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A COMPARATIVE ANALYSIS OF HYDROLOGICAL AND DIFFUSION METHODS OF FLOOD ROUTING

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ABSTRACT

As a part of the study of flood routing methods as applicable to Indian rivers, an attempt was made to apply the Conventional Muskingum method, the Constant and Variable Parameter Muskingum Cunge methods, the Kulandaiswamy General Storage Equation method and the Variable Parameter Diffusion method to a reach each in the Rivers Narmada and Cauvery. A total of ten floods in Narmada and eight in Cauvery were analysed. The three Muskingum methods produced very similar results with which the general storage equation method also agreed. Any improvement in one or other method was only marginal. It was found that the diffusion method can also be applied with improved results. However, one experiences considerable difficulty in the model parameter calculations of all the methods.

INTRODUCTION

A large number of flood routing methods are known to be successfully applied to various rivers and floods. These methods are classified into two groups, namely hydrological or storage routing and hydraulic routing. The hydrological methods are based only on the continuity equation whereas the hydraulic methods employ both the continuity and momentum equations (i.e. the St. Venant equations).

The storage routing techniques have been widely used, especially in developing countries, for their simplicity. Also, the hydraulic methods involve complicated methods of solution and high requirements of computing facilities. However, if certain simplifications are made to the momentum equation, the hydraulic method will lead to a simpler version than the original one, in which case both the solution and computation are simple and easy. The routing techniques based on a diffusion wave model developed by Hayami (1951), Thomas and Wormleaton (1970) and Price (1973) are the best examples for the simplified hydraulic methods.

This study attempts to find the applicability of some of the well known storage routing methods and the Variable Parameter Diffusion method due to Price (1973) to two of the Indian rivers. The reaches of Mortakka-Garudeshwar of the River Narmada in Central India and Kodumudi-Musiri of the River Cauvery in Peninsular India were used.

METHODS OF ROUTING

Storage routing techniques are based on the continuity equation which in its simplest form can be written by equating the difference between inflow and outflow to the rate of change of storage as

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$$Q_i - Q_{i+1} + q = dS/dt \qquad (1)$$

where Q_i = inflow into the reach; Q_{i+1} = outflow from the reach; q = lateral inflow entering the reach; and S = storage in the reach.

The relationship between the storage, inflow and outflow makes it possible to determine the outflow for a known inflow if the storage is known. A popular method is the Muskingum equation developed by McCarthy (1938), which linearly relates the storage to inflow and outflow as

$$S = K[\varepsilon Q_i + (1 - \varepsilon) Q_{i+1}] \qquad (2)$$

where K = storage coefficient; and ε = weighting parameter.

The conventional method of finding the parameters K and ε is by a trial and error graphical procedure. Karmegam et al (1985) have brought out the difficulties in arriving at suitable values of K and ε for a practical situation.

Although the Muskingum method is widely used, one of the major criticisms is that the storage is linearly related to the flow. In an attempt to improve the method, Cunge (1969) proved that the solution of the finite difference forms of equation (1) and (2) can be approximated to the solution of the convection-diffusion equation

$$\partial Q/\partial t + \omega\, \partial Q/\partial x = \mu\, \partial^2 Q/\partial x^2 + \omega\, q_e \qquad (3)$$

if $$K = \Delta x/\omega \qquad (4)$$

and $$\varepsilon = 1/2\,[1 - (Q/\Delta x.\omega.B.S_o)] \qquad (5)$$

where Q = discharge; Δx = reach length; ω = convection parameter; μ = diffusion parameter; B = average top width of flow; and S_o = average bed slope of the river; and q_e = lateral inflow in m^3/sec/m.

Equations (4) and (5) enable the Muskingum parameters to be calculated according to the physical and hydraulic characteristics of the reach under study. The above equations then form the basis for what is known as the Muskingum-Cunge method.

Ponce and Yevjevich (1978) presented the Muskingum-Cunge method with variable parameters, allowing both K and ε to vary with discharge.

Defining

$$\omega = dQ/dA|_{i,j} \qquad (6)$$

and $$q_w = Q/B|_{i,j} \qquad (7)$$

where A = area of flow; q_w = discharge per unit width; i = space section; and j = time level, they worked out ω and q_w by iteration using a four point average for which the values at (i+1, j+1) obtained by the average of those at the three points (i,j), (i,j+1) and (i+1,j) served as the first guess for the iteration. Their result shows that the Variable Parameter Muskingum-Cunge method takes into account the non-linearity of the phenomenon.

A storage equation, very much general in nature, has been derived by Kulandaiswamy (1964) as

$$S = \sum_{n=1}^{N} a_{(n-1)}\, d^{n-1} Q_{i+1}/dt^{n-1} + \sum_{m=1}^{M} b_{(m-1)}\, d^{m-1} Q_i/dt^{m-1} \qquad (8)$$

where the a and b coefficients are either constants or functions of variables involving Q_i and Q_{i+1}. The values of N and M depend on the order of time derivatives of Q_i and Q_{i+j} that need to be considered. If N and M are both unity, equation (8) reduces to the Muskingum equation (2) with

$$b_o = K\varepsilon \qquad (9)$$

and $a_o = K(1-\varepsilon)$ (10)

Equation (8) was originally developed for a study of rainfall-runoff relationship. It was, however, later used for channel routing in a limited way by Kulandaiswamy et al (1967) and elaborately by Karmegam and Wormleaton (1983).

Limiting the values of N and M to 2, from practical point of view, equation (8) becomes

$$S = a_o Q_{i+1} + a_1 dQ_{i+1}/dt + b_o Q_i + b_1 dQ_i/dt \qquad (11)$$

Depending on whether the influence of rates of changes of outflow and inflow are to be studied, either the second or fourth of both the terms on the right hand side can be dropped. The routing procedure, in fact, is similar to that of the Muskingum, except the fact that equation (2) is replaced by equation (11) and the resulting second order differential terms are expanded numerically using an appropriate finite-difference net in the x-t plane. The complete procedure is given by Karmegam (1981) and Karmegam and Wormleaton (1983). The advantage with the general equation over the Muskingum is that it takes into account the rate of change of inflow and outflow for the storage calculation.

Following Hayami (1951) it is customary to use equation (3) in diffusion routing methods, identifying ω as the velocity of the kinematic flood wave and μ as the diffusion coefficient. Thomas and Wormleaton (1970) derived a numerical flood routing technique for constant values of ω and μ and found that these two parameters cannot be ascertained for a wide range of floods in a reach.

Price (1973) overcame the difficulty due to the uncertainity in the values of ω and μ by deriving a Variable Parameter Diffusion method based on the equation

$$\partial Q/\partial t + C(Q)\, \partial Q/\partial x = Q\, \partial/\partial x\, (\phi\, \partial Q/\partial x) + Q/S_o\, dS_o/dx\, \partial Q/\partial x + 3/5\, \phi_c (\partial Q/\partial x)^2 + C\, q_e \qquad (12)$$

where φ = diffusion coefficient; and C = speed of the flood wave. Price introduced $\overline{C}(Q)$ as the average value of C(Q,x) along the reach, to define the equivalent river model in the place of a natural river. He formed the basic equation for Variable Parameter Diffusion method as

$$\partial Q/\partial t + \overline{C}\, \partial Q/\partial x = Q\, \partial/\partial x\, (\alpha/L\, \partial Q/\partial x) + 3/5\, \alpha_c/L\, (\partial Q/\partial x)^2 + \overline{C}\, q_e \qquad (13)$$

where α = attenuation parameter; and L = length of the river reach. Price derived the value of the attenuation parameter as

$$\alpha(Q) = 1/2 [1/L \sum_{m=1}^{M} P_m/S_m^{1/3}]^{-3} \sum_{m=1}^{M} P_m^2/L_m S_m^2 \quad (14)$$

where P_m = area in plan of the inundated flood plain and channel in the mth subreach with L_m and S_m being the corresponding length and bed slope of the channel. For a small inbank flood

$$\alpha = 1/2\, W_c [1/L \sum_{m=1}^{M} L_m/S_m^{1/3}]^{-3} \sum_{m=1}^{M} (L_m/S_m^2) \quad (15)$$

where W_c = mean width of the channel. $\alpha(Q)$ for a known peak flow can be calculated if data regarding flood plain inundation is available and intermediate values of α obtained from a Q Vs α curve judiciously drawn with the two computed values of α. Price also found that the $\overline{C}$ could be calculated from a semi empirical formula

$$\overline{C} = \omega + Q^* \, d/dQ \, (L/T_p) \quad (16)$$

$$\text{where } \omega = L/T_p - 2\alpha/L^2 \, Q^* \quad (17)$$

with Q* as the attenuation of flood peak.

Ignoring the insignificant terms in equation (13), the Variable Parameter Diffusion method can be based on the equation

$$\partial Q/\partial t + \overline{C}\, \partial Q/\partial x = \alpha/L \; Q \; \partial^2 Q/\partial x^2 + \overline{C}\, q_e \quad (18)$$

The routing is effected by solving equation (18) written in a finite difference form using the implicit Crank-Nicholson scheme. Rangapathy et al (1986) describe the procedure in a detailed manner.

APPLICATION AND RESULTS

With a view to assess the suitability of the methods for application to Indian rivers, two river reaches were chosen. The reach Mortakka-Garudeshwar of the River Narmada with a riverine distance of 268 Km runs along Central India in the state of Madhya Pradesh. The average bed slope of this reach is 0.00053. The second reach is the Kodumudi-Musiri section of the River Cauvery in Tamil Nadu. The reach length is 68 Km with an average bed slope of 0.00057.

Ten floods between the year 1971 and 1979 for the River Narmada and eight floods between 1975 and 1981 for River Cauvery were considered. The Conventional Muskingum (CM), the Constant Parameter Muskingum Cunge (CPMC), the Variable Parameter Muskingum Cunge (VPMC), the Kulandaiswamy General Storage Equation (KGSE) and the Variable Parameter Diffusion (VPD) methods were applied for all the floods chosen for this analysis.

The results are presented in two tables. Tables 1 and 2 relate to the Narmada and the Cauvery respectively.

DISCUSSION

Price (1973) applied the CPMC and VPD methods for hypothetical floods in a prismatic rectangular channel and compared the results with the results obtained by solving the full St. Venant equations using the leap-frog explicit finite difference scheme. Both the methods predicted the peak flow and time to peak accurately. He also found that the VPD method was more successful in predicting the shape of the hydrograph thus bringing out its superiority over the CPMC method. This conclusion can now be verified for floods in Indian rivers.

Comparing the three Muskingum methods applied to the River Narmada, it can be seen from Table 1 that the errors in the computed peak flows are less in CPMC and VPMC than in the CM with an exception of one or two stray cases. The standard deviation values show improvement for the CPMC and VPMC over CM indicating that the Cunge methods predict the shape of the hydrograph closely. The percentage errors in the mass conservation also indicate that the Cunge versions are better than the CM.

However, the results in Table 2 of the River Cauvery indicate that there is nothing to choose between the three methods. In fact, three floods have been routed better by the CM than the CPMC and VPMC methods.

It may be recalled that the parameters K and ε for the CM method are calculated by a trial and error procedure by plotting the weighted flows $[\varepsilon Q_i+(1-\varepsilon)Q_{i+1}]$ against the respective storages (S). The measured inflows (Q_i) and outflows (Q_{i+1}) in a reach may be accurate, but there may not be any data relating to the storage. One is forced to resort to an approximate procedure for the computations. Also the loop form of the relationship between S Vs $[\varepsilon Q_i+(1-\varepsilon)Q_{i+1}]$ may never become a straight line as required by the method. However, the parameters for the CPMC method are calculated based on the hydraulic and geometric characteristics of the reach under study. For the VPMC method, in addition to the above, the parameters are allowed to vary with the flow in the reach. This means that the CPMC and VPMC are expected to produce better results than the CM.

It may appear that the results presented in Tables 1 and 2 may not commensurate with the above conclusion. This might be due to fact that the computation of parameters for the CPMC and VPMC requires relationship between discharge and area (Q Vs A) and between discharge and top width (Q Vs B). Such data must be representative of the whole reach under study, which are not available for Indian rivers. Here again, one is forced to use the data available for the upstream and downstream sections which are mostly gauging stations with controlled site conditions. The cross-section details of such stations cannot be considered to reflect the characteristics of the entire reach between them.

The KGSE method compares favourably with the Muskingum methods in both the rivers. In fact, it provides marginally improved results. It is probable that the rates of changes of inflow and outflow when included may indirectly bring in the effect of the hydraulic and geometric characteristics of the reach. But, it is observed from the results that the four parameters (a_0, a_1, b_0 and b_1) involved in the routing are different for different floods (not included in the tables for want of space). Though a definite relationship cannot be immediately established, it appears that the parameters vary with the flow in the reach. It is difficult to suggest a set of values for the parameters which could be constantly used for all occasions.

The VPD method requires more detailed flood data. The calculations of ω and $\bar{C}$ require observed wave velocities and also attenuation values for different flood peaks. It is even more difficult to obtain $\alpha(Q)$ curve as details of flood inundation for different peak flows are not available. Price could obtain two α values, one for a known high overbank flood and one for an inbank flood. The $\alpha(Q)$ curve was judiciously drawn. The authors could not get details of flood inundation for any peak flow for both the rivers. However, based on some probable values of inundation area, $\alpha(Q)$ curve was constructed, and after elaborate and repeated trials, they could arrive at an $\alpha(Q)$ curve that could route the flood fairly well. Adjustments were also made in $\bar{C}(Q)$ curve as, again, it is difficult to obtain attenuation and flood wave velocity for different floods.

Despite this handicap, it is seen that the VPD method has predicted the floods well. Though the method could not be considered distinctly superior in routing the floods of Narmada (Table 1), it does seem to improve the results in the case of Cauvery. The criterion of standard deviation is much better satisfied and this is reflected in the general shape of the outflow hydrographs as seen from Fig. 3 to 6.

CONCLUSIONS

The Muskingum method and its improvements due to Cunge may produce more or less identical results in the absence of a practical method of determining the Muskingum parameters reflective of the total reach. The CM or the CPMC in most cases, should suffice and the VPMC may not improve the results.

The KGSE model can be equally agreeable if a satisfactory relationship between inflow and the model parameters could be obtained.

The VPD method can also be attempted, even in the absence of relevant inundation data, by judiciously constructing the (Q) and $\bar{C}(Q)$ curves. The method is particularly useful when the prediction of the shape of the hydrograph is the requirement.

None of the methods can be applied without modification of the parameters for any river. Some procedures for optimising the parameters for minimising the errors in routing are essential. Such optimised parameters may then be related to some physical or hydraulic characteristics of the reach. When this is done, the identification of the parameters would be complete. The routing with such parameters would be most acceptable.

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TABLE 1 : MORTAKKA-GARUDESHWAR REACH, NARMADA

Length : 268 Km — Average Bedslope : 0.00053

Year & Period of flood (1)	Routing Technique (2)	Observed outflow peak (Cumecs) (3)	Computed outflow peak (Cumecs) (4)	Error in peak flow (Percent) (5)	Standard Deviation (Percent) (6)	Error in mass conservation (Percent) (7)
1972	CM	44880	49021	-9.23	82.24	9.42
AUG	CPMC		47625	-6.12	81.31	0.80
16-23	VPMC		47531	-5.90	82.82	2.95
	KGSE		44564	0.70	78.96	2.00
	VPD		45538	-1.47	13.64	10.01

TABLE 1 (CONTINUED)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1972	CM	21960	25755	-17.28	47.07	8.50
AUG 29	CPMC		22381	-1.92	47.75	-0.95
SEPT 6	VPMC		22375	-1.89	47.93	-0.61
	KGSE		23268	-5.96	49.64	-0.85
	VPD		20456	6.85	14.06	11.27
1974	CM	34679	34494	0.53	65.02	0.56
AUG 18	CPMC		39512	-13.94	65.76	-1.16
SEPT 26	VPMC		39409	-13.64	65.46	-1.26
	KGSE		35461	-2.25	62.19	-0.25
	VPD		35281	-1.74	13.91	12.55
1975	CM	15130	14427	4.65	42.22	16.93
AUG	CPMC		15661	-3.50	38.89	0.55
21-31	VPMC		15666	-3.54	39.39	1.06
	KGSE		15958	-5.47	39.19	0.70
	VPD		13878	8.26	24.84	18.73
1975	CM	31516	29346	6.89	72.75	15.48
SEPT	CPMC		30676	2.67	71.62	-0.28
8-17	VPMC		30525	3.14	70.29	-0.95
	KGSE		31074	1.40	70.94	-0.62
	VPD		27750	11.95	19.40	15.88
1976	CM	15250	12719	13.03	49.25	36.80
AUG	CPMC		16738	-9.76	33.89	0.30
2-10	VPMC		16597	-8.83	32.35	-1.33
	KGSE		16644	-9.14	33.09	0.15
	VPD		12112	20.58	45.77	38.83
1976	CM	15504	13566	12.50	42.80	26.93
SEPT	CPMC		19447	-25.43	33.39	1.08
1-9	VPMC		18573	-19.79	33.08	1.41
	KGSE		17519	-12.99	31.41	0.29
	VPD		13800	10.99	31.83	28.88
1977	CM	22108	20502	7.26	72.43	3.54
AUG	CPMC		23786	-7.59	70.47	2.46
5-13	VPMC		23310	-5.49	73.07	6.41
	KGSE		21124	4.45	68.87	2.95
	VPD		21420	3.11	17.31	16.93
1977	CM	20508	17870	12.86	35.61	15.13
AUG 28	CPMC		21223	-3.49	33.14	-0.87
SEPT 6	VPMC		20642	-0.65	31.90	-2.72
	KGSE		19861	3.15	30.53	-1.94
	VPD		17762	13.39	18.36	4.91
1979	CM	39206	24079	38.58	64.06	31.10
AUG	CPMC		31972	18.45	56.95	-0.13
8-16	VPMC		32010	18.35	54.89	-1.54
	KGSE		36240	7.56	65.39	-0.75
	VPD		23782	39.34	46.00	33.21

TABLE 2 : KODUMUDI-MUSIRI REACH, CAUVERY

Length : 68 Km — Average Bedslope : 0.00057

Year & period of flood	Routing Technique	Observed outflow peak (Cumecs)	Computed outflow peak (Cumecs)	Error in peak flow (Percent)	Standard Deviation (Percent)	Error in mass conservation (Percent)
1975 SEPT 17-26	CM	2564	2318	9.6	31.80	2.20
	CPMC		2338	8.8	33.44	3.05
	VPMC		2344	8.6	33.10	2.80
	KGSE		2385	7.0	30.13	0.85
	VPD		2564	0.0	17.17	-11.63
1975 NOV 2-7	CM	2331	2115	9.3	52.12	4.19
	CPMC		2063	11.5	53.13	5.30
	VPMC		2067	11.3	51.75	5.30
	KGSE		2164	7.2	53.04	3.33
	VPD		2398	-2.9	29.01	-23.83
1979 AUG 11-18	CM	4139	4303	-4.0	58.51	1.04
	CPMC		4347	-5.0	57.45	1.07
	VPMC		4337	-4.8	57.01	1.28
	KGSE		4265	-3.0	56.86	0.91
	VPD		4460	-7.8	11.33	1.17
1980 JULY 1-17	CM	5320	5215	2.0	60.90	-6.10
	CPMC		4952	7.3	60.53	0.57
	VPMC		4783	10.0	30.09	0.78
	KGSE		5041	5.2	60.49	0.65
	VPD		5006	5.9	10.14	-0.64
1980 JULY 29 AUG 5	CM	2778	2668	4.0	45.71	0.34
	CPMC		2700	2.8	45.18	0.23
	VPMC		2700	2.8	45.54	0.40
	KGSE		2598	6.5	44.03	0.25
	VPD		2821	1.6	9.96	-8.50
1981 AUG 17-26	CM	4521	4066	10.0	48.95	1.83
	CPMC		3871	14.4	48.65	2.12
	VPMC		3857	14.7	48.18	2.09
	KGSE		3863	14.6	49.51	1.15
	VPD		3907	13.6	17.64	7.71
1981 SEPT 9-24	CM	4710	4134	12.3	44.59	0.60
	CPMC		3919	16.8	44.73	0.88
	VPMC		3464	15.8	44.10	0.88
	KGSE		4360	7.4	45.13	0.49
	VPD		3871	17.8	22.87	6.98
1981 OCT 27 NOV 4	CM	1925	1549	18.0	28.70	-0.75
	CPMC		1653	14.1	26.27	-1.20
	VPMC		1671	13.2	26.63	-1.15
	KGSE		2259	-17.4	32.07	-0.02
	VPD		1726	10.4	13.97	7.39

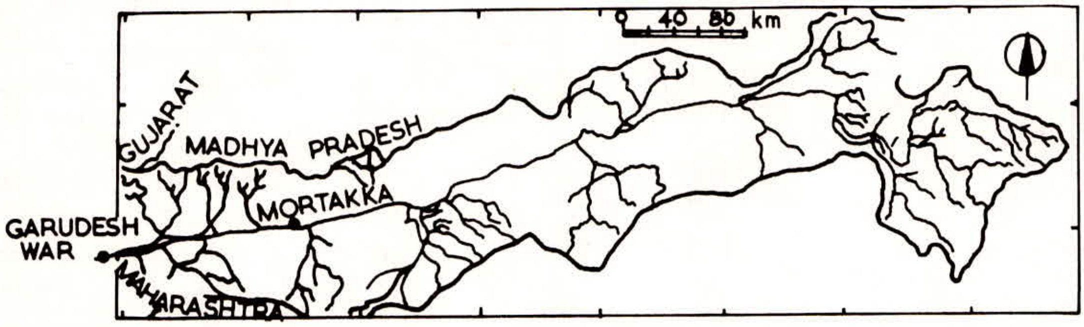

FIG.1. PART OF NARMADA BASIN SHOWING MORTAKKA-GARUDESHWAR REACH

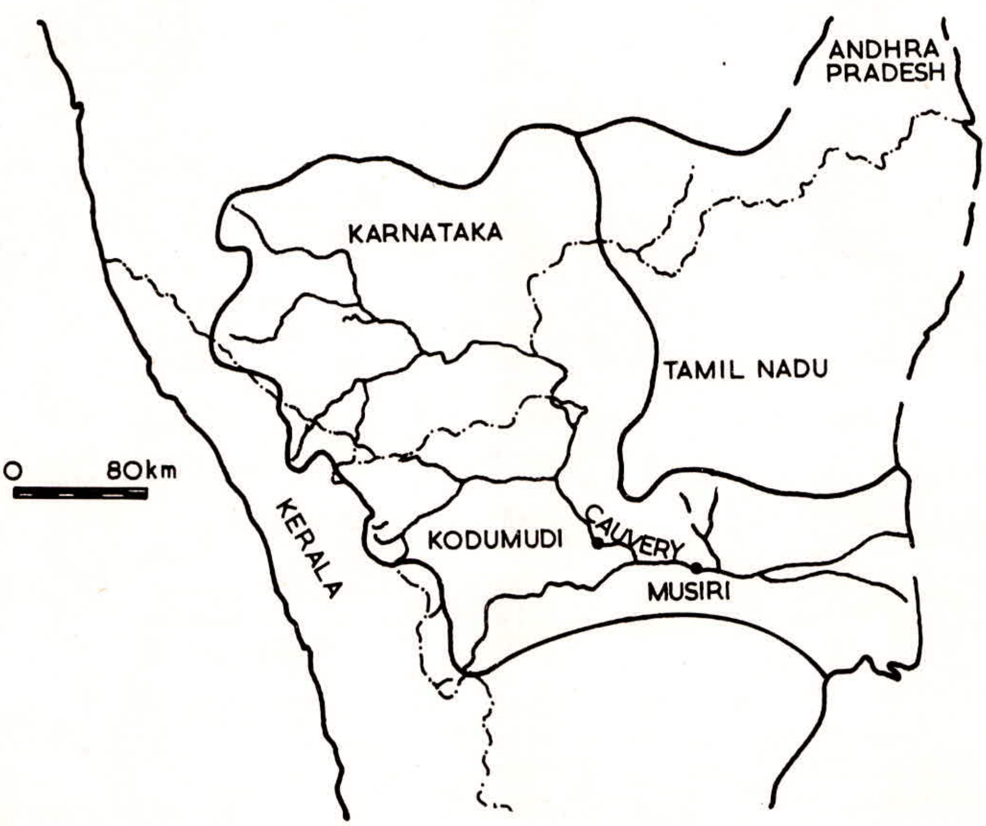

FIG.2. CAUVERY BASIN SHOWING KODUMUDI-MUSIRI REACH

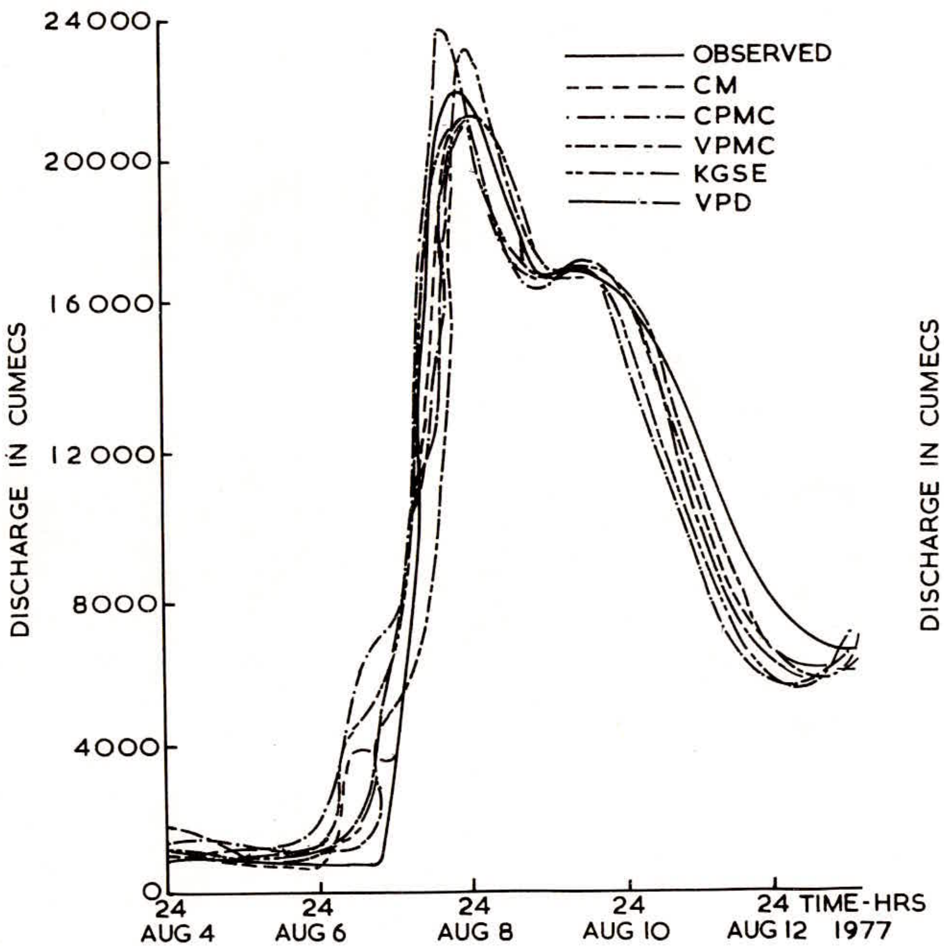

FIG.3

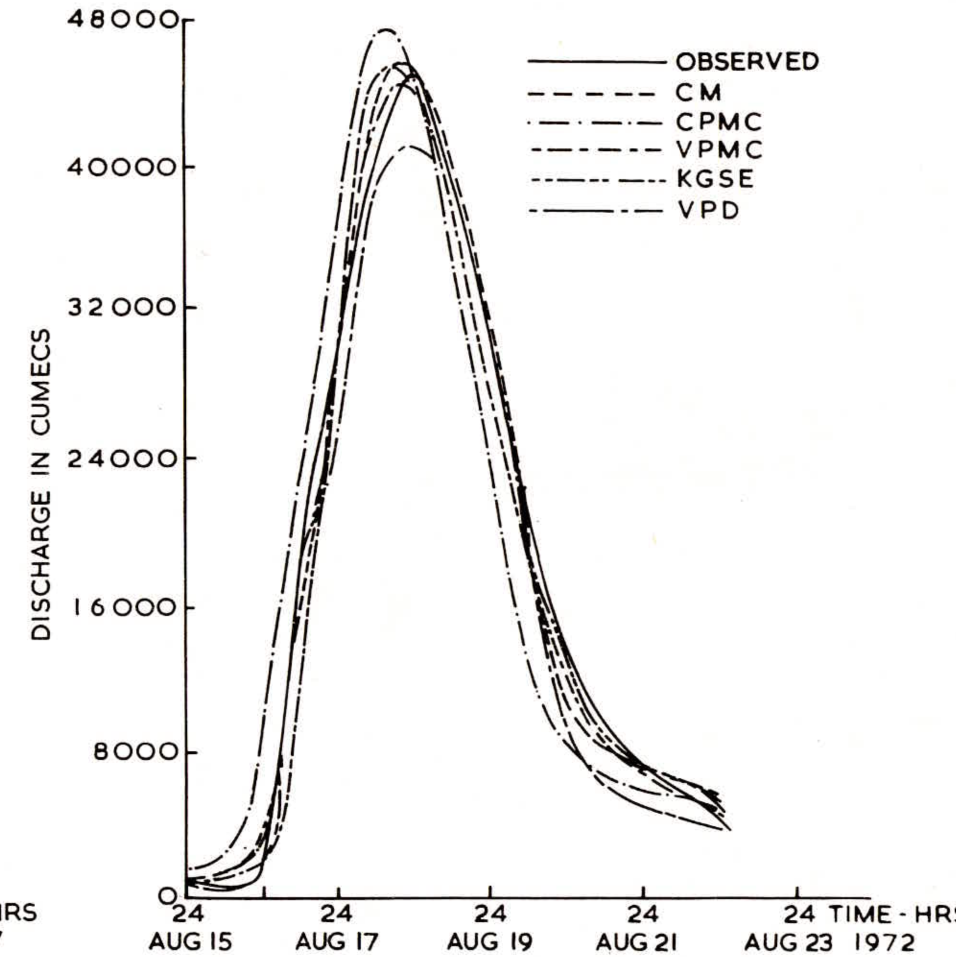

FIG.4

OBSERVED AND COMPUTED HYDROGRAPHS AT GARUDESHWAR

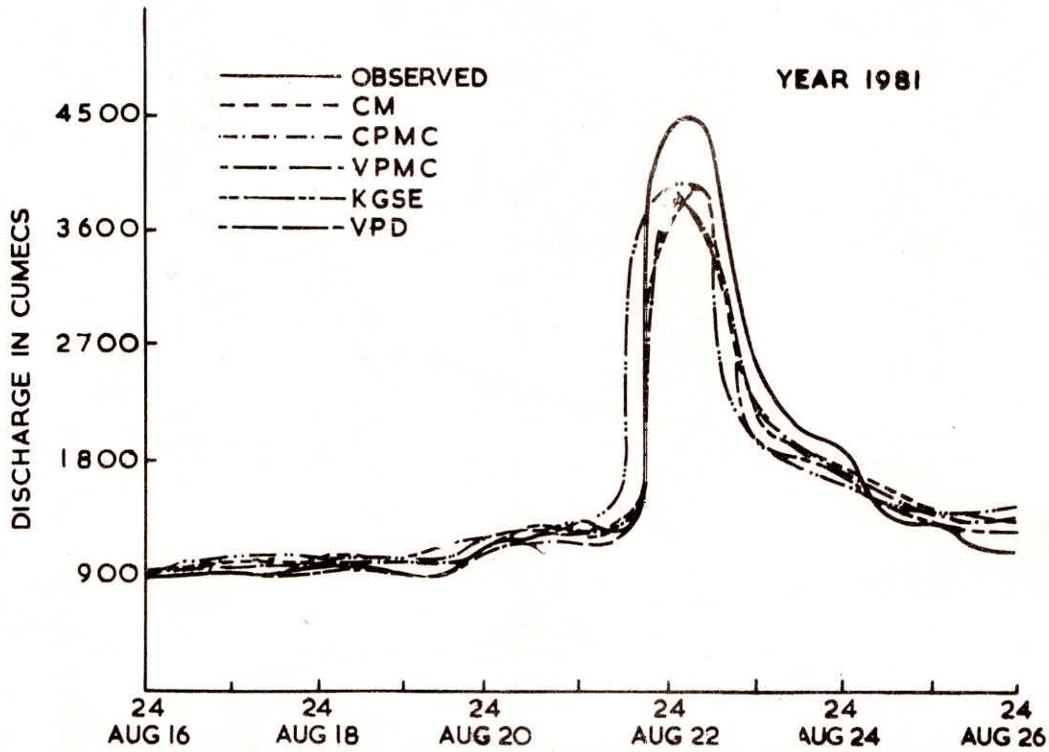

FIG.5

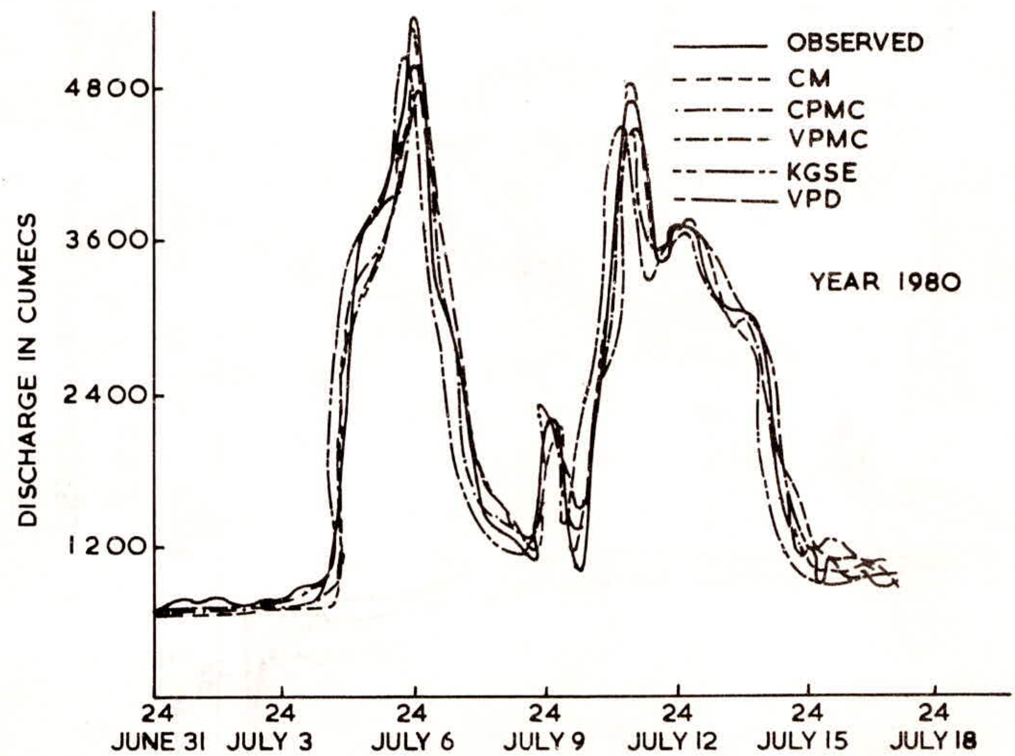

FIG.6

OBSERVED AND COMPUTED HYDROGRAPHS AT MUSIRI