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UNSTEADY FLOW TO A LARGE - DIAMETER WELL INFLUENCED BY A RIVER AND A NO FLOW BOUNDARY

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1983-84

CONTENTS

LIST OF SYMBOLS

m_p	-	number of time steps upto which pumping is carried out
$Q_A(t)$	-	aquifer contribution during time t
$Q_p(t)$	-	pumping rate during time t
$Q_w(t)$	-	well storage contribution during time t
Q	-	constant pumping rate
r_1	-	distance of pumping well from the impermeable boundary
r_2	-	distance of the pumping well from the river boundary
r_c	-	radius of the well casing
r_w	-	radius of the well screen
r	-	radial distance from the centre of the well
$s_w(t)$	-	drawdown at the well face
s	-	drawdown
T	-	transmissivity of the aquifer
t_p	-	duration of pumping
t	-	time
α	-	a parameter equal to $\phi(r_w/r_c)^2$
β	-	a parameter equal to T/ϕ
ϕ	-	storage coefficient

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ABSTRACT

Large-diameter wells are extensively used in many parts of the world. The cheapness and simplicity of construction and operation of these wells are often the main reasons for their use. Besides, large-diameter wells are suitable for shallow aquifers with low transmissivity. In many situations an impervious boundary or a recharge boundary is encountered in the vicinity of the well. In the present report a general but simple mathematical tool has been developed to analyse unsteady flow to a large-diameter well located near a river and an impervious boundary. The analysis has been done using image well theory and discrete kernel approach.

Variations of drawdown at the well point with time have been presented in non-dimensional form for various durations of pumping and for different values of storage coefficient for specific positions of the hydrologic boundaries. The influence of the hydrologic boundaries on drawdown at well point has been analyzed. The recovery of the well storage when the well has been pumped for different durations have been determined for different values of storage coefficient. Making use of the graphs, the time required for 90 percent recovery of the well storage can be known. It is seen that as the value of storage coefficient decreases the time span for 90 percent recovery increases.

1.0 INTRODUCTION

Large-diameter wells are extensively used in many parts of the world. The cheapness and simplicity of construction and operation of these wells are often the main reasons for their use (Jain, 1977). Another important advantage is that large-diameter wells are suitable for shallow aquifers with low transmissivity. Since ancient time, people in India and in other South Asian Countries have used large-diameter shallow dug wells, tapping mostly the phreatic and in some cases the semiconfined or confined near surface aquifers. Dug wells of this type continue to be the primary source of ground water in rural India. According to Baweja (1979), of the total 9.5 million wells in India, 79% were dug wells with large-diameter, 18% were shallow tubewells in the hard rocks and soft rocks, and the remaining 3% were deep tube wells in alluvial basins. Lahiri (1975) has estimated that about 71% of the ground water abstracted in the year 1971 was from large-diameter wells. The farming community in hard rock areas is heavily dependent on these type of wells as a supplemental source for irrigation and domestic water. A better understanding of large-diameter well is therefore important for an optimum development of ground water resources. In the present report a solution is given for unsteady flow to a large-diameter well in a confined aquifer located near a fully penetrating river and a no-flow boundary.

2.0 REVIEW

Several investigators have analyzed the effect of pumping from a nonflowing well of large-diameter in an aquifer of infinite areal extent (Papadopulos and Cooper, 1967; Lai et al, 1973; Lai and Wusu, 1974; Boulton and Strėltsova, 1975; Fenske, 1977; Seethapathi, 1978; Rushton and Redshaw, 1979; Rushton and Holt, 1981; Herbert and Kitching, 1981; Patel and Mishra, 1983; Rushton and Singh, 1983). Foremost among the solutions is that of Papadopulos and Cooper. The solution given by Papadopulos and Cooper for a flow to a large-diameter well in a confined aquifer is based on the solution given by Carslaw and Jaeger (1959) for an analogous problem in heat flow. The evaluation of aquifer response by Papadopulos and Cooper's method requires numerical integration of an improper integral involving Bessel's function. The numerical integration therefore involves large computations. The family of type curves given by Papadopulos and Cooper depicting response of the aquifer at the well point during continuous pumping contains straight line portions which are parallel. These straight portions of the type curves correspond to the period when most of the water is pumped from the well storage. If a short duration pumping test is conducted in a large-diameter well, the time drawdown curve matches with any of the straight line portions of the type curves. Although a unique value of transmissivity can be obtained, the evaluation of storage coefficient with short duration pump test data is questionable. Papadopulos and Cooper (1967)have stated that the determination of transmissivity is not so sensitive to the choice of the type curve to be matched, where as the determined value of storage coefficient will change by an order of magnitude when the data plot is moved from one type curve to another. According to them, the well storage dominates the time drawdown curve up to a time t given by $t = (25\ r_c^2)/T$, where r_c is radius of the well casing and T is the transmissivity of the aquifer. For accurate determination of storage coefficient the well should be pumped beyond this time which is quite long for aquifer with low transmissivity. Large-diameter wells are generally constructed in shallow aquifers with low transmissivity and long duration pumping test in such wells is therefore not practicable (Herbert and Kitching, 1981). Under these circumstances, analysis of the unsteady flow during recovery and evaluation of aquifer parameters with the help of recovery data need due consideration. Rushton and Holt (1981), and Herbert and Kitching(1981) used numerical methods to analyze flow to a large-diameter well during the abstraction phase and the recovery phase.

Generally, in India centrifugal pumps are used by farmers for withdrawal of water from a large-diameter well. When centrifugal pumps are used the abstraction rate falls off significantly with increasing drawdown. Though analytical expressions have been derived by Lai et al for a linear or exponential decrease in abstraction rate with time, the characteristics of centrifugal pumps require a reduction in abstraction rate with the unknown well drawdown. Using numerical model, Rusthon and Singh (1983) developed type curves for large-diameter wells when the abstraction rate decreases with drawdown.

In reality no aquifer is of infinite areal extent. Hydrologic boundary such as a river or an impermeable boundary is often encountered in the vicinity of a well. Flow to well located near hydrologic boundary has been analyzed conveniently by image well theory. According to the image-well theory hydrologic boundaries are replaced by imaginary wells which produce the same effects as that of the boundaries. Boundary problems are thereby simplified to consideration of an aquifer of infinite areal extent where-in the real and the image wells operate simultaneously. The effects of real and image well are computed making use of the principle of superposition.

In the last decade, many complex ground water problems have been analyzed by the discrete kernel approach (Maddock, 1972; Morel-Seytoux and Daly, 1975; and Morel-Seytoux, 1975). Discrete kernel approach has been found to be convenient for analysing unsteady flow to large-diameter well in a homogeneous confined aquifer (Patel and Mishra, 1983). In the present report the unsteady flow to a large-diameter well influenced by an impermeable boundary and a perennial fully penetrating river has been analyzed during and after stoppage of pumping, using discrete kernel and image well theory. The well storage has been taken into consideration in the analysis.

3.0 PROBLEM DEFINITION

Figure 1 shows a schematic cross-section of a large-diameter well in a confined aquifer located near a fully penetrating perennial river and an impervious barrier. The well screen has a radius equal to r_w. Radius of the unscreened part of the well is r_c. The aquifer is homogeneous, isotropic, and initially at rest condition. Pumping is carried out at a uniform rate upto time t_p. It is necessary to determine the drawdown in the piezometric surface at the well face and at any distance r from the centre of the well at time t after the onset of pumping.

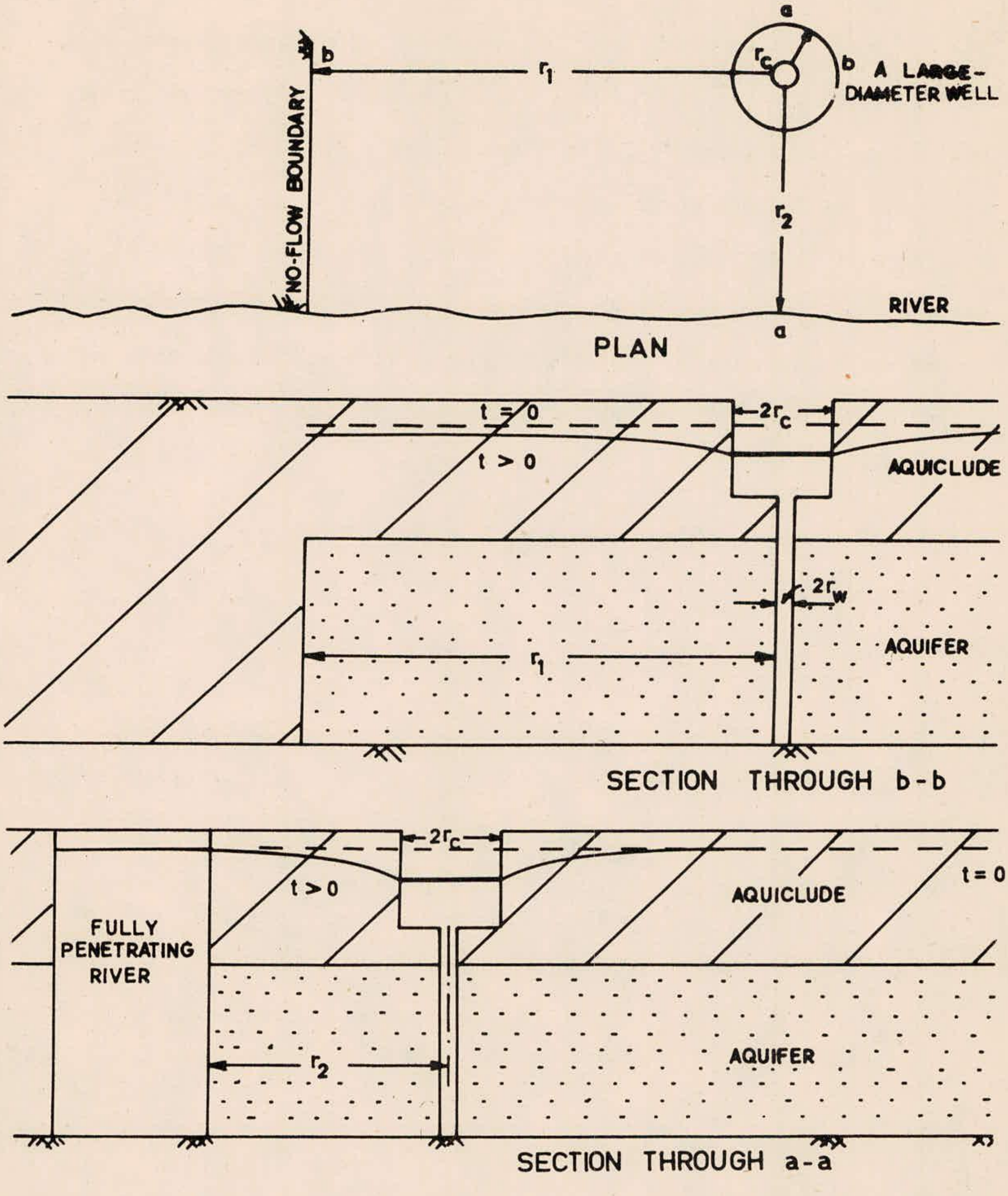

FIGURE 1 - A LARGE-DIAMETER WELL NEAR A RIVER AND A NO-FLOW BOUNDARY

4.0 METHODOLOGY

The following assumptions have been made in the analysis:

1. At any time the drawdown in the aquifer at the well face is equal to that in the well.
2. The time parameter is discrete. Within each time step, the abstraction rate of water derived from well storage and that from aquifer storage are separate constants.

The basic differential equation for axially-symmetric, radial, unsteady ground-water flow in a homogeneous, isotropic, confined aquifer of uniform thickness is given by

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{\phi}{T}\frac{\partial s}{\partial t} \qquad \ldots (1)$$

where, s = drawdown; r = distance from the centre of the well, t = time, ϕ = storage coefficient, and T = transmissivity of the aquifer.

For the initial condition $s(r,0) = 0$ and the boundary condition $s(\infty, t) = 0$, solution to the above differential equation when a unit impulse quantity is withdrawn from the aquifer is given by (Carslaw and Jaeger, 1959)

$$s(r,t) = \frac{e^{-\frac{r^2}{4\beta t}}}{4\pi T t} \quad ; \quad \beta = T/\phi \qquad \ldots (2)$$

Defining a unit impulse kernel

$$k(t) = \frac{e^{-\frac{r^2}{4\beta t}}}{4\pi T t} \qquad \ldots (3)$$

drawdown for a variable pumping rate can be written in the form

$$s(r,t) = \int_o^t Q_A(c) \cdot k(t-c)dc, \qquad \ldots (4)$$

where , $Q_A(c)$ is the variable discharge rate from the aquifer at time c.

Dividing the time span into discrete time steps and assuming that the aquifer discharge is constant within each time step but varies from step to step, the drawdown at the end of time step n can be written as (Morel-Seytoux, 1975)

$$s(r,n) = \sum_{\gamma=1}^{n} \delta_r(n-\gamma+1) \cdot Q_A(\gamma), \qquad \ldots (5)$$

where, the discrete kernel coefficient $\delta_r(m)$ is defined as

$$\delta_r(m) = \int_0^1 k(m-c)dc = \frac{1}{4\pi T}\{E_1(\frac{r^2}{4\beta m}) - E_1(\frac{r^2}{4\beta(m-1)})\} \qquad \ldots (6)$$

where, $E_1(\ \ )$ is an exponential integral defined as

$$E_1(X) = \int_X^\infty \frac{e^{-u}}{u} du$$ (Abramowitz and Stegun, 1970).

The discrete kernel coefficient $\delta_r(m)$ is the drawdown at the end of the m^{th} time period at a distance r from the pumping well in response to withdrawal of a unit quantity of water from the aquifer storage during the first time period. A unit time period may be 0.1 day, 1 day or 1 week etc.

The disturbance created by the river and the no-flow boundary on drawdown can be simulated by image well theory. The locations of the image wells which simulate the boundary effect are shown in Fig.2.

Drawdown, $s_w(n)$, at the well face at the end of time step n due to abstraction from well storage is given by

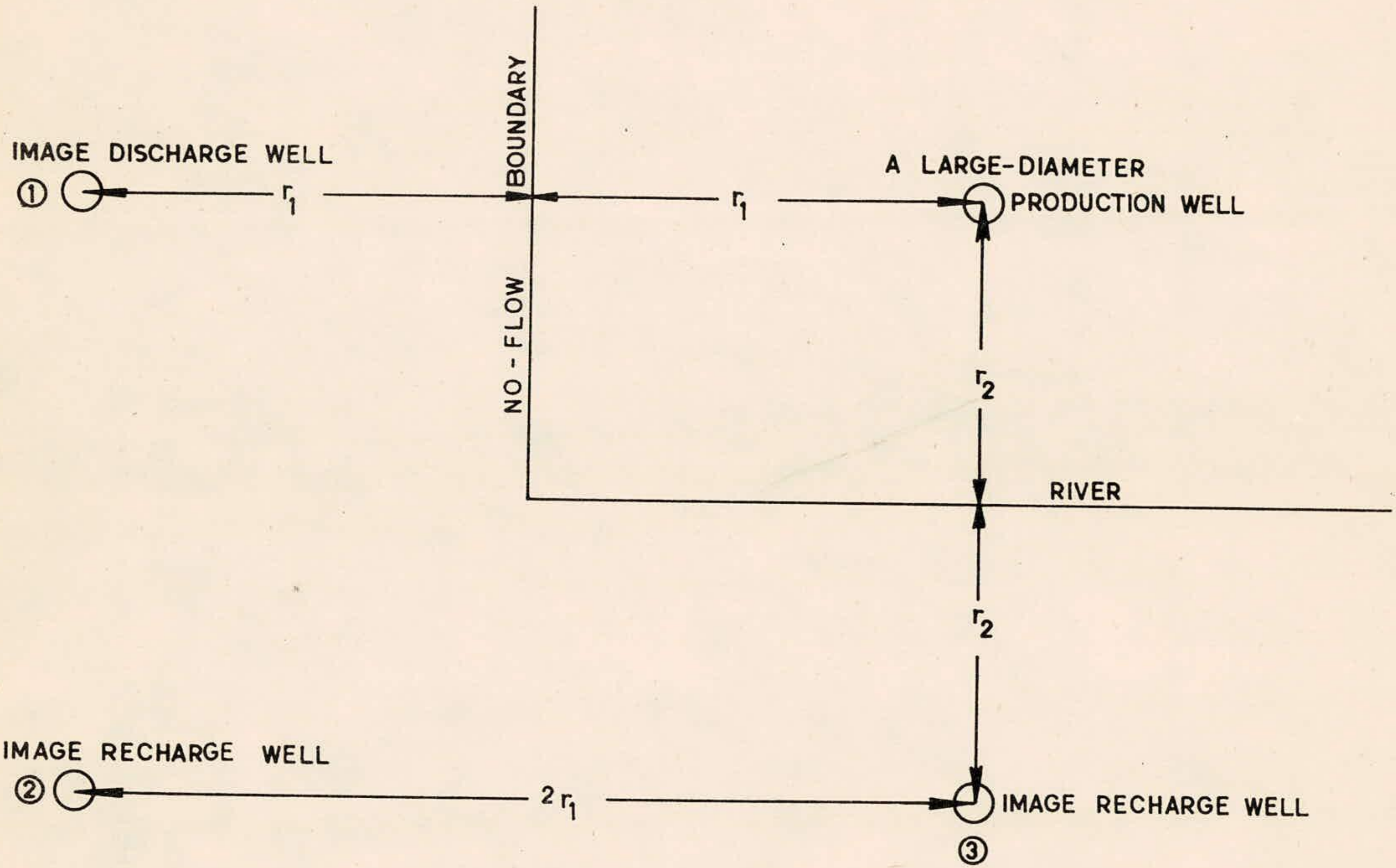

FIGURE 2 - THE IMAGE WELLS CREATING THE SAME BOUNDARY EFFECTS

$$s_w(n) = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_w(\gamma) \qquad \ldots(7)$$

where, $Q_w(\gamma)$ is the withdrawal rate from well storage at time step γ. $Q_w(\gamma)$ values are unknown a priori. A negative value of $Q_w(\gamma)$ means there is replenishment of well storage which occurs during the recovery period.

In addition, drawdown at the well face at the end of time step n due to abstraction from aquifer storage is given by

$$s_A(n) = \sum_{\gamma=1}^{n} Q_A(\gamma) \{ \delta_{rw}(n-\gamma+1)+\delta_1(n-\gamma+1)-\delta_2(n-\gamma+1)-\delta_3(n-\gamma+1) \} \qquad \ldots (8)$$

where

$$\delta_{rw}(m) = \frac{1}{4\pi T} \{ E_1 (\frac{r_w^2}{4\beta m}) - E_1(\frac{r_w^2}{4\beta(m-1)}) \} \qquad \ldots (9)$$

$$\delta_1(m) = \frac{1}{4\pi T} \{ E_1 (\frac{r_1^2}{\beta m}) - E_1 (\frac{r_1^2}{\beta(m-1)}) \} \qquad \ldots(10)$$

$$\delta_2(m) = \frac{1}{4\pi T} \{ E_1 (\frac{r_1^2+ r_2^2}{\beta m}) - E_1 (\frac{r_1^2+ r_2^2}{\beta(m-1)})\} \qquad \ldots(11)$$

$$\delta_3(m) = \frac{1}{4\pi T} \{E_1 (\frac{r_2^2}{\beta m}) - E_1 (\frac{r_2^2}{\beta(m-1)}) \} \qquad \ldots (12)$$

Because $s_w(n) = s_A(n)$, therefore

$$\sum_{\gamma=1}^{n} Q_A(\gamma) \{\delta_{rw}(n-\gamma+1) +\delta_1(n-\gamma+1)-\delta_2(n-\gamma+1) -\delta_3(n-\gamma+1)\}$$

$$= \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n} Q_w(\gamma) \qquad \ldots (13)$$

Rearranging,

$$\{\delta_{rw}(1)+\delta_1(1)-\delta_2(1)-\delta_3(1)\}\ Q_A(n) - \frac{Q_w(n)}{\pi r_c^2} = \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma)$$

$$- \sum_{\gamma=1}^{n-1} Q_A(\gamma)\{\delta_{rw}(n-\gamma+1)+\delta_1(n-\gamma+1)-\delta_2(n-\gamma+1)-\delta_3(n-\gamma+1)\} \quad \ldots(14)$$

At any time the algebraic sum of the quantities of water withdrawn from aquifer storage and from well storage equals the quantity pumped.

Let the total time of pumping be discretised to m_p units of equal time steps. The quantity of water pumped during anytime step n can be written as

$$Q_A(n) + Q_w(n) = Q_p(n) \quad \ldots(15)$$

in which,

$Q_A(n)$ = water withdrawn from aquifer storage, and

$Q_w(n)$ = water withdrawn from well storage.

For $n > m_p$, $Q_p(n) = 0$. Otherwise $Q_p(n) = Q$, where Q is the pumping rate for unit time period.

Equations (14) and (15) can be written in the following matrix form:

$$\begin{bmatrix} \delta_{rw}(1)+\delta_1(1)-\delta_2(1)-\delta_3(1), & -\frac{1}{\pi r_c^2} \\ 1 & , \quad 1 \end{bmatrix} \cdot \begin{bmatrix} Q_A(n) \\ Q_w(n) \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_w(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\{\delta_{rw}(n-\gamma+1)+\delta_1(n-\gamma+1)-\delta_2(n-\gamma+1)-\delta_3(n-\gamma+1)\} \\ Q_p(n) \end{bmatrix}$$

Hence,

$$\begin{bmatrix} Q_A(n) \\ \\ Q_W(n) \end{bmatrix} = \begin{bmatrix} \delta_{rw}(1)+\delta_1(1)-\delta_2(1)-\delta_3(1), & -\frac{1}{\pi r_c^2} \\ \\ 1 & , \quad 1 \end{bmatrix}^{-1} \cdot$$

$$\begin{bmatrix} \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\{\delta_{rw}(n-\gamma+1)+\delta_1(n-\gamma+1)-\delta_2(n-\gamma+1)-\delta_3(n-\gamma+1)\} \\ \\ Q_p(n) \end{bmatrix}$$

... (17)

Thus, $Q_A(n)$ and $Q_W(n)$ can be solved in succession starting from time step 1.

In particular, for time step 1

$$\begin{bmatrix} Q_A(1) \\ \\ Q_W(1) \end{bmatrix} = \begin{bmatrix} \delta_{rw}(1)+\delta_1(1)-\delta_2(1)-\delta_3(1), & -\frac{1}{\pi r_c^2} \\ \\ 1 & , \quad 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ \\ Q_p(1) \end{bmatrix}$$

... (18)

Once $Q_A(n)$ values are solved, the drawdown at any point can be found using the following equation:

$$s_r(n) = \sum_{\gamma=1}^{n} Q_A(\gamma)\{\delta_{R_1}(n-\gamma+1)+\delta_{R_2}(n-\gamma+1)-\delta_{R_3}(n-\gamma+1)-\delta_{R_4}(n-\gamma+1)\} \quad \ldots (19)$$

in which,

R_1 = distance between the production well and the point at which drawdown is to be known,

R_2 = distance between the image discharge well and the point,

R_3, R_4 = distance of image recharge wells from the point,

$$\delta_{R_1}(m) = \frac{1}{4\pi T} \{E_1(\frac{R_1^2}{4\beta m}) - E_1(\frac{R_1^2}{4\beta(m-1)})\}.$$

Equation (17) is a general equation. When any one of the hydrologic boundaries or both are at infinity, the contribution of aquifer storage and well storage to pumping can be derived from equation (17) as follows:

Let the perennial river is at a finite distance from the well and the impermeable boundary is at infinity. In such case $r_1 \to \infty$. Consequently $\delta_1(m)$ and $\delta_2(m)$ tend to zero and equation (17) reduces to

$$\begin{bmatrix} Q_A(n) \\ Q_W(n) \end{bmatrix} = \begin{bmatrix} \delta_{rw}(1)-\delta_3(1), & -\frac{1}{\pi r_c^2} \\ 1 & , \quad 1 \end{bmatrix}^{-1} \cdot$$

$$\begin{bmatrix} \frac{1}{\pi r_c^2} \sum_{\gamma=1}^{n-1} Q_W(\gamma) - \sum_{\gamma=1}^{n-1} Q_A(\gamma)\{\delta_{rw}(n-\gamma+1)-\delta_3(n-\gamma+1)\} \\ Q_p(n) \end{bmatrix} \quad .. (20)$$

5.0 RESULTS

The discrete kernel coefficients $\delta_{rw}(m)$, $\delta_1(m)$, $\delta_2(m)$, and $\delta_3(m)$ are generated and stored for known values of aquifer parameters (T,ϕ), radius of the well screen (r_w), and distances of the hydrologic boundaries from the large-diameter well (r_1, r_2) for different values of time (m). After generating the discrete kernel coefficients, $Q_A(n)$ and $Q_W(n)$ are found by solving eqs. (14) and (15) for known values of r_c and duration of pumping (t_p). The drawdown at the well face is then obtained with the help of equation (7).

The variation of $\{s_w(t)\}/\{Q/4\pi T\}$ with $\{4Tt/\phi r_w^2\}$ for a specific location of the hydrologic boundaries for different values of α, where $\alpha=\phi\{r_w/r_c\}^2$, is shown in Figures 3. $s_w(t)$ is the drawdown at the well face at time t and $\{s_w(t)\}/\{Q/4\pi T\}$can be regarded as the well function for a large-diameter well located near a river and a no-flow boundary. The type curves shown in Figure 3 contain the response of the aquifer during the abstraction as well as the recovery phase. Each of the recovery curves is characterized by a non-dimensional time factor, $\{4Tt_p\}/\{\phi r_w^2\}$,at which it leaves the time drawdown curve of the abstraction phase. This non-dimensional time factor can be used to check the accuracy of the aquifer parameters determined by curve matching.

The variations of $\{s_w(t)\}/\{Q/4\pi T\}$ with $\{4Tt\}/\{\phi r_w^2\}$, for $\alpha = 0.000001$ are shown in Figure 4 for the cases when

i) the aquifer is of infinite areal extent,

ii) a river boundary exists at a distance of $\{r_2\}/\{r_w\} = 50$, and the impervious boundary is at infinity, and

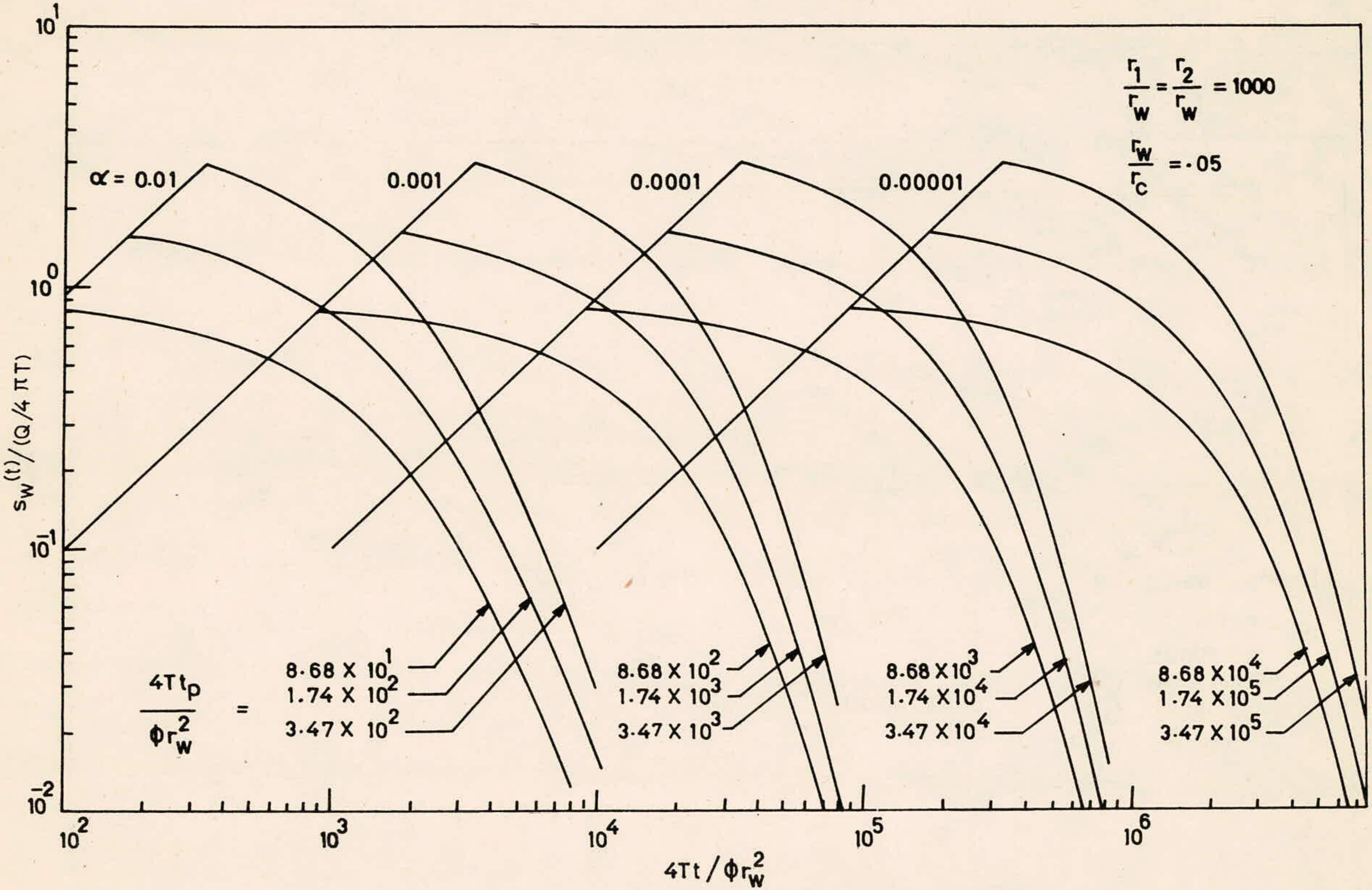

FIGURE 3 - VARIATION OF $\{s_w(t)\}/\{Q/4\pi T\}$ WITH $\{4Tt/\phi r_w^2\}$

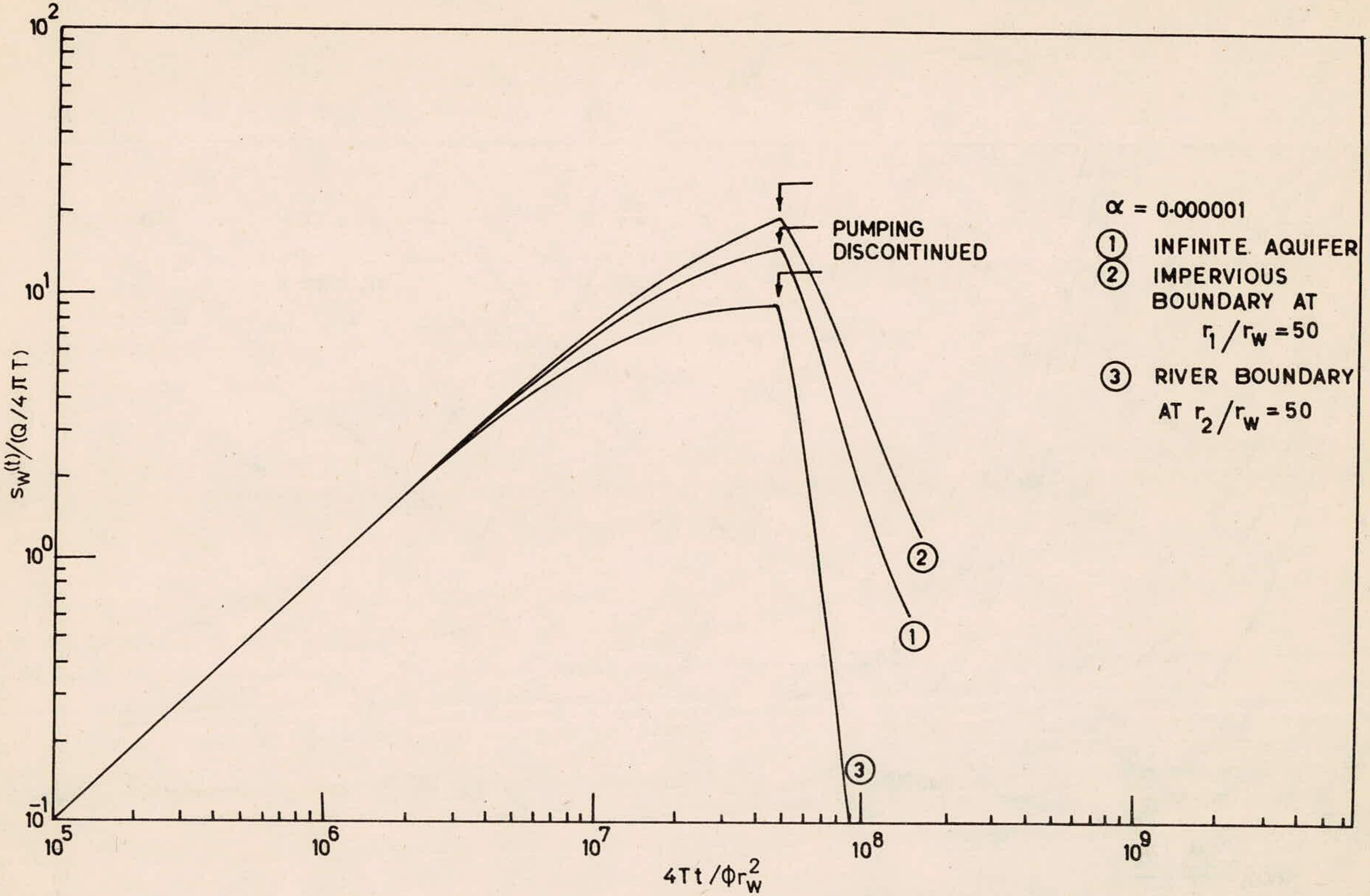

FIGURE 4 - VARIATION OF $\{s_w(t)\}/\{Q/4\pi T\}$ WITH $\{4Tt/\phi r_w^2\}$

iii) a no-flow boundary exists at a distance of $(r_1)/(r_w) = 50$, and the river boundary is at infinity.

It is seen from the figure that when the river is present near the well the recovery of the large-diameter well is very rapid in comparison to the cases where a no-flow boundary exists or none of the boundaries are present. Comparison of drawdowns when pumping stopped shows that because of the presence of the no-flow boundary the drawdown is increased by 16.5 percent. On the contrary if there is a river boundary the drawdown is decreased by 25.6 percent.

The recovery of the well storage has been analyzed for different durations of pumping for various values of storage coefficients. Figure 5 shows the plot of percentage recovery of well storage with non-dimensional time measured since pumping stopped, for different values of storage co-efficient and pumping time. It is seen from the figure that as the storage coefficient decreases the time span for recovery of well storage increases. The duration of pumping has little or no effect on the percentage recovery of well storage for small values of storage coefficient. For example, when $\phi = 0.0001$, the duration of pumping beyond 5 hours of pumping has no influence on the recovery of well storage. This is because of the fact that with smaller value of ϕ, the pump withdraws more water from well storage and the withdrawal from well storage ceases some time after pumping. Within the duration of pumping considered, the contribution of well storage has ceased. On the other hand the recovery of the well storage is influenced by the duration of pumping when the ϕ value is high. For example, for $\phi = 0.1$ when the duration of pumping is 5 hours the non-dimensional time factor for 50 percent recovery of well storage is 31. When the duration of pumping is 20 hours the corresponding value is 39.

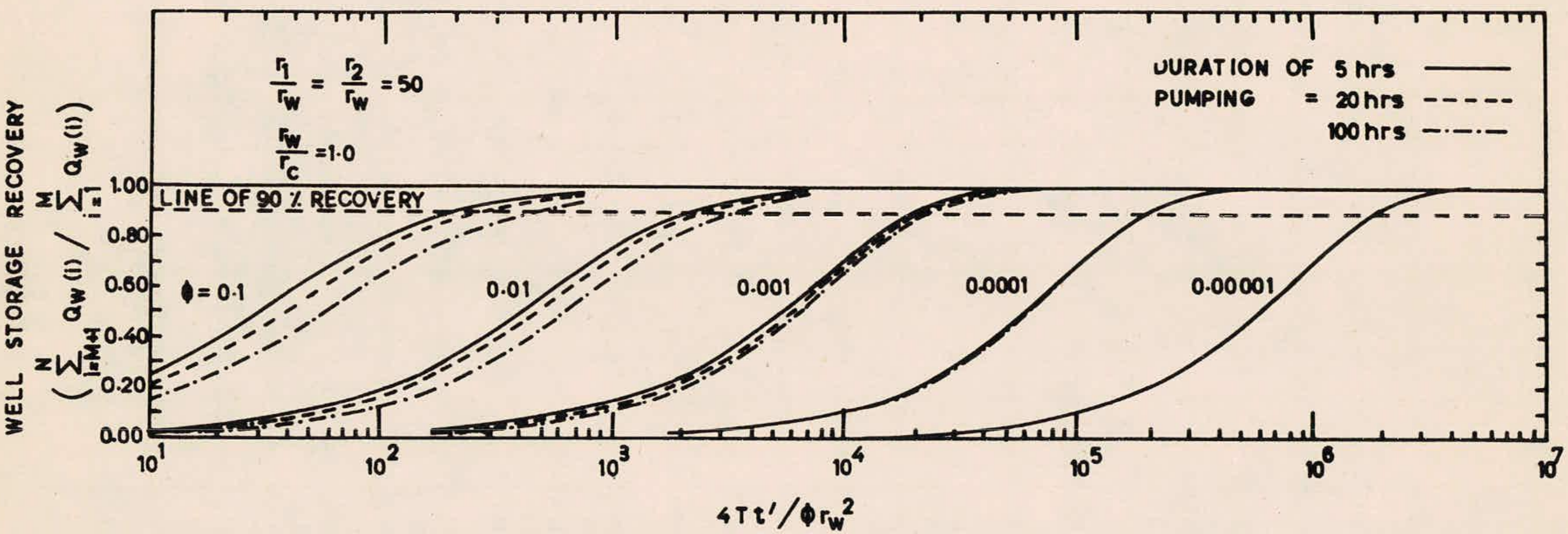

FIGURE 5 - WELL STORAGE RECOVERY WITH TIME

6.0 CONCLUSIONS

A general and simple mathematical model has been described which determines drawdown due to pumping of a large-diameter well located near a no-flow boundary and a river. The recovery of the well storage after stoppage of pumping has been analyzed. From the results, the time span, in which a specific fraction of the well storage is recovered, can be known.

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