APPENDIX—I

LEAST SQUARE METHOD FOR UNIT HYDROGRAPH DERIVATION

The usual way of judging the quality of a unit hydrograph estimate is to examine the difference between the actual measured output and that predicted by the unit hydrograph. These are called the model residuals and are given by :

$$d = Y - Xh$$

wherein,

d is a vector of residuals,

h is a vector of unit hydrograph estimate.

Y is a vector of observed DSRO (output),

X is a matrix of size NRUN × NDUH,

NDUH is no. of unit hydrograph ordinates and

NRUN is no of DSRO ordinates.

For m blocks of rainfall, the structure of the matrix X will be as follows :

$$X = \begin{bmatrix} X_1 0 & 0 \ldots\ldots\ldots 0 \\ X_2 & X_1 \ldots\ldots\ldots 0 \\ X_{m-1} & X_2 \ldots\ldots\ldots 0 \\ X_m & X_{m-1} \ldots\ldots\ldots \\ 0 & X_m \\ 0 & 0 \quad \ldots\ldots\ldots \\ 0 & 0 \quad \ldots\ldots X_1 \end{bmatrix} \quad (\text{NRUN} \times \text{NDUH})$$

Here $X_1, X_2 \ldots\ldots\ldots X_m$ are the blocks of rainfall excess.

If the equations are consistent and h is the exact solution of the equations then all the elements of the vector d are zero. If the equations are inconsistent an estimate of the solution, which makes the vector, d small is sought. This solution may not satisfy any of the equations. The least squares method chooses the UH estimate which minimises the sum of squares of the model residuals i.e. it minimises.

$$\sum_{i=1}^{NRUN} d^2_i = d^T d = (Y - Xh)^T (Y \ldots X h) \qquad (1.3)$$

This is equivalent to

$$d^T d = Y^T Y - h^T X^T Y - Y^T \times b + h^T X^T X h \tag{1.4}$$

Since $h^T X^T Y$ is a scalar variable it is equal to its transpose i.e.

$$h^T X^T Y = Y^T X h \tag{1.5}$$

So $$d^T d = Y^T Y - 2 h^T X^T Y + h^T X h \tag{1.6}$$

Differentiating the above equation with respect to h^T

$$\frac{\partial (d^T d)}{\partial h^T} = - 2 X^T Y + 2 X^T X h \tag{1.7}$$

Set this zero to get the least squares. This is the solution of :

$$(X^T X) h = X^T Y \tag{1.8}$$

Some time the unit hydrographs derived using least square method are unrealistic in shape. This is due to the ill conditioning of the matrix $X^T X$. Kuchment (1967) has suggested that the a prior expectation of smoothness in the derived unit hydrograph can be incorporated into the estimation by solving the following equations in place of the normal least square equations :

$$(X^T X + \alpha I) h_s = X^T Y \tag{1.9}$$

Where α is a parameter chosen by the user.