LECTURE 10

REGIONAL UNIT HYDROGRAPH ANALYSIS

OBJECTIVES

This lecture would provide understanding of the various steps involved in developing the regional unit hydrograph relationships. The participant would also be introduced with some of the regional unit hydrograph relationships (synthetic unit hydrographs) developed for specific regions in India.

10.1 INTRODUCTION

Whenever sufficient and reliable records on stream flow and rainfall are available the unit hydrograph for those catchments can be derived from the rainfall-runoff data of storm events using one of the techniques already described in lecture 6 and 8. However, most of the small catchments are generally not gauged and many water resources projects are being planned in those catchments. Therefore, it becomes necessary to have the estimates of floods at the proposed sites in small ungauged catchments. As we know, the unit hydrograph technique is one of the simple and most powerful techniques among others for the estimation of design flood. Thererore, the unit hydrographs for such catchments have to be estimated by using data on climatological, physiographic and other factors of these catchments,

The main purpose of the regional unit hydrograph study is to estimate the unit hydrograph ordinates or the unit hydrograph parameters for basins for which no gauge discharge data are available. The procedure involved in regional unit hydrgraph analysis requires the evaluation of representative unit hydrograph parameters and pertinent physical characteristics for the gauged catchments in the region. Then multiple linear regression analysis is performed, considering one of the unit hydrograph parameters at a time as a dependent variable and various catchment characteristics as independent variables, in order to develop the regional relationship for the unit hydrograph derivation. Further, knowing the catchment characteristic for an ungauged catchment in the region from the available toposheet and climatological data the unit hydrograph for that catchment can be derived using the relationships developed for the region.

In this lecture, the various steps involved in developing the regional unit hydrograph relationships for a hydrometeorologically homogeneous region are described and discussed. Various regional unit hydrograph studies conducted in India as well as abroad are also presented to provide the proper understanding to the participants about the different forms of relationships established.

10.2 BASIC STEPS INVOLVED IN DEVELOPING THE REGIONAL UNIT HYDROGRAPH

The following steps should be followed in executing a regional study to develop regional unit hydrograph relationships for a basin :

(i) Choice of the catchment : In regional study, care should be taken to select those catchments which are indeed similar in hydro-metereological characteristics. The catchments considered for developing the regional unit hydrograph should be able to represent the regional behaviour as close as possible. Further, one should always try to include maximum no. of gauged catchments in the regional study. However, minimum eight to ten catchments are required for the regional study.

(ii) Split sample test for the region : In order to test the performance of the developed regional relationships, the data of at least two to three catchments should be kept independent. It means, those catchments should be treated as ungauged catchments and they should not be considered while developing the regional relationships.

(iii) Rainfall-runoff data : Rainfall-runoff data of different catchments for each of the major past flood events should be considered for analysis. If the catchment underwent to some major changes due to man's influence or landuse changes, then the rainfall-runoff data of only recent past flood events should be considered for analysis.

(iv) Computation of excess rainfall : A suitable technique should be adopted to separate the loss from total rainfall in order to get the excess rainfall hyetograph.

(v) Base flow separation : The base flow should be separated from the streamflow hydrograph using a consistent base flow separation technique, in order to get the direct surface runoff hydrograph.

(vi) Derivation of Unit Hydrograph : The unit hydrograph should be derived by analysin the excess rainfall-direct surface runoff data for each event of different catchment using a suitableg unit hydrograph derivation technique.

(vii) Derivation of representative unit hydrograph : The representative unit hydrograph for each catchment may be derived by averaging the unit hydrograph obtained from different events of the catchment using standard averaging procedure. However, if considerable variations are observed in unit hydrographs derived from different events of a catchment, then the unit hydrograph parameters of each event should be considered, alongwith the catchment and storm characteristics, in the regional study.

(viii) Split sample test for the storms : The performance of the representative unit hydrograph of a catchment should be tested by reproducing the two or three independent storms which are not to be used for deriving the representative unit hydrograph.

(ix) Development of regional unit hydrograph relationship : Step-wise Multiple linear regression analysis can be performed, taking the unit hydrograph parameters of different catchment as dependent variables, and/or climatic characteristics as independent variables to develop the optimal regional unit hydrograph relationships.

(x) Representative Unit hydrograph for ungauged catchments : The regional relationships developed at step (ix) are used for split sample test for the region as described in step (ii)

Further the representative unit hydrograph for the ungauged catchments of the hydrometereologically homogeneous region can be derived using measurable catchment and/or climatic characteristics in the generalized relationships developed in step (ix).

The basic procedure can be summarized as follows :

(i) From records of gauged catchment in a given region, derive the relations between characteristics of the unit hydrograph and the physical characteristics of the catchment. These relations depend on the method used, some of which are described in section 10.4 of this lecture.

(ii) Assume these relations apply to the ungauged catchments in the same region and use them to derive the synthetic unit hydrographs.

10.3 BASIC TECHNIQUES FOR THE DERIVATION OF REGIONAL UNIT HYDROGRAPH

The basic techniques available for regional analysis are correlation techniques which include the following :

10.3.1 Graphical correlation
10 3 2 Simple linear regression, and
10.3.3 Multiple linear regression

10.3.1 Graphical correlation

In graphical correlation technique the unit hydrograph parameters are plotted as a function of the physical characteristics of the catchments. Usually logarithmic graph paper is used, and a best fit line is drawn by eye. Other information such as knowledge of the approximate slope of the curve or limits on some of the parameters can often be used to aid in positioning the curves.

10.3.2 Simple linear regression

In simple linear regression it is assumed that the parameters (or their logarithms) are related to each other by the equation of a straight line. The regression equation is given by

$$Y = \alpha + \beta X \tag{10.1}$$

where

Y = The dependent variables, or its logarithm. In this case the unit hydrograph parameters of different catchments are considered as dependent variable

α = Regression constant

β = Regression coefficient, and

X = The independent variable, or its logarithm. In this case the physical characteristics of the catchments are considered to be the independent variable.

Estimation of Parameters for simple linear regression :

The predicted value of dependent variable Y' is given by

$$Y'_i = a + bX_i$$

The sum of squares of error (M) is given by

$$M = \Sigma\, E^2_i = \Sigma\, (Y_i - Y'_i)^2 \qquad (10.3)$$

or

$$M = \Sigma\, (Y_i - a - bX_i)^2 \qquad (10.4)$$

where Y_i is the observedvalue of dependent variable, a and b can be estimated by taking partial derivatives of M with respect to a and b setting the resulting equation equal to zero.

$$\frac{\partial M}{\partial a} = -2\, \Sigma\, (Y_i - a - bX_i) = 0 \qquad (10.5)$$

$$\frac{\partial M}{\partial b} = -2\, \Sigma\, X_i\, (Y_i - a - bX_i) = 0 \qquad (10\ 6)$$

The solution of Eq. (10.5) and Eq. (10.6) in terms of a and b is as follows :

$$b = (\Sigma\, X_i\, Y_i - \Sigma\, X_i\, \Sigma\, Y_i/N) \,/\, (\Sigma\, X^2_i - (\Sigma X_i)^2/N) \qquad (10.7)$$

$$a = (\Sigma\, Y_i - b\Sigma\, X_i)/N)$$

$$= \overline{Y} - b\,\overline{X} \qquad (10{,}8)$$

Example 10.1

Determine the regression coefficients for following set of data ($N=16$)

Y Values :

13.26, 13.31, 15.17, 15.50, 14.20, 21.20, 7.70, 17.64, 22.91, 18.89, 12.82, 11.58. 15.17, 10.40, 18.02, 16.25.

X Values :

42.39, 33.48, 47.67, 50.24, 43.28, 52.6, 31.06, 50.02, 47.08, 40.89, 37.31, 37.15, 40.38, 45.39, 41.03.

Solution

From Eqn. (10.7) and (10.8) the values of a and b are 13.195 and 0.6480 respectively.

Many programs are available for determining values of a and b that gives a least square best fit to a given set of data. These include programs for hand held or desk top calculators. If the data sets are not large, then hand calculations can also be made.

10.3.3 Multiple linear regression

In general, watershed response is dependent on several watershed parameters. An equation of the following form can be used to provide a mathematical expression that involves several independent variables :

$$X_1 = B_1 + B_2\, X_2 + B_3\, X_3 + \ldots\ldots\ B_m\, X_m \qquad (10.9)$$

Where,

X_1 = the dependent variable (or its logarithm)

B_1, B_2 = regression co-efficients, and

X_2, X_3 = independent variables or their logarithms

This type of analysis is generally known as the multiple linear regression when several watershed parameters are being considered, some of the proposed parameters may have little effect on the dependent variable. These parameters, of course, should be dropped from consideration and the final expression should include only those parameters which significantly affect the result. Thus the objective of a multiple linear regression is to select an optimal equation combining independent variables and coefficients from which a response may be estimated. The optimal equation may be either :

(i) The one which best describes to the actual relationships between the independent and dependent variables (the physical approach), or

(i i) The one which most accurately estimates the dependent variable from the independent ones (the statistical approach), if the true physical relationships are known.

The primary structural difference between these two approaches is that the satistical model tolerates correlation between the independent variables, whereas the physical approach does not.

Since the physical characteristics of the catchment are correlated and the true physical relationships are not known, therefore one has to go for physical approach rather than the statistical approach. Thus the optimal regression equation may be obtained by successive elimination of the independent variables which are statistically least significance in the equation.

If a regression equation with m parameters is fitted to a set of N data points of variable $X_1, X_2 - - X_m$ the number of degress of freedom will be N—m. If the number of parameters is equal to the sample size i.e. N—m = 0, the regression equation will pass through all the points as there is one degree of freedom. This regression can not be used for prediction as the errors of parameters are inversely proportional to the number of degrees of freedom.

(a) Parameter Estimation :

The parameters of the multiple regression are estimated by method of least squares for the sum of squares of residuals

$$M = \sum_{i=1}^{N} \epsilon_i^2 = \sum_{i=1}^{N} (X'_1 - b_1 - b_2X_2 - - - b_m X_m)^2 \tag{10.10}$$

m partial differential equations will be

$$\frac{\partial M}{\partial b_1} = 0, \frac{\partial M}{\partial b_2} = 0 \ldots \frac{\partial M}{\partial b_m} = 0 \tag{10.11}$$

The above m partial differential equations will have m linear equations

$$b_2 \Sigma (\triangle X_2)^2 + b_3 \Sigma \triangle X_3 + - - - b_m \Sigma \triangle X_2 \triangle X_m = \Sigma \triangle X_1 \triangle X_2$$

$$b_2 \Sigma \triangle X_2 . \triangle X_3 + b_3 (\triangle X_3)^2 + \text{---} b_m \Sigma \Delta X_3 \triangle X_m = \Sigma \triangle X_1 \triangle X_3$$

$$b_2 \Sigma \triangle X_2 . \triangle X_m + b_3 \Sigma \triangle X_3 . \triangle X_m \text{---} b_m \Sigma (\triangle X_m)^2 = \Sigma \triangle X_1 \triangle X_m \qquad (10.12)$$

and

$$b_1 = \overline{X_1} - b_2 \overline{X_2} - b_3 \overline{X_3} - b_m \overline{X_m}$$

in which $X_i = x_i - \overline{X}$ with $i = 1$ to m. These equations enable determination of m parameters. b_1 , b_2 — — b_m are the estimates for B_1, B_2 —— B_m. The parameters can be estimated with the help of matrx also.

$$[Y] = [X]\ [B] \quad \text{for} \quad \Sigma \epsilon_i^2 = 0 \qquad (10.13)$$

or

$$[X]^T\ [Y] = [X]^T . [X] . [B] \qquad (10.14)$$

or

$$(X^T X)^{-1} X^T Y = (X^T X)^{-1} X^T B \qquad (10.15)$$

or

$$([X]^T . [X])^{-1} [X]^T . [Y] = ([X]^T . [X])^{-1} [X]^T . [X] . [B] \qquad (10.16)$$

or

$$[B] = [\ [X]^T\ [X]\ ]^{-1} [X]^T\ [Y]$$

where

[B] is the matrix containing m regression coefficient

[X] is a matrix (n Xm) independent variables

[Y] is a matrix of dependent variables (n X 1)

(b) Various statistics used in the Multiple regression

Various statistical terms used in multiple regression are described below :

(i) Mean of the variable (dependent or independent)

$$\text{Mean} = \overline{X} = \frac{\sum_{i=1}^{N} X_i}{N} \qquad (10.17)$$

where N is total number of observations.

(i i) Standard deviation :

$$S = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \overline{X})^2}{N-1}} \qquad (10.18)$$

(iii) Correlation X vs Y :

$$R_{x,\,y} = \frac{\text{Cov}\ (x,\ y)}{S_x S_y} \qquad (10.19)$$

$$= \frac{\sum_{i=1}^{N} (X_i - \overline{X})\,(Y_i - \overline{Y})/n-1}{\sqrt{\frac{\Sigma\,(X_i - \overline{X})^2}{n-1}}\sqrt{\frac{\Sigma\,(Y_i - \overline{Y})^2}{n-1}}} \qquad (10.20)$$

(iv) Regression Coefficients

Regression coefficients are calculated by method of least squares.

(v) Standard error of regression coefficient

$$Sb_i = \frac{S_1}{S_i}\sqrt{\frac{1}{(N-m)}(1-R_i^2)} \qquad (10.21)$$

where

Sb_i = Standard error of i^{th} regression coefficient

S_1 = Standard deviation of residual

S_i = Standard deviation of X_i

N = Number of observations.

m = Number of variables.

R_i = Multiple correlation coefficient of X_i with respect to all variables except to all variable X_i.

(vi) t value :

$$t\ \text{value} = \frac{\text{regression coefficient}}{\text{standard error of regression coefficient}} \qquad (10.22)$$

(vii) Sum of squares due to regression :

$$SSDAR = \sum_{i=1}^{N} (Y'_i - \overline{Y})^2 \qquad (10.23)$$

SSDR is sum of squares due to regression

where

Y'_i is the estimated value of Y (dependent variable)

$\overline{Y}$ is mean of dependent variable.

(viii) Sum of squares from regression

$$SSFR = \sum_{i=1}^{N} (Y_i - Y'_1)^2 \qquad (10.24)$$

SSFR is sum of squares due to deviation from regression.

Y_i and Y'_i are the observed and predicted values.

(ix) Mean squares due to regression :

$$MSDR = \frac{SSDR}{N\,D\,F} \qquad (10.25)$$

where

MSDR is mean squares due to regression

N.D.F. is number of degrees of freedom and is equal to number of independent variables.

(x) Mean squares from the regression

$$\text{MSFR} = \frac{\text{SSFR}}{\text{N.D.F.}} \tag{10.26}$$

where

MSFR is mean squares due to deviation from regression

N.D.F. is number of degrees of freedom and is equal to N — m — 1. Here m is the number of independent variables.

(xi) Multiple correlation coefficient :

This is the square root of coefficient of determination.

Multiple correlatien coefficient $= \sqrt{R^2}$

$$= \sqrt{\frac{\text{sum of squares due to regression}}{\text{sum of square about mean}}}$$

$$= \sqrt{\frac{\sum_{i=1}^{N}(Y'_i - \overline{Y_i})^2}{\sum_{i=1}^{N}(Y_i - \overline{Y}]^2}} \tag{10.27}$$

Sum of squares about the mean is the sum of squares due to regression and sum of square from the regression.

(xii) Standard error of estimate :

Standard error of estimate $= \sqrt{\text{Mean squares from regression}}$

$$= \left(\frac{{}^{N}_{i=1}\,(Y_i'-Y_i)^2}{N-m-1}\right)^{1/2} \tag{10.28}$$

(xiii) F value :

$$\text{F value} = \frac{\text{Mean squares due to regression}}{\text{Mean squares from regression}} \tag{10.29}$$

(*xiv*) *Partial correlation coefficient* : The partial correlation coefficients measure the association of the dependent variable X_1 with any given independent variable X_i. The partial correlation coefficient may be determined by the following equation :

$$r^2_i = \frac{(1-R^2_2) - (1-R^2_1)}{1-R^2_2} = 1 - \frac{1-R_1^2}{1-R_2^2} \tag{10.30}$$

where

r_i is the partial correlation coefficient for ith independent variable

R_1 is multiple correlation coefficient between X_1 and all the independent variables.

R_2 is the multiple correlation coefficient between X_1 and all the independent variables except X_1.

(*xv*) *Beta coefficients*

$$X_1 = B_1 + B_2 X_2 + \dots \dots B_m X_m$$

$$\frac{X_1}{S_1} = \frac{B_1}{S_1} + \frac{B_2}{S_1} X_2 + \dots\dots \frac{B_m}{S_1} X_m$$

or

$$\text{or} \quad \frac{X_1}{S_1} = \frac{B_1}{S_1} + \frac{B_2 S_2}{S_1} \cdot \frac{X_2}{S_2} + \dots \frac{B_m \cdot S_m}{S_1} \cdot \frac{X_m}{S_m} \dots \dots \tag{10.31}$$

$$\text{or} \quad \frac{X_1}{S_1} = \beta_1 + \beta_2 \frac{X_2}{S_2} + \beta_3 \frac{X_3}{S_3} + \dots \beta_m \frac{X_m}{S_m}$$

The effect of individual independent variables on the dependent variable may be measured by a dimensionless form of the regression coefficient, if each variables is expressed in the above form where $\beta_1 \dots, \beta_m$ are the β coefficients.

(c) Measures of Multiple linear correlative Association

The degree of correlation of a dependent variable X_1 to many externally independent variables in a multiple linear association is measured by any of the five parameters, the standard deviation of residuals, the multiple correlation coefficient, the coefficient of determination, the partial correlation coefficient and the beta coefficients.

(*i*) *The standard deviation of residuals* : This is determined as the standard deviation of the differences between the observed and predicted of independent variable values. This parameter is a measure of the closeness with which the observed values approach the estimated values.

If the total number of points, N, is small in comparison of the number of variables, m, involved, the standard deviation of residuals computed by the usual procedure is biased, since the degrees of freedom N-m are much smaller than the sample size N.

(*ii*) *Multiple correlation coefficient* : The correlation coefficient is a measure of the linear association of two variables. In this case the two variables are the estimated and actual values.

(*iii*) *Coefficient of determination* : The square of the multiple correlation coefficient R^2 is called coefficient of Determination. This indicates, part of the variance which has been accounted for by multiple correlation.

(d) Criteria for accepting results of regression analysis

The results of the regression analysis are evaluated by looking at the statistics describing the goodness of fit of the regression equation to the data. The following statistical parameters are generally used as criteria for accepting the results of regression analysis.

(*i*) *Multiple correlation coefficient (R)* : It provides a measure of the percent of variance in the dependent variable explained by the independent variables. The magnitude of these coefficients varies between-1 to 1. The closer the value is to unity the greater the reliability of the estimate.

(*ii*) *Standard error of estimate* (S_e) : It is the standard deviation of the differences between the observed dependent values and the values computed from the regression equation. Therefore, it must be compared with the mean and standard deviation of that variable.

(*iii*) *F-Test* : A test of hypothesis that the regression equation is not explaining a significant amount of variation in dependent variable can be made by calculating the F-value, which is the ratio of mean squares due to regression to mean squares from regression.

The hypothesis is rejected if F value for the regression equation exceeds $F_{(1-\alpha), M-1, N-M}$ where (1—α) is the confidence level. In other words regression equation explains significant amount of variation in dependent variable if F value is greater than $F_{1-\alpha, m-1, N-M}$.

The values of cumulative F distribution for ν_1 (numerator M—1) and ν_2 (denominator, N—M) are given in Haan, (1977).

(*iv*) *T-Test* : A test of hypothesis that the i th independent variable is not contributing significantly to explain the variation in dependent variable is made by calulating the T value The hypothesis is rejected at 1—α probability level if T value exceeds $t_{1-\alpha/2, N-M}$. In other words i th variable is significant at 1—α level if t value is greater than $t_{(1-\alpha/2), (N-M)}$

Percentile values ($t_{\alpha, \nu}$) for the t distribution with degrees of freedom are given in Haan (1977).

10.4 SOME REGIONAL UNIT HYDROGRAH RELATIONSHIPS

10.4.1 Snyder's Approach

Snyder undertook his studies on catchments upto 10,000 square miles in the Appalachian Highlands in the U. S. A. The equations developed were in imperical units. However those equations in metric units are given below :

$$t_p = C_t \, (L L_{ca})^{0.3} \qquad \dots \quad (10.32)$$

$$Q_p = \frac{2.78 \, C_p A}{t_p} \qquad \dots\dots (10\ 33)$$

$$t_r = t_p/5.5 \qquad \dots\dots (10.34)$$

$$r_b = 3 + 3 \, (t_p/24) \qquad \dots\dots (10.35)$$

While Eq (10.35) provides reasonable estimate for large catchments, but it may give excessively large values of the time base for small catchments. Taylor and Schwartz recommend the use of the following equation for the small catchments.

$$t_b = 5 \left(t_p + \frac{t_r}{2} \right) \qquad \dots\dots (10.36)$$

where

t_p = basin lag time in hours

(this represents the difference between the centroid of excess rainfall and the total runoff hydrograph peak as shown in (Fig. 10.1)

L = length of main stream in Km from divide to outlet (Fig 10.2)

L_{ca} = distance from outlet to centre of area of catchment in Km along the stream (Fig 10 2)

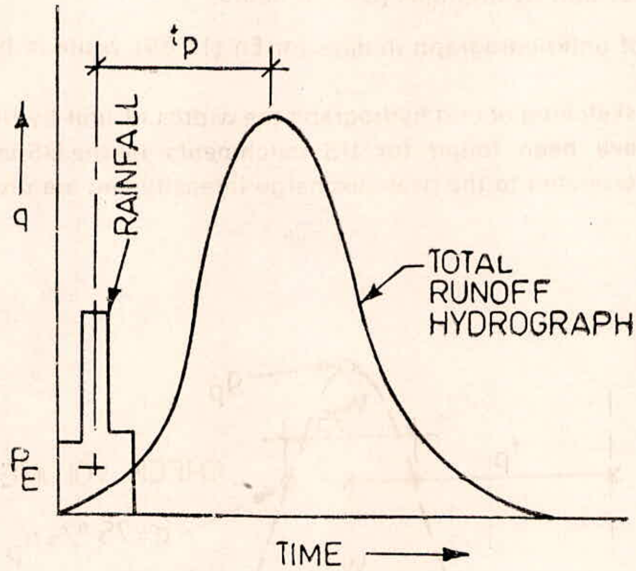

Fig. 10.1 : Basin Lag

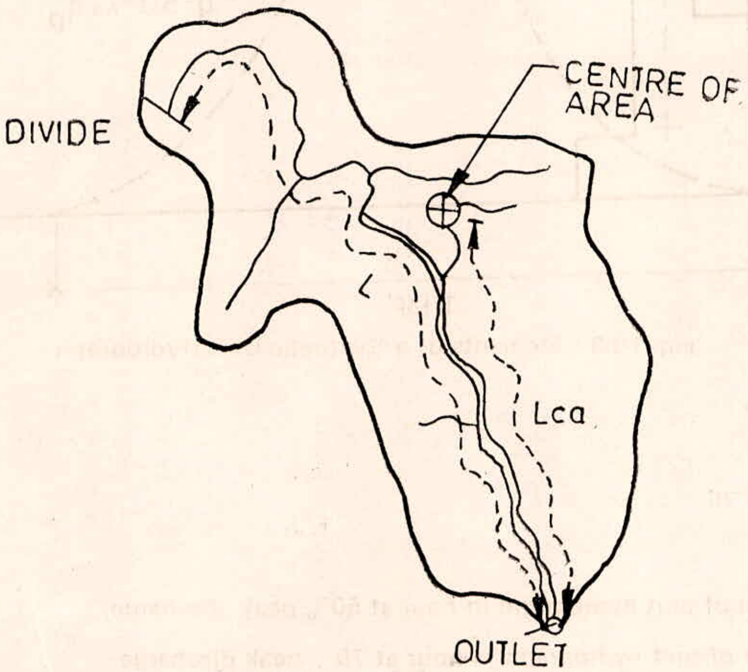

Fig. 10.2 : L and L_{ca} for a typical Basin

C_t=coefficient varying from 0.3 to 0.6 for different regions

Q_p=peak discharge of the unit hydrograph in m³/sec.

A =catchment area above outlet in Sq Km

C_p=coefficient varying from about 0.31 to 0.93

t_r =unit period of unit hydrograph (UH) in hours

t_b=base width of unit hydrograph in days for Eq (10.35) while in hours for Eq. (10.36)

To assist in the sketching of unit hydrograph the widths of unit hydrographs at 50 and 75% of the peak (Fig. 10.3) have been found for US catchments by the US army crops of Engineers. These widths (in hour) correlated to the peak discharge intensity and are given by

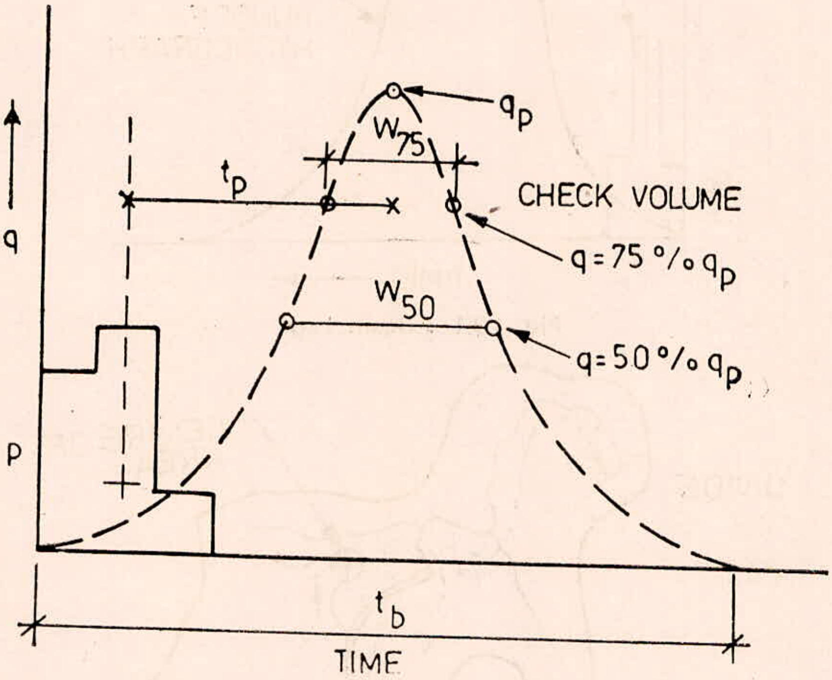

Fig. 10.3 : Elements of a Synthetic Unit Hydrograph

$$W_{50} = \frac{5.87}{q^{1.08}} \qquad \text{......(10.37)}$$

$$W_{75} = W_{50}/1.75 \qquad \text{......(10.38)}$$

where

W_{50}=Width of unit hydrograph in hour at 50% peak discharge

W_{75}=width of unit hydrograph in hour at 75% peak discharge

$q = Q_P/A$ =peak discharge per unit catchment area in m³/s/Km²

Similar form of equations may be derived for different regions with different co-efficients but with same slope. The form of the equations would be :

$$W_{50}=a/q^{1.08} \quad \ldots\ldots (10.39)$$

$$W_{75}=W_{50}/b \quad (10.40)$$

where a and b are the co-efficients varying for different regions

The Snyder Synthetic Unit Hydrograph for an ungauged catchment, located in a specific hydrometreologically similar region, can be derived as follows :

(i) From catchment measure L and L_{ca} in Km.

(ii) Estimate C_t using records of adjoining gauged catchments

(iii) Compute basin lag, t_p in hours using the Eq. (10.32)

(iv) Knowing t_p estimate C_p based on records of adjoining catchments

(v) Compute Q_p (m^3/s) using Eq. (10.33)

(vi) Determine base length t_b in days or in hours using Eq. (10.35) or Eq. (10.36) depending on the size of the catchment.

(v ii) Compute unit period for unit hydrograph, t_r hour using the Eq. (10.34)

(viii) Knowing W_{50} estimate co-efficient 'a' based on records of adjoining catchments

(ix) Knowing W_{75} estimate co-efficient 'b' based on records of adjoining catchments.

(x) Compute W_{50} and W_{75} using the Eq. (10.39) and Eq. (10.40) respectively.

(xi) Plot base, peak, lag time, W_{50} and W_{75} and give them the shape of the unit hydrograph with preserving the unit volume equal to one cm.

In order to construct the unit hydrograph for different duration of effective rainfal t'_r, the basin lag has to be adjusted as :

$$t'_p = t_p + 0.25\,(t'_r - t_r) \quad (10.41)$$

where

t'_p = basin lag in hours for effective duration of t'_r hour

The following relations should be used to compute the peak, time base, W_{50} and W_{75} :

$$Q'_p = 2.78\, C_p\, A/t'_p \quad (10.42)$$

$$t'_b = 3 + 3\,\frac{t'_p}{24} \text{ (days) (for large catchment)} \quad (10.43)$$

$$t'_b = 5\left(t_p' + \frac{t_r}{2}\right) \text{ (hours) (for small catchment)} \quad (10.44)$$

$$W'_{50} = a_1/(q')^{1.08} \quad (10.45)$$

$$W'_{75} = W'_{50}/b \quad (10.46)$$

$$q' = Q'_p/A \quad (10.47)$$

The complete steps to derive a unit hydrograph of duration t'_r other than t_r are :

(i) Measure L and L_{ca} from catchment

(ii) Estimate C_t using records of adjoining gauged catchments

(iii) Compute basin lag t_p in hours using the Eq. (10.32)

(iv) Knowing t_p estimate C_p based on records of adjoining catchments

(v) Compute unit period t_r using Eq. 10.34

(vi) Compute adjusted lag t'_p for a different duration of effective rainfall t'_r using Eq. (10.41)

(v ii) Compute Q'_p from the adjusted lag t'_p using Eq. (10.42)

(viii) Determine base length t'_b using Eq. (10.43) or Eq. (10.44)

(ix) Knowing W'_{50} estimate co-efficient 'a'$_1$ based on records of adjoining catchments.

(x) Knowing W'_{75} estimate co-efficient 'b'$_1$ based on records of adjoining catchments.

(xi) Plot time base, peak, lag time, W'_{50} and W'_{75} and give the shape of the unit hydrograph preserving the unit volume of the unit hydrograph.

Example 10.2

One hour re presentative UH parameters and pertinent physiographic characterics for 21 gauged catchments of Godavari basin subzone 3f are given in Table 10.1 and 10.2 respectively. Using Synder's approach find out the unit hydrograph of one hour duration for an ungauged (Br. No. 24) catchment in the region for which

$A=35$ Sq Km, $L=10.10$ Km, $L_{ca}=7.4$ Km.

Table 10.1 Some Basin Characteristics for Subzone 3f

Sl. No. (1)	Br.No. (2)	A (Km²) (3)	L (Km) (4)	L_{ca} (Km) (5)	LL_{ca} (Km²) (6) = (4) × (5)
1	807	824	67.2	25.8	1733.76
2	875	750	61.1	29.0	1771.90
3	224	750	61.1	23.8	1454.18
4	65	731	92.3	43.1	3978.13
5	228	483	41.8	17.7	739.86
6	15	459	33.1	8.4	278.04
7	184	364	35.2	12.9	454.08
8	604	341	45.0	20.5	922.50
9	269	242	27.7	11.2	310.24
10	881	233	24.1	10.1	243.41
11	969	208	25.0	6.8	170.00
12	57	163	29.0	15.3	443.70
13	36	139	23.0	8.5	195.50
14	566	137	19.6	8.4	164.64
15	494	120	18.2	10.0	182.00
16	51	87	33.7	20.0	674.00
17	59	65	18.0	10.0	180.00
18	20	60	17.7	8.1	143.37
19	161	54	12.2	5.3	64.66
20	4	50	12.2	5.3	64.66
21	491	42	14.7	7.7	113 19

Table 10.2 Representative UH parameters for subzone 3f

Sl. No.	BR. No.	t_p (hours)	Q_p (m^3/s)	t_r (hrs)	t_b (hrs)	W_{50} (hrs)	W_{75} (hrs)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	807	4.5	650	1	17	2.9	1.7
2	875	9.5	290	1	41	6.0	4.0
3	224	9.5	214	1	40	7.5	4.0
4	65	10.5	184	1	40	9.2	5.5
5	228	4,5	280	1	20	3.3	2.0
6	15	5.0	234	1	20	4.2	3.0
7	184	11.5	60	1	44	15.3	7.0
8	604	3.5	228.7	1	12	3.6	2.3
9	269	3 5	140.8	1	14	4.4	3.1
10	881	3.5	190.0	1	14	2.4	1.1
11	969	2.5	179.0	1	12	2.8	1.7
12	57	6.5	65.0	1	24	5.5	3.0
13	37	4.5	80.0	1	15	3.9	2.3
14	566	2.5	190.5	1	9	1.5	0.8
15	494	3.5	65.1	1	13	4.8	3.4
16	51	3.5	65.8	1	11	4.4	2.1
17	59	2.5	66.5	1	6.5	2.1	1.3
18	20	2.0	60.8	1	10	3.4	1.8
19	161	2,5	41.0	1	12	2.9	1.6
20	4	1,5	71.4	1	8	1.6	0.9
21	491	1,5	43,5	1	8	2.3	1.3

Solution :

Derivation of the unit hydrograph of I-hour duration for an ungauged catchment Br. No. 214

The computational steps are :

(i) Compute C_t , C_p , a, and b, values for each gauged catchments and enter them in col, (3), (4), (5), and (6) of Table 10 3 respectively.

(ii) Compute the regional values of C_t and C_p taking the median of those values obtained for gauged catchments at step (i). The regional values are $C_t = 0.62$, $C_p = 0.92$, $a = 2.15$ and $b = 1.71$

(iii) Compute unit hydrograph lag t_p for the ungauged catchment

$$t_p = C_t \, (LL_{ca})^{0.3} = 0.62 \, (10.10 \times 7.4)^{0.30}$$

$$= 2.26 \text{ hours}$$

(iv) Compute unit hydrograph duration :

$$t_r = \frac{t_p}{5,5} = \frac{2.26}{5.5} = 0.41 \text{ hours}$$

(v) Compute adjusted lag t'_p, for $t'_r = 1$ hour

$$t'_p = t_p + 0.25\ (t'_r - t_r)$$

$$= 2.26 + 0.25\ (1.0 - 0.41)$$

$$= 2.41 \text{ hours}$$

(vi) Compute 1 hour unit hydrograph peak using the adjusted lag t'_p

$$Q'_p = \frac{2.78\, C_p\, A}{t'_p} = \frac{2.78 \times 0.92 \times 35}{2.41} \text{ m}^3/\text{S}$$

$$= 37.14 \text{ m}^3/\text{S}$$

Table 10.3 Computation of regional C_t, C_p, a_1 and b_1 values

Sl. No. (1)	Br. No. (2)	$C_t = t_p/(LL_e)^{0.3}$ (3)	$C_p = \frac{Q_p t_p}{2.78A}$ (4)	$a_1 = W_{50}\ (q')^{1.08}$ (5)	$b_1 = \frac{W_{50}}{W_{75}}$ (6)
1	807	0.48	1.28	2.24	1.71
2	875	1.00	1.32	2.15	1.50
3	224	1.07	0.98	1.94	1.88
4	65	0.87	0.95	2.07	1.67
5	228	0.62	0.94	1.83	1.65
6	15	0.92	0.92	2.03	1.40
7	184	1.83	0.68	2.18	2.19
8	604	0.45	0.84	2.34	1.57
9	269	0.63	0.73	2.45	1.42
10	881	0.67	1.03	1.93	2.18
11	969	0.54	0.77	2.38	1.65
12	57	1.04	0.93	2.04	1.83
13	36	0.92	0.93	2.15	1.69
14	566	0.54	1.25	2.14	1.88
15	494	0.73	0.68	2.48	1.41
16	51	0.50	0.95	3.25	2.09
17	59	0.53	0.92	2.15	1.62
18	20	0.45	0.73	3.45	1.88
19	161	0.72	0.68	2.15	1.81
20	4	0.43	0.77	2.35	1.78
21	491	0.36	0.56	2.39	1.85
Median values		0.62	0.92	2.15	1.71

(vii) Compute base length of 1-hour unit hydrograph using Eq. (10.44) valid for small catchment

$$\begin{aligned} t'_b &= 5\,(t'_p + t'_r/2) \\ &= 5\,(2.41 + 1.0/2) \\ &= 5 \times 2.91 \\ &= 14.55 \text{ hours} \end{aligned}$$

(viii) Compute width of unit hydrograph in hour at 50% peak discharge and 75% peak discharge using the Eq. (10.45) to Eq. (10.47) respectively

$$\begin{aligned} W'_{50} &= \frac{a_1}{(q')^{1.08}} = \frac{2.15}{(Q'_p/A)^{1.08}} = \frac{2.15}{(37.14/35)^{1.08}} \\ &= 2.02 \text{ hours} \end{aligned}$$

$$\begin{aligned} W'_{75} &= \frac{W'_{50}}{b_1} = \frac{W'_{50}}{1.71} = \frac{2.02}{1.71} \\ &= 1.18 \text{ hours} \end{aligned}$$

(ix) Plot 1-hour unit hydrograph through the above calculated time base t'_b, time to peak t'_p, peak q'_p, W'_{50} and W'_{74} so that the volume of unit hydrograph comes out to be one cm.

10 4.2 Taylor and Schwartz approach

This procedure is similar to Snyder's approach but introduced another physical characteristic "Mean slope of the catchment". Because of this extra characteristic, it is more cumbersome to derive the co-efficient from adjoining catchment. By the introduction of an additional catchment characteristic other than A, L and L_{ca}, the unit period is made variable.

Unit hydrograph parameters	Catchment characteristics
Lag	L
Peak flow	L_{ca}
Unit period	Area
Base Length	Mean slope of Main steam S
Unit hydrograph width	

Again the constants were derived using imperial units and are presented here in the same units

Lag Relation

$$t_{PR} = C'\, e^{m'} t_R \text{ (hours)} \tag{10.48}$$

where

t_{PR} = Lag for unit period of t_R

$$C' = 0.6/S^{1/2} \tag{10.49}$$

$$m' = 0.212/(LL_{ca})^{0.36} \tag{10.50}$$

$$S = \left[\frac{\Sigma l_i}{\Sigma \frac{l_i}{\sqrt{S_i}}}\right]^2 = \text{Mean slope} \qquad (10.51)$$

where

l_i = sub division length in main stream

$S_i = H_i/l_i$ (feet/feet)

H_i = increase in elevation in l_i measured from catchment map.

PEAK FLOW

$$q_{PR} = C^n e^{m''} t_R \qquad (10.52)$$

where

q_{pR} = Peak flow of unit hydrograph of unit period t_R (cubic feet per second per square mile).

$C^n = 382/(L\, L_{ca})^{0.36}$

$m'' = 0.121\, S^{0.142} - 0.212\, (LL_{ca})^{-0.36} - 0.050$ (10.54)

BASE LENGTH

$$T = 5\left(t_{PR} + \frac{t_R}{2}\right) \text{ (hours)} = \text{Base length} \qquad (10.55)$$

10.4.3 Regional Unit hydrograph relationships for some Indian Basins

CWC derived the regional unit hydrograph relationships for the some Indian Basins relating the various unit hydrograph parameters with pertinent physiographic characteristics.

The representative unit hydrographs of various durations were converted to one hour unit duration for the catchments of lower Godavari subzone (subzone 3 f), Mahanadi subzone (subzone 3 d) and Krishna & Pennar basins (subzone 1 e). Then the parameters of one hour unit hydrograph for the catchments of those subzones were related with different catchment characteristics and the relationships of the following forms were obtained :

$$t_p = a_1 \left(\frac{LL_{ca}}{\sqrt{S}}\right)^{b_1} \qquad (10.56)$$

$$q_p = a_2\, (t_p)^{b_2} \qquad (10.57)$$

$$W_{50} = a_3\, (q_p)^{b_3} \qquad (10.58)$$

$$W_{75} = a_4\, (q_p)^{b_4} \qquad (10.59)$$

$$WR_{50} = a_5\, (q_p)^{b_5} \qquad (10.60)$$

$$WR_{75} = a_6\, (q_p)^{b_6} \qquad (10.61)$$

$$t_B = a_7\, (q_p)^{b_7} \qquad (10.62)$$

The values of the co-efficients in the above equations are given in Table 10.4 for different basins.

Table 10.4 Co-efficient in Regional Relationships

Co-efficients (1)	Lower Godavari basin (subzone 3f) (2)	Mahanadi Basin (subzone 3d) (3)	Krishna & Penner Basin (subzone 1e) (4)
a_1	0.253	1.970	0.258
b_1	0.450	0.240	0.490
a_2	1.968	1.12	1.017
b_2	—0.842	—0.66	—0.52
a_3	2.30	2.195	2.396
b_3	—1.108	—1.1008	—1.08
a_4	1.356	1.221	1.427
b_4	—1.007	—0.95	—1.08
a_5	0.954	0.995	0.750
b_5	—1.178	—0.94	—1.25
a_6	0.581	0.532	0.557
b_6	—1.035	—0.93	—1.12
a_7	4.572	5.72	7.193
b_7	0.90	0.77	0.53

However, for upper Indo-Ganga Plains (subzone 1 e) the parameters of 2 hour unit hydrograph were related with different catchment characteristics and the following relationships were established :

$$q_p = 2.03 \left(\frac{L}{\sqrt{s}} \right)^{0.649} \tag{10.63}$$

$$t_p = 1.858 \, (q_p)^{-1.038} \tag{10.64}$$

$$W_{50} = 2.217 \, (q_p)^{-0.99} \tag{10.65}$$

$$W_{75} = 1.477 \, (q_p)^{-0.876} \tag{10.66}$$

$$WR_{50} = 0.812 \, (q_p)^{-0.907} \tag{10.67}$$

$$WR_{75} = 0.606 \, (q_p)^{-0.791} \tag{10.68}$$

$$t_B = 7.744 \, (t_p)^{-0.779} \tag{10.69}$$

where

L = Length of the main stream in Kilometers

L_{ca} = Length of the main stream from the gauging site to the centre of gravity of the catchment in Kilometers.

S = Statistical stream slope in metres per kilometers

t_p = Time from the centre of unit rainfall duration to the peak of the unit hydrograph in hours.

q_p = Peak discharge of unit hydrograph in cubic metres per second.

t_b = Base width of unit hydrograph in hours.

W_{50} = Width of unit hydrograph measured at discharge ordinate equal to 50% of q_p in hours.

W_{75} = Width of unit hydrograph measured at discharge ordinate equal to 75% of q_p in hours.

WR_{50} = Width of the rising side of unit hydrograph measured in hours at discharge ordinate equal to 50% of q_p in hours.

WR_{75} = Width of the rising side of unit hydrograph measured in hours at discharge ordinate equal to 75% of q_p in hours.

10.4.4 Nash's Approach

Nash (1959) related the first and second moments and IUH with the catchment characteristics for some English basins. He tried various forms of the relationships using different catchment characteristics. However, the following relationships were finally obtained.

$$m_1 = 27.6\, A^{0.3}\, S^{-0.3} \qquad (10.70)$$

$$m_2 = 1.0\, m_1^{-0.2}\, S^{-0.2} \qquad (10.71)$$

where

m_1 is the first moment of IUH about the origin

m_2 is the ratio of the second moment of IUH about the centroid to m_1^2

A is the catchment area (mile^2) and

S is a measure of overland slope.

For ungauged catchment, Eq (10.70) and (10.71) were used to get m_1 and m_2 which are further used to get the parameters of Nash Model, n and K, using theorem of moments.

10 4.5 Regional Unit Hydrograph for Narmada basin based on Clark's Approach

A regional unit hydrograph study has been conducted for Narmada basin at National Institute of Hydrology, Roorkee. In this study the parameters, T_c and R, of Clark Model have been derived for the Narmada sub-basins using HEC-1 programme Package. The values of $(T_c + R)$ and $R/(T_c+R)$ of each of the floods analysed in each of the sub-basins have been averaged for the respective sub-basins. The regional relationship has been presented in the graphical form where average values of (T_c+R) for each sub-basins have been plotted against their respective catchment area as shown in Fig. 10.4. This plot along with the fixed value of $R/(T_c+R)$ has been used to

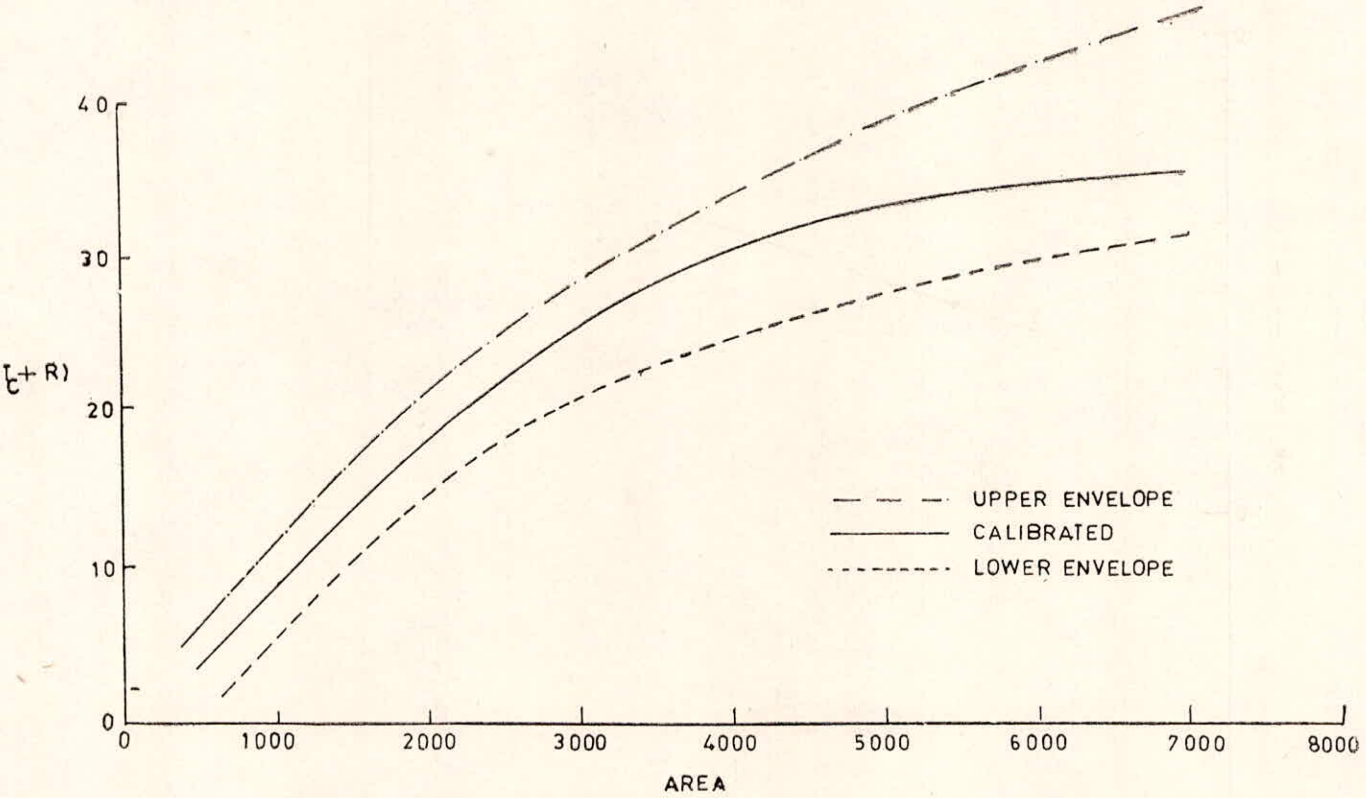

Fig. 10.4 : I_c + Vs Area Relationship

estimate the regional parameters for ungauged catchments. The fixed value of $R/(T_c+R)$ has been taken up around 0.6 for sub-basins up stream of Bermanghat site, while for sub-catchments on downstream of Bermanghat site, the average values of this ratio is around 0.45.

10.4.6 Regional Unit Hydrograph for lower Godavari Basin subzone 3f. based on Nash and Clark's Approach

Singh (1984) developed the regional unit hydrograph relationships relating the physical parameters of five catchments of Godavari basin subzone 3 f. with average peremeters of Nash Model and Clark model for those catchments. Fig. 10.5 and 10.6 illustrate the variation of n K with $(LL_{ca}/\sqrt{S})$ and K with main stream length L respectively. These two plots are used as the regional relationships based on Nash Model.

The Clark's model parameter T_c has been related with $(LL_{ca}/\sqrt{S})$ as shown in Fig. 10.7 A fixed value of the ratio $R/(T_c+R)$ along with T_cV/s $LL_c/\sqrt{S}$ plot is used to establish the regional unit hydrograph relationships based on clark model. Due to non availability of much data for the other bridge catchments of the subzone 3 f. only five catchments have been considered in the regional study. Therefore the study has some what limited scope. However, it provides an encouraging results for further investigation.

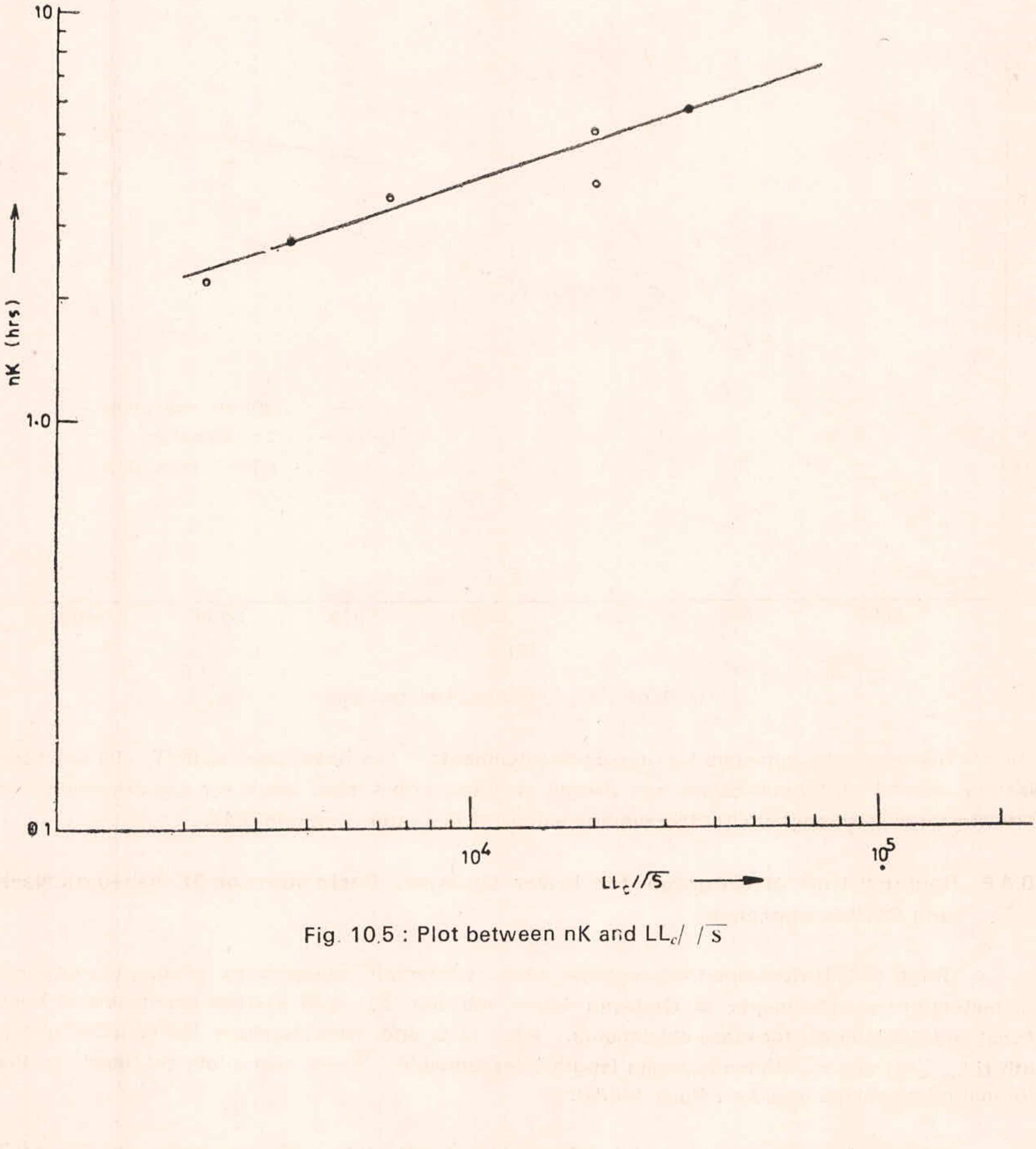

Fig. 10.5 : Plot between nK and $LL_c/\sqrt{S}$

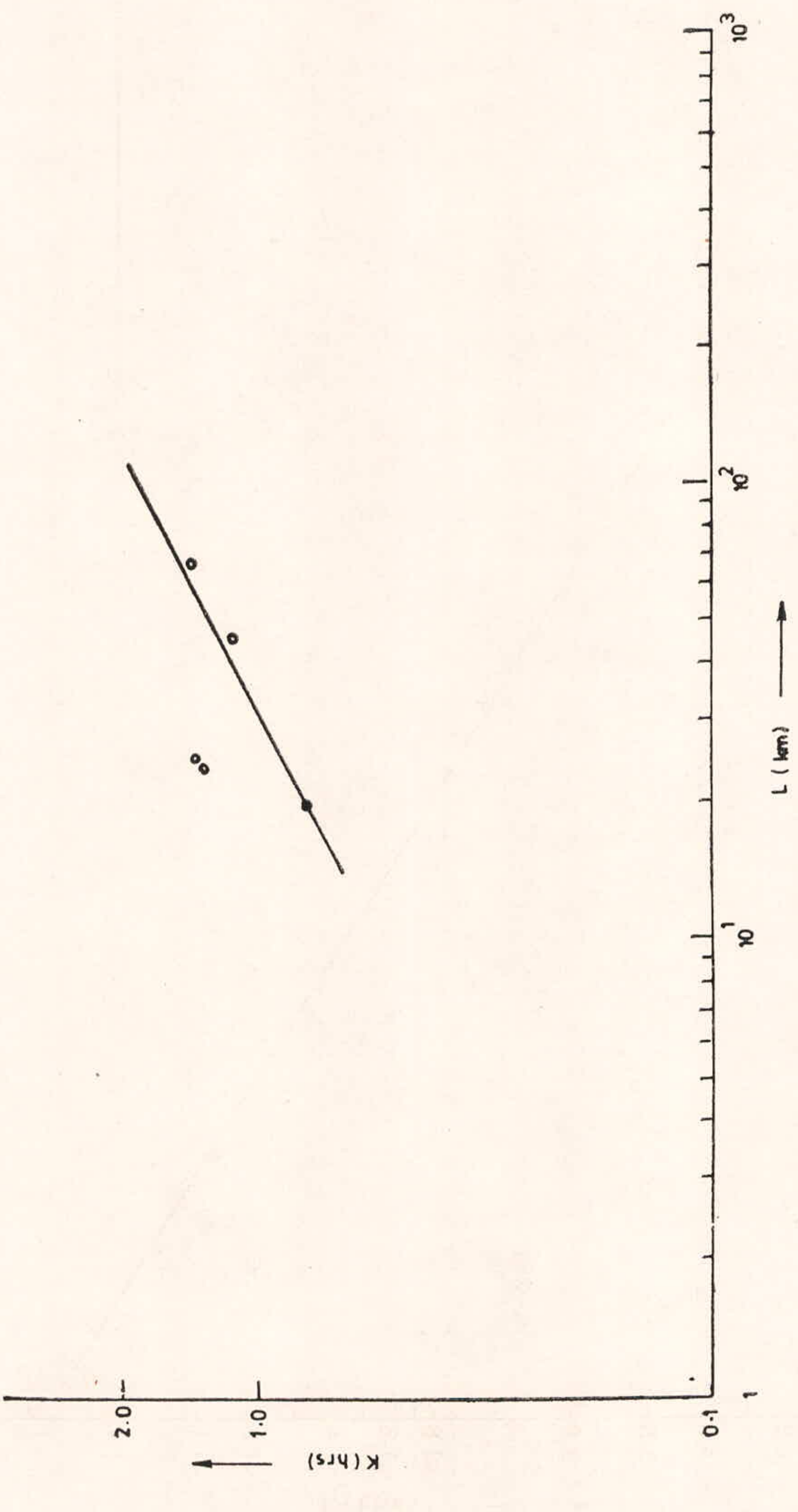

Fig. 10.6 Plot betweeh K and L

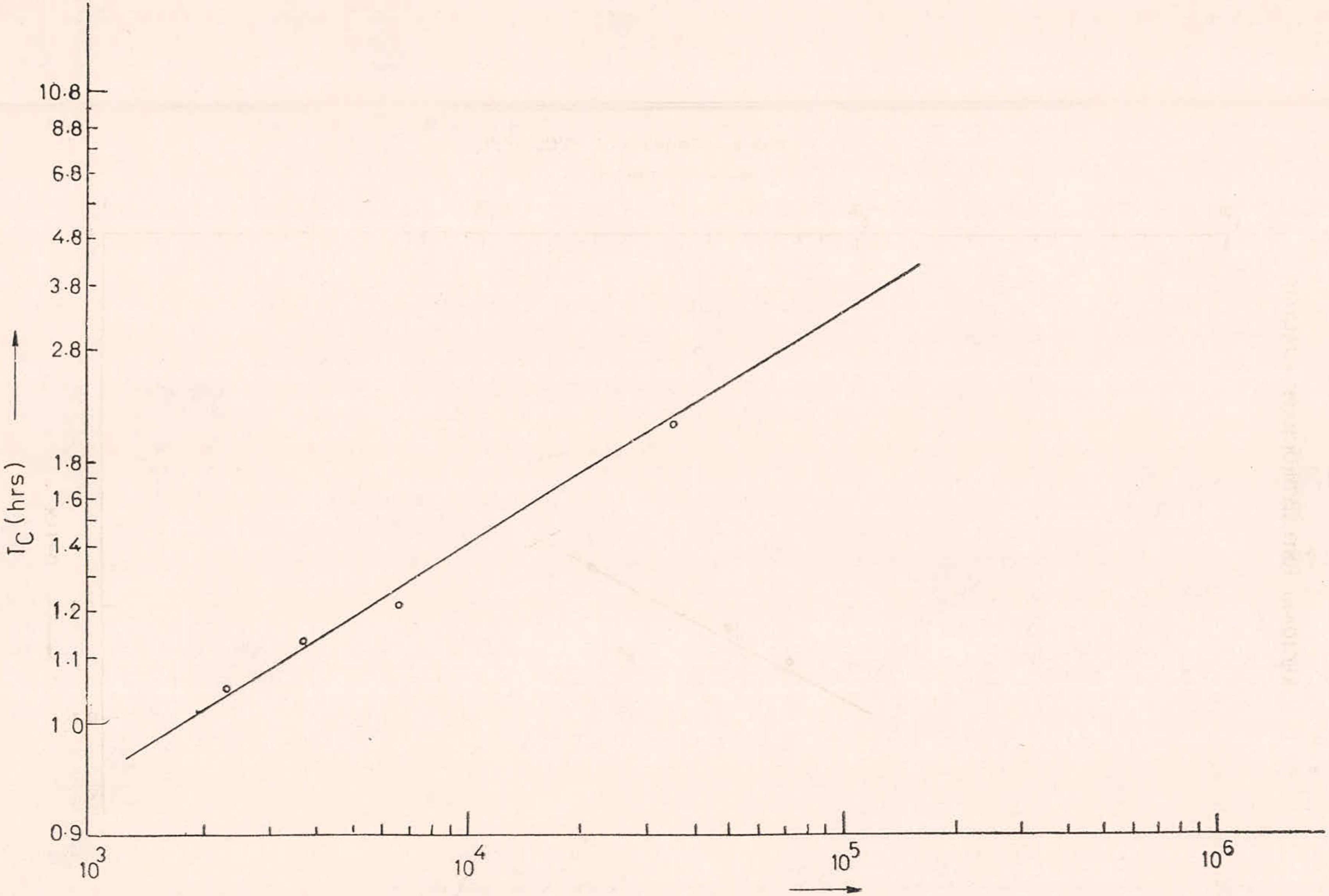

Fig. 10.7 Plot between T_c and LL_c/ /

10.5 SUMMARY

(i) The regional unit hydrograph or synthetic unit hydrograph are a tool to overcome lack of stream flow data at the specific site under investigation.

(ii) It is assumed that the unit hydrograph represents the physiographic characteristics of a catchment.

(iii) Multiple linear regression analysis is a most powerful tool for the regional unit hydrograph analysis.

(iv) Data for unit hydrographs derived from stream flow and rainfall records in catchments adjoining the catchment under study should be used to estimate the constants for use to any regional unit hydrograph study.

(v) Estimated constants in the regional relationships are transposed to the catchment under study for the derivation of unit hydrograph parameters.

(vi) Studies conducted for some typical region in India indicate encouraging results. It demands similar studies in systematic way for different hydrometerologically homogeneous regions in India.

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