LECTURE-8

CONCEPTUAL MODELS FOR IUH DERIVATION

OBJECTIVES

After attending this lecture, the participants would be able to understand the concept of some well known conceptual models such as Conventional Nash model, Integer Nash model and Clark Model for the derivation of Unit hydrograph of a particular duration from a rainfall-runoff event.

8.1 INTRODUCTION

As discussed earlier in lecture no. 6, the direct surface runoff and effective rainfall are assumed to be linearly related with unit hydrograph ordinates. Thus, the ordinates of the unit hydrograph are found through a solution of the set of linear algebraic equations which involves the matrix operations. Even small errors in effective rainfall and direct surface runoff make the general linear solution numerically ill conditioned resulting in unrealistic shape of the unit hydrograph with fluctuating ordinates. The above approach becomes impracticable while relating the unit hydrograph parameters with catchment characteristics. Therefore, it becomes necessary to postulate a general linear hydrologic model (conceptual model) which can be represented by a limited number of parameters. This implies choosing a fixed form or equation for the instantaneous unit hydrograph (IUH) keeping in view the various constraints imposed by the nature of the hydrologic system. However, the number of parameters which is used to define the fixed form of IUH is limited by the number of independent significant relations which can be established with the catchment characteristics. If there are more parameters than this, the extra parameters can not be evaluated from the catchment characteristics for ungauged catchments and must either be given fixed values or related to the other parameters. In either case, the effective number of degrees of freedom is reduced to the number of independent relationships established.

Thus, the following requirements must be fulfilled while choosing a general IUH equation

(i) The IUH ordinates are all positive

(ii) The shape of the IUH is preserved

(iii) The errors in input data should not be amplified during the IUH derivation.

(iv) The number of parameters of the chosen form is limited to the number of independent relation ships established between the responses and the catchment characteristics.

(v) The form should reflect, as far as possible, the physical relationship between the input and output.

(vi) Within the restrictions (iv) and (v), the form chosen should be as flexible as possible, particularly, in regard to fitting the observed events.

Now it is required to evaluate the parameter or parameters used in IUH equation to derive its shape. Moreover, it is desirable to evaluate those parameters from the records. In this lecture some well known conceptual models, namely conventional Nash Model, Integer Nash Model and Clark Model, for IUH derivation are discussed. Methodologies for unit hydrograph derivation are also described with the help of illustrative examples for each model.

8 2 NASH MODEL CONCEPT

Nash (1957) considered that the instantaneous unit hydrograph could be obtained by routing the instantaneous inflow through a cescade of linear reservoirs with equal storage co-efficient. The outflow from the first reservoir is considered as inflow to the second reservoir and so on. The mathematical equation developed from general differential equation for the IUH is given in APPENDIX-II. Fig. 8,1 illustrates the concept of Nash Model.

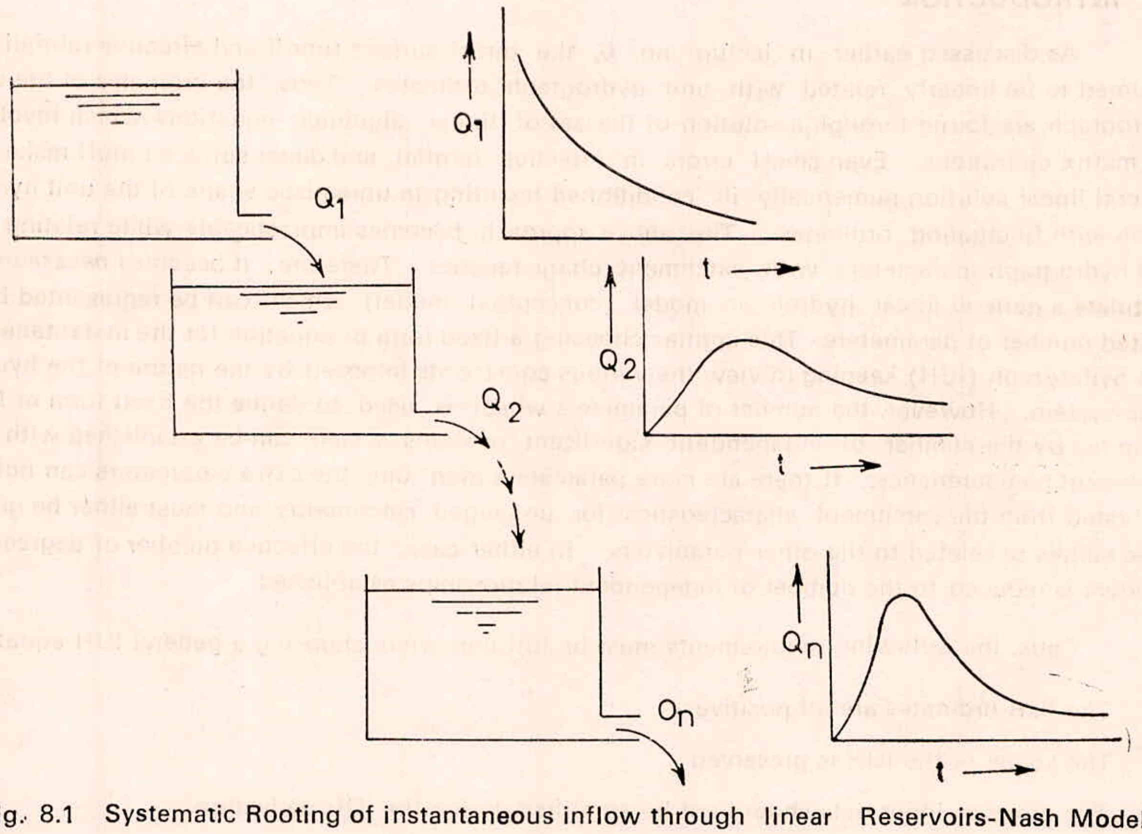

Fig. 8.1 Systematic Rooting of instantaneous inflow through linear Reservoirs-Nash Model

8.2.1 Procedure to Determine Unit Hydrograph Using Nash Model

The procedure to estimate unit hydrograph using Nash model from observed rainfall runoff event, is detailed below :

(i) Estimation of effective rainfall and direct surfaee runoff : The effective rainfall and direct surface runoff may be computed using the procedures outlined in the previous lecture No. 5.

(ii) Estimation of moments of effective rainfall and direct surface runoff : The moments of effective rainfall and direct runoff for use in estimating the parameters of Nash model are to be computed by the following formulae :

$$ {}_rM'_Y = \frac{\sum_{i=1}^{N} \frac{Y_i + Y_{i+1}}{2} \Delta t \, (t_i)^r}{\sum_{i=1}^{N} \frac{Y_i + Y_{i+1}}{2} \Delta t} $$

$$ \text{or} \quad {}_rM'_Y = \frac{\sum_{i=1}^{N} \overline{Y}_i t_i^r}{\sum_{i=1}^{N} \overline{Y}_i} \qquad (8.1) $$

$$ {}_rM'_X = \frac{\sum_{i=1}^{M} X_i \, \Delta t \, t_i^r}{\sum_{i=1}^{M} X_i \, \Delta t} $$

$$ \text{or} \quad {}_rM'_X = \frac{\sum_{i=1}^{M} X_i t_i^r}{\sum_{i=1}^{M} X_i} \qquad (9.2) $$

where

- ${}_rM'_Y$ is the rth moment of the direct surface runoff about the origin.
- ${}_rM_X$ is the rth moment of the excess rainfall hyetograph about the origin.
- Y_i is the ith ordinate of direct surface runoff hydrograph
- X_I is the ith ordinate of excess rainfall hyetograph
- N is the no. of direct surface runoff hydrograph ordinates,
- M is the no. of excess rainfall hyetograph ordinates.
- Δt is the sampling interval (hrs), and
- t_i is the time to the mid point of the ith interval from the origin,

(*iii*) *Estimations of Parameters of Nash Model* : Nash (1958) introduced the theorem of moments relating the moments of excess rainfall hyetograph (input) and direct surface runoff hydrograph (output) with the moments of instantaneous unit hydrograph. The detail description about this theorem is given in APPENDiX—III.

Using the theorem of moments the first and second moment of effective rainfall and direct surface runoff are related with the corresponding moments of IUH as follows :

$$_1M'_u = {_1M'_Y} - {_1M'_X} \quad (8.3)$$

$$_2M'_u - {_3M'_Y} - 2\,{_1M'_X}\,{_1M'_n} - {_2M'_X} \quad (8.4)$$

where

$_1M'_u$ is the first moment of IUH about the origin

$_2M'_u$ is the second moment of IUH about the origin

$_1M'_Y$ is the first moment of direct surface runoff about the origin

$_2M'_Y$ is the second moment of direct surface runoff about the origin

$_1M'_X$ is the first moment of excess rainfall hyetograph about the origin, and

$_2M'_X$ is the second moment of excess rainfall hyetograph about the origin.

The values of $_1M'_Y$, $_2M'_Y$, $_1M'_X$ and $_2M'_X$ can be obtained by using Eq. (8.1) and (8.2). However, $_1M'_u$ and $_2M'_u$ are related with the parameters of Nash Model as follows :

$$_1M'_u = nK \quad (8.5)$$

$$_2M'_u = (n\,K)^2 + n\,K^2 \quad (8.6)$$

(Sea APPENDIX—II for the derivation of Eq. (8.5) and (8.6))

Substituting the values of $_1M'_u$ and $_2M'_u$ of the Eq. (8.5) and (8.6) into Eq. (8.3) and (8.4) the resulting equations may be written as :

$$n\,K = {_1M'_Y} - {_1M'_X} \quad (8.7)$$

$$n\,(n+1)\,K^2 + 2nK\,{_1M'_X} = {_2M_Y} - {_2M'_X} \quad (8.8)$$

In eq. (8.7) and (8.8) only unknowns are n and K. Therefore these two equations can be solved to estimate tne parameters of Nash Model, n and K.

(*iv*) *Estimation of Instantaneous unit hydrograph (IUH)* : Nash's equation for the ordinate of IUH using the two parameters n and K may be given as

$$u\,(t) = \frac{1}{K\,\Gamma n}\,(t/K)^{n-1}\,e^{-t/K} \quad (8.9)$$

(See APPENDIX—II for the derivation of the equation for IUH).

It may be noted that Γn has been used in place of (n—1) ! to account for the non integer values of n as computed from the observed data.

(*iv*) *Estimation of unit hydrograph of T hour duration* : The equation for T-hour unit hydrograph is given as follows :

$$U(T, t) = \frac{1}{T} \left(I(n, t/K) - I(n, (t-T)/K) \right) \tag{8.10}$$

where I (n, t/K) is the incomplete gamma function of order n at (t/K).

(see APPENDIX—II For the description of IUH equation)

To estimate the ordinates of U (T, t), the tables of incomplete gamma functions are available which enable one to compute I (n, t / K) and I (n, (t—T) / K) for known values of n and K. The computer programme may also be used for this purpose.

(vi) The T-hour unit hydrograph obtained from (Eq. 8.10) is converted to the unit of cumec using the following equation :

$$U'(T, t) = U(T, t) * 0.277 * VOL * CA$$

where U' (T, t) is T-hour unit hydrograph ordinates in cumec.

VOL is the unit volume of T-hour UH in mm, and

CA is the catchment area (Sq. km).

8.2.2 Computational steps for deriving the unit hydrograph by Nash Model :

The following steps are involved for deriving the unit hydrograph by Nash Model using the rainfall-runoff data of a particular storm :

(i) Obtain mean rainfall values at each computotional interval taking the weighted mean of the observed values at different stations.

(ii) Estimate direct surface runoff seperating the baseflow from the discharge hydrograph using one of the base flow separation techniques described in lecture no. 5.

(iii) Estimate the excess rainfall hyetograph separating the loss from total rainfall hyetograph using the procedure stated in lecture no. 5.

(iv) Estimate the first and second moment of effective rainfall hyetograph about the origin using the Eq. (8.2).

(v) Estimate the first and second moment of direct surface runoff hydrograph about the origin using Eq. (8.1)

(vi) Solve Eq. (8.7) and (8.8) fot the parameters n and K using the values of moments obtained from step (iv) and (v).

(viii) Estimate the unit hydrograph of duration T hours using Eq. (8.10) and (8.11).

Example 8.1

The ordinate of excess rainfall hyetograph and direct surface runoff hydrograph for a storm of typical catchment of the size 1700 sq. km. are given below. Find out the unit hydrograph of 6-hour duration using the Nash Model procedure.

Time (hrs)	Excess rainfall (mm)	Direct surface runoff (m^3/s)
0	0	0
6	40.209	250
12	100.209	1050
18	60.209	2050
24	0	4350
30	0	4150
36	0	2300
42	0	1070
48	0	450
54	0	120

Solution :

(i) First and second moment of effective rainfall

$$_1M'x = \frac{\sum_{i=1}^{M} X_i\, t_i}{\sum_{i=1}^{M} X_i}$$

$$= \frac{40.209\times3 + 100.209\times9 + 60.209\times15}{40.209 + 100.209 + 60.209}$$

$$= \frac{1925.643}{200.627}$$

$$= 9.598 \text{ hour}$$

$$_2M'x = \frac{\sum_{i=1}^{M} X_i\, t_i^2}{\sum_{i=1}^{M} X_i}$$

$$= \frac{40.209\times(3)^2 + 100.209\times(9)^2 + 60.209\times(15)^2}{40.209+100.209+60.239}$$

$$= \frac{22025.135}{200.627} = 109.765 \text{ hour}$$

(iii) First and second movement of Diroct surfece runoff

$$ {}_rM'_Y = \frac{\sum_{i=1}^{N} \frac{Y_i+Y_{i+1}}{2} t_i^r}{\sum_{i=1}^{N} \frac{Y_i+Y_{i+1}}{2}} = \frac{\sum_{i=1}^{N} (Y_i+Y_{i+1})\, t_i^r}{\sum_{i=1}^{N} (Y_i+Y_{i+1})} $$

$$ {}_1M'_Y = \frac{\frac{250}{2}\times 3 + \frac{(250+1050)}{2}\times 9 + \frac{(1050+2050)}{2}\times 15 + \frac{(2050+4350)}{2}\times 21 + \frac{(4350+4150)}{2}\times 27 + \frac{(4150+2300)}{2}\times 33 + \frac{(2300+1070)}{2}\times 39 + \frac{(1070+450)}{2}\times 45 + \frac{(450+120)}{2}\times 51 + \frac{120}{2}\times 57}{\frac{250}{2} + \frac{(250+1050)}{2} + \frac{(1050+2050)}{2} + \frac{(2050+4350)}{2} + \frac{(4350+4150)}{2} + \frac{(4150+2300)}{2} + \frac{(2300+1070)}{2} + \frac{(1070+450)}{2} + \frac{(450+120)}{2} + \frac{(120)}{2}} $$

$$ = \frac{435720}{15790} = 27.595 \text{ hour} $$

$$ \frac{250}{2}\times 3^2 + \frac{(250+1050)}{2}\times 9^2 + \frac{(1050+2050)}{2}\times 15^2 + \frac{(2050+4350)}{2}\times 21^2 + \frac{(4350+4150)}{2}\times 27^2 + \frac{(4150+2300)}{2}\times 33^2 + \frac{(2300+1070)}{2}\times 39^2 + \frac{(1070+450)}{2}\times 45^2 + \frac{(450+120)}{2}\times 51^2 + \frac{120}{2}\times 57^2 $$

$$_2M'_Y = \frac{}{15790}$$

$$= \frac{13462110}{15790}$$

$$= 852.572 \text{ hour}^2.$$

(iri) Solve the following equations to get the parameters n and K.

$$n\,K = {}_1M'_Y - {}_2M'_X$$

$$n\,(n + 1)\,K^2 + 2n\,K\,{}_1M'_X = 2M'_Y - 2M'_X$$

where

$${}_1M'_X = 9.598 \text{ hour}, \; {}_2M'_X = 109.785 \text{ hour}^2$$

$${}_2M'_Y = 27.395 \text{ hour}, \; {}_2M'_Y = 852.572 \text{ hour}^2$$

$$\therefore nK = 27.595 - 9.598$$

$$= 17.997$$

and

$$n\,(n + 1)\,K^2 + 2\,n\,K \times 9.598 = 852.572 - 109.785$$

$$n\,(n + 1)\,K^2 + 19.196\,n\,K = 742.787$$

$$(n\,K)^2 + n\,K.K. + 19.196\,n\,K = 742.787$$

$$(17.997)^2 + 17.997\,K + 19.196 \times 17.997 = 742.787$$

$$323.892 + 17.997\,K + 345.470 = 742.787$$

$$17.997\,K = 73.425$$

$$K = 4.08$$

$$\therefore n = \frac{17.997}{4.08} = 4.411$$

(iv) Compute unit hydrograph using the following equations :

$$U\,(T, t) = \frac{1}{T}\left[\, I\left(n, t/K\right) - I\left(n, \frac{t-T}{K}\right)\right]$$

$$U'\,(T, t) = U\,(T. t) * 0\,277 * 1700$$

Note ahout the incomplete gamma functian tabie : (APPENDIX—IV)

The incomplete Γ — function is defined as :

$$\Gamma\,(P + 1) = \int_0^x e^{-x}\, x^P\, d\,x$$

Now we term $I\,(x, p) = \frac{\Gamma_x\,(P+1)}{\Gamma_\infty\,(P+1)}$

and it is the evaluation of the I (x, p) function with which we are principally concerned. The I (x, P) always lies between 0 and 1. Thus I (x, P) solves the first difficulty of great range of values involved in tabling $|\overline{X}$ (P + 1).

The second serious difficulty which arises is that of the extreme difference in the value of the argument — x which must be allowed for, if we are able to table I (x. P) from zero up to unity showing seven figures. In other words the required range ot x—since x theoretically runs from 0 to ∞ — varies very much from P = —1 to P = 50. In order to get over this difficulty in place of argument x. $U = \frac{x}{\sqrt{(p+1)}}$ is used (Pearson 1965). The range of U, still considerable, forms one of the difficulties of setting up the table, but the U argument is for more workable than the x—argument. The table therefore deals with I (U, P) not I (x, P).

For the given example :

$$P = n - 1,\ X = \frac{t}{K} \text{ and } U = \frac{X}{\sqrt{(P+1)}}$$

and

$$I(x, P) = \frac{1}{\sqrt{(P+1)}} \int_0^x X^P e^{-x} dx$$

Table 8.1 shows the computation of 6-hrs. unit hydrograph using Nash Model :

Table 8.1 Estimation of unit hydrograph using Nash Model

P=3.411 K = 4.08 hours, T=6 hour, CA=1700 sq. km.

Time (hrs)	X=t/K	$U = \frac{X}{\sqrt{(P+1)}}$	I (U, P) from the *table	I (U',P) from the *table	U (T,t) $\frac{1}{T}$[Col. (4) Col. (5)]	(U, (T, t)) (cumec)* Col. (6) 0.277*CA*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
0	0	0	0	0	0	0
6	1.471	0.7004	0.0377661	0	0 00629	2.97
12	2.941	1.4003	0.2642935	0.0377661	0.03775	17.83
18	4.412	2.1007	0.5643666	0.2642935	0.0500	23.61
24	5.882	2.8006	0.7858772	0.5654666	0.0369	17.43
30	7.353	3.501	0.9076346	0.7858772	0.0203	9.59
36	8.824	4.201	0.9637378	0.9076346	0.0094	4.44
42	10.294	4.901	0.9867319	0.9637378	0.0038	1.79
48	11.765	5.601	0.9954023	0.9867319	0.0014	0.66

Note : In the above table $u' = \frac{X'}{\sqrt{(P+1)}}$ where $X' = \frac{(t-T)}{K}$

* The incomplete gamma function table used for this example is given in APPENDIX—IV.

8.3 INTEGER NASH MODEL

To derive unit hydrograph for a specific duration using conventional Nash Model one has to evaluate the incomplete gamma function. When n is not an integer the incomplete gamma function can only be evaluated either from the incomplete gamma function tebles or the computer programmes. Even the evaluation of ı n also requires the use of complete gamma function tables or the computer programmes. Such type of cemputations make the field practitioners somewhat reluctant to use Nash Model in rainfall-runoff studies. Integer Nash Model, which is a simplified form of the conventional Nash Model, takes the parameter n approximated to the nearest integer and computes the incomplete gamma function using a simplified procedure where the use of pearson table is fully avoided or the use of computer programme is not essential. Thus the field practitioners are able to compute the unit hydrograph very easily by this method using simple scientific calculator. The rainfall-runoff data of a storm event is analysed by this method in the following steps :

(i) Follow the steps (i) to (iv) described in section 8.2.2 for conventional Nash model.

(ii) Consider an integer value of n and modify the values of K to preserve lhe first moment of IUH taking K equal to the ratio of the first moment of IUH about the origin to the integer value of n. However, one may face the problem of deciding the integer 'n' value when the computed value of n, for example, lies somewhere nearer to 3.5. In such circumstances one may adopt a value of 3 and 4 and check for its closeness with the second moment of IUH about the centroid (nK^2).

The integer n which leads to closer nK^2 is considered for integer Nash model.

(iii) Derive the unit hydrograph of T-hour duration using the following equations :

$$U(T, t) = \frac{1}{T} [I(n, y) - I(n, y_1)] \quad (8.12)$$

where, $$I(n, y) = 1 - e^{-y} \sum_{m=o}^{n-1} \frac{y^m}{m!} \quad (8.13)$$

$$I(n, y_1) = 1 - e^{-y_1} \sum_{m=o}^{n-1} \frac{y_1^m}{m!} \quad (8.14)$$

$$y = t/K \quad (8.15)$$

$$y_1 = (t-T)/K \quad (9.16)$$

n = An integer value of n obtained from step (ii)

K = storage co-efficient obtained from step (ii)

(iv) Compute T-hour unit hydrograph ordinates in cumec using the following equation :

$$U'(T, t) = U(T, t) * 0.277 * VOL * CA \quad (8.17)$$

where U' (T, t) is T-hour unit hydrograph in cumec,

VOL is unit volume of UH (mm)

CA is catchment area (Sq. km)

Example 8.2 ; Derive 6-hour unit hydrogragh by Integer Nash Model using the data of example 8.1

Solution : The computational steps are ;

(i) From example 8.1,

$nK = 17.997$ hour

$nK^2 = 73.427$ hour^2

$n = 4,411$

$K = 4.08$ hour

(ii) The value of n as computed from step (i) is 4.411 which is nearer to 4.5. Now adopt $n = 4$ and compute K and nK^2 preserving the first moment of IUH. Hence,

$n = 4.0$

$$K = \frac{nK}{n} = \frac{17\,997}{4.0} = 4.5 \text{ hours}$$

$nK^2 = 80.97$ hours^2

absolute percentage difference in nK $= ((80.97 - 73.427)/73.427) * 100$

$= 10.27\%$

In second case, adopt $n = 5$ and compute K and nK^2 as computed for $n = 4$, Hence

$n = 5$

$$K = \frac{17.997}{5} = 3.60 \text{ hours}$$

$nK^2 = 64.78$ hours^2

absolufe percentage differeuce in $nK^2 = ((73.427 - 64.78)/73.427)*100$

$= 11.78\%$

Since the absolute percentage difference in nK^2 for the later case is more than the former. Hence, the values of parameters adopted for the Integer Nash Model are :

$n = 4$

$K = 4.5$ hours

(iii) Estimate the unit hydrograph ordinates using the Eq. (8.12) to (8.17)

Table 8.2 shows the computation of 6-hour unit hydrograph by Integer Nash Model.

Table 8.2 : Computation of unit hydrograph using Integer Nash Model

Time (Hrs)	y	I (n, y)	y_1	I (n, y_1)	U (T, t)	U' (T, t) (m^3/sec)
(1)	(2)	(3)	(4)	(5)	(6)=[(3)—(5)]/T	(7)
0	0	0	—		0	0
6	1.333	0.0464	0	0	0 0077	3.63
12	2.667	0.2786	1.333	0.0464	0.0387	18.22
18	4.00	0.5665	2.667	0.2786	0.0479	22.56
24	5.333	0.7786	4.00	0.5665	0.03535	16.65
30	6.667	0.1991	5.333	0 7786	0.020	9.42
36	8.000	0.9576	6.667	0.8991	0.009	4.24
42	9.333	0.9832	8.00	0.9576	0.004	1.88
48	10,666	0.9937	9.333	0.9832	0.002	0.94
54	12.000	0.9977	10.666	0.9937	0 0006	0.28

8.4 CLARK MODEL CONCEPT

Clark (1945) suggested that the IUH can be derived by routing the unit inflow in the form of time area concentration curve, which is constructed from isochronal map, through a single linear reservoir. The isochronal map for a typical watershed is shown in Fig. 8.2. However, Fig. 8.3 shows the time area concentration curve constructed from the isochronal map. The concept of Clark Model for IUH derivation is illustrated in Fig. 8.4.

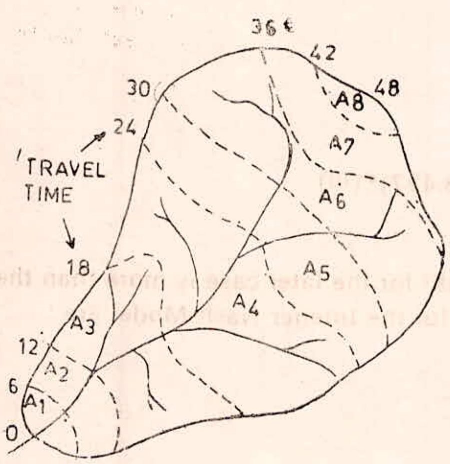

Fig. 8.2 Isochronal Map of a Watershed

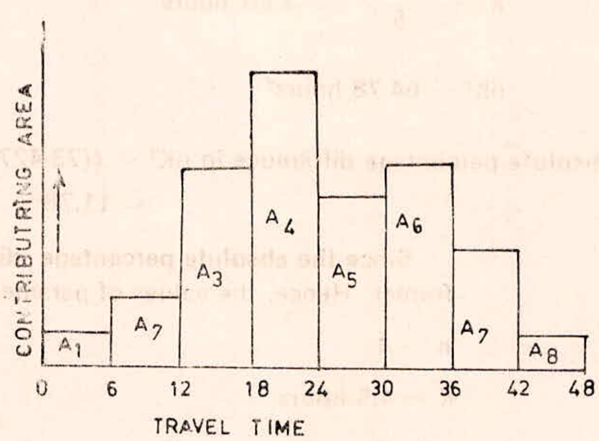

Fig. 8 3 Time Area Curve Constructed from Isochronal Map

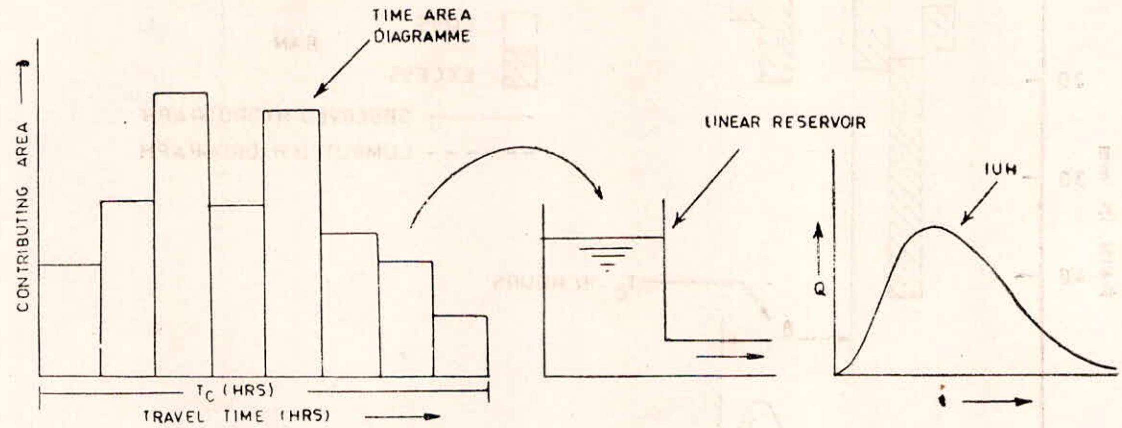

Fig. 8.4 Clark Model

8.4.1 Procedure to determine Unit Hydrograph

The procedure, to determine unit hydrograph using Clark Model, is detailed below :

(*a*) *Estimation of effective rainfall and direct surface runoff :* The effective rainfall and direct surface runoff may be estimated using the procedures discussed in the previous lecture no. 5.

(*b*) *Estimation of parameters for Clark Model :* The Clark model uses two parameters and a time area relation to define the IUH :

(*i*) *Time of concentration (T_c) :* This represents the travel time of a water particle from the most upstream point in the basin to the out flow location. An initial estimate of this lag time is the time from the end of effective rainfall (plus snowmelt if any) over the basin to the inflexion point on the recession limb of the surface runoff hydrograph. This time of concentration is used in developing the time area relation.

(*ii*) *Storage co-efficient (R) :* This is an attenuation constant which has the dimension of time. This parameter is used to account for the effect that storage in the river channel has on the hydrograph. This parameter can be estimated by dividing the flow at the point of inflexion of the surface runoff hydrograph by the rate of change of discharge (slope) at the same time. Another technique for estimating R is to compute the volume remaining under the recession limb of the surface runoff hydrograph following the point of inflexion and divide by the flow at the point of inflexion. In either case, R should be an average value determined by using several hydrographs.

Fig. 8.5 illustrates the above definitions of T_c and R from a typical hydrograph. Since the shapes of hydrographs reflect many irregularities of rainfall and stream patterns, therefore the estimate obtained in this way are usually satisfactory only for the first approximation. Hence the best fit parameters should be determined for the best reproduction of hydrographs using some standard optimization techniques.

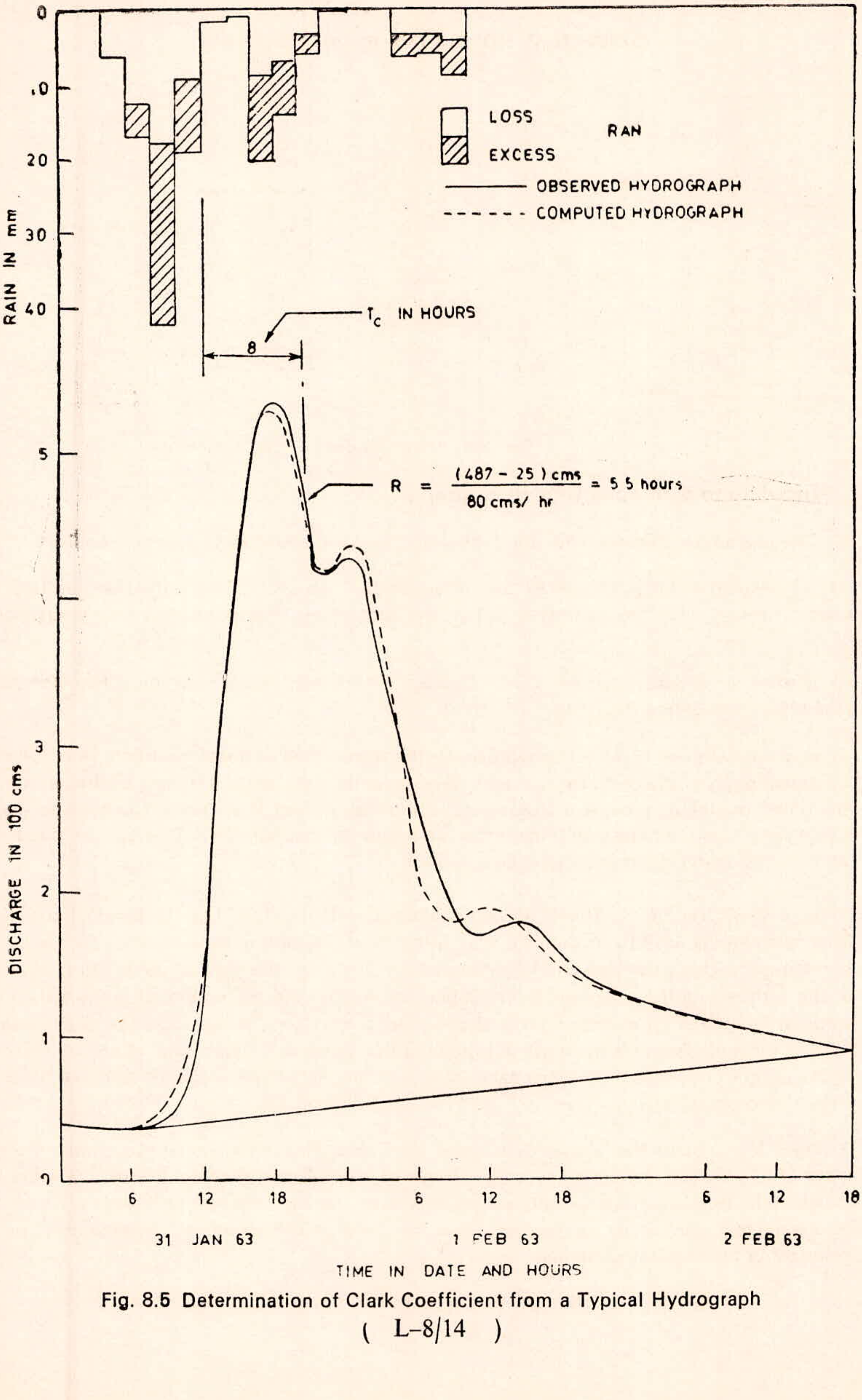

Fig. 8.5 Determination of Clark Coefficient from a Typical Hydrograph

For better performance of the method :

- The durations of unit rainfall excess and computation interval Δt must be shorter than t_c. Preferably shorter than one third of T_c.
- The storms selected for study should be several times longer than the computation interval Δt in order to provide representative basin coverage of rainfall.

(*iii*) *Time-area diagrams :* The other necessary item to compute an IUH using Clark model is the time-area relation. When T_c has been determined the basin is divided into incremental runoff producing areas that have equal incremental travel times to the outflow location. The computational steps involved, in constructing the time-area curve, are :

- Measure the distance from the most upstream point in the basin to the outflow location along the principal water course.
- Estimate the time of travels as the ratio of $L/\sqrt{S}$ along the water course where L is the length of a segment and S is the slope of the segment.
- Laid out the isochrones representing equal times of travel to the outflow location after establishing the location of lines using the ratio $L/\sqrt{/S}$ of different segments.
- Measure the area between the isochrones and tabulate them in upstream segments versus the corresponding incremental travel time for the each incremental area.

The increment of time used to sub-divide the basin need only be small enough to adequately define the areal distribution of runoff while the time period selected as the computation interval must be equal to or less than the unit duration of excess. Since the former is frequently larger than the later, a plot percent of time of concentration versus accumulative area is useful in determining time-area relationships (Fig. 8.6). Such a curve facilitates rapid development of unit hydrographs for various computation intervals and unit duration of excess.

(*c*) *Estimation of IUH :* The resulting shape of time area diagramme is routed through a linear reservoir to simulate the storage effects of the basin and the resulting outflow represents the IUH.

Before going for linear reservoir routing, the runoff from the contributing areas (between the isochrones) which would be translated to the outflow location, should be expressed in proper unit. The conversion to proper units of discharge can be made through the relationship.

$$I_i = K\, a_i / \Delta t \tag{8.17}$$

where I_i = Ordinate in proper units of discharge of the translation hydrograph at the end of period i.

a_i = Ordinates in units of area-depth of excess of the translation hydrograph at the end of period i

K = Conversion factor to convert a_i to i_i

Δt = Time period of computation interval in hours.

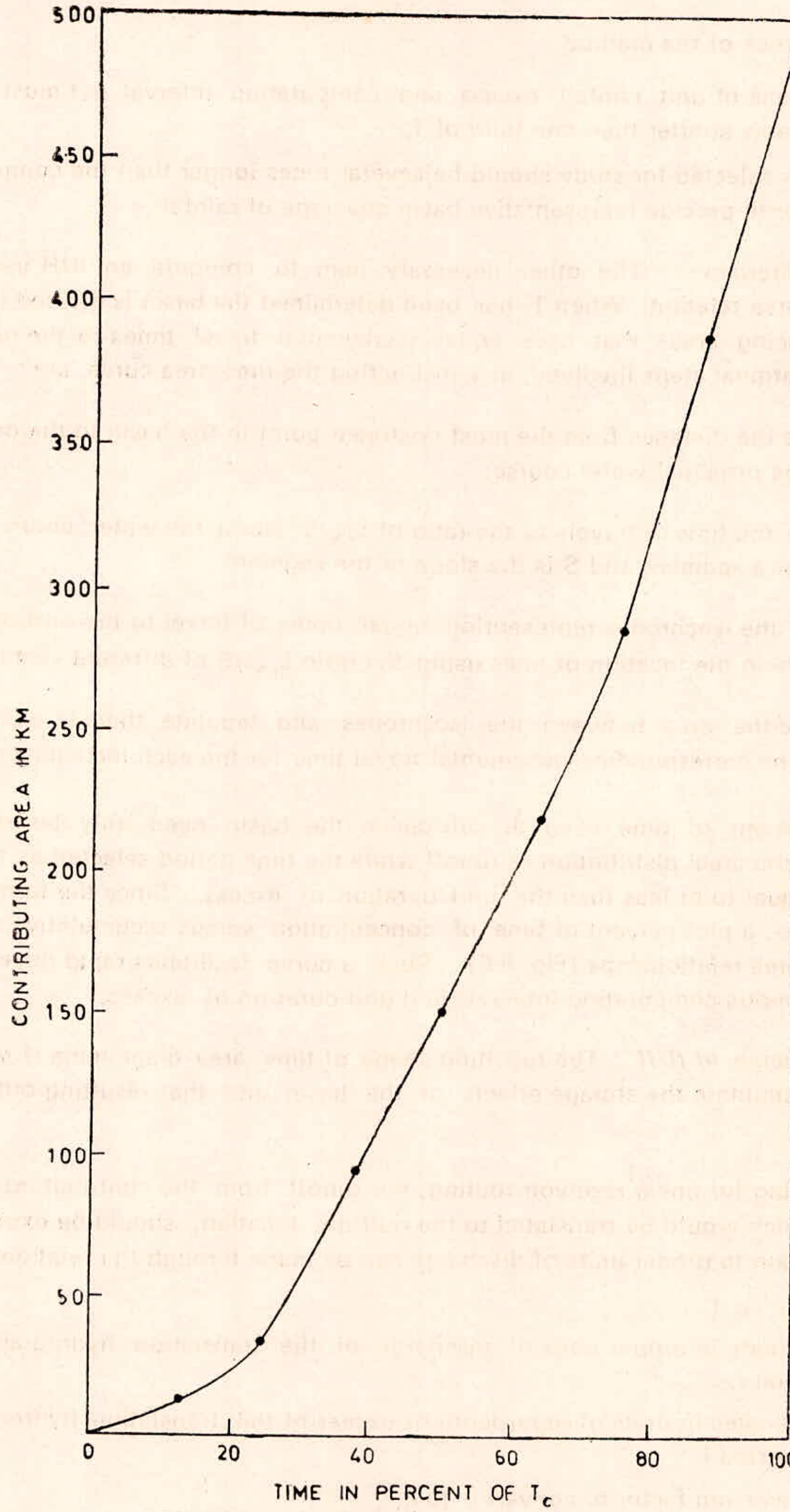

Fig. 8.6 Watershed Time Area Relation

Then the linear reservoir routing is accomplished using the general equation, (see APPENDIX—V)

$$U_i = C\, I_i + (1-C)\, U_{i-1} \tag{8.18}$$

where C and (1—C) are routing co-efficients

U_i is the IUH at the period i

U_{i-1} is the IUH at the period i—1

$$C = \frac{\Delta t}{R + 0.5\, \Delta t} \tag{9.19}$$

(d) Estimation of Unit Hydrograph : The hydrograph that results from routing these flows from the incremental areas is the IUH. The IUH can be converted to a unit hydrograph of unit rainfall duration Δ t by simply averaging the two ordinates of IUH spaced at an interval Δ t apart as follows :

$$U\,H_i = 0.5\, (U_i + U_{i-1}) \tag{9.20}$$

The IUH can be converted to a unit hydrogtaph of some unit rainfall duration other than Δ t, provided that it is an exact multiple of Δ t, by the following equation :

$$U\,H_i = \frac{1}{n}\, [0.5\, U_{i-n} + U_{1-n+1} + \ldots + U_{i-1} + \ldots 0.5\, U_i] \tag{8.21}$$

where

$U\,H_i$ = Ordinate at time i of unit hydrograph of duration D-hour and computational interval Δ t.

$$n = \frac{D}{\Delta\, t}$$

D = Unit hydrograph duration (hours)

Δt = Computational interval (hours)

U_i = Ordinate at time i of IUH

8.4.2 Computational Steps :

The Computational steps involved in Clark Model are :

(i) Make first estimate of Clark Model parameters, T_c and R, from the excess rainfall hyetograph and direct surface runoff hydrograph computed using the procedure described in lecture no. 5.

(iii) Construct the time-area curve, taking the T_c value obtained from step (i), using the procedure described in the early part of this lecture.

(iii) Measure the area between each pair of isochrones by planimeter.

(iv) Plot the curve of time versus cumulative area. Note that the abscissa is expressed in percent of T_c. Tabulate increments between points that are at computational interval Δ t apart.

(v) Convert the units of inflow using the Eq. (8.17).

(vi) Route the inflow obtained from step (v) using the Eq. (8.18) and (8.19) to get IUH ordinates.

(vii) Compute the unit hydrograph of the excess rainfall duration using Eq. (8.21) and (8.22).

Example 8.3 : Clark Model parameters, T_c and R, derived from a short duration storm rainfall excess hyetograph and direct surface runoff hydrograph, are 8 and 7.5 hours respectively for a typical catchment of area 250 Km². The ordinates of time area diagramme are :

Hour :	1	2	3	4	5	6	7	8
Area (Km²)	10	23	39	43	42	40	35	18

Derive 2-hour unit hydrograph using Clark Model

Solution :

Computational steps are : (Ref. Table 8.3)

(i) Convert the units of inflow using the equation

$$I_i = K\, a_i / \Delta t$$

where

a_i = the ordinates of time area curve in Km².

I_i = the ordinates of the same time area curve in m³/sec.

K = conversion factor

Δt = computationul interval (hours).

= 1 hour (for this example)

If the volume under the time area curve is taken equivalent to 1 cm rainfall excess then the conversion factor K would be :

$$K = \frac{1 \times 10^6 \times 10^{-2}}{3.6 \times 10^3} = \frac{10}{3.6} = 2.778 \text{ m}^3/\text{s/sq. km.}$$

Therefore the above equation may be written as :

$$I_i = 2.778 \ a_i/1 = 2.778 \ a_i$$

(ii) Compute the routing co-efficient C using the equation :

$$C = \frac{\Delta t}{R + 0.5 \Delta t} = \frac{1}{7.5 + 0.5} = 0.125$$

$$\therefore 1 - C = 1 - 0.125 = 0.875$$

(iii) Route the time-area curve obtained from step (i) Using the Eq. (8.18) ; (see Col. (3), Col. (4) and Col. (5)).

$$U_i = CI_i + (1 - C) U_{i-1}$$

$$U_i = 0.125 I_i + 0.875 U_{i-1}$$

where U_i = the ith ordinate of IUH in m³/sec.

(iv) Compute 2-hour unit hydrograph using the Eq. (8.21) (see col. (6)).

$$UH_i = 1/n\ [0.5 U_{i-n} + U_{i-n+1} + \dots\dots\ U_{i-1} + 0.5 U_i]$$

For this example :

D = 2 hours

Δt = 1 hour

$$\therefore n = \frac{D}{\Delta t} = 2$$

$$\therefore UH_i = \tfrac{1}{2}\ (0.5 U_{i-2} + U_{i-1} + 0.5 U_i]$$

See Table 8.3 for the detailed computation

Table 8.3 Clark Model Computation

Time (hrs)	Time area Diagramme (Km²)	0.125 I=2.78 × 0.125 × Col. (2) (m³/s)	0.875 × Col (5) (m³/s)	Col. (3)+(4) IUH Ordinates (m³/s)	2-hour unit hydrograph ordinates (m³/s)
(1)	(2)	(3)	(4)	(5)	(6)
0	0	0	0	0	0
1	10	3.5	0	3.5	0.875
2	23	8.0	3.1	11.1	4.525
3	39	13.5	9.7	23.2	12.225
4	43	14.9	20.3	35.2	23.175
5	42	14.6	30.8	45.4	34.750
6	40	13.9	39.6	53.5	44.875
7	35	12.1	46.8	58.9	52.825
8	18	6.2	51.4	57.6	57.225
9	0	0	50.5	50.5	56.150
10	0	0	44.1	44.1	50.675
11	0	0	38.6	38.6	44.325
12	0	0	33.8	33.8	38.775
13	0	0	29.6	29.6	33.950
14	0	0	25.9	25.9	29.725
15	0	0	22.7	22.7	26.025
			etc.	etc.	etc

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