TUTORIAL-5

UNIT HYDROGRAPH DERIVATION BY CONCEPTUAL MODELS

Problem :

(a) Using the data of rainfall runoff for the storm considered in Tutorial no. 2, find out one hour unit hydrograph by conventional Nash Model.

(b) Find out one hour unit hydrograph by Integer Nash model using the rainfall runoff data of the same storm.

(c) If Clark model parameters, T_c and R, derived from the excess rainfall hyetograph and direct surface runoff hydrograph of the storm computed in Tutorial no. 2, are 3 hours and 3 2 hours respectively, find out one hour unit hydrograph using Clark Model. The ordinates of time area diagram are given below.

Time (h)	0—0.5	0.5—1.0	1.0—1.5	1.5—2.0
Area (km²)	74.12	140.02	197.67	181.20
Time (h)	2.0—2.5	2.5—3.0		
Area (km²)	148.25	82.36		

Solution :

(a) Derivation of one-hour unit hydrograph using Conventional Nash Model :

(i) Compute the first and second moments of excess rainfall about the origin using the following equations respectively :

$$_1M'_X = \frac{\sum_{i=1}^{m} X_i t_i}{\sum_{i=1}^{m} X_i}$$

$$_2M'_X = \frac{\sum_{i=1}^{m} X_i t_i^2}{\sum_{i=1}^{m} X_i}$$

For the given problem,

$$ {}_1M'_X = \frac{3.58 \times 0.5 + 4.07 \times 1.5 + 2.54 \times 2.5}{3.58 + 4.07 + 2.54} = \frac{14.245}{10.19} $$

$$ {}_1M'_X = 1.398 \text{ hrs} $$

and $$ {}_2M'_X = \frac{3.58 \times (0.5)^2 + 4.07 \times (1.5)^2 + 2.54 \times (2.5)^2}{3.58 + 4.07 + 2.54} = \frac{25.93}{10.19} $$

$$ {}_2M'_X = 2.543 \text{ hrs}^2 $$

(ii) Compute the first and second moment of direct surface runoff hydrograph about the origin using the following equations respectively :

$$ {}_1M'_Y = \frac{\sum_{i=1}^{N} \frac{Y_i + Y_{i+1}}{2} t_i}{\sum_{i=1}^{N} \frac{Y_i + Y_{i+1}}{2}} $$

$$ {}_2M'_Y = \frac{\sum_{i=1}^{N} \frac{Y_i + Y_{i+1}}{2} t_i^2}{\sum_{i=1}^{N} \frac{Y_i + Y_{i+1}}{2}} $$

Table T 5.1 shows the intermediate calculations involved in computation of ${}_1M'_Y$ and ${}_2M'_Y$ for the given problem. Hence

$$ {}_1M'_Y = \frac{16226.62}{2331.335} $$

$$ = 6.96 \text{ hrs} $$

$$ {}_2M'_Y = \frac{136233.37}{2331.335} $$

(iii) Compute the parameters, n and K, of Nash Model solving the following equations :

$$ n\,K = {}_1M'_Y - {}_1M'_X $$

Table T 5.1

Computation of First and Second Moment of Direct Surface Runoff

No.	Y_i	$\frac{Y_i \times Y_{i+1}}{2}$	t_i	t_i^2	$\frac{Y_i + Y_{i+1}}{2} t_i$	$\frac{Y_i + Y_{i+1}}{2} t_1^2$
1	0	0	0.5	0.25	0	0
2	0	23.685	1.5	2.25	35.528	53.291
3	47.37	79.210	2.5	6.25	198.025	496.063
4	111.05	240.395	3.5	12.25	841.383	2944.484
5	369.74	391.580	4.5	20.25	1762.110	7929.495
6	413.42	375.265	5.5	30.25	2063.958	11351.766
7	337.11	276.450	6.5	42.25	1796.925	11680.012
8	215.79	215.130	7.5	56.25	1613.475	12101.062
9	214.47	183.815	8.5	72 25	1562.428	13280.633
10	153.16	140.00	9.5	90.25	1330.08	12635.00
11	126.84	121.285	10.5	110 25	1272.44	13360.65
12	115.53	99.87	11.5	132.25	1148.51	13207.81
13	84.21	68.55	12.5	156.25	856.88	10710.94
14	52.89	42.24	13.5	182.25	570.24	7698.24
15	31.58	25.92	14.5	210.25	375.84	5449.68
16	20.26	17.11	15.5	240.25	265.21	4110.68
17	13.95	13.29	16.5	272.25	219.29	3618.20
18	12.63	11.98	17.5	306.25	209.65	3668 88
19	11.32	5.66	18.5	392.25	104.71	1937.14
20	0	0				
21	0	0				
22	0	0				
23						
		2331.335			16226.602	136233.37

$n\,(n+1)\,K^2 + 2n\,K.{}_1M'_X = {}_2M'_Y - {}_2M'_X$

Putting the computed valves of ${}_1M'_X$, ${}_2M'_X$, ${}_1M'$, ${}_2M'$ in the above equations, we get

$$nK = 6.960 - 1.398$$

$$= 5.562$$

$$n\,(n+1)\,K^2 + 2\,n\,K \times 1.398 = 58.434 - 2.543$$

$$n\,(n+1)\,K^2 + 2\,n\,K \times 1.398 = 55.891$$

$$n^2 K^2 + n\,K^2 + 2\,n\,K \times 1.398 = 55.891$$

$$(n\,K)^2 + n\,K.K + 2\,n\,K \times 1.398 = 55.891$$

$$(5.562)^2 + 5.562\,K + 2 \times 5.562 \times 1.398 = 55.891$$

$$K = \frac{55.891 - (5.562)^2 - 2 \times 5.562 \times 1.398}{5.562}$$

$$K = 1.690 \text{ hrs}$$

$$n = \frac{n\,K}{K}$$

$$= \frac{5.562}{1.690}$$

$$= 3.291$$

(iv) Compute one hour unit hydrograph ordidates by conventional Nash Model using the following equations :

$$U\,(T,\,t) = 1/T\,[1\,(n,\,t/K) - 1\,(n,\,(t-T)\,/\,K)\,)$$

$$U'\,(T,\,t) = 0.2778 \times VOL \times U\,(T,\,t)$$

Where the values of $1\,(n,\,t/K)$ and $1\,(n,\,t-T)/K)$ are computed using the incomplete gamma function table given in APPENDIX-IV for the range of values required in the problem.

Table T 5.2 shows the computation of one hour unit hydrograph using the above equation where the required values of incomplete gamma functions are interpolated from the given table in APPENDIX-IV.

Tabte T 5.2

Estimation of Unit Hydrograph Ordinates

P = 2.30, K = 1.690. T = 1 hour, CA = 823 62 Sq. Km

Time (hrs) (t)	x = t/k	U = $\frac{x}{\lvert P+1}$	1 (U.P) (from the *table	1 (U',P) (from the *table)	U (T, t) [Col. (4) —Col. (5)]	U (T, t) (cumecs) Col. (6)* 0.2778 *CA
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0.592	0.321	0.0132	0	0.0132	3.02
2	1.183	0.651	0.08192	0.0132	0.06872	15.72
3	1.775	0.977	0.20384	0.08192	0.12192	27.90
4	2 367	1.303	0.35093	0.20384	0.14709	33.65
5	2 959	1.629	0.4965205	0 35093	0.14559	33.31
6	3 550	1.954	0 62433	0.4965205	0.12780	29.24
7	4.142	2.280	0.728928	0.62433	0.10459	23.93
8	4.734	2.606	0.8095304	0.728928	0.08060	18.44
9	5.325	2.931	0.868711	0.8095304	0.0592	13 55
10	5.917	3.257	0.9078615	0.868711	0.0392	8.97
11	6.509	3.583	0.9410507	0.9078615	0.0332	7.59
12	7.101	3.909	0.9613378	0.9410507	0.0203	4.64
13	7.692	4.234	0.9748704	0 9613388	0 0135	3.09
14	8 284	4.560	0.9837636	0.9748704	0.009	2 04
15	8 876	4.886	0 9886508	9 9837636	0.049	1.12
16	9.467	5.211	0.9935124	0.9886508	0.00486	1.11
17	10.059	5.537	0.9959276	0.9935124	0.00240	0 549
18	10.651	5.863	0.9974632	0.9974632	0 00150	0 343
19	11.24	6.189	0 9984312	0.9974632	0.00096	0.22

* Incomplete Gamma function table is given in APPENDIX IV

(b) Computation of one hour unit hydrograph using Integer Nash Model

The computational steps are :

(i) Follow the step (i), (ii), and (iii) as for conventional Nash Model. It provides the following values of the parameters for the given problem :

$n K = 5.562$ hrs

$n K^2 = 9.400$ hrs^2

$n = 3.291$

$K = 1.690$ hrs

(ii) Now take the integer values of n nearer to the actual value of n and modify the value of K to preserve the first moment of IUH about the origin. For this problem :

n (Integer) = 3

$$K = \frac{5.562}{3}$$

$= 1.854$ hour

$nK^2 = 10.31$ hr^2

(iii) Estimate the unit hydrograph ordinates using the following equations :

$$U(T, t) = 1/T\,[I(n, y) - I(n, y_1)]$$

$$I(n, y) = 1 - e^{-y} \sum_{m=0}^{n-1} \frac{y^m}{m!}$$

$$I(n, y_1) = 1 - e^{-y_1} \sum_{m=0}^{n-1} \frac{y_1^m}{m!}$$

$$y = t/K$$

$$y_1 = (t-T)/K$$

$$U^*(T, t) = 0.2778 \times VOL \times U(T, t)$$

For the given problem,

$n = 3$

$K = 1.854$ hours

$T = 1$ hour

VOL = 1 mm

Table T 5.3 shows the computation of unit hydrograph using Integer Nash Model.

Table T 5.3 Computation of one hour unit hydrograph using Integer Nash Model

Time (hrs)	y	I (n, y)	y_1	I (n, y_1)	U (T, t)	U' (T, t) (m^3/S)
(1)	(2)	(3)	(4)	(5)	(6)=[(3)—(5)]/T	(7)
1	0.539	0.0175	—	—	0.0175	4.09
2	1.079	0.0954	0.539	0.0175	0.0779	17.82
3	1.618	0.2213	1.079	0.0954	0.1259	28.81
4	2.157	0.3657	1.618	0.2213	0.1444	33.04
5	2.697	0.5056	2.157	0.3627	0.1399	32.01
6	3.236	0.6276	2.697	0.5056	0.122	27.91
7	3.776	0.7272	3.236	0.6276	0.0996	22.79
8	4.315	0.8045	3.776	0.7272	0.0773	17 69
9	4.854	0.8625	4.315	0.8045	0.0580	13.27
10	5 393	0.9048	4.854	0.8625	0.0423	9.67
11	5.933	0.9349	5 393	0.9048	0.0301	6.89
12	6.472	0.9561	5.933	0.9349	0.0212	4.85
13	7.012	0.9706	6.472	0.9561	0.0145	3.31
14	7.551	0.9805	7.012	0.9706	0.0099	2.27
15	8.091	0.9872	7.551	0.9805	0.0067	1.53
16	8.626	0.9916	8.091	0.9872	0.0044	1.01
17	9 169	0.9946	8.629	0 9916	0.0030	0.69
18	9.709	0.9965	9.169	0.9946	0.0019	0.43
19	10.248	0.9977	9.709	0.9965	0.0012	0.27

(a) Computation of one hour unit hydrograph using Clark model

The computational steps are :

(i) Convert the units of inflow using the equation

$t_i = K a_i \triangle t$

where a_i = the ordinate of time area curve in km^2

I_i = the ordinates of the same time area curve in m^3/sec.

K = conversion factar

$\triangle t$ = computational interval (hours)

= 0.5 hours (for this example).

If the volume under the time - area curve is taken equivalent to 1 mm rainfall excess, then the conversion factor K would be :

$$K = \frac{1 \times 10^6 \times 10^{-3}}{3.6 \times 10^3} = \frac{1.0}{3.8} = 0\ 2778\ m^3/s/sq.\ km$$

Therefore the above equation may be written as :

$$I_i = 0.2778\ \frac{a_i}{0.4} = 0.5556\ a_i\ m^3/s/sq.\ km.$$

(ii) Compute the routing co-efficient C using the equation

$$C = \frac{\triangle t}{R + 0.5\ \triangle t} = \frac{0.5}{3\ 2 + 0.5 \times 0.5} = 0.145$$

(ii) Route the time area curve obtained from step (i) using the equetion (see col. (3), Col. (4) and Col. (5) of the table 1.7).

$$U_i = C\ I_i + (1 - C)\ U_{i-1}$$

$$U_i = 0.145\ I_i + 0.855\ U_{i-1}$$

where U_i = the ith ordinates of IUH in m^3/s

(iv) Compute 1-hour unit hydrograph using the equation (see Col. (6) of table 1.7)

$$UH_i = 1/n\ (0.4\ U_{i-n} + U_{i-n+1} + \ldots\ldots\ U_{i-1} + 0.5\ U_i)$$

For this example :

D = 1 hour

$\triangle t$ = 0.5 hour

$$\therefore\quad n = \frac{D}{\triangle t} = \frac{1}{0.5} = 2$$

$$\therefore\quad UH_i = \tfrac{1}{2}\ (0.5\ U_{i-2} + U_{i-1} + 6\ 5\ U_i)$$

See Table T 5,4 for detailed computation

Table T 5.4 Clark Model Computation

Time (hrs)	Time Area Diagramme (km^2)	0.145 I= 0.145 × 0.556 × Col. (2) (m^2/s)	0.855 × Col. (5)	Col (3) + Col (4) IUH ordinates (m^3/s)	1-hour UH ordinates (m^3/s)
(1)	(2)	(3)	(4)	(5)	(6)
0	0	0	0	0	0
0 5	74.12	5.976	0	5.976	1.49
1.0	140,02	11.288	5.109	16.397	7.09
1.5	197.67	15.936	14.019	29.955	17.18
2.0	181.20	14.608	25.612	40.220	29.13
2.5	148.25	11.952	34.388	46.340	39 19
3.0	82.36	6.639	39.620	46.259	44.79
3.5			39.551	39.551	44.61
4.0			33.82	33.82	39.80
4.5			28.92	28.92	34.03
5.0			24.73	24.73	29.09
5.5			21.14	21.14	24.88
6.0			18.07	18.07	21.27
6 5			15.45	15.45	18 19
7.0			13.21	13.21	15.55
7.5			11.29	11.29	13.29
8.0			9.65	9.65	11.36
etc.			etc.	etc.	etc.