TN-6

ESTIMATION OF SEEPAGE FROM CANAL USING TRACER TECHNIQUE

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CONTENTS

LIST OF SYMBOLS

b	–	Width of the canal at water surface
b'	–	bottom width of the canal
C	–	a constant
c	–	concentration of tracer at any time t
c_o	–	concentration of tracer at t equal to zero
D	–	diameter of the bore hole
D_w	–	depth to water level measured from bed level of canal at a distance l from the centre of the canal
d	=	distance of the borehole from centre of the canal
F	–	cross-section of the measuring volume perpendicular to the direction of flow
H	–	maximum depth of water in the canal
H'	–	borehole depth below water table
h_o	–	piezometric head outside the borehole
h_i	–	piezometric head inside the borehole
K	–	coefficient of permeability
K_i	–	hydraulic conductivity inside the borehole
K_o	–	hydraulic conductivity of undisturbed aquifer
l	–	distance from the centre of the canal at which water table is measured
m	–	slope of the graph of $\log_e c/c_o$ vs t
p	–	pressure
q_i	–	total inflow of water into the borehole
q	–	total seepage loss per unit length of the canal
q'	–	seepage loss occurring through bed of the canal per unit length

R - radius of the borehole

t - time

V - borehole volume

V_a - horizontal component of filtration velocity or Darcy velocity

w - complex potential

x, y - cartesian coordinates

α - a correction factor for hydrodynamic field deformation

γ_w - unit weight of water

θ - Zhukovsky function defined as $\theta = z + i\, w/K$

θ' - angle made by tangent to the phreatic line with vertical

ϕ - velocity potential function

ϕ_i - velocity potential function inside the bore hole

ϕ_o - velocity potential function outside the borehole

ψ - stream function

LIST OF FIGURES

ABSTRACT

Loss of water due to seepage from irrigation canals constitutes a substantial percentage of the total usable water. By the time the water reaches the field, it has been estimated that the seepage losses are of the order of 45 percent of the water supplied at the head of the canal. Of the various factors which influence seepage loss from a canal, the most important are the boundary conditions of the flow domain and permeability of the medium. Only after a correct assessment of the coefficient of permeability and boundary conditions, the seepage losses can be estimated either by numerical method or by analytical technique. Needless to say that it is very difficult to determine the insitu coefficient of permeability of the porous medium. In order to avoid the difficult task of estimating the coefficient of permeability and the prevailing subsurface boundary conditions experimental techniques like ponding method and inflow-outflow method have been used for estimation of seepage losses. In recent years, tracer technique has also been used to estimate seepage loss from water bodies because of its comparatively easy operation in respect to other experimental methods.

The seepage velocities at various sections of Deoband canal, a branch of Upper Ganga Canal, have been found by U.P. Irrigation Research Institute using nuclear tracer technique. These seepage velocities have been used subsequently in a formula derived by U.P.I.R.I. to estimate the seepage losses from the canal. In a study carried out at NIH, the U.P.I.R.I. formula used for estimating the seepage loss from tracer technique test data, has been rederived starting from the fundamentals. The losses estimated by the rederived formula using tracer technique data are found to agree with seepage losses measured by other experimental methods. Conformal mapping analysis has also been carried out to determine seepage loss from the canal.

From the analysis of seepage, it is found that the losses are only taking place through bed of the canal and the canal banks are relatively impervious. The deviation of the observed phreatic line from the locus of the phreatic line obtained analytically for a homogeneous flow domain justifies the above statement.

1.0 INTRODUCTION

Loss of water due to seepage from irrigation canals constitutes a substantial percentage of the total usable water. By the time the water reaches the field, it has been estimated that the seepage losses are of the order of 45 percent of the water supplied at the head of the canal. Of the various factors which influence seepage loss from a canal, the most important are the boundary conditions of the flow domain and permeability of the medium. Only after a correct assessment of the coefficient of permeability and boundary conditions, the seepage losses can be estimated either by numerical method or by analytical technique. Needless to say that it is very difficult to determine the insitu coefficient of permeability of the porous medium. In order to avoid the difficult task of estimating the coefficient of permeability and the prevailing sub-surface boundary conditions experimental techniques like ponding method and inflow outflow method have been used for estimation of seepage losses. In recent years, tracer technique has also been used to estimate seepage loss from water bodies because of its comparatively easy operation in respect to other experimental methods. The temporal variation of concentration of a tracer is observed in a bore hole near a canal which furnishes the Darcy velocity of flow. Making use of the Darcy velocity and assessing the area of flow, seepage loss from the canal is estimated. In the present study estimation of seepage loss from canal by tracer technique has been analysed.

2.0 REVIEW

Ground water flow velocity can be evaluated using Darcy's formula when the hydraulic gradient and the permeability are known. Using a tracer it is possible to measure directly the ground water velocity. The method consists of injecting tracer into a well and then measuring its concentration decrease with time. The principle applies for any tracer. Compared to any other tracers radioactive isotopes are easier to be detected insitu with high degree of accuracy at very low concentrations.

The horizontal filtration velocity V_a of water is given by

$$V_a = - \frac{V}{\alpha F} \log_e \left(\frac{c}{c_o} \right)$$

where V is the bore hole volume in which dilution takes place, F is the cross-section of the measuring volume perpendicular to the direction of the undisturbed ground water flow, t is the time interval between measurement of concentration c_o and c, and α is a correction factor accounting for the distortion of the flow lines due to the presence of the borehole. The above expression is a particular solution of the differential equation describing dilution rate of the tracer. Knowing the filtration velocity, the seepage from water bodies can be estimated.

The seepage losses from Ganga canal and Deoband canal(a branch of Upper Ganga Canal) have been determined at various sections using tracer technique by U.P.I.R.I. (Garg et al, 1983; Bhargava et al, 1983). Also the seepage losses from the minor and distributaries of Deoband branch system have been determined by ponding method. It is found that the losses per unit area of canal surface, estimated by tracer technique are much higher than that of the losses determined by ponding method.

The same discrepancy has been found while studying seepage loss from Salawa distributary. In the present report the formula used for estimating the seepage loss from tracer-technique-test-data has been rederived starting from the fundamental. The losses estimated by the corrected formula using tracer technique data is found to agree with seepage losses measured by other experimental methods.

3.0 PROBLEM DEFINITION

A canal is running in an area where the ground water table is encountered at shallow depth. A borehole is made at a distance 'd' from the centre of the canal as shown in figure 1. In the borehole a tracer is added and its concentration is measured with respect to time. It is aimed to find the seepage loss from the canal making use of the variation of the concentration of tracer in the borehole with time.

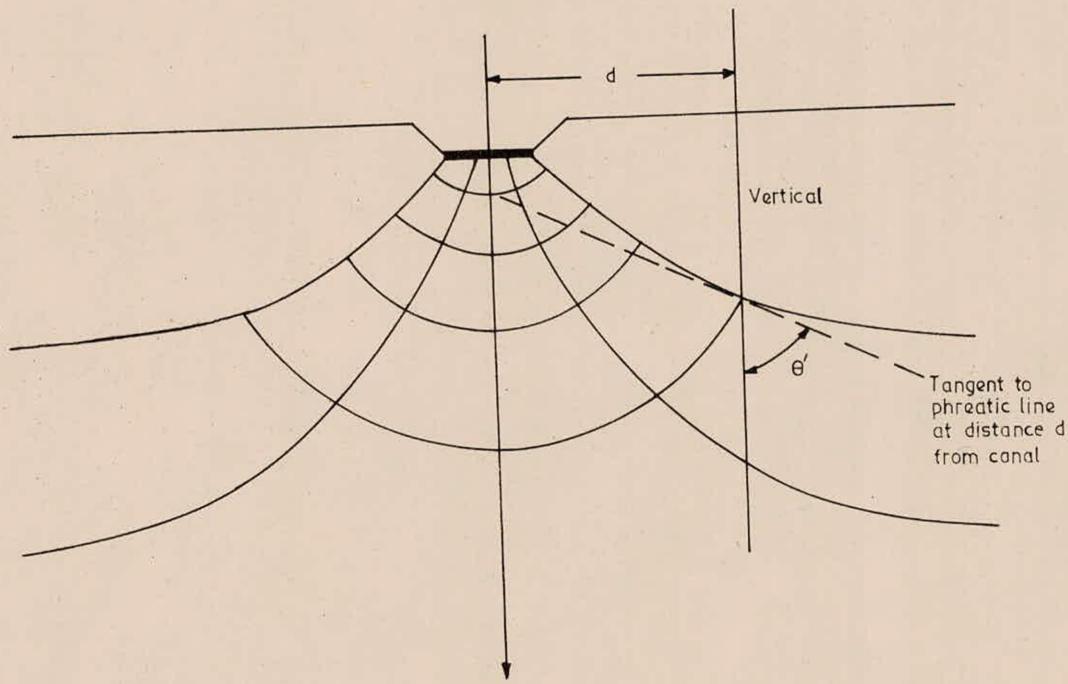

FIGURE 1 - IDEALIZED FLOW CONFIGURATION

4.0 METHODOLOGY

The following assumptions are made while deriving the formula:

i) The canal is treated as a line source.

ii) The flow lines are radial and equipotential lines are circular.

iii) The porous medium at any reach has constant permeability which may vary from reach to reach.

iv) The plane of flow is normal to the canal axis.

Let the variation of concentration of the tracer with time is measured in the vertical borehole which is located at a distance 'd' from the centre of the canal. Let the diameter of the borehole is D and the borehole is taken to a depth H below the water table. The volume of water V in the borehole is equal to $\pi D^2 H/4$. Let c is the concentration of the tracer at any time t. Assuming that the water entering to the borehole is free from the tracer used, the mass balance for a period from t to $t+\Delta t$ can be written as:

Total inflow of tracer to the borehole in time Δt = total outflow of tracer from the borehole in time Δt+ change in concentration of the solution in borehole x volume of solution in borehole.

If q_i is the total inflow of water into the borehole in unit time, the mass balance equation can be written as:

$$0 = q_i c.\Delta t + V.\Delta c$$

$$\text{or } \frac{V}{q_i}\frac{dc}{c} = -\,dt \qquad \ldots (1)$$

Let the time is measured from the instant the tracer is added to the water in the borehole. Integrating and applying the initial condition that at t=0, $c=c_o$, the following expression can be derived:

$$t = \frac{V}{q_i} \log_e \frac{c_o}{c} \qquad ...(2)$$

Assuming that the distortions in the flow pattern due to presence of the borehole are occurring in a plane normal to borehole axis and that there is no distortion below the bottom of the borehole, the total flow rate q_i entering to the borehole can be approximated to (McWhorter et al, 1975, Appendix I).

$$q_i = D V_a H' \alpha \qquad ...(3)$$

in which

V_a = Darcy velocity in a direction normal to the canal axis at larger distance from the borehole.

α = a factor for hydrodynamic field deformation.

The above relation has been derived on the assumptions that water flows with a uniform gradient at large distance from the borehole and the aquifer has infinite areal extent. Replacing the value of q_i and V in equation 2,

$$\log_e \frac{c}{c_o} = \frac{4\alpha V_a}{\pi D} t \qquad ...(4)$$

Thus when logarithm of concentration is plotted at various times, the resulting graph is a straight line with slope equal to $\frac{4\alpha\ V_a}{\pi D}$.

Let m is the slope of the graph. Hence,

$$V_a = \frac{m\pi D}{4\alpha} \qquad ...(5)$$

Knowing m, the Darcy velocity V_a can be determined. Once the Darcy velocity is determined the seepage loss from the canal can be derived as follows:

An idealized configuration of stream lines and equipotential lines is shown in Figure 1. Let θ' is the angle made by the tangent to the phreatic line with the vertical at the borehole. θ' can be approximately known by knowing positions of water table at two near-by points. In the absence of the borehole the water will move in the radial direction. Since V_a is the component of velocity along horizontal direction, the superficial velocity in radial direction is given by V_a cosec θ'.

The total segmental circular area = $2.d.\theta'$.

Hence the total seepage loss from the canal per unit length is given by

$$q = 2.d.\theta'.\text{cosec}\ \theta'\ V_a \qquad \ldots(6)$$

Replacing V_a by equation (5)

$$q = \frac{\pi}{2\alpha}.m.d.\theta'.\text{cosec}\ \theta' \qquad \ldots(7)$$

Using equation (7), the seepage loss from a canal can be determined after knowing the value of m from tracer studies.

5.0 RESULTS

The seepage loss at various chainage has been found using equation (7) and the tracer study data published by UP IRI,and are presented in Table 1.

The accuracy of the determination of the apparent velocity by the point dilution method depends strongly on the value of the factor α . The determination of this factor may be carried out experimentally. The disturbance around the borehole during construction will modify the permeability of the surrounding soil and consequently the factor α. The maximum value of α may be equal to 2 as obtained from McWhorter's analysis given in Appendix I. As reported by Gaspar with great permeabilities of experimental soil and apparent velocity as high as about 10^{-3}cm/s, the use of a filter bed around the borehole should reduce the factor in the range from .5 to 0.75 (Gaspar, 1969). The value of α has been taken as 1 while calcuating seepage loss from Deoband canal.

Conformal mapping technique can also be used (Appendix II) to determine seepage loss from canal provided the value of K is known correctly. $\frac{q}{KH}$ evaluated by using conformal mapping technique at chainage 6.4 km from Deoband canal is found to be 3.35 (Fig.2), where q = the total seepage loss per unit length of canal, K = the coefficient of permeability and H is the maximum depth of water in the canal. Seepage loss calculated from tracer technique using equation (7) is 1.1763 $m^3/sec/10^6 m^2$. The corresponding $\frac{q}{KH}$ value is 2.319. If q' is the part of seepage occurring through bed, it is found from conformal mapping that $\frac{q'}{KH}$ = 2.4948. This value is in close agreement with the

TABLE 1 - SEEPAGE LOSS FROM DIFFERENT SECTIONS OF DEOBAND CANAL CALCULATED USING TRACER TECHNIQUE

Chainage in Km.	Location	Darcy Velocity V_a 10^{-7} m/s	Dist. d in m.	Angle in radian	Seepage calculated using data on right bank $m^3/sec/10^6m^2$	Seepage calculated using data on left bank $m^3/sec/10^6m^2$.	Average seepage loss $m^3/sec/10^6m^2$
1	2	3	4	5	6	7	8
2.4	RB	2.13	21.60	1.5359			
	RC	2.27	27.60	1.5533	0.8628		
	RD	2.01	33.60	1.5706			0.7711
	LB	1.54	21.60	1.5533			
	LC	1.76	27.10	1.5521		0.6794	
	LD	1.76	32.60	1.5706			
6.4	RM	2.52	11.00	1.0472			
	RN	2.44	20.00	1.5533	0.7905		
	RO	2.13	29.00	1.5533			
	RP	2.35	38.00	1.5706			
	LM	2.55	11.00	0.8727			0.7782
	LN	2.52	21.00	1.5184			
	LO	2.27	28.00	1.5309		0.7658	
	LP	2.21	36.00	1.5706			
8.8	RQ	1.48	22.70	1.4835			
	RS	1.65	32.70	1.5535	0.6397		
	RT	1.60	42.70	1.5706			

1	2	3	4	5	6	7	8
	LQ	1.60	22.70	1.5359			0.5992
	LS	1.12	32.70	1.5533		0.5587	
	LT	1.46	42.70	1.5706			
23.3	RN	1.76	18.20	1.5359			
	RO	1.68	25.20	1.5533	0.6668		
	RP	1.57	32.20	1.5621			
	LN	1.04	18.30	1.4835			0.6101
	LO	1.57	25.90	1.5010		0.5533	
	LP	1.37	34.00	1.5184			
24.4	RB	1.43	18.75	1.5359			
	RC	1.34	23.75	1.5533	0.5030		
	RD	1.23	28.75	1.5533			
	LA	1.65	25.75	1.5184			0.7092
	LB	1.88	31.75	1.5621		0.9153	
	LC	1.54	37.75	1.5706			
	LD	1.48	44.75	1.5706			
49.1	RM	1.20	18.00	1.5621			
	RN	1.09	23.00	1.5706	0.4784		
	RO	1.15	28.50	1.5706			0.3983
	LM	0.95	15.00	1.5533		0.3181	
	LN	0.92	20.00	1.5706			
	LO	0.81	25.00	1.5706			

1	2	3	4	5	6	7	8
50.0	RA	1.18	9.00	1.5533			0.3296
	RB	1.01	18.50	1.5706	0.3004		
	RC	0.87	23.50	1.5706			
	LB	1.01	18.50	1.5010		0.3587	
	LC	0.95	23.50	1.5184			
73.7	RA	0.59	7.30	1.5359			
	RB	0.76	11.30	1.5621	0.4172		
	RC	0.48	15.30	1.5706			
	RD	0.48	19.30	1.5706			0.4577
	LA	0.48	12.50	1.5359		0.4983	
	LB	0.56	17.50	1.5533			
	LC	0.48	22.50	1.5621			

value obtained by tracer technique. Therefore, it is argued that the seepage is only taking place through bed of the canal and the canal banks are relatively impervious. The deviation of the observed phreatic line from the locus of the phreatic line obtained analytically for a homogenous flow domain which is shown in Fig.3 justifies the above statement.

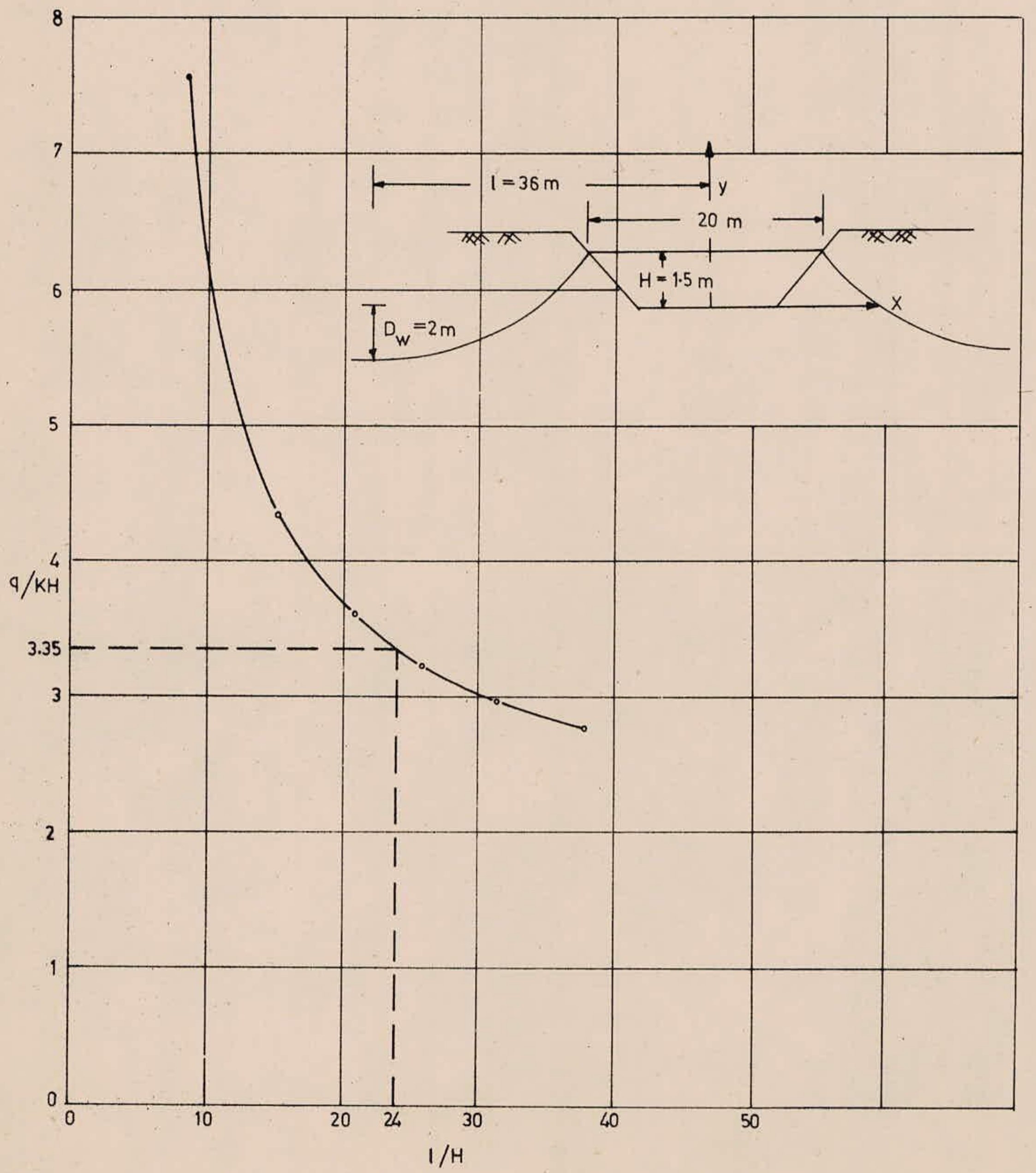

FIGURE 2 - VARIATION OF q/KH WITH l/H FOR b=10 m, H=1.5 m AND D_W = 2m.

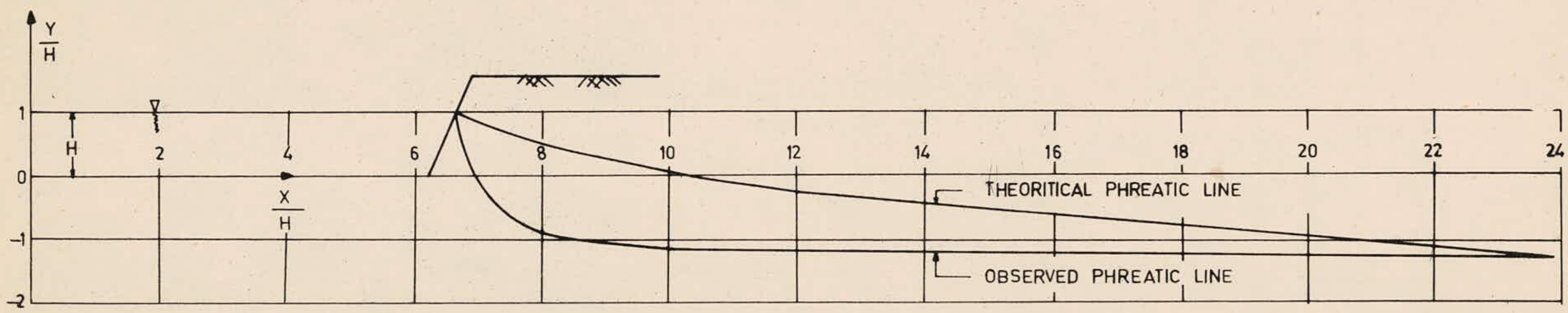

FIGURE 3 - LOCATION OF THEORETICAL AND OBSERVED PHREATIC LINE AT CHAINAGE 6.4 KM OF DEOBAND CANAL

6.0 CONCLUSIONS

Seepage losses from a canal have been estimated using tracer technique. It is found that the seepage losses estimated using tracer technique compare well with that estimated by conformal mapping. The seepage loss from Deoband canal, a branch of Upper Ganga Canal, is found to vary along its length from 0.3296 to 0.7782 $m^3/sec/10^6 m^2$. It is seen, in general, that the seepage losse from the canal is decreasing with increasing distance from the head of the canal and most of the seepage losses are occurring through the bed.

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APPENDIX -I

DISRUPTION IN FLOW PATTERN DUE TO PRESENCE OF BOREHOLE

The presence of borehole causes a disruption in the original flow pattern. The actual flow pattern in a disrupted aquifer is quite complex. It depends upon the original flow pattern, time, geometry of the disrupted area, the location of the disrupted area with respect to aquifer boundaries, the distribution of hydraulic conductivity in the disturbed and undisturbed portions of the aquifer. A first approximation to the post borehole flow pattern can be derived (McWhorter et al, 1975) by assuming that:

1) the aquifer is very large in areal extent,
2) the flow in the aquifer is uniform and one-dimensional at large distance from the borehole,
3) the borehole is a disturbed zone with very high permeability and the flow pattern in the presence of borehole can be obtained as a limiting case,
4) the hydraulic conductivity K_o of the undisturbed portion of the aquifer is a constant,
5) the geometry of the distrubed portion of the aquifer is that of a cylinder with axis normal to the plane of the flow, and
6) the flow is two-dimensional and steady.

The distribution of piezometric head outside and inside the borehole area are given by

$$h_o = - \frac{V_a x}{K_o} \{1 + (\frac{K_o - K_i}{K_o + K_i}) (\frac{R^2}{x^2 + y^2})\} \qquad \ldots (1)$$

and

$$h_i = \frac{-2V_a x}{K_o + K_i} \qquad \ldots(2)$$

respectively. In equations 1 and 2, h_o is the piezometric head outside the borehole, h_i is the piezometric head inside the borehole, V_a is the Darcy velocity in the x-direction at large distance from the borehole, R is radius of the borehole, K_o is the hydraulic conductivity outside the borehole, K_i is the hydraulic conductivity inside the borehole, x and y are coordinate directions with the origin at the centre of the borehole. h_o and h_i both satisfy the Laplace equation i.e.

$$\frac{\partial^2 h_o}{\partial x^2} + \frac{\partial^2 h_o}{\partial y^2} = 0$$

and

$$\frac{\partial^2 h_i}{\partial x^2} + \frac{\partial^2 h_i}{\partial y^2} = 0$$

At the periphery of the circle i.e. at $x^2 + y^2 = R^2$, $h_i = h_o$. Using equations 1 and 2, the velocity potential function for inside and outside the borehole area can be written as

$$\phi_i = K_i \frac{2 V_a x}{K_o + K_i} \qquad \ldots(3)$$

$$\phi_o = V_a x \{1 + \left(\frac{K_o - K_i}{K_o + K_i}\right) \left(\frac{R^2}{x^2 + y^2}\right)\} \qquad \ldots(4)$$

Using the Cauchy Reimann conditions i.e.

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \qquad \ldots(5)$$

and

$$\frac{\partial \phi}{\partial y} = - \frac{\partial \psi}{\partial x} \qquad \ldots(6)$$

where ψ is the stream function. ψ_i from equation (3) is found to be

$$\psi_i = \frac{2 V_a K_i y}{K_o + K_i} \qquad \ldots(7)$$

The total quantity of flow to the borehole per unit depth is thus

$$q = \frac{4 V_a K_i R}{K_o + K_i} \qquad \ldots(8)$$

In the limiting case when $K_i \rightarrow \infty$, the quantity of flow entering to the borehole is given by

$$q = 4 v_a R \qquad \ldots(9)$$

Thus because of the disruption the normal quantity of flow is doubled in the disrupted zone.

APPENDIX II

DETERMINATION OF SEEPAGE LOSS FROM DEOBAND CANAL BY CONFORMAL MAPPING

Water table exists at shallow depth in the command area of Deoband canal. Therefore seepage from a canal when water table exists at shallow depth has been analysed by conformal mapping. An approximation to the canal profile is made in order to have an easy solution. In this approximation the specified canal width and depth are untouched.

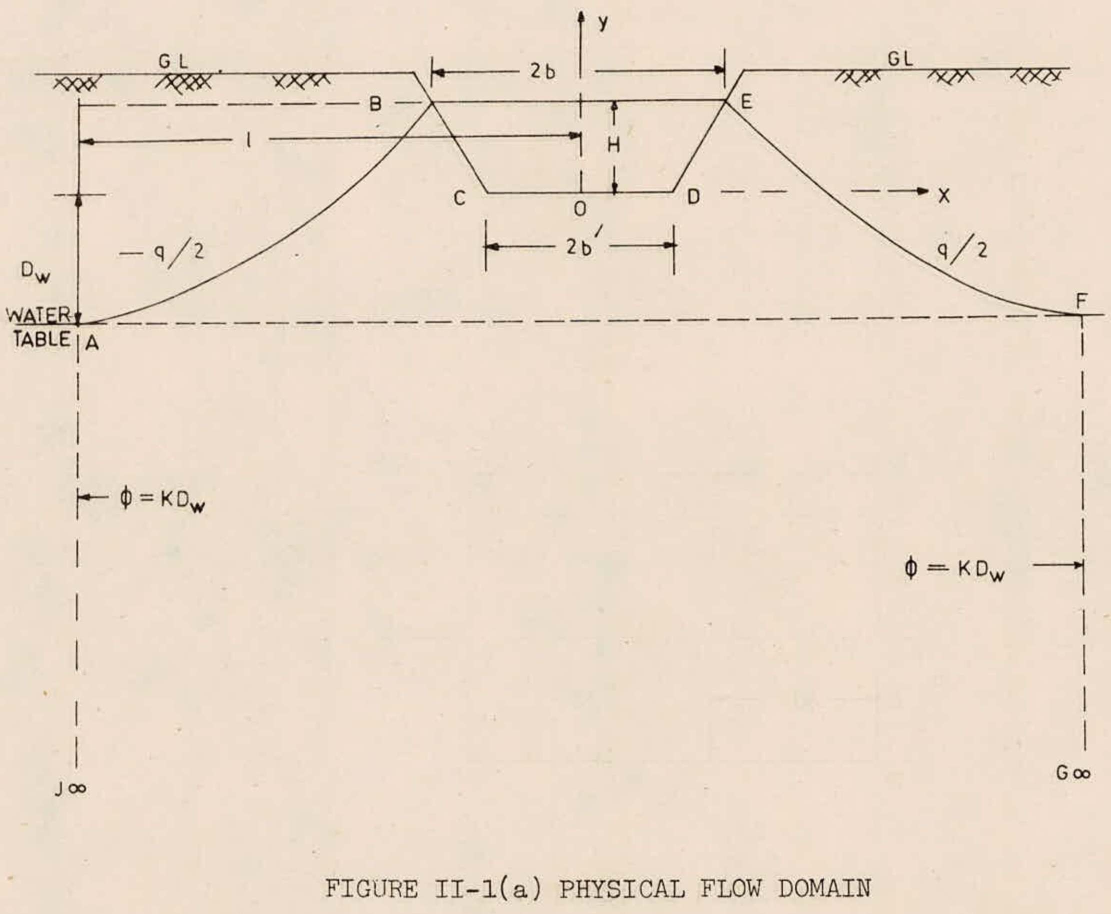


FIGURE II-1(a) PHYSICAL FLOW DOMAIN

Figure II-1(a) shows a schematic crosssection of a trapezoidal canal in z plane. At a large distance l from the centre of the canal the water table in the aquifer is at a depth D_w below the bed level of the canal. It is assumed that the aquifer is homogeneous, isotropic, and semi infinite. It is required to find the quantity of seepage and position of phreatic lines.

The pertinent complex potential plane w, when $w = \Phi + i\psi$ is shown in figure II-1(b) in which ψ is stream function and Φ is the velocity potential function defined as:

$$\Phi = -K\left(\frac{p}{\gamma_w} + y\right) + c \qquad \ldots(1)$$

in equation 1,

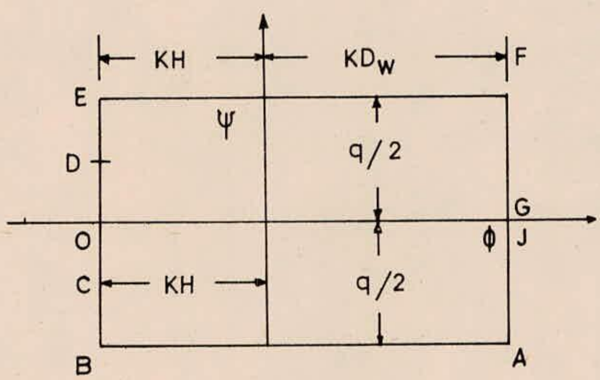
K = coefficient of permeability,

p = pressure

γ_w = unit weight of water

y = elevation head and

c = constant which has been taken to be zero.



II-1(b) w-PLANE

The flow domain consists of phreatic lines which are curvilinear. Therefore conformal mapping can not be applied directly to the flow domain. One of the ways of analysing unconfined flow problem is to transform the

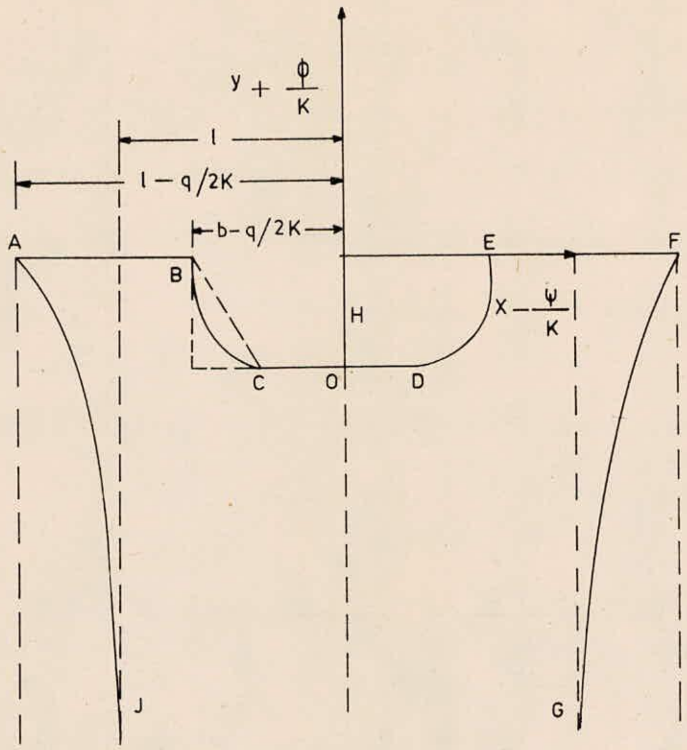


FIGURE II-1(c) TRUE − θ PLANE

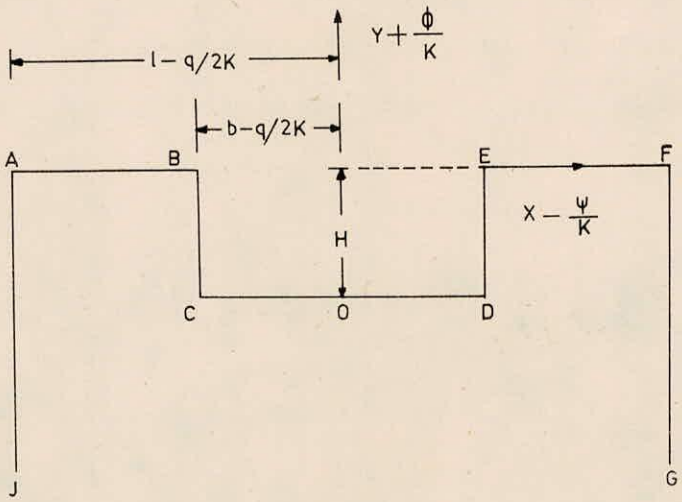


FIGURE II-1(d) IDEALIZED − θ PLANE

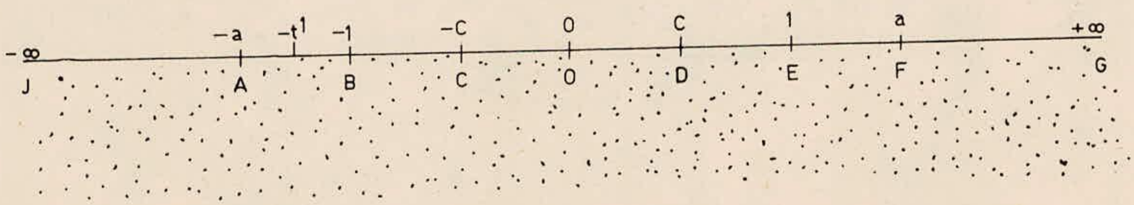


FIGURE II-1(e) t-PLANE

FIGURE − II-1 − STEPS OF MAPPING

flow domain in z plane to the Zhukovsky's θ plane and subsequently apply conformal mapping technique. The pertinent Zhukovsky's θ plane where $\theta = Z + i\,\frac{w}{K}$ is shown in figure II-1(c). Existence of vertical equipotential lines are assumed at the distance 1 from the canal while drawing the θ plane. In figure II-1(d) the idealised θ plane has been shown.

The conformal mapping of θ plane to the lower half of the auxiliary t plane shown in figure II-1(e) is given by

$$\theta = M \int_{-a}^{t'} \frac{(t^2 - c^2)^{\frac{1}{2}}\, dt}{(a^2 - t^2)^{\frac{1}{2}}\ (t^2-1)^{\frac{1}{2}}} - 1 + \frac{q}{2K} \qquad \ldots(2)$$

the vertices J, A, B, C, D, E, F and G being mapped onto points $-\infty$, $-a$, -1, $-c$, c, 1, a, ∞ respectively on the real axis of t plane. M is a constant to be evaluated. From the relations between θ and t planes at the different vertices the following relations are obtained starting from equations (2).

$$1 - b = M\, I_1 \qquad \ldots(3)$$

$$H = M\, I_2 \qquad \ldots(4)$$

$$b' - \frac{q'}{2K} = b - \frac{q}{2K} \qquad \ldots(5)$$

$$b' - \frac{q'}{2K} = M\, I_3 \qquad \ldots(6)$$

where,

$$I_1 = \int_{-a}^{1} \frac{(t^2 - c^2)^{\frac{1}{2}}\, dt}{(a^2 - t^2)^{\frac{1}{2}}\ (t^2-1)^{\frac{1}{2}}} \qquad \ldots(7)$$

$$I_2 = \int_{-1}^{-c} \frac{(t^2 - c^2)^{\frac{1}{2}} \, dt}{(a^2 - t^2)^{\frac{1}{2}} (1-t^2)^{\frac{1}{2}}} \qquad \ldots(8)$$

$$I_3 = \int_{-c}^{0} \frac{(c^2 - t^2)^{\frac{1}{2}} \, dt}{(a^2 - t^2)^{\frac{1}{2}} (1-t^2)^{\frac{1}{2}}} \qquad \ldots(9)$$

The mapping of the complex potential plane w to the real axis of the t plane is given by

$$w = M' \int_{-a}^{t'} \frac{dt}{(a^2 - t^2)^{\frac{1}{2}} (t^2 - 1)^{\frac{1}{2}}} - \frac{iq}{2} + KD_w \qquad \ldots(10)$$

Making use of the relations between w and t planes at the vertices B and E the following relations are obtained starting from equation (10);

$$- K (H + D_w) = M' \frac{2}{1 + a} F\left(\frac{\pi}{2}, \frac{a - 1}{a + 1}\right) \qquad \ldots(11)$$

$$q = K (H + D_w) \frac{F\left(\frac{\pi}{2}, \frac{2\sqrt{a}}{1+a}\right)}{F\left(\frac{\pi}{2}, \frac{a - 1}{a + 1}\right)} \qquad \ldots(12)$$

Applying the condition at point c for which $w = -KH \frac{- iq'}{2}$ the following expression is obtained:

$$\frac{q'}{KH} = \frac{q}{KH} - 2\left(1+ \frac{D_w}{H}\right) \frac{F\left(\sin^{-1}\sqrt{\left\{\frac{(1+a)\ (1+c)}{2\ (1+c)}\right\}}, \frac{2\sqrt{a}}{1+a}\right)}{F\left(\frac{\pi}{2}, \frac{a-1}{a+1}\right)} \qquad \ldots(13)$$

The following simplified relations are obtained making use of equations (3), (4), (5) , and (12):

$$\frac{b}{H} = \left(1 + \frac{D_w}{H}\right) \frac{F\left(\frac{\pi}{2}, \frac{2\sqrt{a}}{1+a}\right)}{2F\left(\frac{\pi}{2}, \frac{a-1}{a+1}\right)} + \frac{I_3}{I_2} \qquad \ldots(14)$$

$$\frac{l}{H} = \frac{b}{H} + \frac{I_1}{I_2} \qquad \ldots(15)$$

For assumed values of a and c, and for given values of H and D_w, the corresponding $\frac{b}{H}$ and $\frac{l}{H}$ can be obtained from equation (14) and (15) respectively. q and q' for the calcuated $\frac{b}{H}$ and $\frac{l}{H}$ values can be obtained from equations (12) and (13). Result for particular value of $\frac{b}{H}$

can be obtained further using an iteration procedure.

The locus of the phreatic lines are found as follows:

For $-a \leq -t' \leq -1$

$$(x - \frac{\psi}{K}) + i\,(\frac{\phi}{K} + y) = M \int_{-a}^{-t'} \frac{(t^2 - c^2)^{\frac{1}{2}}\,dt}{(a^2 - t^2)^{\frac{1}{2}}\,(t^2 - 1)^{\frac{1}{2}}} - 1 + \frac{q}{2K} \qquad \ldots(16)$$

For the phreatic line AB $\psi = \frac{-q}{2}$, and $\phi + Ky = 0$.

Substituting value of M from equation (4) in equation (16) and simplifying

$$\frac{x}{H} = \frac{I_7}{I_2} - \frac{1}{H} \qquad \ldots(17)$$

in which

$$I_7 = \int_{-a}^{-t'} \frac{(t^2 - c^2)^{\frac{1}{2}}\,dt}{(a^2 - t^2)^{\frac{1}{2}}\,(t^2 - 1)^{\frac{1}{2}}} \qquad \ldots(18)$$

For $-a \leq -t' \leq -1$, relation between w and t plane is given by

$$\phi + i\psi = M' \int_{-a}^{-t'} \frac{dt}{(a^2 - t^2)^{\frac{1}{2}}\,(t^2 - 1)^{\frac{1}{2}}} - \frac{iq}{2} + KD_w \qquad \ldots(19)$$

substituing the value of M', ψ, and $\phi = -ky$ in above equation and simplifying

$$\frac{y}{H} = (1 + \frac{D_w}{H}) \frac{F(\mathrm{Sin}^{-1}\sqrt{\{\frac{(1+a)\,(a-t')}{(a-1)\,(a+t')}\}},\ \frac{a-1}{a+1})}{F(\frac{\pi}{2},\ \frac{a-1}{a+1})} - \frac{D_w}{H} \qquad \ldots(20)$$

in which

F (,) is elliptic integral of first kind.

For an assumed value of t', the $\frac{x}{H}$ and the corresponding $\frac{y}{H}$ for the phreatic line can be obtained from equations (17) and (20) respectively.