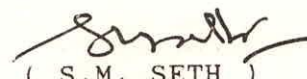# DISPERSION OF POLLUTANTS IN STREAMS

PREFACE

Water is a precious gift by the environment to the mankind. Preservation of waters and conservation of their quality thus become the responsibly of the users. The planning and management of preserve waters are not merely a question of ensuring the availability in the right quantity at the right time for diverse purposes, but also ensuring the right quality for different uses.

Streams which are in many areas serving the basic needs of water, have traditionally been used as a convenient disposal sites for various industrial and municipal wastes. Disposal of liquid effluents from sewage treatment plants and incidental releases of wastes from thermal and nuclear power stations are common in many rivers stretches in the country. Hardly, we think before the disposal about their environmental hazards to be caused to the downstream users of water. Consequences of such disposal may lead to the serious health hazards, problem to the aquatic biota, etc,. Many instances in the country e.g. river Yamuna at Delhi, river Ganga at Kanpur, varanasi etc., to mention few, are evidences of such hazards. These instances eventually tell us to control water pollution and conserve the water quality even if necessary, at the source of pollution, if needed, paying higher cost.

Effluent discharges into a river or any other waterbodies get mixed and diluted with the receiving waterbodies mainly due to the longitudinal, transverse and vertical mixing and radioactive decay. All these properties again depend on river hydraulic characteristics and varies from river to river. The study of mixing phenomenon of pollutants a rivers and variation of concentration of pollutants at different time and space are very much necessary to take preventative measures for safeguarding the downstream users from incoming pollution. The main parameter which characterizes the mixing and dispersion of pollutants along the cross section is dispersion co-efficient. Determination of dispersion coefficient is important for accurate estimation of pollution level.

Realizing the need of the study, as a first step in this direction, the Environmental Hydrology Division of the Institute has taken up the study to compile all possible information about the studies carried out on dispersion of pollutants in streams. The report titled "Dispersion of pollutants in Stream", prepared by **Shri N.C. Ghosh, Scientist C,** *Environmental Hydrology Division* contains a review of studies and the basic mechanism of dispersion, and methodology for predicting dispersion, is a contribution of work assigned to him for year 1993-'94.

The report has been prepared under the guidance of Dr. G.C.Mishra, Scientist F and Dr. K.K.S.Bhatia, Scientist F & Head Environmental Hydrology Division.

(S.M. SETH)
Director

CONTENTS

LIST OF NOTATIONS

Notation wherever it is first appeared, has been clarified. However, the most commonly used notations have further been explained below:

A .... : cross sectional area

a_0 .... : radius of the pipe

C .... : concentration of tracer mass or organic matters

$\bar{C}$ .... : Avg. concentration of tracer mass or across the stream

C' .... : deviation of tracer or organic matter from stream avg. concentration.

D .... : molecular diffusion co-efficient.

d .... : depth of flow.

g .... : acceleration due to gravity.

K .... : longitudinal dispersion co-efficient.

k .... : organic decay co-efficient.

L_m .... : mixing length

l_f .... : characteristic length, the distance from the point of maximum thread of velocity to the most furthest bank.

M .... : mass of tracer or pollutants.

Q .... : rate of flow or discharge.

q .... : solute mass.

r .... : hydraulic radius.

S .... : slope of the energy gradient line.

T_E .... : Eulerian time scale

T_L .... : Lagrangian time scale.

t .... : time

u_0 .... : maximum velocity.

u .... : velocity along the direction of flow.

$\bar{u}$ .... : average velocity across the flow direction.

u' .... :deviation of velocity from the mean flow velocity.

u_* .... :shear flow velocity.

W .... :width of the channel or stream.

x,y,z .... :cartesian co-ordinate.

α,β .... :conastant of proportionality

ε .... :eddy co-efficient.

ε_t .... :transverse mixing co-efficient.

ε_v .... :vertical mixing co-efficient.

$\varkappa$ .... :von Karman constant.

ξ .... :transformed co-ordinate of distance along direction of flow.

τ .... :transformed co-ordinate of time.

σ .... :variance of tracer particles.

$\bar{\sigma}$ .... :variance of ensemble particles.

τ_ρ .... :shear stress.

ρ .... :density of fluid.

ABSTRACT

A traditional concern is stream disposal of liquid wastes from municipal, industrial and sewage treatment plants. Beside these ,another common tendency of disposing blow down discharges from fossil or nuclear power stations to the nearest perennial open channel. For environmentalists the real concern of the problem is, how pollutants transport with the river water and when the river regenerates its assimilative capacity . The most important aspect of river transport phenomenon is; how quickly pollutants mix with river water and how concentration distribution varies in the river water. From environmental considerations, the quality and quantity of effluents need to be so regulated that at every section between the point of release and till pollutants reached a safer limit, a reasonable wide zone of uncontaminated water to be made available available for movement of aquatic biota, if necessary after allowing the dilution of effluents.

An important parameter which characterize the ability of streams in disbursing the pollutants across the depth, width and towards the direction of flow is "longitudinal dispersion co-efficient" or simply "dispersion co-efficient". It seems that dispersion is relative unimportant for steady-state, one-dimensional flows, and usually considered to be important for unsteady flows and loads which are very common in rivers. Study of dispersion of pollutants in stream has many application starting from monitoring and ending with modelling of water quality parameters for both conservative and non-conservative materials. Determination of dispersion co-efficient is not a task but requires lot of data and due care of analysis. The algorithm which describes the dispersion process is analogous to the Fickian

diffusion equation. However, there were numbers of opinion, some investigators have depicted that the Fickian diffusion equation can not be applied in the convective period (convective period has been described as the time required for pollutants to reach from point of release to the point where it is completely mixed).while others opined that Fickian equation is not the correct description for mixing pattern mainly in natural streams where stream irregularities exist. Moreover, lot many opinions are also observed for mixing pattern and about the determination of dispersion co-efficient. It is clear from the studies that dispersion co-efficient is an empirical parameter derived from eddy-diffusivity assumption.

It is attempted in the report to describe the dispersion processes in stream in detail. Efforts have been made to elaborate the algorithm which describes the dispersion process, and how it can be used and had been used to determine the dispersion co-efficient has been highlighted. Studies related to dispersion of pollutants in stream have also critically been reviewed in this report.

1.0 INTRODUCTION

Traditionally, streams have been treated as convenient disposal sites for various industrial and municipal wastes. Besides these, one of the most common means for the disposal of liquid effluents from sewage treatment plants and blow down discharges from fossil or nuclear power stations is to discharge them to the nearest perennial open channel. In recent years, more and more concern has been about the potential environmental hazards of such disposal. This is obvious because excessive disposal beyond the cleansing capacity of the river may cause serious environmental hazards to the downstream users and aquatic biota. Thus, for effective control of pollution in streams, it is necessary to understand quantitatively the mixing and transport phenomenon of pollutants receive by the streams and how and when the river regenerates its purifying capacity.

Miscible effluents advected through a perennial stream get mixed and diluted with the receiving water mainly due to longitudinal, transverse and vertical mixing, and radioactive decay with time. More precisely, when an organic waste is discharged into a stream, it is assumed that the natural processes of purification will further reduce objectionable material to a more stable form through the action of biological oxidation, dilution, and natural decay. It is obvious that the natural processes can be efficiently utilized only if the dilution process is aided by adequate natural mixing and dispersion of the wastewater after they have been introduced into the stream takes place. Generally, dilution is associated with a mixing length that is defined as the longitudinal distance required for the effluent to get completely mixed with the river flow. For large perennial streams, concentration of the contaminants beyond the mixing length is not a major environmental concern . However, potential

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contamination may exist between the points of discharge and complete mixing of effluents which may create serious environmental hazards to the aquatic biota . It needs to be seen that the quality and quantity of effluents are so regulated that at every section between the point of release and complete mixing, a reasonable wide zone of uncontaminated water is available for the movement of aquatic biota after allowing for the dilution of effluents. For this, a general guideline is necessary to limit the mixing zone to 25% of the cross sectional area or volume of flow or to 50% of the width of the stream (Prakash, 1977).

To understand the transport processes of pollutants and their mixing phenomena, it is necessary to study the behavior of pollutants when they mix with the receiving waters. Advection and dispersion are the main factors govern the transport of pollutants in the river. In case of advection, materials move with the fluid, hence their response in the downstream depend upon flow characteristics only and easy to estimate. While in case of dispersion, materials move within the fluid, as a result, concentration gradient of pollutants need to exist for their movement. Thus, the study of dispersion phenomena is important to ascertain the concentration of pollutants before the complete mixing takes place. The study of dispersion processes find application in regulating pollution sources and evaluating risks from accidental releases of pollutants. The other applications of studying dispersion are pollution forecasting, prediction of water temperature variation downstream of the source of thermal discharges, in predicting salt water intrusion into tidal estuaries, and for predicting the variation of the reaeration rates of rivers.

The report highlights the phenomena of mixing processes supported by the theory govern the process , and when and how

complete mixing takes place and their estimation procedure. A brief review of the works carried out on dispersion has also been highlighted.

2.0   DISPERSION ?

The scattering of particles or cloud of contaminants by the combined effects of shear and transverse diffusion, is called dispersion. More elaborately, when soluble or fine suspended matter is introduced into a flowing stream , it will be transported downstream, but at the same time the original contours of the zone containing the substance spreads out in every direction. The resulting pattern of the average cross-sectional concentration resembles a probability distribution along the flow axis. It is highly skewed at first, characterized by high concentration values within a short zone and tapering tail in the upstream direction . With increasing flow times the patterns and trends approach to a normal distribution as the substance spreads in both the upstream and downstream directions, as shown in Fig. 1. This process is called longitudinal dispersion.
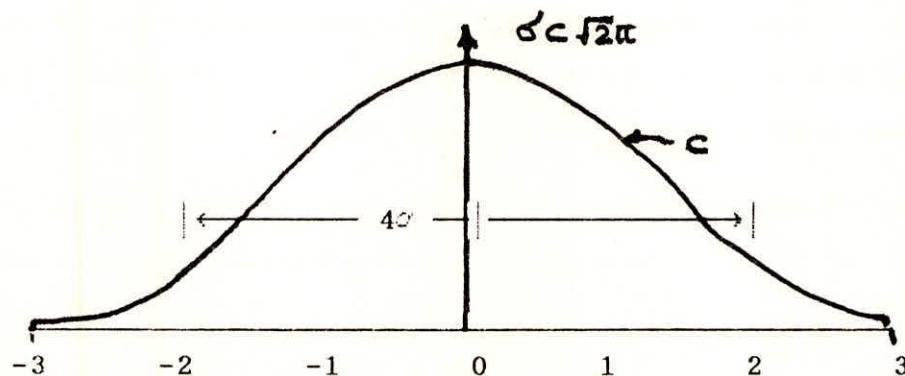

Fig. 1 : The Normal Distribution of Cloud

3.0 NECESSITY OF STUDYING DISPERSION IN STREAMS

Understanding ability of streams to disperse pollutants is essential for effective abatement of pollution in streams. Unless the mixing phenomena of pollutants and their concentration distribution across the cross-section over the distance and time is known, it would not be possible to predict reliably the occurrence of pollution to the downstream users. The important parameter which characterize this ability of stream is the co-efficient of longitudinal dispersion or simply the dispersion co-efficient. This co-efficient combines the effect of diffusion (mixing produced by turbulence) and makes it possible to spread the pollutants over a long distance along streams.

There are many applications of dispersion co-efficient. An application is pollution forecasting for an accidental discharge of lethal chemical or radioactive materials into river. Whatever the cause of accident, it is important to provide immediate information to downstream water users about the expected pollutant concentration in the river at various time and locations, which can be possible through computer analysis provided dispersion co-efficient is known.

Another application of dispersion co-efficient is the prediction of water temperature variation downstream of the source of thermal discharges, such as, from a steam or from a nuclear power plant uses river water for cooling.

The third major area in which dispersion co-efficient is needed is, in predicting salt water intrusion for tidal estuaries.

Other applications include predicting the variation of the reaeration rates in rivers, point source pollution etc..

In short, it can be said that study of dispersion co-efficient for river is absolutely important to derive the time rate of change of variation of pollutants in the river.

4.0 · ·CRITICAL REVIEW

The details review of studies carried out by different researchers have been discussed in section 9.0. A summary of the studies is included in this section.

The first study on dispersion was reported by Taylor (1953) who had described the dispersion process by one dimensional Fickian diffusion equation. Subsequently in year 1954, Taylor related longitudinal dispersion co-efficient in pipes to the shear velocity and hydraulic radius (area/wetted perimeter). Using the same reasoning, Elder (1959) extended his analysis to infinitely wide channels using the logarithmic vertical velocity profile and ignored the effect of lateral velocity gradients for one dimensional dispersion. Taylor (1954) and Elder (1959) formula for dispersion co-efficient can by no means be applied to open uniform channels and natural streams since they had been derived for different flow conditions.

After Taylor and Elder, it was actually Fischer (1966,1967,1968,1973, 1975) who greatly contributed theoretically and practically on phenomena of dispersion process and made it clear that effect of the lateral velocity gradient is very important and seems to be more important than the vertical velocity gradient. Fischer had first indicated that Fickian diffusion equation is the correct description for dispersion prediction only after the complete mixing of pollutants. He clearly defined the convective period and mixing length. Applying the Taylor's and Elder's concept, Fischer derived the algorithm for dispersion co-efficient and verified with experimental and practical data. Differences of dispersion co-efficient in natural streams had also been explained by him.

A number of laboratory studies and field studies had been attempted by different researchers namely, Krenkel (1960),Glover

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(1964), Patterson & Gloyna (1965), Thackston (1966), Hays (1966), Crunch (1967), Edward & Krenkel (1967), Sooky (1969), Bansal (1970), Gofrey and Frederick (1970), Chatwin (1970,1971), McQuivery and Keefer (1974), Day (1974,1975), Jain (1976), Liu (1977), Beltaos (1980), and Dorbran (1982) came out with different degree of success in predicting dispersion co-efficient. The reported values of dimensionless dispersion co-efficient (K/du_*) observed by different researchers were ranged between 1.75 to 7500. Most of their analysis were either based on the Taylor's concept or on the derivation given by Fischer. But very little effort had been done to advance the understanding the dispersion process. The studies of McQuivery and Keefer (1974), and Liu (1977) were some thing different and they had tried to co-relate the other channel parameters with the dispersion co-efficient.

It was clearly noted by several investigators that the assumption regarding the Fickian diffusion definition of the dispersion coefficient may not be universally valid and offered supplemental corrections for other effects that are neglected or poorly represented. Fischer (1966,1967,1968) noted that Fickian dispersion is not valid description for initial mixing period. Hays et al. (1966) and other investigators had attempted to include the corrections for dead zone. Jayawardana and Lui (1983, 1984) developed a time dependent dispersion model based on the lagragian correlation. In addition, a number of different conceptual approaches have been proposed to explain the non-Fician behavior observed in the study.

Despite the impressive efforts to discover appropriate phenomelogical descriptions to cover a wide range of stream conditions or to cover narrow and important classes of conditions, there are little real guidance presently available. A number of investigators seem to imply that well-defined guidance cannot be

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reasonably derived by suggesting that the best approach is to measure dispersion coefficients in the field.

This eventually indicate that the dispersion co-efficient would be an empirical parameter which could be derived from stream hydraulic and geometric characteristics.

5.0   THEORY OF DISPERSION

The phenomenon of dispersion is strongly analogous to the process of molecular diffusion. Though, molecular diffusion by itself is not of direct consequence in environmental problems, except on the microscopic scale of chemical and biological reactions. However, it is necessary to understand the process of molecular diffusion and thereafter, it can be seen how it is related with the dispersion phenomena of rivers.

5.1   Diffusion Process

The diffusion process was, first described by German physiologist, Adolph Fick, which is known as Fick's law. Fick's law says that the flux of solute mass, i.e., mass of a solute crossing a unit area per unit time in a given direction, is proportional to the gradient of solute concentration in that direction.

For one dimensional diffusion process, Fick's law can be represented mathematically as;

$$q = - D \, \partial C/\partial x \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

where,   q = solute mass

D = co-efficient of proportionality ($length^2/time$) or, molecular diffusion.

$\partial C/\partial x$ = solute concentration gradient in the direction of flow.

Considering, a one dimensional transport process in which

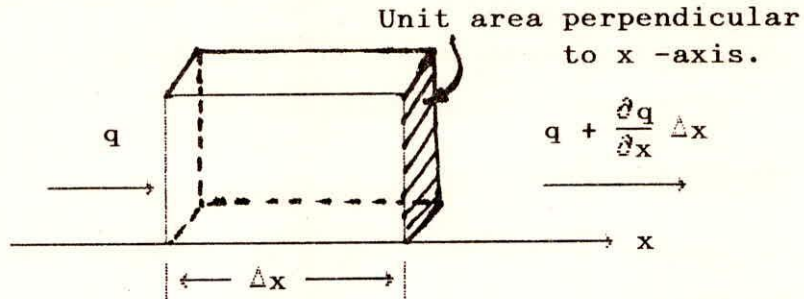mass is being transferred in the x direction as illustrated in Fig. 2.

Fig. 2 : Control Volume

Using mass balance theory for the control volume, one can have, i.e.,(Molecules in - Molecules out) of the volume defined by the boundary source = time rate of change of mass in the volume, i.e.,

$$\frac{\partial C}{\partial t} = - \frac{\partial q}{\partial x} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (2)$$

For molecular diffusion processes we also have Fick's law Equ. (1) which can be substituted in equation (2) to give,

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Equation (3) is known as diffusion equation and describe how mass is transferred by Fickian diffusion processes. In full cartesian coordinate system, the equation can be written as;

$$\frac{\partial C}{\partial t} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) \dots\dots\dots\dots\dots\dots(4)$$

For the most fundamental solution to equation (3), consider an initial slug of mass M introduced at time zero at x origin, and the slug spreading by diffusion only. Since the process is linear, C is proportional to the mass introduced and can be a function of M,x,t and D. Applying the dimensional analysis with appropriate

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dimensions of the dependent variables, we have,

$$C = \frac{M}{(4\Pi D t)^{1/2}} f\left(\frac{x}{(4 D t)^{1/2}} \right) \quad \ldots\ldots\ldots\ldots(5)$$

Transforming equation (3) into an ordinary differential equation by defining $\eta = x/(4 D t)^{1/2}$ and substituting equation (5) into equation (3) to obtain;

$$\frac{df}{d\eta} + 2 \eta f = 0, \text{ which has the solution, } f = C_0 e^{-\eta^2} \quad \ldots(6)$$

The total mass contained in the system can be found by integrating the concentration along the whole x- axis, i.e,

$$\int_{-\infty}^{\infty} C \, dx = M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Substituting, equation (5) and (6) in equation (7), and performing the integration with $C_0 = 1$ for all times;

$$C(x,t) = \left[\frac{M}{(4\Pi D t)^{1/2}} \right] \exp \left(- \frac{x^2}{4 D t} \right) \quad \ldots\ldots\ldots\ldots(9)$$

Equation (9) is the required fundamental solution of the diffusion equation (3).

5.2   Combined effect of Advection and Diffusion

Equation (3) and (4) represent the diffusion equation for the fluid in stationary condition and that the mass transport is by diffusion only. Suppose the fluid itself is moving with a mean velocity u. Then transport of material will also take place by the motion of the fluid, which is called advection, besides the diffusion processes. To represent and analyze the process of advection and diffusion of materials transported by the fluid, one must define the variables affecting the concentration and transportation. Let us assume, a mass of pollutant (QC) is transported through an area (A) in the yz plane with a mean

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velocity u. From the principle of conservation of mass, the amount of material passing any point will equal the amount flowing past by normal displacement Q.C, minus the amount of material exchanged as a result of diffusion of the material which is D.A $(\partial C/\partial x)$, $(\partial C/\partial x)$ is the change in concentration with distance. The total mass transfer across the section considered is $(Q.C - D.A.\partial C/\partial x)$. The increase in material contained in an element of volume A.dx during time dt must equal the quantity of material entering less that leaving, and is equal to A.dx.dt.$\partial c/\partial t$. The amount of material flowing into the element will be Q.C. dt $- D.A.\partial C/\partial x$. The amount of material flowing out of the element is :

$$Q.C.dt + Q.(\partial C/\partial x) \, dt.dx - D.A.\frac{\partial}{\partial x}\left(C + \frac{\partial c}{\partial x}\, dx\right) dt$$

$$= Q.C.dt + Q.\frac{\partial C}{\partial x}.dt.dx - D.A.dt\left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2}.\, dx\right)$$

The amount of material passing any point;

$$\frac{\partial C}{\partial t}.\, A\, dx.dt = Q.C.dt - D\, A.\frac{\partial C}{\partial x}\, dt - \left[Q.C.dt + Q.\frac{\partial C}{\partial x}\, dt.dx\right.$$

$$\left. - D.A.dt\left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2}.\, dx\right)\right]$$

from which,

$$\frac{\partial C}{\partial t} + u\, \frac{\partial C}{\partial x} = D.\frac{\partial^2 C}{\partial x^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots(10)$$

$$\downarrow \qquad\qquad \downarrow$$

Advective   Disffusive
term   term

Equation (10) is the fundamental one dimensional advection diffusion (A-D) equation for the fluid transporting solute. For full cartesian coordinates, the equation (10) can be written as;

$$\frac{\partial C}{\partial t} + u\, \frac{\partial C}{\partial x} + v\, \frac{\partial C}{\partial y} + w\, \frac{\partial C}{\partial z} = D\left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right]\dots(11)$$

5.3   Basic Process of Dispersion

Referring definition of dispersion which indicates that dispersion takes place by the combined effects of shear and transverse diffusion. It is, therefore, necessary to understand the shear effect on the molecular diffusion. Taylor in his work published in 1953 for dispersion in laminar shear flow described that if two molecules are carried in the flow one at the center and one near the wall, the rate of separation caused by the difference in advective velocity will exceed that caused by molecular motion. The difference in rate of separation beyond the advective velocity had been found to be because of their thermal molecular motion which is called molecular diffusion.

It was also described that the velocity of any single molecule is equal that of the stream line on which it is located; a function of the cross sectional position. Because of the molecular diffusion each molecule moves at random back and forth across the cross section and after some long time "forgetting time" its location is independent of the location at which it started. Therefore, its velocity is independent of its initial velocity. Thus, the motion of a single molecule rely on the sum of the series of independent steps of random length. If we consider the transformed co-ordinate system moving at the mean velocity the random steps are, with respect to the moving co-ordinate system, equally likely to be backward as forward, since the mean motion is zero. Since, the step length and time increment are different from those of molecular diffusion, it is better to find a different value of diffusion co-efficient which is also known as dispersion co-efficient for shear flow description rather than step length and time increment.

To understand the dispersion process in the shear flow, let us begin with its behavior in laminar flow and then see how it changes in the turbulent flow and finally in streams.

5.3.1 Dispersion in Laminar Shear Flow

Let us consider, the two dimensional flow guided between parallel walls by a distance d, so that all flow lines are parallel to the walls. The velocity variation between the walls is given by u(y) (as illustrated in Fig. 3), and the mean velocity is u.
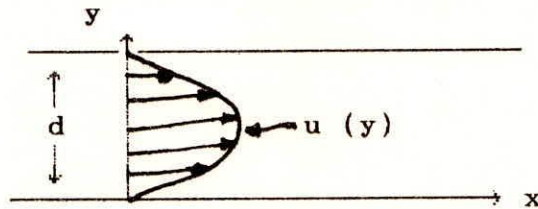

Fig. 3 : Velocity Variation

The mean velocity can be found out by integration;

$$\bar{u} = \frac{1}{d} \int_0^d u \, dy \qquad \dots\dots\dots\dots\dots\dots\dots\dots(10)$$

and the deviation of the velocity from the cross-sectional mean is defined as; $u'(y) = u(y) - \bar{u}$.

Similarly, the mean concentration at any cross section of flow for the solute concentration C (x,y) would be;

$$\bar{C} = \frac{1}{d} \int_0^d C \, dy \dots\dots\dots\dots\dots\dots\dots\dots(11)$$

deviation from the mean is $C'(y) = C(y) - \bar{C}$.

Since the flow in the x direction , the diffusion equation (10) can be written as;

$$\frac{\partial}{\partial t} (\bar{C} + C') + (\bar{u} + u') \frac{\partial}{\partial x} (\bar{C} + C') = D \left[\frac{\partial^2}{\partial x^2} (\bar{C} + C') + \frac{\partial^2 C}{\partial y^2} \right]$$

$$\dots\dots\dots\dots(12)$$

Equation (12) can be simplified by transformation to a coordinate system whose origin moves at the mean flow velocity (as illustrated in Fig. 4).

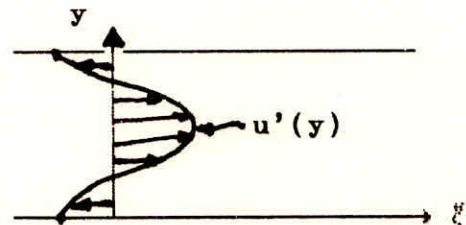Let, $\xi = x - \bar{u}.t$, $\tau = t$

Fig. 4 : Velocity Variation in transformed coordinate.

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by 'chain rule' of calculus,

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x} \cdot \frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial x} \cdot \frac{\partial}{\partial \tau} = \frac{\partial}{\partial \xi}$$

$$\frac{\partial}{\partial t} = \frac{\partial \xi}{\partial t} \cdot \frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial t} \frac{\partial}{\partial \tau} = - \bar{u} \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \tau}$$

Replacing the above value in Eq. (12) to give

$$\frac{\partial}{\partial \tau} (\bar{C} + C') + u' \frac{\partial}{\partial \xi} (\bar{C} + C') = D \left[\frac{\partial^2}{\partial \xi^2} (\bar{C} + C') + \frac{\partial^2 C'}{\partial y^2} \right] \dots (13)$$

The transformation to the ξ, τ system allows to view the flow as an observer moving at the mean velocity. In the moving co-ordinate system one will have only observeable velocity u'. If we consider the Taylor's observation that the rate of spreading along the flow direction caused by the difference in advective velocity should greatly exceed that due to molecular diffusion the longitudinal diffusion term can be neglected in that case , the equation (13) then reduces to :

$$\frac{\partial}{\partial \tau} \bar{C} + \frac{\partial}{\partial \tau} C' + u' \frac{\partial \bar{C}}{\partial \xi} + u' \frac{\partial C'}{\partial \xi} = D \frac{\partial^2 C'}{\partial y^2} \dots \dots \dots (14)$$

Solution of Eq.(14) in general is very diificult to find, however, Taylor obtained the solution of the equation by discarding first two and fourth terms . This is only possible if the cross sectional deviation C' is everywhere on the cross-section much less than the mean value $\bar{C}$, the term $\partial \bar{C}/\partial \tau$ and u' $\partial C'/\partial \xi$ are much smaller than u'$\partial \bar{C}/\partial \xi$ and may be neglected. For steady state condition, the variation of concentration is also very small, therefore, $\partial C'/\partial \tau$ is possible to ignore. After discarding all these terms, the terms left with only advective and diffusion term (as given below) and appeared that they are balanced. This is what Taylor derived and find out the following differential equation;

$$u' \frac{\partial \overline{C}}{\partial x} = D \frac{\partial^2 C'}{\partial y^2} \quad \text{with} \quad \frac{\partial C}{\partial y} = 0 \;, \text{ at } y = 0, h \;\; \ldots\ldots(15)$$

The Eq. (15) has the solution ;

$$C'(y) = \frac{1}{D} \frac{\partial \overline{C}}{\partial x} \int_0^y u' \int_0^y u' \, dy \, dy + C'(0) \;\ldots\ldots\ldots(16)$$

For the mass transport in the stream wise direction relative to the moving coordinate axis is given by;

$$M = \int_0^d u'C' \, dy = \frac{1}{D} \frac{\partial \overline{C}}{\partial x} \int_0^d u' \int_0^y \int_0^y u' \, dy \, dy \, dy \;\ldots\ldots(17)$$

The extra term $\int_0^d u'\{C'(0)\} \, dy = 0$, because $\int_0^d u' dy = 0$

[as there is no deviation of velocity from mean cross sectional velocity]

The equation (17) indicates that the total mass transport in the stream wise direction is proportional to the concentration gradient in the stream wise direction. Same concept is also applicable for molecular diffusion. The Eq. (17) reflects the integrated sense for diffusion in the flow direction due to the whole field of flow.

The bulk transport coefficient, or "dispersion" coefficient, in analogy to the molecular diffusion coefficient could be described as;

$$M = - K \, d \, \frac{\partial \overline{C}}{\partial x} \quad \ldots\ldots\ldots\ldots\ldots(18)$$

The dispersion coefficient K expresses the diffusivity property of the velocity distribution and is generally known as the "longitudinal dispersion coefficient". Equation (17) and (18) give the relation of dispersion coefficient as follows;

$$K = \frac{-1}{dD} \int_0^d u' \int_0^y \int_0^y u' \, dy \, dy \, dy \;\ldots\ldots\ldots(19)$$

K plays the same role for the whole cross section as does

D, the molecular diffusion coefficient, on a microscopic scale. Thus the diffusion equation for cross sectional averages with moving coordinate system could be written as;

$$\frac{\partial \overline{C}}{\partial \tau} = K \frac{\partial^2 \overline{C}}{\partial \zeta^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (20)$$

which in fixed co-ordinate system indicate;

$$\frac{\partial \overline{C}}{\partial t} + \overline{u} \frac{\partial \overline{C}}{\partial x} = K \frac{\partial^2 \overline{C}}{\partial x^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (21)$$

Equation (21) is known as the "one dimensional dispersion equation" and it is used widely in the analysis of dispersion in environmental problems, like rivers and estuaries. Which for an instantaneous source extending throughout the cross section at x=0, has the solution,

$$\overline{C} = \frac{M}{(4 \Pi A^2 K t)^{1/2}} \exp \left[- \frac{(x - \overline{u} t)^2}{4 K t} \right] \quad \dots\dots\dots\dots (22)$$

Equation (22) is the expression for Gaussian Distribution. If the mass is unity, it is known as the normal distribution , as shown in Fig. 1.

5.3.2 Dispersion in Turbulent Shear Flow

Because of the difference of velocity profile in the turbulent flow than the laminar flow in the same cross section , the cross sectional turbulent mixing co-efficient will play the role of molecular diffusion as in the laminar flow i.e, $\varepsilon(y)$, cross sectional mixing coefficient or "eddy co-efficient", will be in the place of D, molecular diffusion coefficient. The only significant difference in mathematics is that the cross sectional mixing coefficient $\varepsilon(y)$ which is a function of cross sectional position y. Therefore, the diffusion equation as described in equation (15) can be written for unidirectional turbulent flow

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between parallel plates as given below;

$$\bar{u}' \frac{\partial \bar{C}}{\partial x} = \frac{\partial}{\partial y} \epsilon(y) \frac{\partial C'}{\partial y} \quad \dots\dots\dots\dots\dots\dots\dots(23)$$

and, the corresponding dispersion coefficient;

$$K = \frac{-1}{d} \int_0^d u' \int_0^y \frac{1}{\epsilon} \int_0^y u' \, dy \, dy \, dy \quad \dots\dots\dots\dots\dots(24)$$

5.3.3 Physical Meaning of Balancing Advection and Dispersion (as obtained in equation (15))

Equation (15) indicates that the advective and diffusion term balance at one place, when this actually occur. Let us consider that at some initial time t=0 a line source of tracer is depicted in the flow as shown in Fig. 5 (a)
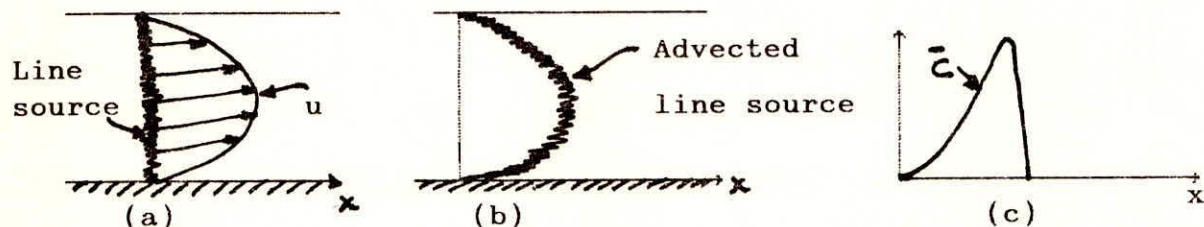
Fig. 5 : Longitudinal Distribution by Advection of Line Source.

The actual initial distribution of the tracer could easily visualize as a line source. The initial line source is advected and distorted by the velocity profile and the distorted line source begins to diffuse across the cross section, as shown in Fig. 5 (b). During this period advection and diffusion, are by no means in balance. The cross sectional average concentration distribution is highly skewed during this period (Fig. 5 (c)).

If we wait a much longer time than it could be observed that cross sectional average concentration ($\bar{C}$) varies slowly along the channel, and $\partial \bar{C}/\partial x$ is approximately constant over a long period of time. C' becomes small because cross sectional diffusion evens out cross sectional concentration gradients. This is the

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position where advective and diffusion balance and described by the Taylor as shown in equation (15). Once the balance is established further spreading follows by the longitudinal dispersion as described by the equation (22), whose solution after sufficient long time is normally distributed cloud moving at the mean speed $\bar{u}$ and continuing to spread according to $\frac{1}{2}(d\sigma^2/dt) = K$.

6.0   Phenomena of Mixing of Pollutants in River

Consider a stream of effluent discharged (as shown in Fig.6 (a)). The mixing and dispersion processes what happens in the river can be divided into three zones. In the first stage (near the discharge - as indicated by mark A), the mixing is accomplished by the action of buoyant jets, a phenomenon governed by the momentum and buoyancy of the discharge rate of dilution. As the waste is diluted, the effects of the initial momentum and buoyancy are also diluted, leading to the second stage in which the waste is mixed across the receiving channel primarily by turbulence in the receiving stream.(point B). Finally, when the effluent is fully mixed (point C) across the stream the process of longitudinal shear flow dispersion will tend to erase any longitudinal concentration variations. This last stage is, like the zone where Taylor's analysis of longitudinal dispersion in pipes is applicable, and one can find that there is an equivalent analysis for longitudinal dispersion in rives. Sometimes, the first stage may extend over the entire channel, effectively eliminating the second stage.
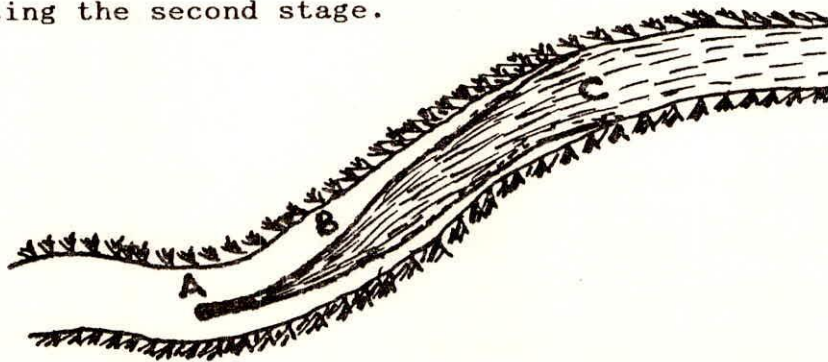
Fig 6 : Three Stages in the mixing of an Effluent into a River.

Lateral and vertical mixing are also predominant in the turbulent mixing. For one dimensional stream water quality modelling the calibration errors in the longitudinal dispersion co-efficients arise because lateral and vertical mixing effects lumped together with longitudinal mixing. This can be illustrated by the Figure 7 (a) and (b).
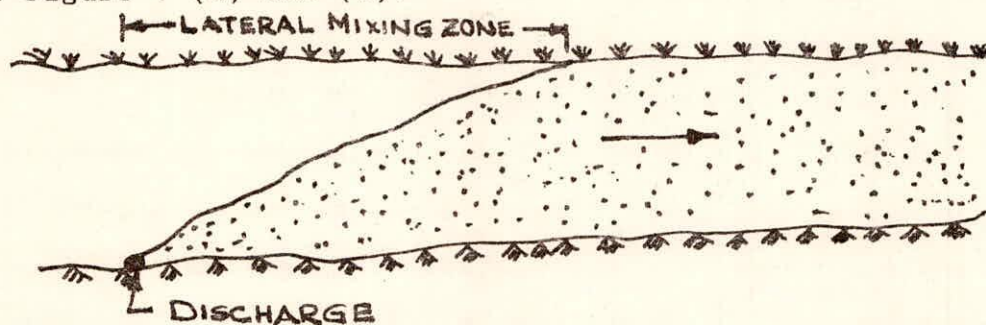
Fig. 7 (a) : Aerial View of the River
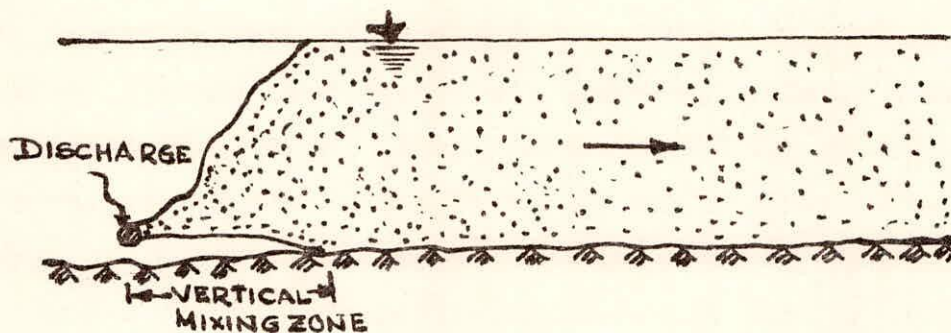
Fig. 7 (b) : Side View of the River

Fig 7 : Zone of lateral and Vertical mixing for discharges into a Stream.

6.1   Vertical Mixing

Vertical mixing is mainly because of the variation of vertical velocity profile and turbulent causes by bottom shear stress. The length requires to develop the complete vertical mixing depends on the width ,depth of flow and the density of flowing water. In a study it has been shown that if B/W u_*^3 (B = $(\Delta\rho/\rho)$g.Q), where, ρ = density of the receiving water, $\Delta\rho$ = difference of density between receiving water and effluent waters, g = acceleration due to gravity, Q = effluent discharge, W = width

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of the flow in the channel, $u_* =$ shear velocity) is less than one, vertical mixing will be independent of density effects. Or in otherward, one can say if $B/W \, u_*^3$ is very large, an effluent will spread rapidly across the channel in the form of a density driven current and will likely be to form a layer at the surface. Also, if $B/W \, u_*$ is large, the vertical mixing of the surface or bottom layer will be slow. The coefficient of vertical mixing can be derived from the log-arithmetic law velocity profile as derived by Elder (1959) as given below;

$$\epsilon_v = \varkappa \, d \, u_* (\, y/d) \, [1 - (y/d)] \dots\dots\dots\dots\dots (25)$$

[$\varkappa$ = von Karman constant, y = vertical co-ordinate of the flow]

Jobson and Sayre (1970) observed from their experimental study of the vertical mixing of dye in a flume that ϵ_v can be represented as;

$$\epsilon_v = 0.067 \, d \, u_* \, , \text{ for } \varkappa = 0.4. \dots\dots\dots\dots\dots (26)$$

6.2   Transverse Mixing or Lateral Mixing

Transverse mixing is caused by the bottom generated by the turbulence and it is the most dominated factor in dispersion . As there is no transverse velocity profile in an infinitely wide channel it is not possible to establish a transverse analogy to determine the transverse mixing coefficient. As a result, lot of experimental studies were conducted to co-relate the transverse coefficient with the channel hydraulic properties. Okoye (1970) gave a plot of ϵ_t/du_* versus width to depth ratio (W/d) and found the most scattered values. In an another study (Lau and Krishnappan, (1977)) it was possible to minimize the scatter of large data points by plotting ϵ_t/Wu_* versus W/d. However, for practical purposes these parameters are difficult to determine. Fischer (1977) developed the transverse mixing coefficient for a straight rectangular channel. In almost all cases the

nondimensional transverse mixing coefficient, ε_t/du_* has been in the range of 0.1 - 0.2. For practical purposes in straight rectangular channels the following expression had been suggested which can be applied within an error bound of approximately ± 50%.

$$\varepsilon_t = 0.15 \, d \, u_* \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

6.3 Longitudinal Mixing

Turbulence causes longitudinal mixing at about the same rate as transverse mixing because there is an equal lack of boundaries to inhibit motion. However, in any case, longitudinal mixing by turbulent eddies is unimportant because the shear flow dispersion coefficient caused by the velocity gradient is much higher than the mixing coefficients caused by the turbulence one. Downstream of the mixing zone, the longitudinal mixing is dominant because nonuniform lateral and vertical velocity profiles cause water in the center of the stream away from the bottom to move faster and overtake slower moving water downstream. The same process occurs in the mixing zone but, in addition, mixing due to vertical and lateral gradients are also superimposed. After a tracer has become adequately mixed across the cross section, the final stage in the mixing process is the reduction of longitudinal gradients by longitudinal dispersion.

6.4 Turbulent Mixing in Rivers

To understand the turbulent mixing, let us consider, mixing in the idealized case of an infinitely wide channel of constant depth, and then see how the results can be applied to compute rates of mixing across the real stream.

Turbulent mixing can be described by the diffusion equation with a turbulent mixing coefficient in place of the molecular diffusion coefficient and such a coefficient in turbulent diffusion can be applied only after the diffusion particles have

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been spread and covered a distance larger than the Lagrangian
time scale (a measure of how far a particle travels before it
forgets its initial velocity). The turbulent mixing coefficient is
given by ;

$$\varepsilon_{\kappa} = T_{L} \ [\ \langle U^2 \rangle \ ]^{1/2} \dots\dots\dots\dots\dots\dots\dots\dots\dots (28)$$

where, T_{L} = Lagrangian time scale

$\langle U \rangle$ = intensity of turbulence

It has been shown (Laufer ,1950) that turbulence intensity
in any wall shear flow is proportional to the shear stress on the
wall. The flow of water in a straight channel of constant depth
and larger width meets all of the requirements for the use of
turbulent mixing co-efficient. If the side walls are very far
apart, the width of the flow plays no role, and, in such cases,
depth is the important length scale. A cloud of tracer deposited
in such a flow will grow until it fills the depth, and then will
continue to grow in the directions of the length and breadth. The
Lagrangian length scale will be some multiple or fraction of the
depth. Experiments showed that it is approximately equal to the
depth. Therefore, once the cloud extends over several depths, the
requirements for the use of a constant turbulent mixing
coefficient will be fulfilled. Thus, for the idealized flow,
turbulent mixing co-efficient must be proportional to the product
of depth and shear velocity. This indicates that there will be one
coefficient for vertical mixing and another for transverse and
longitudinal mixing. The presence of horizontal boundaries at
surface and bottom means that the turbulence will not be
isotropic.

Natural channels differ from uniform rectangular ones
basically for three important aspects: i) the depth may vary
irregularly, ii) the channel is likely to curve, and iii) large

side wall irregularities. None of these factors have much influence on the rate of vertical mixing, since the vertical motion is limited by the local depth. On the other hand, the rate of transverse mixing is strongly affected by the channel irregularities because they are capable of generating a wide variety of transverse motions. Bends and side wall irregularities are common to many channels and have major effect on transverse mixing. The bigger the irregularity the faster the transverse mixing.

The channel curvature has also effect on transverse mixing. Higher value of ε_t/du_* had been observed in sharply curveying channels. When a flow rounds a bend the centrifugal forces induce a flow towards the outside bank at the surface, and compensates reverse flow near the bottom. To predict the transverse dispersion coefficient based on the shear flow, Fischer (1969) described that the result can be written as

$$\frac{\varepsilon_t}{du_*} = \left[\frac{\bar{u}}{u_*}\right]\left[\frac{d}{R}\right] \dots\dots\dots\dots\dots(29)$$

R = the radius of the curve.

In case of straight, uniform channel the values of ε_t/du_* were reported in the range of 0.1 - 0.2 with an average of 0.15 (for practical purposes). Curves and sidewalls irregularities increase the coefficient in natural streams to 0.4. If the stream is slowly meandering and sidewall irregularities are moderate, ε_t/du_* values varies between 0.4- 0.8. For practical purposes, one can use the following formula for all cases;

$$\varepsilon_t/du_* = 0.6 \pm 50 \text{ \%} \dots\dots\dots\dots\dots\dots(30)$$

6.5   Mixing Time and Mixing Length

Mixing time and mixing length are the two important factors related to the dispersion phenomenon in natural stream. The diffusion equation has the limit to apply in places where tracer follow the Gaussain distribution.

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As mentioned in the foregoing discussion, the spreading of marked fluid particles occur by the combined action of a nonuniform velocity distribution and diffusion. Downstream of the injection point, the cross sectional distribution of particle concentration will eventually become nearly uniform and independent of the geometrical configuration of the source. The time required from injection to become pollutants fully mixed i.e., the point where cross sectional distribution become uniform, is called mixing time. Or in otherwords, the measure of the length of the convective period that is initial period in which longitudinal dispersion is not properly described by the one-dimensional diffusion equation. The distance corresponding to the this time scale from the source is called mixing length.

6.5.1 Determination of Mixing Time

The time scale can be derived by two ways; a) by expressing the time required for cross sectional mixing which is called Eulerian Time Scale, and b) by evaluating the Lagrangian integral time scale.

For the cross sectional mixing i.e Eulerian time scale, the time scale is proportional to the square of the length divided by the mixing coefficient. Mathematically,

$$T_E = l^2/\varepsilon_v \qquad \dots\dots\dots\dots\dots\dots (31)$$

To describe the dispersion by diffusion equation, it is necessary for the motion of each tracer particle not to correlate with its initial velocity. The time requires for this to occur is measured by Lagrangian time scale,(a measure of how far a particle travels before it forgets its initial velocity) defined as;

$$T_L = \int_0^\infty \psi(\tau) \, d\tau \qquad \dots\dots\dots\dots\dots\dots (32)$$

in which, $\psi(\tau) = \dfrac{\langle U(t).U(t+\tau)\rangle}{\langle U\rangle^2}$

$\psi(\tau)$ = Lagrangian auto correlation function.

The ensemble average $\langle U(t).U(t+\tau)\rangle$ means the average over a large number of trials of the product of the velocity of a single particle, τ is any value of time which is different than t. If a statistically large number of tracer particles are distributed uniformly over a cross section at $\xi = 0$ and at t = 0 . Taylor has shown that at any later time the statistical distribution of particles is given by;

$$\langle X^2(t)\rangle = 2 \langle U^2\rangle \int_0^t (t-\tau)\, \psi(\tau)\, d\tau \quad \ldots\ldots\ldots\ldots\ldots(33)$$

in which $\langle X^2\rangle$ is equivalent of the variance of the tracer particle cloud, $\sigma^2 \langle X\rangle$, defined by;

$$\sigma^2 \langle X\rangle = \dfrac{\int_{-\infty}^{\infty} \overline{C}\, \langle X^2\rangle\, dX}{\int_{-\infty}^{\infty} \overline{C}\, .dx} \quad \ldots\ldots\ldots\ldots\ldots\ldots(34)$$

If the one dimensional dispersion equation is a correct description of the dispersion process, the co-efficient may be evaluated as;

$$K = -\dfrac{1}{2}\ .\dfrac{d}{dt}\ \sigma^2 \langle X^2\rangle \quad \ldots\ldots\ldots\ldots\ldots(35)$$

Differentiating equation (33), the dispersion coefficient yields,

$$K = \langle U^2\rangle \int_0^t \psi(t)\, d\tau \quad \ldots\ldots\ldots\ldots\ldots(36)$$

For open channel flow, the mean turbulence level is sufficiently small compared to the deviations within the cross section of time averaged velocity so that

$\langle U^2\rangle = \overline{u}^2$

Assuming, t $\longrightarrow \infty$, equation (36) yields

$K = \overline{u}^2 .T_L \quad \ldots\ldots\ldots\ldots\ldots(37)$

From the above derivation, the time scale for both the

condition can easily be determined for any flow condition.

6.5.1.1 Two Dimensional Flow

i) Eulerian Time Scale : For the Eulerian scale the characteristics length is the depth of flow (d). The average value of the mixing co-efficient is vertical mixing co-efficient which is calculated from the distribution of shear and Reynolds analogy. For the two dimensional flow with logarithmic velocity profile as used by the Elder, the vertical mixing co-efficient is given by;

$$\varepsilon_v = \frac{\varkappa}{6} . d u_* \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (38)$$

$\varkappa$ = von Karman constant.

Putting the value in equation (31), the time scale for cross sectional mixing or Eulerian time scale is given by;

$$T_E = \frac{6}{\varkappa} . \frac{d}{u_*} \quad \dots\dots\dots\dots\dots\dots\dots\dots (39)$$

ii) Lagrangian Time Scale : Average velocity distribution in case of logarithmic law is ;

$$\bar{u}^2 = (\frac{u_*}{\varkappa})^2$$

Lagrangian time scale from equation (37)

$$T_L = \frac{K}{\bar{u}^2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (40)$$

Elder (1959) described the dispersion co-efficient using the von Karman constant as;

$$K = \frac{0.404}{\varkappa^3} . d u_* \quad \dots\dots\dots\dots\dots\dots (41)$$

Using the value of K in equation (40), Lagrangian time scale is given by ;

$$T_L = \frac{0.404}{\varkappa} . d u_* \quad \dots\dots\dots\dots\dots (42)$$

Comparing the Eulerian and Lagrangian time scales , i.e., equation (39) and (42), one can have;

$$T_E = 14.8 \ T_L \quad \dots\dots\dots\dots\dots\dots\dots\dots (43)$$

which indicates that Eulerian time scale is 14.8 times more than the Lagrangian time scale.

6.5.1.2 For Natural Streams

For natural streams, transverse mixing is predominant, thus transverse mixing co-efficient governs the Eulerian time scale. The important length, in that case, is the distance over which mixing takes place to establish a uniform distribution ; for symmetric channels length scale is half width. Since most channels are not symmetric, the characteristic length, l_f, which is defined as the distance between the thread of maximum velocity and the furthest distant point within the cross section, would be approximately the distance from the point of maximum surface velocity to the most distant bank. The characteristic mixing co-efficient will,therefore, be the transverse mixing co-efficient. The local depth could be replaced by the hydraulic radius (r).

The Eulerian time scale in that case would be,

$$T_E = \frac{l_f^2}{\varepsilon_t} = \frac{l_f^2}{0.23 \, r \, u_*} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(44)$$

where, $l_f = w/2$ (w = width of the channel)

$\varepsilon_t = 0.23 \, ru_*$ (turbulent mixing co-efficient as obtained by Elder (1959) for a flow of 1 cm deep down a water table.)

The corresponding Lagrangian time scale from equation (43)

$$T_L = T_E / 14.8 = 0.30 \frac{l_f^2}{r \, u_*} \quad \dots\dots\dots\dots\dots\dots\dots\dots(45)$$

The dispersion co-efficient corresponding to this time scale is given by (putting the value in equation (37)).

$$K = 0.30 \, \bar{u}^2 \frac{l_f^2}{r \, u_*} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(46)$$

Dorban (1982) postulated that in natural streams large and small errors can occur in calculation of characteristic length due

to the deviation of maximum surface velocity, mainly for the nonuniform and wandering channel. In order to obtain more faithful value of the characteristic length for a reach of a turbulent stream, he suggested that the value of deviation from the axis of the stream be bound taking into account the sign which can either be plus (+) or minus (-).

The average deviation of the mainstream from the axis of the stream allows one to obtain the new characteristics length for the given reach as follows :

$$l_a = l_d + l_{avg.} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(47)$$

where, l_a = new characteristics length.

l_d = avg. deviation of the mainstream from the axis of the stream (the point of maximum surface velocity)

l_{avg} = avg. value of the distance of the stream axis from the bank (i.e.,half width of the stream).

6.5.3 Determination of Mixing Length

The mixing length is the channel length required for the tracer to completely mix through out the cross section. The reasonable criterion for the distance require for complete mixing is given by;

$$L_m = \alpha \, \bar{u} \, l_f^2 \, / \varepsilon_t \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots(48)$$

where, α = dimensionless time scale = t/T_L

Fischer (1968) suggested that for reasonably practical criterion for the end of the convective period, α = 0.4 , while chatwin (1972) suggested that $\alpha \geqq 1.0$ is required for the concentration distribution to approach Gaussain.

The characteristics lengths (l_f) as suggested are:

For center line discharge:

l_f = 0.5 W (both by Fischer & Chatwin)

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For side injection of tracer :

l_f = twice the center line discharge = W (by Fischer)

= 1.5 times of the center line discharge = 0.75 W

(by Chatwin).

W = width of the channel

The mixing length for both the conditions are;

$$L_m = 0.4 \ \bar{u} \ l_f^2 \ /\varepsilon_t \qquad (\text{ Fischer, 1968 }) \ \ldots\ldots\ldots\ldots \ (49)$$

$$L_m = 1.0 \ \bar{u} \ l_f^2 / \varepsilon_t \qquad (\text{ Chatwin, 1970 }) \ \ldots\ldots\ldots\ldots \ (50)$$

where, $\varepsilon_t = 0.23 \ d \ u_*$.

7.0 Longitudinal Dispersion in Rivers

After a tracer has adequately been mixed across the cross section, i.e., the stage of complete mixing, the final stage in the mixing process is the reduction of longitudinal gradients by longitudinal dispersion. If an effluent is discharged at a constant rate into a river whose discharge is also constant, there is no need to be concerned about longitudinal dispersion.

There are, however, practical cases where longitudinal dispersion is important. For the accidental spill of a quantity of pollutant, for example, release of radioactive material from a river side nuclear power station, daily cyclic variation of output from a sewage treatment plant; determination of longitudinal dispersion in such cases are important. The governing equation that allows to determine the longitudinal dispersion co-efficient (K) in a natural stream is analogous to the diffusion equation as derived in section 3, equation (21).

$$\frac{\partial \bar{C}}{\partial t} + \bar{u} \ \frac{\partial \bar{C}}{\partial x} = K \ \frac{\partial^2 \bar{C}}{\partial x^2}$$

The dispersion co-efficient for unidirectional turbulent flow between parallel plates is given in equation (24). The general form for the longitudinal dispersion co-efficient can be derived by introducing the dimensionless quantities ; $y' = y/d$, $u'' = u'/(\overline{u'}^2)^{1/2}$ and $\varepsilon' = \varepsilon/E$, where, y = position variable of depth, E = the cross sectional average of ε. $(\overline{u'}^2)^{1/2}$ = the intensity of the velocity deviation (it is not the turbulent intensity but a measure of how much the turbulent averaged velocity deviates throughout the cross section from its cross sectional mean). Substituting these dimensionless term in equation (23), we have;

$$K = -\frac{1}{d}\int_0^d u''\,(\overline{u'}^2)^{1/2}\int_0^{y'd}\frac{1}{\varepsilon'E}\int_0^{y'd}d^2 u''\,(\overline{u'}^2)^{1/2}dy'.dy'.dy'$$

$$= -\frac{d^2\,\overline{u'}^2}{E}\int_0^1 u''\int_0^{y'}\frac{1}{\varepsilon'}\int_0^1 u''\,dy'.dy'.dy' \quad \ldots\ldots\ldots\ldots (51)$$

$$= -\frac{d^2\overline{u'}^2}{E}\,I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

where, $\quad I = -\int_0^1 u''\int_0^{y'}\frac{1}{\varepsilon'}\int_0^{y'} u''\,dy'.dy'.dy'$

The longitudinal dispersion co-efficient described in equation (24) and its general form as given in eq. (51,52) for unidirectional turbulent flow, would show wide variation in results in natural stream because of the transverse variation of velocity across the stream. The above mentioned equations were derived from the vertical velocity variation along the cross section. However, in natural stream transverse mixing co-efficient is more important in producing longitudinal dispersion than vertical mixing co-efficient. The transverse mixing co-efficient is ten times higher than vertical mixing co-efficient [comparing eq. (26) and eq. (27)]. This indicates that transverse mixing time is 90 times the vertical mixing time [comparing the Eulerian time scale, $T_E = 1^2/\varepsilon_v$]. Thus, for computation of dispersion co-efficient , transverse mixing co-efficient (ε_t) is predominant.

The effect of vertical mixing can be taken in terms of velocity variation along the depth.

As such, there is no transverse velocity profile, as a result, it is not possible to establish a transverse analogy. However, Fischer (1967) has derived a transverse profile by plotting the cross stream variation of the depth averaged velocity, and described mathematically as given below;

$$\bar{u}^z(y) = \frac{1}{d(y)} \int_{-d(y)}^{0} u(y,z) \, dz \quad \dots\dots\dots\dots\dots\dots(53)$$

where, $\bar{u}^z(y)$ = shear flow velocity profile extending over the width of the stream W.

u(z) = logarithmic velocity profile as used by Elder [$u = \bar{u} + (u_*/x) \{ 1 + \ln (z+d)/d\}$] along the depth of flow d.

In real stream, transverse velocity profile is more important than the vertical velocity profile in determination of longitudinal dispersion. This can be made more clear, if we look into the eq. (52), where dispersion co-efficient is proportional to square of the distance over which the shear flow profile extends. For natural stream, characteristic length is the function of width of stream. Therefore, more the ratio of width to depth, the more dominance of transverse velocity profile would be observed. For example, for width to depth ratio in the range of ten or greater, which is usually found in the case of river, the transverse profile of velocity would be 100 or more times important in producing longitudinal dispersion than the vertical profile.

Thus, a quantitative estimate of the dispersion co-efficient in real stream can be obtained neglecting the vertical profile entirely and applying Taylor's analysis to the

transverse velocity profile. Let us consider, a slice of thickness dx moving at the mean flow velocity $\bar{u}$, as illustrated in Fig. 8. Let, transverse velocity variation $u'(y) = \bar{u}(y) - \bar{u}$.
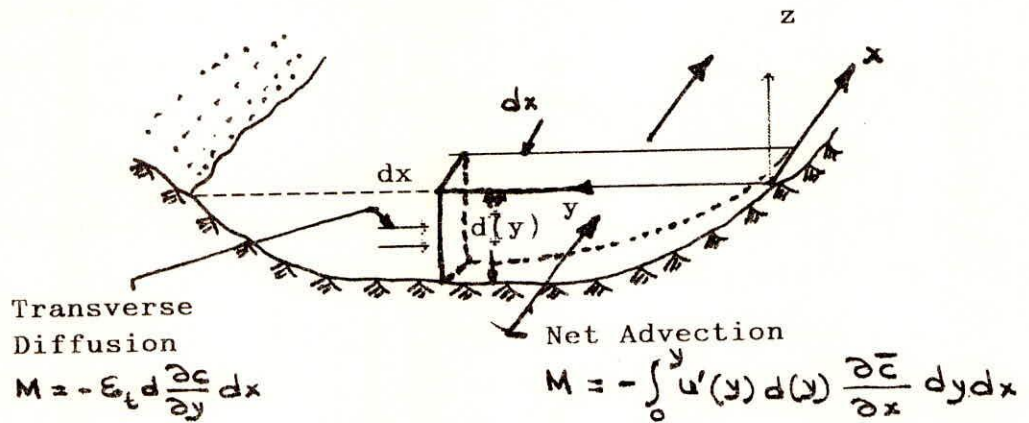

Fig. 8 : Illustration of the balance of Advection and Diffusion

Taylor observed that the longitudinal dispersion starts from the point where the balance of advection and diffusion occur, in this case, a balance of transverse diffusive mass transport through the vertical face in the xz plane at left edge of the slice versus the net advective mass transport through the vertical faces in the yz plane is occurred. Using the Taylor's findings as derived in eq. (15) and subsequently derived in eq. (23) for turbulent shear flow, the equivalent of the first integral of eq. (23) is given by;

$$\int_0^y u'(y)\, d(y)\, \frac{\partial \bar{C}}{\partial x}\, dy = d\, \varepsilon_t\, \frac{\partial C'}{\partial y} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(54)$$

Implicit in this formulation is $C'(y)$ and $u'(y)$ and assumed to vary only across the stream. Vertical variations are neglected because of their small effect.

Integrating eq. (54) to find $C'(y)$;

$$C'(y) = \frac{\partial \bar{C}}{\partial x} \int_0^y \frac{1}{\varepsilon_t \cdot d(y)} \int_0^y u'(y)\, d(y)\, dy.\, dy. + C'(0) \quad ..(55)$$

For the mass transport in the stream wise direction,

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$$M = \int_0^W U^{'}(y) \cdot d(y) \cdot C'(y) \, dy$$

$$= \frac{\partial \bar{C}}{\partial x} \int_0^W u'(y) \, d(y) \int_0^y \frac{1}{\varepsilon_t \, d(y)} \int_0^y u'(y) \, d(y) \, dy.dy.dy \quad \ldots\ldots(56)$$

The extra constant term, $C'(0) = 0$

As the bulk transport co-efficient or dispersion co-efficient, is analogous to the molecular diffusion co-efficient as defined in eq. (18) , i.e.,

$$M = - A K (d\bar{C}/dx) \text{ , where, } A = \text{cross sectional area of the slice.}$$

$$K = - \frac{1}{A} \int_0^W u' \, d \int_0^y \frac{1}{\varepsilon_t \cdot d} \int_0^y u' \, d \, dy.dy.dy \quad \ldots\ldots\ldots\ldots(57)$$

Equation (57) is the expression for determination of dispersion co-efficient in natural stream developed by the Fischer (1967). It is similar to the equation (24) and differs only in the inclusion of variable depth across the stream width. The lateral turbulent mixing co-efficient ε_t need to be determined by experiment. However, Fischer observed that Elder's (1959) experimental result for $\varepsilon_t = 0.23 \, d \, u_*$ in a flow 0f 1 cm deep down a water table could be a well match for using in eq. (57).

The eq. (57) can be used in a channel of larger width to depth ratio (preferably 6 or greater) and in which there are significant lateral variations in downstream velocity.

7.1   Variability of Dispersion Co-efficient in Real Streams

Equation (57) describes the dispersion co-efficient for uniform flows. However, real streams may have bends, sandbars, side pockets, pools, bridge piers etc.. Every irregularity. contributes to dispersion which may be trailing or additive. To apply the Taylor's analysis, the limitation is , it can only be

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applied when advective term and diffusive term is balanced, which is called the point of complete mixing.

The irregularities in real streams increase the length of the initial period and usually lead to the production of a long "tail" on the observed concentration distributions. One way to treat this tail is to ignore it because this may not contain much dye. If tail is not ignored the variance increases unreasonably. There were number of experiments to compute the effect of long tail on longitudinal dispersion co-efficient. However, no specific conclusions were arrived. It has been postulated by several researchers [Valentine and Wood, (1975), Valentine, (1978), Day (1975)] that "dead zones" which create by the side pockets , increase the length of initial period and the magnitude of longitudinal dispersion co-efficient.

Bends have also effect on dispersion co-efficient. Bends increase rate of transverse mixing, and thereby to some extent reduce the dispersion co-efficient. This is because of the tendency to induce transverse velocity profile. At any natural bend the high velocities will be concentrated toward the outside bank and low velocities toward inside one. Thus in a meandering stream the velocity differences across the stream are accentuated and the dispersion co-efficient is likely to be much greater than in a stream which is straightened.

Fischer (1969) quantified that for bends of sufficiently long, the steady- state concentration profile will establish depending on the ratio of the cross sectional diffusion time to the time required for flow to round the bend, i.e., the ratio,

$$\gamma = (W^2/\varepsilon_t)/ (\bar{u}/L), \quad \text{where, } L = \text{length of the curve.}$$

These eventually tell upon that it is very difficult to predict the dispersion co-efficient with high accuracy in real streams.

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Mixing theory is equally applicable for non-conservative substances such as Bio-chemical Oxygen Demand in a sewage effluent or in a power station effluents. Since many of the non-conservative substances undergo first-order decay, it is worthwhile to see the effluent on the decay term in analysis of dispersion. Let us consider, substances whose rate of decay in a water body is given by;

$$\frac{dC}{dt} = - kC \quad [k = \text{first order decay rate}]$$

and suppose that M units of mass per unit time of this substances are discharged into a river whose discharge is Q. Downstream of the initial mixing zone, i.e., the point where the advective and diffusion term balance, the diffusion equation becomes;

$$\bar{u} \frac{\partial C}{\partial x} = K \frac{\partial^2 C}{\partial x^2} - kC \quad \dots\dots\dots\dots\dots\dots (58)$$

with boundary condition; $C \longrightarrow 0$ as $x \longrightarrow \infty$. The solution of the above equation is;

$$C = C_0 \exp \{ - (kx/\bar{u}) [(2/\alpha) (\alpha + 1)^{1/2} - 1)]\}$$

$$= C_0 \exp [- (\bar{u}x/2K) (\alpha + 1)^{1/2} - 1)] \quad \dots\dots\dots\dots (59)$$

where, $\alpha = \dfrac{4K\, k}{\bar{u}^2}$, $C_0 =$ constant of integration.

If $\alpha = 0$, the dispersion co-efficient is eliminated. If α tends to zero the quantity in the square bracket in equation (59) tends to one and the solution is simple first order decay, i.e., $C = C_0$

The constant of integration can be determined from the condition at which mass entering the stream must equal the rate at which it is being removed by decay. If the source is located at x = 0 and the initial mixing distance has length L_m, we can write for the reach where the one dimensional equation applies, as given below;

$$\int_0^{\alpha} k\, C\, A\, dx = M\, (L_m) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (60)$$

$M(L_m)$ is the mass transported through cross section at $x = L_m$. Substitution of C from eq. (59) into eq. (60) permits computation of C_0 for given $M(L_m)$. However, $M(L_m)$ undergoes some decay occurs during the initial mixing process. How much is that quantity, that can only be computed from detailed investigation of concentration in the initial zone. So an analytical expression for C_0 based one dimensional theory is not possible. For practical purposes, the problem is solved by neglecting the existence of initial zone, i.e.,putting L_m = 0. The integration of equation (60) for L_m = 0 gives;

$$C_0 = \frac{M}{Q}\left[\frac{2}{\alpha}(\alpha + 1)^{1/2} - 1)\right] \quad \dots\dots\dots\dots\dots\dots (61)$$

where, M = rate of addition of mass at source.

In equation (49) it has been shown that for steady state flow in a river, the distance required for complete mixing is;

$$L_m = 0.4\, \bar{u}\, l_f^2\, /\, \varepsilon_t \quad [\ l_f = W \text{ for side injection }]$$

The distance requires for decay of the substance to a factor e^{-1} is $L_d = \bar{u}/k$ [for first order decay t = f(C)/k].

Comparing the equations for L_m and L_d , we have;

$$L_d /\, L_m = 2.5\, \varepsilon_t /\, k\, W^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots (62)$$

The quantity α may be written approximately as ;
$$\alpha = 0.024\, W^2\, k\, /\varepsilon_t \quad [\text{ using } \varepsilon_t/du_* = 0.6 \text{ and } K = 0.011\, \bar{u}^2 W^2/du_*]$$

If $L_d > L_m$ i.e., decay distance is higher than the cross sectional mixing distance , α must be less than approximately 0.06. The corresponding value of [$(2/\alpha)(\alpha + 1)^{1/2} - 1)$] is 0.985, which indicates that the solution is exactly that for the first order decay neglecting the dispersion co-efficient. There may be two

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possibilities for such conditions, either a) the material decays before it mixes across the cross section, Or, b) the longitudinal dispersion has negligible effect on the decay of non-conservative substances and hence can be dropped. If $L_d < L_m$, the concentration distribution need to be computed numerically. If $L_d > L_m$, and if the river and effluent discharges are steady, the downstream concentration can reasonably be computed by the first order decay solution, i.e., $C = (M/Q) \exp(-kx/\bar{u})$. However, effluent discharges are hardly ever steady. The typical daily fluctuation in output from sewage treatment plant leads to the gradients of concentration of discharged material into river, and these gradients are subsequently leveled out by the longitudinal dispersion. Therefore, daily fluctuation cloud only be handled easily by the numerical models.

These eventually tell upon that dispersion of non-conservative substances do not much depends on longitudinal dispersion co-efficient but on the first order decay of the substances irrespective of the position of decay distance and mixing length.

9. Estimating Dispersion Co-efficient in Real Streams

The foregoing discussions reflect that it would not make any sense if one try for too high accurate prediction of dispersion co-efficients in real streams. Because it would not possible to include exact effect of irregularities of the stream.

Fischer (1975) gave a formula which was derived from the general expression of dispersion co-efficient as given in eq. (52). Using $I = 0.07$, $d = 0.7W$, $\bar{u'}^2/\bar{u}^2 = 0.2$, and $E = \mathcal{E}_t = 0.6d u_*$, the dispersion co-efficient is given by;

$$K = \frac{0.011 \bar{u}^2 W^2}{d u_*} \quad \dots\dots\dots\dots\dots\dots (63)$$

It was reported that prediction of dispersion co-efficient by equation (63) has been found to agree with observations within a factor of four or so. Moreover, it was also observed that calculation of observed values from field data usually accurate within a factor of two or so, thus the prediction by the above eq. (63) could be accepted for real streams. The advantage in using the above formula is; it requires only width, depth, and velocity data of a stream.

Dispersion co-efficient from observed values is usually computed by one of the two methods; i) change of moment method, ii) routing procedure.

9.1 Change of Moment Method : in which the rate of growth of variance of the tracer cloud is computed, mathematically,

$$K = \frac{1}{2} \frac{d}{dt} \sigma^2$$

σ^2 = the variance of the concentration distribution w.r.t distance along the stream

9.2 Routing Method : in which input C-t curve is routed anlog the flow direction to obtain the downstream C-t curve.

The difficulty in using the change of moment method is that the long "tails" on observed distribution make it difficult to compute meaningful value of variance. However, the routing procedure avoids this problem by matching a downstream observation of passage of a tracer cloud predicted based on upstream observation.

In this procedure, the upstream observed curve is used as the initial tracer distribution, and a concentration -time curve for the down stream station is predicted by the one-dimensional dispersion model as given in eq. (21). The predicted and observed

downstream station curves are compared. If the comparison is not adequate, a new dispersion co-efficient is selected, and the calculation is repeated until the best possible comparison is obtained. The compared approximation of dispersion co-efficient should be equal to the value of K as in eq. (63).

Mathematically, the procedure is as follows: if at some time $t = t_0$ and initial distribution of tracer $C_0(\xi, t_0)$ is observed, then according to the bulk diffusion equation, the distribution of tracer at any later time is;

$$C(\xi, t) = \int_{-\infty}^{\infty} C_0(\xi', t_0) \frac{\exp\left[-\frac{(\xi - \xi')^2}{4K(t - t_0)}\right]}{\left[4\Pi K (t - t_0)^{1/2}\right]} d\xi'$$

To use this formula, time concentration data need to be converted to distance concentration data by assuming that no dispersion takes place during passage of tracer cloud past the measuring station.

$C(\xi, \overline{t_0}) = C(X_0, t)$, in which, $\xi = \overline{u}(t_0 - t)$, and

t_0 = mean time of passage past a station located at longitudinal station X_0.

To predict a curve for station X_1, $C(X_1, t)$ from one measured at X_0. The $C(\xi, t_0)$ is replaced by $C(X_1, t)$, inside the integral ξ is replaced by $\overline{u}(t_1 - t)$, t_1 = mean time of passage at X_1. $C_0(\xi', t_0)$ is repalced by the $C_0(X_0, \tau)$ and $\xi' = \overline{u}(\overline{t_0} - \tau)$ and $t - t_0$ is replaced by the difference in mean passage times, $\overline{t_1} - t_0$, and the element of intergration $d\xi' = \overline{u} d\tau$.

$$C(X_1, t) = \int_{-\infty}^{\infty} C(X_0, \tau) \frac{\exp\left[-\frac{\{\overline{u}(\overline{t_1} - \overline{t_0} - t + \tau)\}^2}{4K_1(\overline{t_0} - t)}\right]}{4\Pi K (\overline{t_1} - t_0)^{1/2}} \overline{u}\, d\tau \quad ..(64)$$

The routing procedure can also be used to predict downstream distribution once the value of K is known. This produce

was found to be reasonably accurate if the dimensionless time of the upstream observation is greater than 0.4.

10.   Studies Related to Dispersion
       During 50's decade

The first published analysis of longitudinal dispersion was by Taylor in 1953.

Based on the concept developed by him as also given in eq.(17), Taylor (1953) analyzed the dispersion of solute in laminar flow in a tube assuming that soute has been in the tube long enough to become well distributed over the cross section so that axial symmetric is observed, and found that longitudinal dispersion coefficient is inversely proportional to the molecular diffusion coefficient and related in the following form ;

$$K = a^2 u_0^2 / 192.D \qquad \dots\dots\dots\dots\dots\dots\dots(65)$$

where, a = radius of the pipe.

   u_0 = maximum velocity on the center line.

   D = molecular diffusion co-efficient

Subsequently in year 1954, Taylor studied the dispersion in turbulent shear flow and indicated that primary mechanism for dispersion in shear flow was because of the variation in convective velocity within the cross section,and the process had been described by a one dimensional Fickian diffusion equation.

Taylor restricted his analysis to asymmetries flow in a long, straight circular pipe and described the turbulent dispersion co-efficient as ;

$$K = 10.1 .a . u_* \qquad \dots\dots\dots\dots\dots\dots(66)$$

in which, u^* the shear velocity = $(\tau_p/\rho)^{1/2}$, where τ_p = the wall shear, and ρ = the density of fluid.

In year 1956, Aris had also conducted a study for laminar shear flow and showed that it would be possible to obtain Taylor's

main results without stipulating the features of the concentration distribution. He proved that the spreading by longitudinal molecular diffusion is directly additive to that caused by the velocity profile. The approach adopted is often referred as the "concentration moment" method.

Using the same reasoning as considered by Taylor ,Elder (1959) analysed a flow down for two dimensional flow in an infinitely wide inclined plane using the von Karman logarithmic velocity profile and described the dispersion coefficient as follows :

$$K = 5.93 \ d \ u_* \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(67)$$

using , von Karman constant , $\varkappa$ = 0.41

During 60's Decade

Much of the studies related to dispersion were during sixties decade. Studies reported by Fischer during the decade were appreciated from different corners.

Fischer (1966) conducted study for application Elder's concept in Taylor's analysis and to verify the effect of lateral velocity variations on dispersion co-efficient in natural stream. He postulated that the one dimensional diffusion equation is a valid description of dispersion of dispersing cloud of tracer particles only after an initial, convective- dominated period. However, no methods had been given for estimating the duration of the convective period. It was further suggested that the method of moments is the better description for determining the longitudinal dispersion co-efficient whenever a tracer cloud is behaving according to a diffusion equation, irregardless of initial concentration distribution and irregardless of whether distance concentration or time concentration curves are used. The formula

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suggested for computing the dispersion co-efficient would be from the variances of C-t curve, as given below;

$$K = \frac{\bar{u}^2}{2} \frac{\sigma_2^2 - \sigma_1^2}{t_2 - t_1} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(68)$$

Where, σ_2^2 and σ_1^2 are the variances of the time concentration curves measured after the initial period when the lateral and vertical mixing is completed. t_1 and t_2 are the mean passage time.

Subsequently in year 1967,Fischer justified his earlier concept and described that for natural streams dispersion of a cloud of tracer particles from a point source could be divided into two periods: a) the convective period, during which the movement of tracer particles is still dependent on their initial convective velocity, and (b) the diffusive period or "Taylor" period during which the bulk motion of the cloud is described by a one dimensional diffusion equation in the flow direction. The duration of the convective period in terms of distance downstream from a point or line source,(the criterion for use of the Taylor one dimensional diffusion equation) would be;

$$L_m > 1.8 \frac{l_f^2}{r} \frac{\bar{u}}{u_*} \quad \dots\dots\dots\dots\dots\dots\dots \dots (69)$$

In addition to the above , Fischer had given an approximate functional relationship which relates the dispersion co-efficient to bulk parameters of the channel as described in eq. (57)and given below . An analytical description of the time scale for complete mixing had also been obtained as described in eq.(39, 40, 42, 43, 44, 45). He also observed that dispersion coefficient is proportional to the channel width, and inversely to the depth. This was in contradiction to a previous study by Elder where it

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was said that the coefficient is proportional to the depth and independent of width. The dispersion coefficient observed by the Fischer for large streams was very much greater than what it obtained from the Elder's concept. Fischer concluded that for natural streams of reasonbly uniform cross section, his theory expected to be accurate within a factor of approximately 2. The method suggested for predicting the dispersion co-efficient in natural streams (only variations in the transverse (y) direction) was ;

$$K = - \frac{1}{A} \int_0^W q'(y)\, dy \int_0^y \frac{1}{\varepsilon_t\, d.(y)}\, dy \int_0^y q'(y)\, dy \quad \ldots\ldots (70)$$

$$\text{in which, } q'(y) = \int_0^{d(y)} u'(y,z)\, dy$$

$$\varepsilon_t = 0.23\, d\, u_*$$

Later, Fischer (1968) validated his methods of predicting diffusion co-efficient and convective time period for different natural streams data. A procedure had been described for calculating dispersion coefficient from tracer observations in natural streams. A comparative estimation of dispersion coefficients by various methods had shown that the procedure suggested by him was accurate. He further justified that the length of the dispersing cloud and the decay rate of the peak concentration remains within a factor of 2 and in uniform streams the dispersion coefficient could usually be predicted within an error of 30%.

The other applications of Taylor's analysis during the late sixties to predict the dispersion in natural streams and channels were attempted by Krenkel (1960), Glover (1964), Patterson & Gloyna (1965), Thackston (1966), Hays (1966), Crunch (1967), Edward and Krenkel (1967), Sooky (1969) and in year 1970 , Bansal , Gofrey and Frederick , Yotsukura et al.. They came out with

mixed success with wide variety of dimensionless dispersion co-efficient ranging from 1.75 to 7500. The largest value of dimensionless dispersion co-efficient ($K/d\ u_*$) was observed by Yotsukura et al (1970) for Missouri River.

During 70's Decade

Because of the large variation in the longitudinal dispersion co-efficient in natural streams , there were lot of efforts during the seventy's decade to study the phenomenon of dispersion in natural streams for different shape factor and stream hydraulic parameters. The studies reported during seventies were; Chatwin (1971), McQuivey & Keefer (1974), Fischer (1973,1975), Todorovic (1975), Day (1975), Jain (1976), Liu (1977).

Chatwin (1971) conducted a study for two dimensional uniform flow and argued that for large times, where Taylor's model is valid, the dispersion cloud become normally distributed approximately a time d^2/K after its injection and the skewed longitudinal distribution is produced in period $t < 0.4(d^2/K)$. When $t > 0.4\ d^2/K$, the variance of the dispersion cloud grows linearly with time, and the initial skew degerates into normal distribution in the period $0.4 < (tk/d^2) < 1.0$. He proposed that an adequate approximation for dispersion could be;

$$\left[t \log_e \left(\frac{B}{C\ t^{1/2}} \right) \right]^{1/2} = \left[\frac{x}{2\ K^{1/2}} - \frac{\bar{u}\ t}{2\ K^{1/2}} \right] \quad \ldots\ldots(71)$$

where B = constant, proportional to the amount of the diffusion material.

Chatwin determined the value of constant 'B' by plotting left hand side of the eq. 69 against time (t) for several values of B and accepting the correct one which gave the smoothest curve near peak and determined the dispersion co-efficient corresponding to that value of B.

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McQuivery and Keefer (1974) had proposed a simplified method for predicting dispersion co-efficient for use in stream, based on the uniform flow analysis. The analogy was observed between the linear one dimensional solute dispersion equation and the linear one dimensional flow equation and developed the relation between them to predict the longitudinal dispersion coefficients as given below;

$$K = 0.058 \ \frac{Q}{S \ W} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \ (72)$$

S = the slope of the energy gradient line.

Todorovic (1975) attempted to develop a general stochatic model for longitudinal dispersion, studying the bed sediment particles released in a flow in various manner over a certain period of time and observed that the stochastic model appears to be a particular case of the random entry of variables like distance and time.

Day (1975) attempted to characterize the longitudinal mixing in rivers by the dispersion co-efficient and related this co-efficient to bulk flow and channel geometry. He postulated that it would be incorrect to apply Taylor's analysis to a non-Fickian dispersion process. And also observed that spread or standard deviation of any initially concentrated mass increases linearly with distance and concentration distribution maintain a persistent asymmetry. He further observed that channel irregularities, mainly tapering, increases the value of dispersion co-efficient continually.

Fischer (1973) compared results of some of the earlier studies carried out for natural streams with theoretically predicted dispersion co-efficient and observed that in some cases the error was in the order of 4.5 times. He described the differences because of the non uniform cross section and unsteady

flow which were not taken care of in the theory. He also studied the effect of channel irregularities on dispersion co-efficient and stated that meanders cause nine fold reduction in the dispersion co-efficient.

Later in 1975, Fischer suggested a simple formula to compute the dispersion co-efficient for natural streams of any nature which was considered to be more accurate than other findings for two dimensional flow;

$$K = \frac{0.011 \, \bar{u}^2 \, W^2}{d \, u_*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(73)$$

Jain (1976) presented a simplified method for estimating dispersion coefficient for natural streams (as given in eq. 74) incorporating the shape factor of the channel cross sections. He developed the method based on the Fischer's (1967) analytical formulation, as given below:

$$K = \beta \, \frac{\bar{u}^2 \, W^3}{\phi \, A \, u_*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(74)$$

ϕ = constant, value varies from 0.1. and 0.2, and,

$$\beta = - \frac{d \, u_*}{\varepsilon} \int_0^1 (d_0 - q_0) \int_0^y (d_0)^{-2} \int_0^y (d_0 - q_0) dy_0 \, dy_0 \, dy_0 \quad \ldots(75)$$

where, $y_0 = y/W$, $d_0 = d/r$, $q_0 = q/(u_* \cdot r)$

Liu (1977) suggested that since longitudinal dispersion mainly vary because of the transverse velocity gradient (Fischer,1967) rather than vertical velocity gradient it would be more appropriate to use new dimensionless dispersion co-efficient, β, which depends on the channel shape factor and shape of the velocity distribution across the cross section, rather than the conventional dimensionless dispersion co-efficient α as used by Elder (1959). The dimensionless co-efficient ,β, is similar to that chosen by Jain (1976).For wide channel , Liu observed that

the appropriate expression for dispersion co-efficient would be ;

$$K = \beta \frac{Q^2}{u_* r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

where β is analogous to an expression derived by Fischer as given in eq. (70), and similar to eq. (75).

Experimentally, the relation of β was observed as;

$$\beta = 0.18 \left(\frac{g \, r \, s}{u_*} \right) ,$$

where, g = acceleration due to gravity.
s = slope of the energy gradient line.

During 80's Decade

Beltaos (1980) derived a dispersion model incorporating two dispersion parameters λ and L_m (described below) and suggested a method for prediction of dispersion when the process is not Fickian.

where, $\lambda = \bar{u}'^2 \, T_L / (L_m \, \bar{u})$

u' = deviation of the local (time averaged) velocity.

T_L = Lagrangian time scale, $\bar{u}$ = cross sectional avg. velocity

For large streams, it was further shown that within stream lengths of practical interest, the dispersion process will never be Fickian, with the exception of small streams.

Dorbran (198.) had attempted a study to compute the behavior of dispersion co-efficient in mountainous natural stream and proposed a new method for calculating characteristics mixing length for non-uniform winding stream. He observed that :

Fischer's methods ; a) the method of flow parameters and methods based on tracer study (1967,1968), b) routing procedure, and c) the method of variance change represent (1966), are the most convenient methods for calculating dispersion co-efficient for natural stream even for non-uniform streams.

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Dorbran proposed that lagrangian time scale need to be adjusted for winding non-uniform stream because of the variation of the thread of transverse velocity profile from the axis of the stream in the winding. His adjusted time scale has been discussed in eq. (47).

Because of the variation of dispersion co-efficient with respect to time particularly in the early stages of dispersion, accurate estimation of dispersion co-efficient becomes a difficult task. Jayawardena and Lui (1983) proposed a simple time-dependent model for the dispersion co-efficient based on the study conducted in few selected narrow streams in Hong Kong. The model was developed based on the Fickian concept consists of three stages; first stage is for $t/T_L \leq 0.1$, the second stage is for $0.1 < t/T_L < 20$ and the third stage is for $t/T_L \geq 20$. Where, t is passage time and T_L is lagrangian time scale for dispersion. The dispersion co-efficients for these three stages proposed as:

For, $t/T_L \leq 0.1$, $\qquad K = \dfrac{\overline{u}^2 r}{u_*} (b_1 t' + b_2)$

For, $0.1 < t/T_L < 20$, $\quad K = b_0 \dfrac{\overline{u}^2 r}{u_*} \left[1 - \dfrac{b_3}{t'} \left[1 - \exp \left(- \dfrac{t'}{b_3} \right) \right] \right]$

For, $t/T_L \geq 20$, $\quad K = \gamma K_F$

$\overline{u}$ = cross sectional mean velocity, u_* = shear velocity,

r = hydraulic radius, t' = dimensionless time

γ = constant lies between 0.95 and 1.0

b_0 , b_1 , b_2 , and b_3 = dimensionless constants

K_F = dispersion co-efficient during Fickian period.

It was also observed by them that for sufficiently large times, an accurate estimate for T_L is pre-requisite for accurate prediction of dispersion co-efficient.

In 1984, Jayawardena and Lui attempted a study for

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numerical solution of the dispersion equation for variable dispersion co-efficients as derived in their earlier study. The numerical model was developed using finite elements in space and finite differences in timefor the solution of convective period. The numerical model developed by them had been compared with dispersion equations for conservative and non-conservative materials and found to argee closely with the respective analytical solutions. Attempt was also made to apply the numerical solutions for practical applications and found a reasonable fit.

11.   Scope for Studies

There are different opinions for application of Fickian diffusion equation in stream dispersion modelling. Some described that Fickian diffusion is the correct description for dispersion prediction after the convective period, however, some others observed that flow do not become Fickian even during the stream length of interest mainly for large streams. Moreover, mixing pattern in rivers mainly depend on the velocity variation both vertical and horizontal depending upon the river geometry and hydraulic characteristics, shear velocity, depth of flow, width of the rivers as well as the river irregularities. These be the factors vary from river to river, scopes always persist for studying the dispersion phenomenon of each river and prediction of dispersion co-efficient for each river.

One point is very clear from the studies carried out by different researchers that dispersion co-efficient is an emperical parameter derived from eddy-diffusivity assumption. Hence, further study is necessary for better approximation in prediction of dispersion co-efficient for more closure and accurate value of the co-efficient.

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12.   REMARKS

Study of dispersion of pollutants in stream has significant importance in controlling pollution hazard of stream or to safegaurd the downstream users from the danger of accidental or sudden release of pollution. Dispersion has relatively unimportant where steady state of flows and loads are attained.

It is clear from the various studies that Fickian diffusion equation is the correct description of longitudinal dispersion only after the convective period and where the complete mixing takes place. Wide range of variation of dispersion co-efficient and the basic A-D equation indicate that Dispersion Co-efficient is an emperical parameter derived from the eddy-diffusivity assumption. Better the assumption and derivation of parameters will lead to the closure value of dispersion co-efficient.

Non-conservative material have less influence by the longitudinal dispersion co-efficient. The mixing of the non-conservative material take place by the first order decay co-efficient and the concentration of contaminants need to be determined by the first order decay equation irrespective of position of decay distance.

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STUDY GROUP

DIRECTOR   : DR. S.M. SETH

DIVISION   : ENVIRONMENTAL HYDROLOGY DIVISION

HEAD   : DR. K.K.S. BHATIA, Sc. 'F'

SCIENTIST   : Sh. N.C. GHOSH, Scientist 'C'