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TRADE OFF BETWEEN ACCURACY AND GRID SIZES IN A GROUNDWATER MANAGEMENT MODEL

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ABSTRACT

In water resources systems, most of the models are optimum simulation type in which management of resource is the main objective. Generally groundwater management models use governing partial differential equation and the problem is formulated using either Finite Element Method (FEM) or Finite Difference technique (FD).

To a certain extent it is seen that in FD as the number of grid points increase the results are more accurate. Accuracy may depend on the parameters of the model, their range, type of formulation, grid spacing, dimensions, etc.

Here an attempt is made to formulate a groundwater system as a distributed parameter model using FD approximations of the governing partial differential equation. The resulting system of simultaneous equations are embedded in a Linear Programming model for management. The problem is formulated for different boundary condition and grid sizes and correlation is made for grid sizes to the accuracy.

INTRODUCTION

Modelling of groundwater system involves several hundreds of parameters thus the model is complex and needs more CPU time. Groundwater systems are modelled using FEM or FD. In FD upto a certain point, as the number of grid points increase the results are more accurate. Thus if the grid points are more, the results are accurate but requires more CPU time. The smaller number of grid points require less CPU time but results are less accurate. We have to sacrifice one for the other.

In the present study[4] a distributed parameter groundwater management model is formulated using FD for different boundary conditions, grid points ranging from 4 to 400, and trade off between them is made. This will help in selecting a suitable grid size for the required accuracy.

LITERATURE REVIEW

Aguado E. and Remson I.[1] (1974) used LP for different types of flow in which the decision variables are groundwater variables. He combined LP with FP approximation.

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Gorelock S. (1983)[2] reviewed the distributed parameter groundwater management modelling method. He had classified management models as hydraulic management models and policy evaluation models.

Yeou Koung Tung and Christine E[6] (1985) discussed some computational experiences using embedding technique for groundwater management.

PROBLEM FORMULATION

A hypothetical aquifer in the reference 5 is considered. The following assumptions are made:

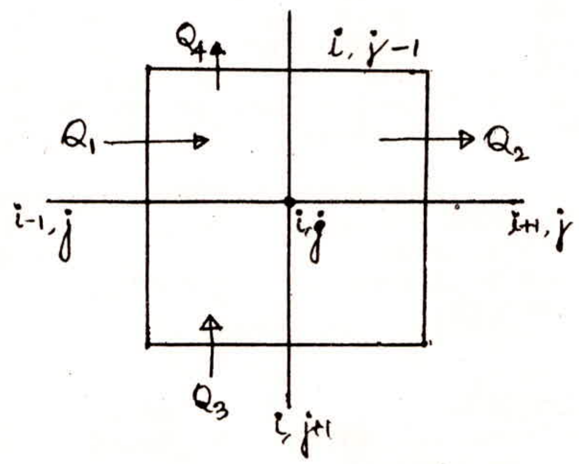
i) Aquifer is bounded by regular sides
ii) The boundary conditions are assumed constant throughout the planning period.
iii) Two dimensional flow is considered.
iv) The flow is assumed to be steady.

The partial differential equation describing 2-D, unsteady flow, in a heterogeneous anisotropic aquifer can be written as

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) = S \frac{\partial h}{\partial t} + Q \quad \text{-- (1)}$$

The above equation is incorporated as a set of constraints in the LP model using embedding technique. The above equation can be converted into a set of algebraic equations using a Finite Difference Scheme.

Consider the following nodel representation:



Let $T_{i,j,1}$ = aquifer transmissivity between the nodes i,j, and i, j+1,

$T_{i,j,2}$ = aquifer transmissivity between the nodes i,j,and i+1, j.

Now the equation (1) can be discritised as

$$h_{i,j}(T_{i-1,j,2} + T_{i,j,2} + T_{i,j,1} + T_{i,j-1,1} + S_{i,j,} (\frac{\Delta x^2}{t})$$
$$- T_{i-1,j,2}\, h_{i-1,j} - T_{i,j,2}\, h_{i+1,j} - T_{i,j,1}\, h_{i,j+1}$$
$$- T_{i,j-1,1}\, h_{i,j-1} + Q_{i,j} = S_{i,j} (\frac{\Delta x)^2}{\Delta t} h^{\emptyset}_{i,j} \qquad (2)$$

The equation (2) when written for each node located at the centre of each cell will result in a set of linear algebraic equations.

The minimisation of drawdown (or maximization of head) is used as the objective function in the model.

$$\text{Maximize } Z = \sum_{K=1}^{NP} \sum_{t-1}^{NT} h_{Kt} \qquad (3)$$

where, NP = Total number of nodes in FD grid
NT = Number of planning periods.

The following constraints are used in the model:

i) The partial differential equation of groundwater flow when converted into algebraic equation can be embedded as a set of linear constraints in the model

ii) $h_i \geqslant H_1$

iii) $h_i \leqslant H_2$

iv) $Q_i \leqslant 5\ Mm^3$/year

v) $\Sigma Q_i \geqslant 7Mm^3$/year

The above LP model is solved for different boundary condition and different levels of discritisation. The number of grid points used are 4, 9, 16, 25, 100, 225 and 400. Fig.1 shows the nodal representation and different boundary conditions. The following data are used in the problem:

The aquifer area is 10 km x 10 km

The transmissivity is assumed as 1000 m^2/day

It is assumed that the aquifer is naturally replenished at the rate of 100 mm/year.

The problem is solved by using a LP package called ZX3LP on DEC1090 Computer system at IIT/Kanpur.

DISCUSSION OF RESULTS

Table 1 shows the comparative study of initial heads for various gridsizes for boundary condition 1. In this table average values are written for all grids keeping 400 grid as a standard one. It is seen that mean error increases as the grid number is reduced. The constant head boundary is more sensitive.

Table 2 shows the comparative study of optimum pumpages for boundary condition 1.

Figure 2 shows the above variations graphically.

Figures 3,4 and 5 show the variation of accuracy to grids for boundary conditions, 2,3,and 4 respectively.

Table 3 shows the summary of mean error and CPU time for various conditions.

CONCLUSIONS

The groundwater management model is formulated as LP optimization model with flow equations embedded as one of the constraints. The problem is solved for 4 types of boundary conditions and at different levels of discritisation.

The accuracy increases as the number of grid increases but the incremental increase is reduced. For the problem considered 100-150 grids may be optimum.

CPU time seems to be experimentally increased as the grid points increases. For 100 grids CPU time is 62 seconds whereas it is about 2000 seconds for 400 grids.

The mean error for 100 grid is about 15%, The values of heads are more sensitive near constant head boundaries. It is advisible to have more grids near constant head boundary. Nature of boundary is appreciable for heads as well as pumpages.

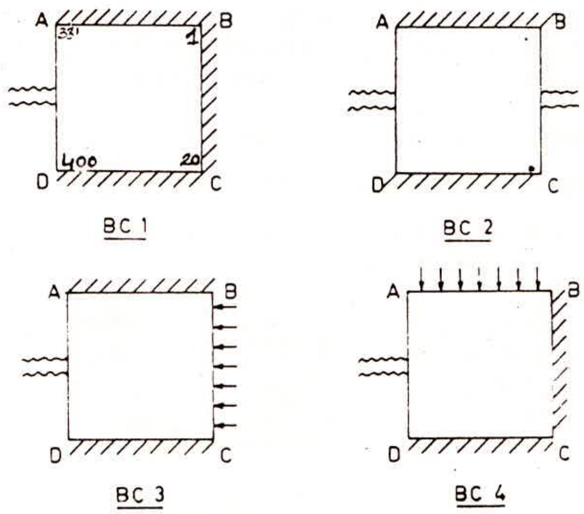


FIG.1 BOUNDARY CONDITIONS

Table 1 Comparative Study of Initial Heads for BC1

Grid point w.r.t. 400	400	225	% error	100	% error	25	% error	16	% error	9	% error	4	% error
1-20	13.54	13.48	0.43	13.69	1.10	13.88	2.50	13.69	1.10	13.69	1.10	13.69	1.10
21-40	13.50	13.40	0.74	13.69	1.40	13.88	2.90	13.69	1.40	13.69	1.40	13.69	1.40
41-60	13.42	13.40	0.15	13.42	0.00	13.88	3.42	13.69	2.01	13.69	2.01	13.69	2.01
61-80	13.35	13.30	0.38	13.42	0.30	13.88	3.97	13.69	2.54	13.69	2.54	13.69	2.54
81-100	12.63	12.50	1.02	12.87	1.90	12.77	1.10	13.69	8.39	13.69	8.39	13.69	8.39
101-120	12.24	12.31	5.70	12.87	5.20	12.77	4.33	11.98	2.12	13.69	13.69	13.69	13.56
121-140	11.96	12.31	2.90	12.05	0.45	12.77	6.77	11.98	0.16	13.69	14.46	13.69	14.46
141-160	10.87	12.18	12.05	12.05	14.99	12.77	17.40	11.98	10.20	10.65	2.02	13.55	25.90
161-180	10.68	10.53	1.40	10.95	2.50	10.55	1.21	11.98	12.17	10.65	0.28	13.69	28.10
181-200	10.59	10.03	5.03	10.95	4.00	10.55	0.37	11.98	13.17	10.65	0.56	13.69	29.30
201-220	9.87	10.03	1.62	9.58	3.03	10.55	6.38	8.36	13.20	10.65	7.90	6.50	34.14
221-240	8.94	9.69	8.38	9.58	7.16	10.55	17.90	8.56	4.25	10.65	19.10	6.50	27.30
241-260	7.74	7.44	3.87	7.94	2.58	7.22	6.71	8.56	10.60	10.65	37.50	6.50	16.08
261-280	6.62	6.82	3.22	7.94	19.90	7.22	9.64	8.56	30.00	10.65	60.80	6.50	1.81
281-300	5.68	6.82	20.00	6.02	6.00	7.22	27.10	8.56	52.10	4.56	19.70	6.50	14.43
301-320	4.94	6.39	29.30	6.02	21.80	7.22	46.10	3.42	30.70	4.56	7.69	6.50	31.50
321-340	3.84	3.23	15.88	3.85	0.26	2.79	27.10	3.42	10.93	4.56	18.75	6.50	69.30
341-360	2.76	2.32	15.90	3.85	39.40	2.79	1.08	3.42	24.00	4.56	65.20	6.50	135.56
361-380	1.87	2.32	24.10	1.36	27.20	2.79	50.00	3.42	83.00	4.56	143.00	6.50	247.58
381-400	1.06	1.75	65.21	1.36	28.00	2.79	163.00	3.42	222.00	4.56	330.00	6.50	544.13
Mean error			6.36		8.98		16.3		21.38		36.70		62.05
Mean error excluding last 20 grid			3.27		7.70		8.50		10.82		21.20		36.60

Table 2 Comparative Study of Pumpages for BC1

Grid point w.r.t. 400	400	225	% error	100	% error	25	% error	16	% error	9	% error	4	% error
81-100	0	0	0	0	0	0	0	0.41	0	0	0	0	0
101-120	0	0	0	0	0	0	0	0.41	0	0	0	0	0
121-140	0.74	0	0	0	0	0	0	0.41	44.5	0.72	0	0	0
141-160	0	0	0	0	0	0	0	0.41	0	0.25	0	0	0
161-180	0	0	0	0	0	0.25	0	0.41	0	0.25	0	0	0
181-200	0	0.45	0	0	0	0.25	0	0.41	0	0.25	0	0	0
201-220	1.14	0.45	60.50	0.50	56.10	0.25	78.00	0.75	35.08	0.25	78.12	0.88	22.80
221-240	0.44	0.20	54.50	0.50	13.60	0.25	43.18	0.75	70.45	0.25	43.08	0.88	100.00
241-260	0	1.22	0	1.50	0	1.50	0	0.75	0	0.25	0	0.88	0
261-280	0.92	0.60	34.70	1.50	63.40	1.50	63.00	0.75	18.40	1.60	73.90	0.88	4.34
281-300	2.62	0.60	77.00	1.50	42.70	1.50	42.70	0.75	71.30	1.60	38.46	0.88	66.41
301-320	1.33	3.46	160.00	1.50	12.70	1.50	12.70	0.75	43.60	1.60	20.30	0.88	33.88
Mean error			7.1		8.90		30.80		23.50		23.20		25.10

Table 3 Summary of Mean Error and CPU Time for Various Grid Sizes

Number of grids		4	9	16	25	100	225	400 (assumed)
Mean % error w.r.t. head	BC1	62.05	36.70	21.38	16.30	8.98	5.36	0
	BC2	171.00	89.70	63.40	45.00	15.40	5.79	0
	BC3	88.50	63.80	46.10	72.70	6.32	4.08	0
	BC4	77.80	51.90	43.40	54.80	5.20	4.63	0
Mean error w.r.t. pumpage	B1	25.10	23.20	23.50	30.80	8.90	7.70	0
	B2	28.70	26.40	25.00	21.20	19.70	10.90	0
	B3	46.70	16.40	51.50	9.75	7.48	33.40	0
	B4	34.78	31.90	25.46	17.67	22.09	25.76	0
CPU time in secs		0.13	0.21	0.69	2.00	62.00	900.00	2100.00

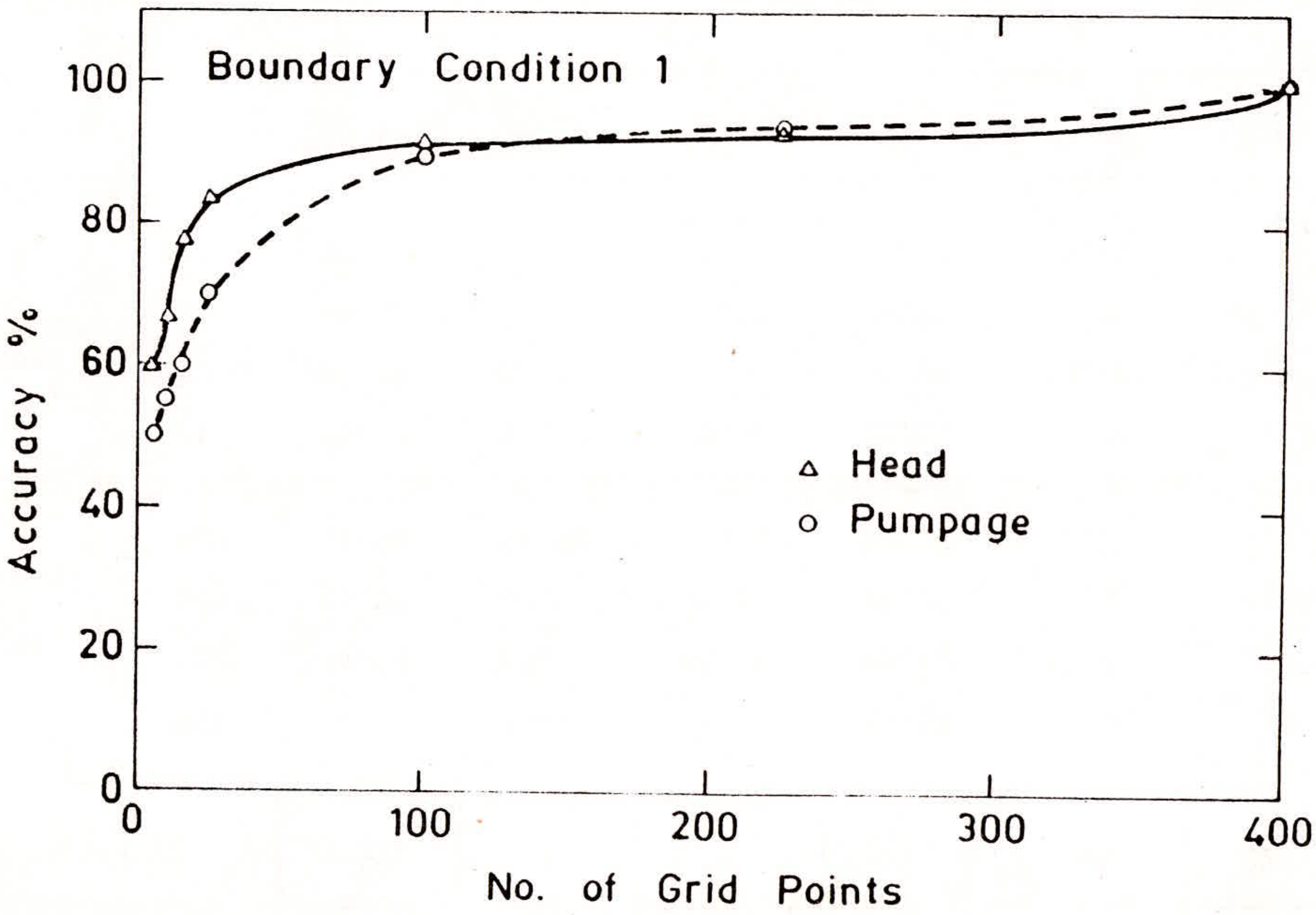


Fig. 2 Variation of Accuracy with Number of Grid Points.

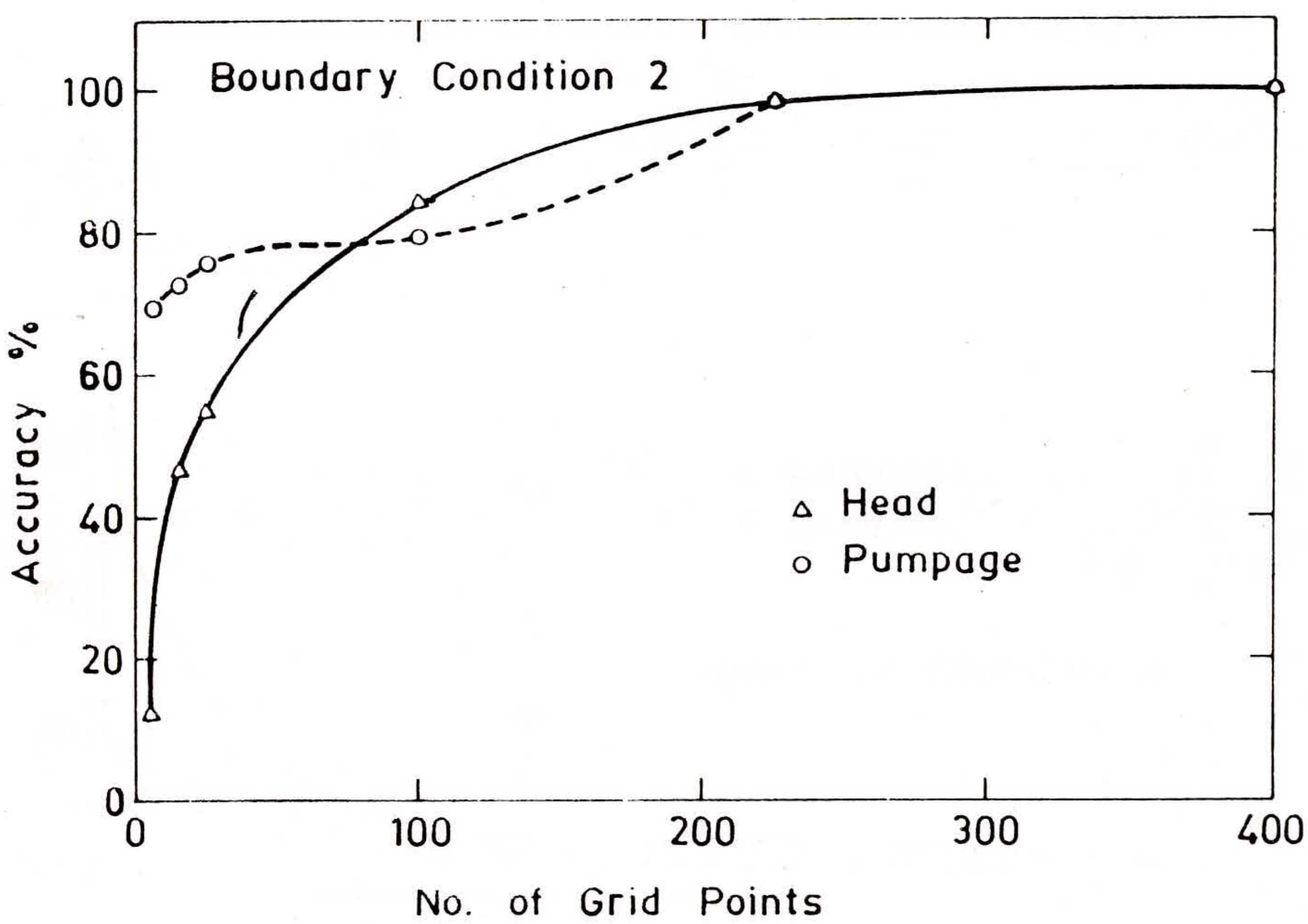


Fig. 3 Variation of Accuracy with Number of Grid Points.

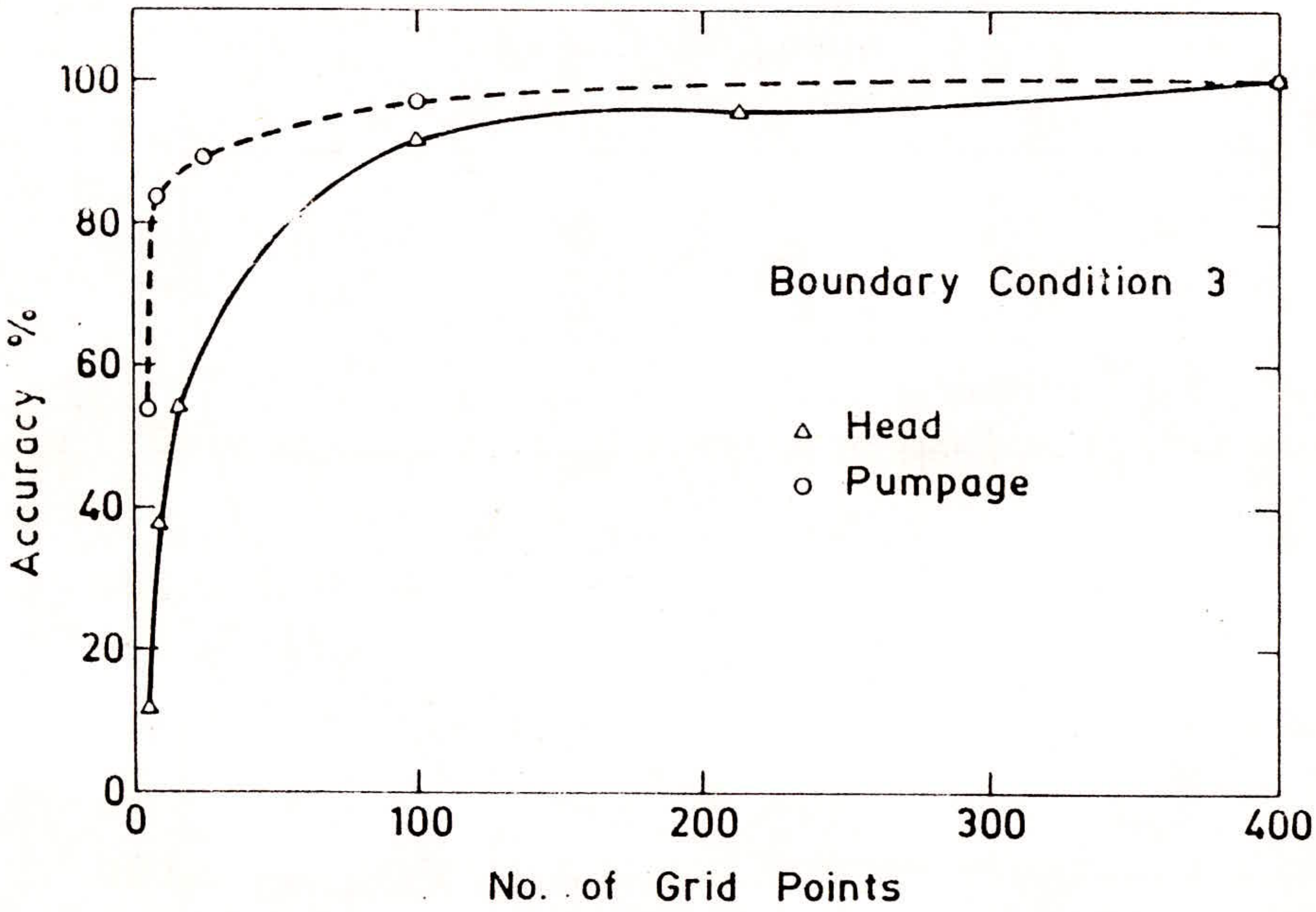


Fig. 4 Variation of Accuracy with Number of Grid Points.

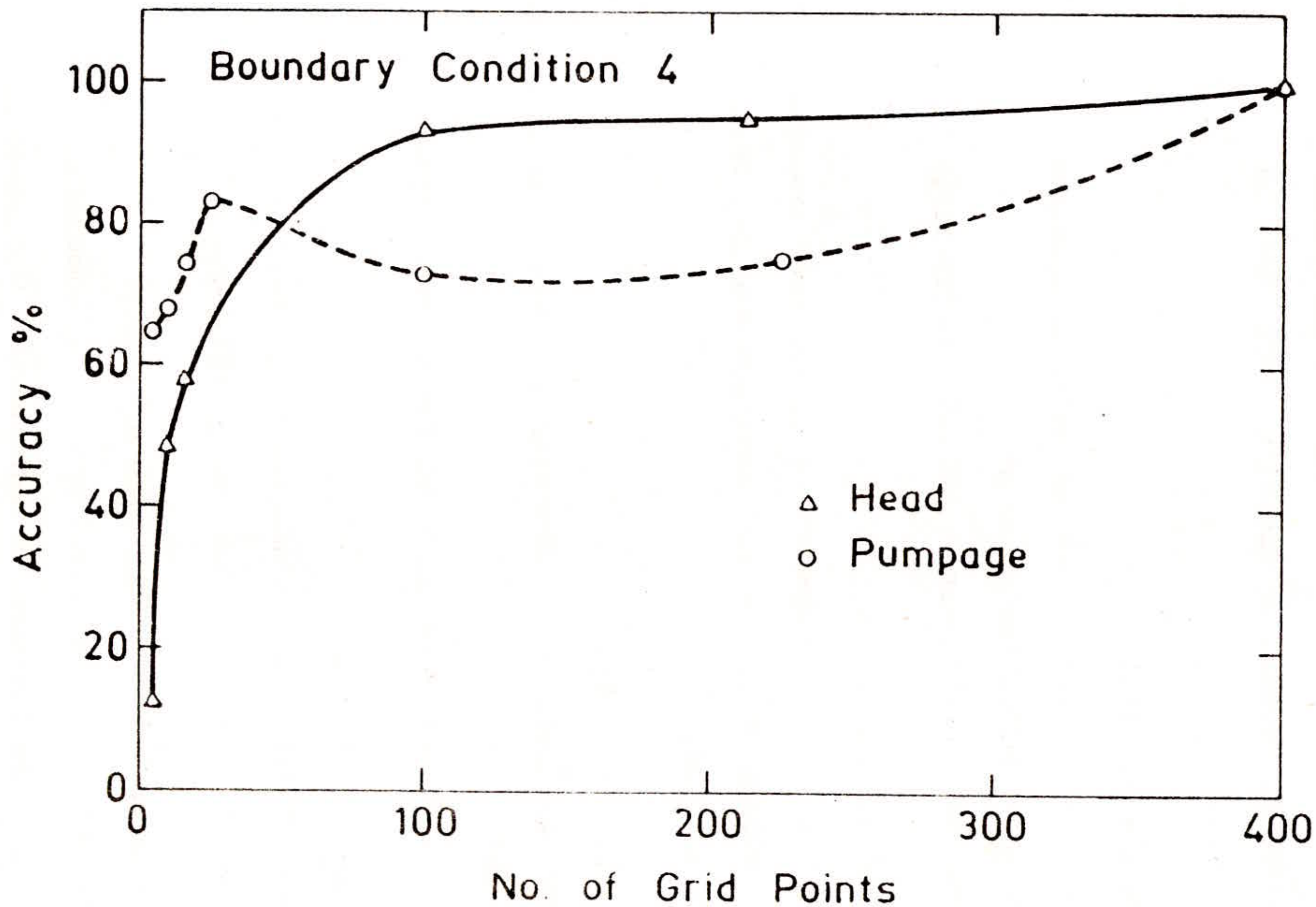


Fig. 5 Variation of Accuracy with Number of Grid Points

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