OPTIMIZATION MODEL FOR CONTROLLING LAKE WATER POLLUTION

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ABSTRACT

Surface water bodies are mainly polluted by non-point sources such as runoff from cropped lands. The pollution levels in the lake water can be maintained within acceptable levels to obtain maximum possible profit from the agricultural crops. In this paper, the formulation and solution of a nonlinear optimization model is presented. The optimization model maximizes the net returns for determining the optimal areas of different crops with restrictions imposed on minimum acreage for each crop and minimum level of fertilizer pollutant in a surface water body. The applicability of the presented optimization model is shown with the help of an example. The presented methodology can be used for controlling the pollution of lake water from any source of pollution.

INTRODUCTION

There are different sources of irrigation water viz. rivers, canals, lakes, tanks, open wells, tube wells etc. All these sources of irrigation water receive nitrogen and phosphorous directly or indirectly as pollutant resulting from fertilizers applied to agricultural fields for increasing crop production. Many researchers have worked and are still working for deciding the optimal fertilizer application rates. Of course optimal rates of fertilizers will not produce the maximum crop yields, but undesirable effects on the environment for example pollution of surface water and groundwater resources may be restricted.

A major portion of the nitrogen and phosphorous loads in lake water comes through runoff from agricultural fields, grass and forest surfaces, and industrial waste of cities. The nitrogen and phosphorous losses are less from forest and grasslands than from the cultivated lands (Olson et. al., 1981). Fertilizers can enrich the material that is eroded, but they can reduce the amount of erosion by contributing to better vegetative cover. Also, fertilizers increase production on the land thus decreasing the cultivated area and keeping more land to permanent grass and forest cover. Several researchers have shown the relationship between the yield, profit and fertilizer application rate (Colwell, 1994). Most of them are in either graphical, tabular or equation form. This information can be used to decide the optimal rate of fertilizer application for maximizing the profit under desired constraints. This paper presents formulation and solution of a nonlinear optimization model for deciding the area under different crops to maintain the acceptable levels of pollutants in the lake water.

OPTIMIZATION MODEL

The profit from a cropped land can be represented by a nonlinear function of cropped area (Colwell, 1994). The nonlinear relation may be a polynomial or a power function. The coefficients of these functions will vary according to the type of soil, type of crop, agricultural practice, the use of fertilizers, irrigation and field conditions. Initially the yield/net return increases nonlinearly with increasing rate of fertilizer application. After attaining the maximum value the return starts decreasing with the further increase in the rate of fertilizer application. In addition, the excess use of fertilizers causes pollution of water bodies. With some compromise on the profit, the pollution levels in the water bodies may be kept within desirable limits. Therefore, this type of problem can be formulated and solved as optimization problem. Using a power equation, the nonlinear optimization model can be described as:

Objective Function:

$$Max\ Z = \sum_{i=1}^{N} a_i X_i^{b_i} \tag{1}$$

where i = 1, 2, . . ., N represent crop number/type with N being total number of crops, a_i and b_i are coefficients for the i^{th} crop, and X_i is the area under the i^{th} crop (ha).

Constraints:

Area constraints: If A is the total cropped area of the watershed (ha), and L_i is the minimum area under the i^{th} crop, then the following constraints should be satisfied.

$$\sum_{i=1}^{N} X_i \le A \tag{2}$$

$$X_i \ge L_i \tag{3}$$

Water quality constraint: If p_a is the allowable level of a pollutant (fertilizer) in the water body, p_i is the application rate of fertilizer in the i^{th} crop and α_i is the fraction of applied fertilizer reaching to the water body from the area under the i^{th} crop, then to maintain the acceptable levels of pollutant concentrations in the surface water bodies the following constraints should be satisfied.

$$\sum_{i=1}^{N} \alpha_i p_i X_i \le p_a \tag{4}$$

The applicability of the above optimization model is demonstrated with the help of an example using data from Haith (1982).

EXAMPLE

A watershed having a cropland area (A) of 100 ha discharges its runoff to a nearby lake. There are three crops grown in the watershed and phosphorous is applied to the crops. The rate of phosphorous applied to each crop, the percentage of phosphorous reaching to the lake in the runoff water, the minimum area under each crop and the net returns obtained from each crop as functions of area are given in Table 1. It is desired to maintain the total input of phosphorous to the lake from cropland runoff below (p_a) 800 kg/year. Develop an optimization model to maximize the net returns maintaining the desired level of pollution in the lake water.

Table 1. Data for optimization of phosphorous level in lake from cropped land

Crop no. (i)	**Rate of Application of P (p_i) (kg/ha)**	**Percentage of P Reaching to Lake (α_i)**	**Minimum Area of Crop Required (L_i) (ha)**	**Net Return (Z_i) (\$/year)**
1	40	10	0	$1000\ X_1^{1/2}$
2	50	12	15	$3000\ X_2^{1/3}$
3	60	8	5	$1200\ X_3^{1/2}$

SOLUTION

Using the given data, the optimization model was formulated with objective of maximizing the net returns from the cropped area. Substituting the values of $a_{i's}$ and $b_{i's}$ from Table 1 in Equation 1, the objective function was written as:

$$Max\ Z = 1000X_1^{1/2} + 3000X_2^{1/3} + 1200X_3^{1/2} \tag{5}$$

Substituting the values of A, and $L_{i's}$ in Equations (2) and (3), the area constraints were written as:

$$X_1 + X_2 + X_3 \leq 100 \tag{6}$$
$$X_1 \geq 0 \tag{7}$$
$$X_2 \geq 15 \tag{8}$$
$$X_3 \geq 5 \tag{9}$$

Substituting the values of $\alpha_{i's}$, $p_{i;s}$ and p_a in Equation (4) the water quality constraint was written as:

$$0.1 \times 40 \times X_1 + 0.12 \times 50 \times X_2 + 0.08 \times 60 \times X_3 \leq 800 \tag{10}$$

The optimization problem was solved using LINGO. The optimal value of net return was obtained as \$ 22409.1 and the optimal values of areas of three crops were obtained as 27.2 ha, 33.7 ha and 39.1 ha, respectively.

The same optimization problem was also solved with different values of allowable limits on phosphorous levels in the lake water. The effect of the change in the value of p_a on net return and areas are given in Table 2. It is evident from Table 2 that phosphorous level can not be maintained below 430 kg/yr. The optimal solution and the optimal returns are affected when the value of allowable level of phosphorous in the lake is changed from 430 to 500. Further, it was found that there was no effect of allowable level of phosphorous on the optimal solution when the value of p_a was increased beyond 500. It can be inferred from the solution that lower levels of phosphorous can be maintained without compromising the net return. This range of pollution level and its effect on optimal solution is applicable to this example problem only. However, there may be specific range of pollution level affecting the optimal solution for different cases. Therefore, it may be advisable to decide the pollution level limit after carrying out this type of analysis to ensure the minimum possible pollution level with the maximum possible returns.

Table 2. Effect of allowable limit of phosphorous on optimal solution

S. No.	Allowable Limit (p_a) (Kg/year)	Optimal Return (Dollar)	Optimal Area (ha)		
			X_1	X_2	X_3
1.	430	Not Feasible	-	-	-
2.	435	19272.8	78.7	15.0	6.3
3.	440	20156.0	72.5	15.0	12.5
4.	460	21662.4	50.0	16.6	33.4
5.	490	22374.6	31.4	29.3	39.3
6.	495	22403.0	28.9	31.8	39.3
7.	500	22409.1	27.2	33.7	39.1

CONCLUSION

This paper presents the formulation and solution of a nonlinear optimization model to maximize profit for maintaining the desired level of a pollutant in lake water receiving runoff from cropped lands. The optimal values of different cropped areas and their net returns are only affected to a specific range of a pollutant level to be maintained in the lake water. It will not be possible to maintain the pollutant level in the lake water below a certain minimum value. Also, there will be a particular level of pollutant above which the optimal solution will not change. These levels of pollutant can be easily determined with the help of the presented optimization model. The presented model can be easily modified and extended for controlling the pollution of lake water from any source of pollution.

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