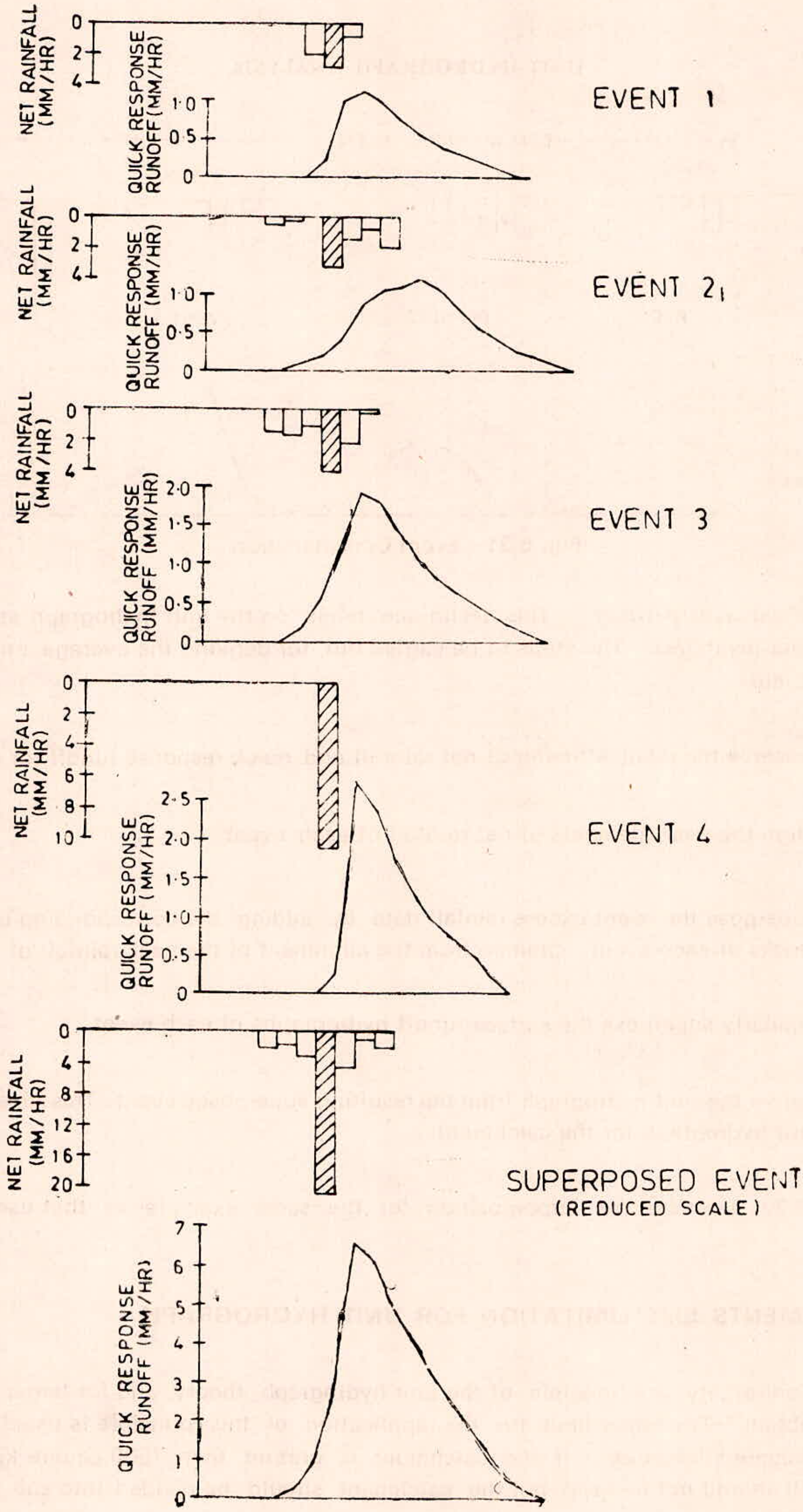

Fig. 6.22 Event Superposition

6.19 APPLICATION OF UNIT HYDROGRAPH

Having derived a unit hydrograph the question arises as to what use do we put it ? Some of the current application are :

— Estimation of design flood

— Estimation of runoff from ungauged catchments

— Filling up the missing records

— Flood forecasting to estimate flood flows from rainfall records in real time.

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