RESERVOIR ROUTING

by

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INTRODUCTION

The passage of flood hydrograph through a reservoir is an unsteady flood phenomenon. The equation of continuity is used in all hydrologic routing methods as primary equation. According to this equation, the difference between the inflow and outflow is equal to the rate of change of storage, i.e. :

$$I - Q = \frac{dS}{dt} \qquad (1)$$

where, I = inflow, Q = outflow, S = storage, and t = time.

Alternatively, in a small time interval Δt, the difference between the total inflow volume and total outflow volume in reach is equal to the change in storage in that reach :

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S \qquad (2)$$

where, $\bar{I}$, and $\bar{Q}$ denote average inflow and outflow during time Δt, and ΔS denotes change in storage during Δt. Assuming

$$\bar{I} = (I_1 + I_2)/2, \; \bar{Q} = (Q_1 + Q_2)/2$$

$$\Delta S = S_2 - S_1$$

where, suffixes 1 and 2 denote the beginning and end of time interval Δt, equation (2) is written as :

$$\left(\frac{I_1 + I_2}{2}\right) \Delta t - \left(\frac{Q_1 + Q_2}{2}\right) \Delta t = S_2 - S_1 \qquad (3)$$

Here the time interval Δt must be sufficiently small so that the inflow and outflow hydrographs can be assumed to be linear in that time interval. Further, Δt must be shorter than the time of transit of flood wave through the reservoir.

Equation (3) can be rearranged as :

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1\Delta t}{2}\right) = \left(S_2 + \frac{Q_2\Delta t}{2}\right) \tag{4}$$

The schematic representation of reservoir routing is given in Fig. 1.

Reservoir routing requires the relationship between the reservoir elevation, storage and discharge to be known. This relationship is a function of the topography of reservoir site and the characteristics of the outlet facility. It is important to view the relationship between elevation, storage and discharge as a single function because changes can take place in the topography and the elevation-discharge characteristics will get affected. McCuen(1989) has discussed this aspect in detail.

Using the above basic equation, several methods for routing a flood wave through a reservoir have been developed, namely:

- The Mass Curve Method
- The Puls Method
- The Modified Puls Method
- The Wisler-Brater Method
- The Goodrich Method
- The Steinberg Method
- The Coefficient Method

A brief description of each of these methods follows.

THE MASS CURVE METHOD

This is one of the most versatile methods of reservoir routing, various versions of which include: (i) direct, (ii) trial and error, and (iii) graphical. Here the trial and error version is being described in detail.

For solution by trial and error method, equation (2) can be rewritten as:

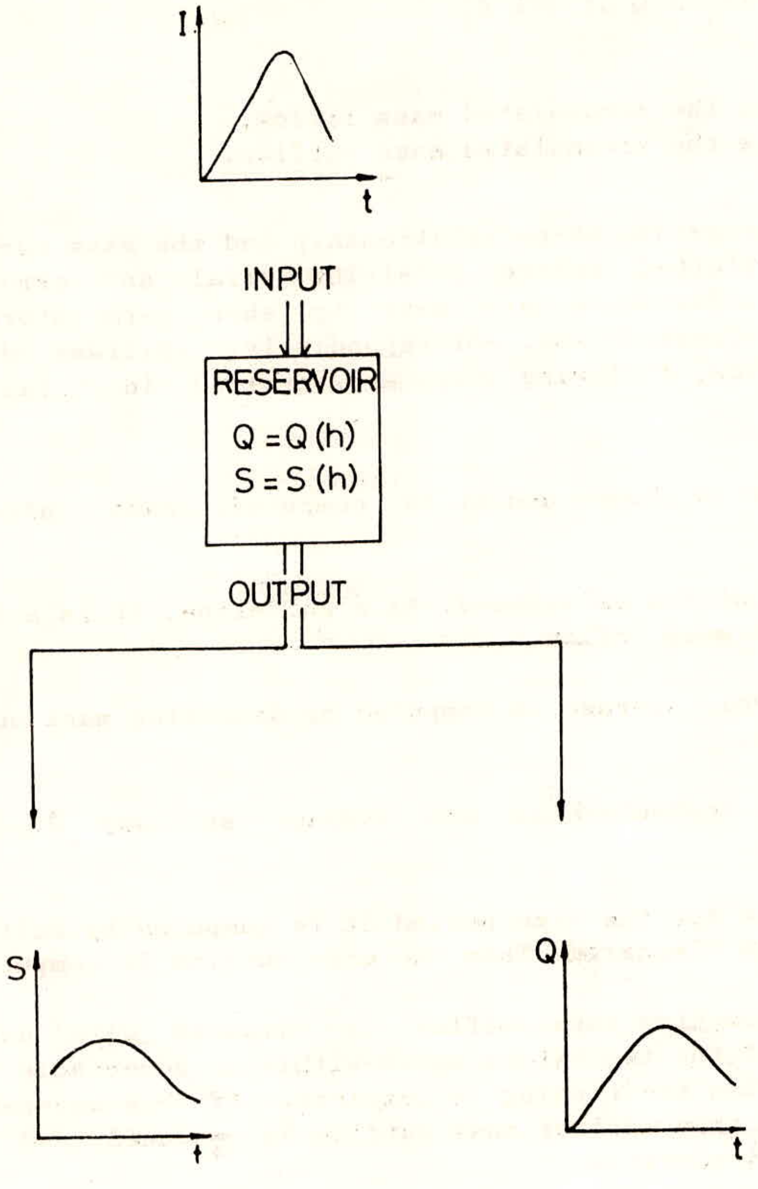

FIG. 1. SCHEMATIC REPRESENTATION OF RESERVOIR ROUTING

$$M_2 - (V_1 + \bar{Q}\,\Delta t) = S_2 \qquad (5)$$

where, M is the accumulated mass inflow,
V is the accumulated mass outflow.

A storage-discharge relationship and the mass curve of inflow should be plotted before obtaining trial and error solution. Necessary adjustments are made to show zero storage at the beginning elevation and, correspondingly, spillway discharge is obtained. Now, following steps are involved in trial and error solution :

(a) A time is chosen and Δt is computed. Mass inflow is also computed.

(b) Mass outflow is assumed. As a guideline, it is a function of accumulated mass inflow.

(c) Reservoir storage is computed by deducting mass outflow from mass inflow.

(d) The instantaneous and average spillway discharges are calculated.

(e) Outflow for the time period Δt is computed by multiplying Δt with average discharge. Then the mass outflow is computed.

(f) Now, computed mass outflow is compared with assumed mass outflow. If the two values agree within an acceptable degree of accuracy, then the routing is complete. If this agreement is not acceptable, then another mass outflow is assumed and the above procedure is repeated.

THE PULS METHOD

In the Puls method, the continuity equation is expressed as:

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1}{2}\Delta t\right) = \left(S_2 + \frac{Q_2}{2}\Delta t\right) \qquad (6)$$

The computations are performed as follows. At the starting of flood routing, the initial storage and outflow discharge are known. In equation (6) all the terms in the left hand side are known at the beginning of time step Δt. Hence the value of the function $\left[S_2 + \frac{Q_2 \Delta t}{2}\right]$ at the end of the time step is calculated by equation (6). Since the relation $S = S(h)$ and $Q = Q(h)$ are known, $\left\{S_2 + \frac{Q_2 \Delta t}{2}\right\}$ will enable one to determine the reservoir elevation and hence the discharge at the end of the time step. This procedure is repeated to cover the full inflow hydrograph.

THE MODIFIED PULS METHOD

This is also referred to as the Storage-Indication Method. This method represents equation (1) in finite difference form :

$$S_2 - S_1 = (I_1 + I_2)\frac{\Delta t}{2} - (Q_1 + Q_2)\frac{\Delta t}{2} \qquad (7)$$

in which Q may incorporate controlled discharge Q_c as well as uncontrolled discharge Q_s,

$$Q = Q_c + Q_s$$

Separating the known quantities from the unknown ones and rearranging :

$$(I_1 + I_2) - \left(Q_{c_1} + Q_{c_2}\right) + \left(\frac{2S_1}{\Delta t} - Q_{s_1}\right) = \frac{2S_2}{\Delta t} + Q_{s_2} \qquad (8)$$

The left side contains the known terms and the right side is unknown. The inflow hydrograph is known. Q_c, which may pass through the turbines, outlet works, or over the spillway is also known. The uncontrolled discharge, Q_s, goes freely over the spillway. It depends upon the depth of flow over the spillway and spillway geometry. Further, the depth of flow over the spillway depends upon the level of water in the reservoir. Therefore :

$$S = S(Y)$$

$$Q_s = Q_s(Y)$$

where, Y represents the water surface elevation. The right side of equation (8) can be written as :

$$2S/\Delta t + Q = f(Y)$$

In order to utilize equation (8), the elevation storage and elevation-discharge relationship must be known. For simplicity Q_c is assumed to be negligible and Q can be taken to imply Q_s. Before routing, the curves of $(2S/\Delta t \pm Q)$ versus Q are constructed. The routing is now very simple and can be performed using the above equation.

THE WISLER-BRATER METHOD

In this method, storage is expressed as a function of sum of inflow and outflow and storage curves for (I+Q) versus (2S/Δt + I + Q) are constructed. The basic equation of reservoir routing can be expressed as :

$$2S_1/\Delta t + I_1 + 2I_2 - Q_1 = 2S_2/\Delta t + I_2 + Q_2 \tag{9}$$

In the above equation every term on left hand side is known for a given routing period and hence the right side can be computed. Then the value of $I_2 + Q_2$ can be read from the storage curves. Since, I_2 is known, Q_2 is obtained. This procedure is repeated for subsequent routing periods.

The above procedure can also be extended to the case where storage is a function of weighted sum of inflow and outflow.

THE GOODRICH METHOD

In this method, the continuity equation is expressed as :

$$2S_1/\Delta t + I_1 + I_2 - Q_1 = 2S_2/\Delta t + Q_2 \tag{10}$$

Goodrich method involves construction of a family of routing curves for $[(2S/\Delta t) \pm Q]$ against Q for various values of I. As all the terms on the left side of the above equation are known, the right side can be obtained for a routing period Δt. The value of Q_2 can now be read from the routing curves against $[2S_2/\Delta t + Q_2]$ and then S_2 can be computed. The routing can be carried out for subsequent time periods in a similar manner.

THE STEINBERG METHOD

The Steinberg method expresses equation (1) as :

$$\Delta t(I_1 + I_2 - Q_1)/2 + S_1 = \Delta t/2\, Q_2 + S_2 + K \qquad (11)$$

The term $K = S\ (Q\Delta t/2)$ is called the storage factor. Curves are plotted for storage factor which are termed as K-curves. The storage curves, showing storage as a function of inflow and outflow, are superimposed over K-curves. All left side terms of above equation are known and so the right side is obtained. Then Q_2 and S_2 are read from the superimposed curves. The routing is similarly carried out for subsequent time periods.

THE COEFFICIENT METHOD

The coefficient method represents the reservoir by a single conceptual storage element, where storage S is directly proportional to outflow Q.

$$S = K\,Q \qquad (12)$$

where K is a proportionality factor equal to the reciprocal of the slope of the storage curve that can be a constant or a variable function of outflow. If K is constant, then the reservoir is linear, otherwise the reservoir is non-linear.

For flood routing, a finite difference approximation is normally employed. Equation (1) and (2) can be combined and written as :

$$(I_1+I_2)\frac{\Delta t}{2} - (Q_1+Q_2)\frac{\Delta t}{2} = K(Q_2 - Q_1)$$

Rearranging the terms,

$$Q_2 = Q_1 + C(I_1 - Q_1) + 0.5C(I_2 - I_1) \qquad (13)$$

in which,

$$C = \Delta t / (K + 0.5\Delta t) \quad (14)$$

If K is variable, then C can be derived and plotted as a function of Q. For each routing period, the appropriate value of C must be obtained corresponding to the outflow under consideration. Then, by using equation (13) flood routing can be performed.

GENERAL COMMENTS

Selection of a proper routing time interval Δt in all flood routing problems is very important. Its value should be neither too long nor too short. If it is too long and exceeds the travel time through the reservoir, then the crest segment of outflow containing the peak discharge could pass through the reservoir between time intervals and could, therefore, not be computed. If on the other hand, it is too short, then it takes longer to perform flood routing. Further, Δt is assumed so that I is approximately linear during this period. As a guideline, Δt should be one-third to one-half the travel time through the reservoir. Furthermore, the routing interval Δt can be either variable or constant. However, it is more realistic to use a variable Δt, keeping it small for a large change in mass inflow and large for a small change therein.

The merits and demerits of different methods are enumerated below :

The routing operation performed by trial and error solution of the Mass Curve method is simple and easily done. This can be efficiently adapted to complex routing problems.

The Puls method and the Modified Puls method, both have two shortcomings. First, the assumption that the outflow begins at the same time as the inflow implies that the inflow passes through the reservoir instantaneously regardless of its length. Second, it is difficult to choose an appropriate Δt since negative outflow occurs during recession whenever $\Delta t > 2S_2/Q_2$ or $Q_2/2 > S_2/\Delta t$. The former drawback is not a serious one if the ratio of T_t/T_m is less than or equal to 1/2, where T_m denotes time to peak of inflow hydrograph and T_t denotes travel time. T_t is defined as L/u, with L being the length of the reach and u being average steady state velocity. The latter weakness can be circumvented by plotting discharge versus [(2S/Δt)+Q] curve on a log-log paper and comparing the plot with the line of equal values. If the plotted values lie above the line of equal values, drawn figure must be abandoned and a new value of Δt must be selected. Further, negative outflow can be avoided usually by taking Δt less than T .

The Wisler Brater method requires observed basin for routing computations and so this can best be simulated in controlled conditions. Due to this reason, its use is very must restricted for practical purposes.

The Steinberg method requires K-curves and their superimposition over storage curves, thereby, it involves a lot of graphical work before actual routing computations are carried out.

DATA REQUIREMENTS

For obtaining solution of a reservoir routing problem, following data have to be known :

(i) Storage volume Vs elevation curve for the reservoir,

(ii) Water surface elevation Vs. outflow and hence storage Vs. outflow discharge,

(iii) Inflow hydrograph,

(iv) Initial values of storage, inflow and outflow,

(v) For coefficient method, the value of proportionality constant K, which is the reciprocal of the slope of the storage curve, is also needed.

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