SA-2

RESERVOIR OPERATION STUDIES

G N YOGANARSIMHAN
PROFESSOR
WRDTC
UNIVERSITY OF ROORKEE
ROORKEE

NATIONAL INSTITUTE OF HYDROLOGY
JAL VIGYAN BHAVAN
ROORKEE-247667 (UP)
INDIA

1983-84

PREFACE

This state-of-art report deals with problems associated with operation of a single purpose/multi-purpose reservoir. Starting with heuristic procedures of reservoir operation, the general concepts of rule curve and zoning of reservoir operation have been discussed. The report also includes discussion on the operation of reservoirs serving hydro-power plants when used in conjunction with thermal power plants, for various cases like bounded and unbounded reservoirs. The complexity of reservoir operation problem increases when one is to deal with multi-reservoir systems. In such cases , application of mathematical digital models involving use of simulation or optimization techniques becomes necessary. These aspects have been discussed in detail in the present report, which also gives a comprehensive list of bibliography.

CONTENTS

LIST OF FIGURES

1.0 RESERVOIR OPERATION

1.1 INTRODUCTION

Studies on the operation of reservoirs have drawn the attention of many researchers and reservoir managers in recent years due to one or more of the following reasons:

(i) Realization that under proper management the the structural and non-structural elements of a water resource systems mobilize the latent usefulness of natural water resources while allaying their potential destructiveness.

(ii) Development of concepts of economy and value functions (short term production functions of water for irrigation, power, flood control, and other purposes).

(iii) Multireservoirs operated as a system offer a great variety of operating alternatives and thereby providing flexibility in operation.

(iv) Development of mathematical programming techniques, computational algorithms and increased computer capabilities to handle large complex systems.

(v) Growth and diversification of water uses and public demand for maintaining a high quality environment and long term stability reducing risk and uncertainties and increased reliability of services (irrigation water Energy etc.).

(vi) As an effort to reappraise and enhance where possible the value of existing water systems through better management practices.

(vii) A challenge to solve the complex and multifaceted problem.

(viii) The operation plans designed at planning stage are coarse in nature and need refinement during the actual operation once the system comes into being and needs to be continuously updated as and when additional data interms of input and information on system performance accumulate. Further the introduction of new interventions changes the character of interaction among the subsystems.

(ix) Objective drift or the changed needs compared to the needs for which the systems was designed originally.

(x) To quantify decision and to evolve a rational approach to the operation of such systems.

The literature review reveals, that there exists no general procedure or algorithm for the solution of reservoir optimization problems. This is due to extreme complexity of the system and diverse nature of its configuration, management, operation, political and economic goals, and environmental interactions.

1.2 Problem Defined

Given a reservoir or multiple reservoirs (independent or inter connected), the problem of reservoir operation is to specify releases from each of the reservoirs to satisfy the demand with regard to the release, energy generation, and the storage space, the stored water and stored potential energy. The complexity of the problem increases from single reservoir single purpose to interconnected multiple reservoirs with multipurpose as the later offer a variety of release combinations. The time dimension is most important in two different ways. The decision problem is not a one time water release problem as the benefit accrued from such releases is a function of such releases in the near past and also of the near future encompassing a period of one flood period or one irrigation season or an hydrologic year or multiples of these. Further the continuous character of the decision problem requires planning of operation in terms of on-line operation (continuous operation), medium term operation and the long term operation integrated properly to meet the overall system objective. Each of these operations has to satisfy the characteristic specification demand of the type of operation. For example in a long term operation, conservation, amount of potential energy and the expansion plan may be of interest whereas in the case of on-line operation, in addition to the long term requirements, the rate of release and storage change may also be of interest. It seems appropriate and logical to analyze the problem in terms of on-line, medium and long term problems and integrate them. Since the system demands are so varying, it may not be possible to evolve a universal approach to the problem of reservoir management although some common algorithms often useful in solving such problems can be identified.

1.3 Complexity of The Problem

The complexity of any system is determined in terms of the number of variables or components and their interactions. Higher the interactions, higher is the complexity of the system. The high degree of complexity of reservoir operation scheduling problem can be explained in terms of the following components and their interactions, environment and modelling.

1. Multi-unit, Multi-purpose Characteristcs

The water stored in any reservoir has to serve both conflicting and complementary multiple purposes. These purposes include:

(i) Water supply for municipal, industrial and agricultural (irrigation) needs.

(ii) Water quality improvement by releasing higher quality water to dilute and transport downstream wastes.

(iii) Flood control through moderation by providing storage capacity to absorb floods during periods of floods and release control to take advantage of the downstream channel capacity to reduce likelihood of flood damage.

(iv) Hydropower production by operation of reservoirs so as to minimize loss of energy and meet energy and power requirements.

(v) Navigation by insuring sufficient depth of water in navigation channels and sufficient watersupply for lockages.

(vi) Recreation, whose benefits, while sometimes difficult to quantify in monetary terms, are nonetheless often present if appropriate pool levels and limits on level fluctuations are maintained.

(vii) Fish and wild life enhancements through maintenance of desirable pool levels of flows during critical periods in the year for greater fish and wild life production and fishing and hunting benefits.

These purposes when met by multireservoirs offer varieties of options in terms of on-line, medium terms and long term operations.

2. Economic interaction

Some of the purposes served by the reservoirs are to be integrated with other alternatives serving the same purposes. For example the most important purpose of power production is to be integrated with the total electric utility service. When such integrated operations are considered, the problem of non-separable objection functions leads to complexity.

The evaluation of hydropower involves both non-linearity and non separability. A function f(x) is called separable, if it can be expressed as a sum of single-variable functions as :

$$f(x) = \sum_{i=1}^{n} f(x_i) \qquad \ldots (1)$$

otherwise it is non-separable. Mathematically the separability may be described in terms of Hessian having all its off diagonal elements equal to zero i.e.

$$\frac{\partial^2 f}{\partial x_i \quad \partial x_j} = 0 \qquad i \neq j \qquad \ldots (2)$$

The definition gives more insight into and brings out the importance of the nonseparability of objective function in interconnected or non-interconnected multiple reservoirs operated integrally with a thermal system. The first order partial derivative $\partial f / \partial x_i$ is the marginal hydropower benefit(f) from a unit release (x_i) from reservoir i in any period t. The second partial derivative $\partial^2 f / \partial x_i \ \partial x_j$ represents the rate of change in the marginal benefit obtained from reservoir i as a function of release from reservoir j. The existence or otherwise of non-diagonal elements of the Hessian simply describes the existence or otherwise of economic interaction of reservoirs. The existence of economic interaction is quite evident whether the multiple reservoirs are hydrologically connected or not when operated integrally with thermal system.

1.4 Uncertainty

The explicit treatment of uncertainties in the exogenous flows of a reservoir system is of course a major theme in water resources research. In addition the future demand and unit availability are also subject to uncertainty. But the later is unlikely to have as great an impact as uncertainty about inflows. The most desirable scheduling model would be one which handles uncertainty using non-anticipative stochastic control. This means that the operating policies produced by the model have the property that, in each time period, the release decisions depend on no more information than will be available to the system manager at that time. Ignoring uncertainties in the future inflow may result in unrealistic reservoir release scheduling policies.

In a reservoir scheduling context non-anticipativity means that release decisions in each period must be based solely on observation on inflows for previous periods, including of course any information these observations may yield about future inflows. Though the difference between the deterministic and the stochastic problem is small and is in terms of inflows described in terms of probabilities, the stochastic problems is computationally large.

Approaches to the problem

The reservoir scheduling problem has been presented as a highly complex stochastic problem with non-separable objective function with no universal approach to the solution. The solution to real stochastic problem is not in sight. However, different approximations have been advanced. The solution to problems range from common sense rules to modelling involving sophisticated mathematical techniques. This report is framed to discuss initially the conventional operation policy followed by some of the rules developed by experience from operating the reservoir. This is followed by mathematical modelling applied to reservoir operation problems. A brief account of the various mathematical techniques used in reservoir operation are discussed and is concluded by a comparative bibliography of reservoir scheduling models. The direction of future work is discussed in the last chapter. The report provides an extensive bibliography on the topic.

2.0 HEURISTIC PROCEDURES

2.1 Introduction

Conventionally the reservoirs are designed on some yield models based on mass balance technique or its variations making use of historic streamflows or through probabilistic analysis making use of generalized statistical properties of the historic flows. In all the cases, built in there, is the conventional reservoir operation policy which is graphically represented in the figure.

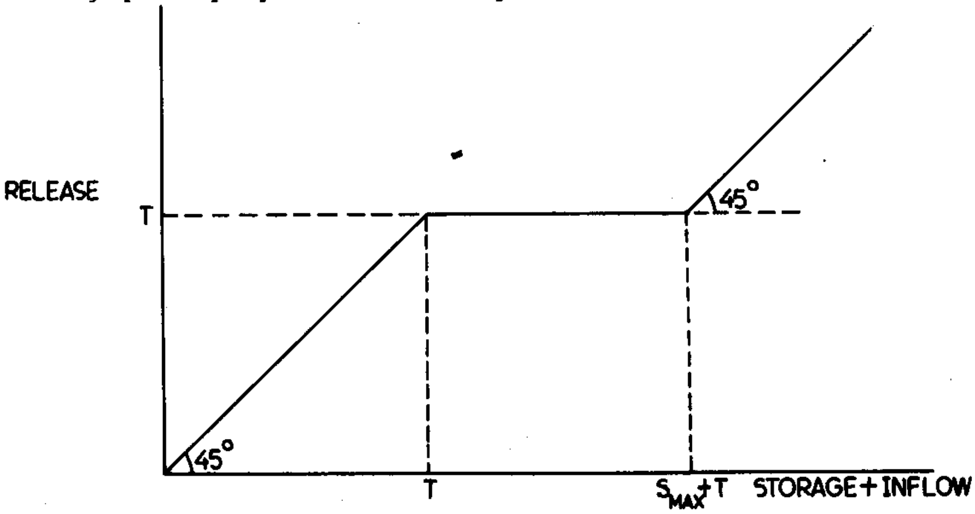

FIG.1 CONVENTIONAL RESERVOIR OPERATION POLICY

The classic operation policy has two conditional statements:

(i) If there is not enough water to meet the target, release all the water.

(ii) If there is more than enough water release enough to meet target output unless there is more water than can be stored, in which case the excess is also released. The policy stated above is a one time operation policy without relation to the release of water at any other time. This type of time isolated releases of water is neither beneficial nor desired. Second the water beyond the target output in any period has no economic value and that the economic loss due to shortage of water varies linearly. These are true in a restricted situations and in general there is always a marginal water use available or a market for surplus energy.

The classic reservoir policy is very rigid and restrictive in operation and so not used in practice.

Under the assumption that it is possible to define ideal storage levels and the releases or diversions therefrom for all periods short or long throughout the hydrologic period that satisfy all water users, reservoir operating policies are needed to guide the operator or the reservoir manager when it is not possible to satisfy these ideal conditions. The ideal storage volumes or levels in individual reservoirs are typically defined by 'rule-curves'. Due to varying hydrologic conditions the occurrance of deviations from these rule-curves is quite normal and hence a detailed set of instructions are necessary to take advantages of the excess water or to salvage the situation under deficient conditions.

Rule curve specifies the reservoir levels and releases for various purposes as a function of time and is to be updated as new data accumulate. For finding rule curves among the various reservoirs in the system, the criteria used is that the critical conditions should not be attained simultaneously in all the reservoirs. In the case of two reservoirs in series, the upstream reservoir release schedule will bias the development of rule-curve for the reservoir located on the downstream.

Rule curves must be understood as guidelines under stationary conditions (in the probabilistic sense). The purpose of the operating policy is to distribute any necessary deviations from ideal conditions in a manner that satisfies mandate laws or regulations and/or that minimizes the perceived discomfort or hardship to all water users in the system.

The existing operating policies vary from a rigid rule curve to those that precisely define management of deviations from the rule cuves for all combinations of hydrologic and reservoir levels, used in actual operation are quite instructive.

2.2 Some Operating Policies

These operational policies, which are developed by intuition are anticipatory in character and are quite useful in the case of single purpose single reservoir or multiple reservoirs as the case may be. These are useful specifically to municipal water supply reservoirs wherein conservation of water is the sole objective. The rules are different for series or parallel configuration of reservoirs.

(i) Reservoirs in series: The operating rule in this case is that downstream releases are met by the immediately upstream reservoirs till it is completely drawn down and before using any upstream reservoirs.

In the figure 2, releases D_2 and D_3 are to be met from reservoir 2 till it is completely drawn down .When it is not possible to meet the demand D_2 and D_3 from reservoir 2,then only water from reservoir 1 is to be released to meet the demands D_2 and D_3.This rule makes use of available storage fully and ensures conservation of avoidable spills from downstream reservoirs.

(ii) Reservoirs in Parallel: In this configuration two procedures are commonly used.

One involves discharging water first from reservoirs with relatively larger drainage areas or potential inflows per unit storage capacity.

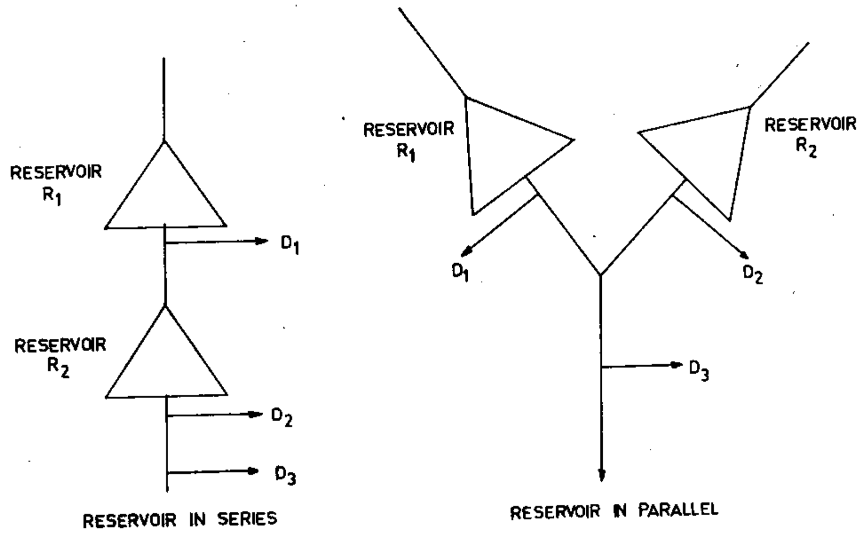

Fig.2 CONFIGURATION OF RESERVOIRS.

In the figure 2, the drainage areas to storage volume capacity ratios for two reservoirs are compared(assuming the runoff per unit of drainage area is same). The reservoir with the larger ratio is used to supply diversion D_3 before the other reservoir is drawn down. Discharging water first from the reservoir having the largest drainage to storage volume capacity ratio will usually result in a reasonable conservation of water. Another and more precise procedure involves drawing in tandem from each reservoir. This requires monitoring storage volumes and estimating future inflow. Such a policy minimizes expected water wastage. The later rule is referred to as a space rule. In terms of probability when parallel reservoirs are operated by this rule, the objective is to equalize the probability that the reservoirs will have filled at the end of drawn-down refill cycle. Actually all the reservoirs will be full and spilling, full and not spilling, or partly full, the unoccupied storage space being proportioned to inflows during the drawdown refill cycle.

Mathematically the sapce rule can be stated as :

$$\sum_{j}^{m} \frac{S_{max\,j} - S_{ijk} - Q_{jk} + R_{jk}}{(S_{maxj} - S_{ijk} - Q_{jk}) + R_T} = \frac{Q_{i\,,\,n-k}}{\sum_{j}^{m} Q_{j,\;n-k}} \qquad \ldots (3)$$

Where S_{maxj} is the full capacity of the jth in a series of m parallel reservoirs. S_{ijk} is the initial contents of the jth reservoir in the kth of a series of n months; Q_{jk} is the flow into jth reservoir in the kth month ; R_{jk} is the release from the jth reservoir in the kth month; R_T is the sum total of releases required to fulfill target outputs; and $Q_{j,\;n-k}$ is the prescribed flow into the jth reservoir for the remaining n-k months of the drawdown refill cycle. We can solve the above equation for R_{jk},the release from the jth reservoir in the kth month.

$$R_{jk} = S_{ijk} + Q_{jk} + [\; \sum_{j}^{m}(S_{maxj} - S_{ijk} - Q_{jkj}) + R_T \;]$$

$$x(Q_{j,\;n-k}/\sum_{j}^{m} Q_{j,n-k}) - S_{maxj}$$

Subject to the constaint $0 < R_{jk} < (S_{ijk} + Q_{jk})$.

A numerical example of the space rule is provided by Maass et al in their book Design of Water Resources System, which is reproduced for purposes of clarity.

Example: Application of the space rule to two parallel reservoirs. The reservoirs are operated for a typical month, during which the target output for irrigation is 250 x 10^3 acre ft. No unregulated inflow below the reservoirs and above the point of diversion is available.

The space rule is very useful for situations where inflow forecasting is very reliable as in the case of runoff from snowmelts. For other types of streamflows the effectiveness of the space rule would be a function of the coefficient of variation of the mean monthly flows, the correlation between flows on adjacent streams and the reliability of flow forecasts.

	I T E M.	Capacities, contents, inflows and releases 10^3 acre ft.		
		Reservoir A	Reservoir B	Total reservoir A & B
1.	Max reservoir capacity	150.0	2500.0	2650.0
2.	Reservoir contents at the beginning of month	50.0	1500.0	1550.0
3.	Available storage space at the beginning of month(1)-(2)	100.0	1000.0	1100.0
4.	Inflow during the month	90.0	400.0	490.0
5.	Total target output or release required during month	-	-	250.0
6.	Total space available at the end of month (3) -(4) +(5)	-	-	800.0
7.	Predicted inflow between end of month and end of refill cycle	110.0	1100.0	1210.0
8.	Proportion of required space at end of month	$\frac{110.0}{1210.0}$	$\frac{1100.0}{1210.0}$	$\frac{1210.0}{1210.0}$
9.	Allocation of space at the end of month (6) x (8)	78.2	781.8	860.0
10.	Reservoir contents at end of month (1) -(9)	71.8	1718.2	1790.0
11.	Release during month(2)+ 4)-(10)	68.2	181.8	250.0

In modified form the space rule can also be used to apportion releases among reservoirs for flood control, based on 6-hr or other short intervals of time. It is valid too, when each unit of water is of equal value in a given reservoir but not of equal value in different-reservoirs. However, the space rule must be modified in form to deal with this situation. Unequal values of water are created in a system for instance when a fixed head power plant downstream from reservoir B generates firm energy from irrigation releases, but no such plant exists below reservoir A. In an application of the modified space rule, the economic value of system output was maximized by preventing or minimizing spills of the higher valued water from reservoir B at the expense of spills lower valued water from reservoir A. Had the water within either of the two reservoirs not possessed a fixed value, (as in the case of variable-head power plant) application of the rule would have become much more difficult.

(iii) Pack Rule:

The Pack rule makes use of the streamflow forecasts and tries to avoid spills by additional releases of water in advance(say for dump energy generation). The Pack rule is so called because the possible future spill is packed as tightly as possible into future space turbine capacity.

In mathematical terms:

$$R_d = Q_{n-k} - (S_{max} - S_{TK}) - P_{n-k} \qquad \ldots (5)$$

Where R_d are the additional releases for the current month, k, for the generation of dump energy; Q_{n-k} is the predicted flow into the reservoir for the remaining(n-k) months of the drawdown refill cycle; S_{max} is the full reservoir capacity; S_{TK} is the reservoir contents in the current month after current flows have been added and releases made to meet the target output for energy and P_{n-k} is the useful water capacity of the turbines for the remaining n-k months of the drawdown refill cycle. If the right hand side of the equation is not positive $R_d = 0$ and the equation is further subject to constraint $P_c - R_d \leq S_{TK}$ where P_c is the useful water capacity of the turbines in the current month after releases have been made through turbines to meet the target output for energy. Predictions as they are seldom perfect that the reservoir may not refill or spill.

Pack rule can be applied whenever releases beyond specified output requirements are of value. The rule can be of assistance in making decision involving time for simple systems or for parts of complex systems, it can not be used to apportion water among purposes of reservoirs.

The Hedging Rule

The concept of hedging is to distribute, the shortage if anticipated,uniformly so that the intensity of shortage is minimized. It is sometimes economical to accept a small current deficit in output so as to decrease the probability of a more severe water or energy shortage.

Economically a hedging rule can be justified only if the proposed uses of water have nonlinear loss functions. If the marginal values of water for specific uses are constant, the economic losses from shortages must be linear and,because streamflows are stochastic, it follows that it is optimal to postpone shortages as long as possible, inspite of a high probability of a severe deficit later. Therefore, an assured full supply now is preferable to a definite deficit now with a lesser probability of a heavy deficit later. If severe deficit are penalized proportionately more than mild deficits, however, it may pay to reduce the probabilities of suffering heavy deficits.

2.3 Storage Zoning

The zoning of the storage space and rules governing the maintenance of the storage levels in any specified range is based on the reasoning that there exists, for a specified time, a desired storage level in each reservoir. The operators are expected to maintain these levels in each of the reservoirs as closely as possible while generally trying to satisfy various water needs downstream. The releases are adjusted upward or downward depending on the reservoir condition being above or below the specified level respectively. These release rates may not be specified but will depend in part on any maximum or minimum flow requirements and on the expected inflow.

Typically a rule curve exactly aims at this type of operation. The desired storage level for single purpose reservoir is easy to arrive at, whereas for a multipurpose reservoir it is worked out as a compromise among recreational, fish and wild life, flood control, hydropower and water supply interests. These rule curves are derived either based on historical stream flow records or through simulation studies. Still left in this type of operation is the large flexibility in day to day operation to adjust releases and storage levels within the specified limits. This type of operation needs a very experienced personnel to decide day to day operation taking into consideration the trade offs among purposes and at the same time minimizing the deviations from desired storage levels and discharges over time and space.

The operation in few cases is further assisted by multiple zoning. Various storage zones often identified are:

(i) Conservation Zone: This is the zone, the water stored in which has to satisfy the release demands for various conservation purposes including recreation and environmental needs.

(ii) Flood control Zone : This is the storage space exclusivdely earmarked for absorbing floods during high flood periods. The releases are increased as necessary when the water stored in the reservoir falls in this zone.

(iii) Spill and Surcharge Zone: This storage space is above the flood control zone and corresponds to the flood rise during extreme floods and spilling. This space is occupied mostly during high flows and the releases downstream (i.e.spills) are at or near maximum.

(iv) Buffer Zone: This is the storage space between conservation zone and the dead Zone and the reservoir level will be brought down to this zone under extreme draught situations. When the reservoir is in this zone, the release from the reservoirs caters only to essential needs.

(v) Inactive Zone or Dead Storages: This space is normally meant to absorb some of the sediment entering the reservoir. The water in this zone may be utilized only under extreme dry conditions and the withdrawls are limited to absolute minimum.

Figure 3 shows such zones in a reservoir. These zones may vary throughout the year. The release policy will be to release as large as possible when the reservoir is in spill zone and to release as maximum as possible without causing flood damages downstream when the reservoir content is in the flood control zone, and try to bring the reservoir to the top of the conservation zone at the earliest possible time. The release from the conservation zone is governed by the requirements of water for various purposes intended to be met by the stored water and the day to day releases may be adjusted based on the inflow anticipated and the future requirements upto the end of the operating horizon. When the amount of water is anticipated to be short compared to the demand, releases may be curtailed. It must again be noted that the limits of zones may vary from month to month. This type of defining of zones varying over months is useful in multipurpose reservoirs.

In the context of multiple reservoirs,zoning offers some flexibility in operation of individual reservoirs. Such guidelines for multiple reservoirs together with rules for individual reservoirs offer additional guidance to operators of multiple reservoir systems. A further guidance is provided some times by defining subzones within conservation zone. The way it is commonly managed is to balance the storage level in different reservoirs, that is at any time all the reservoirs are maintained in the same zone(meaning the percentage filling of the zone is equal in all the reservoirs called equal function policy) or the reservoirs releases is based on some sort of ranking or priority concept(that is water in a particular zone is released from low priority reservoir first, and then from the next higher priority reservoir and so on) or by maintaining a storage lag between reservoirs(meaning withdrawls from the same zone from different reservoirs are effected with some time lag, but still keeping the percentage difference of zone volume of all reservoirs same within the lag cycle). This type of operation is needed to provide corrections the reservoir balancing after an expected or extreme hydrologic event.

These multiple zones and sub-zones and operating rules are prescriptive in character as compared to simple curve and this is a desirable feature as it reduces the enormous flexibility available in operating reservoirs with simple rule curves.

Further guidance in operation with regard to rate of release of water from storage is provided by defining flow ranges, which are defined in terms of normal range, extended range and the extreme range. These nomenclatives are very clear. Normal releases correspond to normal reservoir conditions, the range is to provide adjustments in deviations from the rule curve contingent on the actual stream flow. Extended range corresponds to extreme storage conditions.

Further, sometimes, wherever it is possible to forecast floods or snow melts, conditional rule curves can be defined. These rule curves may be presented either in the form of tables or graphs and show the desired reservoir levels as a function of the expected inflows. Conditional rule curves may be defined for the entire operation cycle time or for a part of it.

Many of the heuristic procedures do not take explicitly economic considerations to account, though indirectly do so in terms of water conservation and release controls and aim at reliability and resilence.

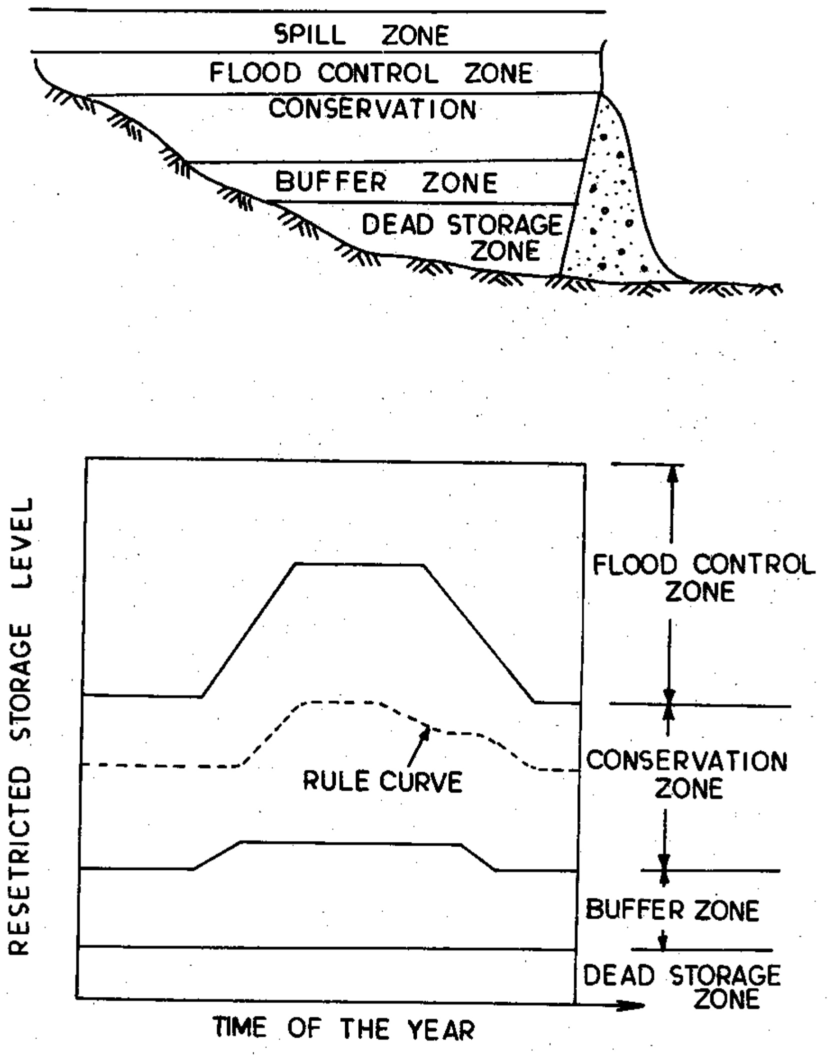

FIG. 3 - VARIOUS RESERVOIR ZONES AND THEIR VARIATIONS WITH TIME

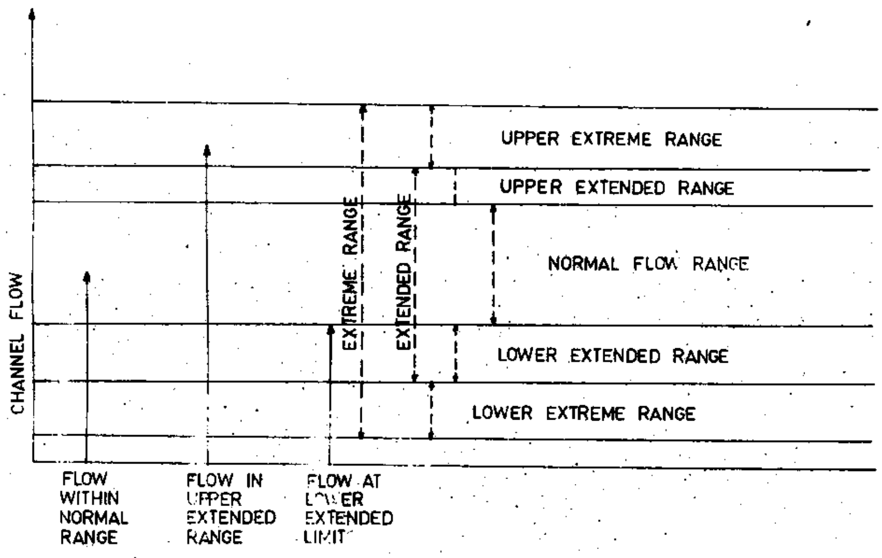

FIG.4 CHANNEL FLOW RANGES BASED ON STORAGE

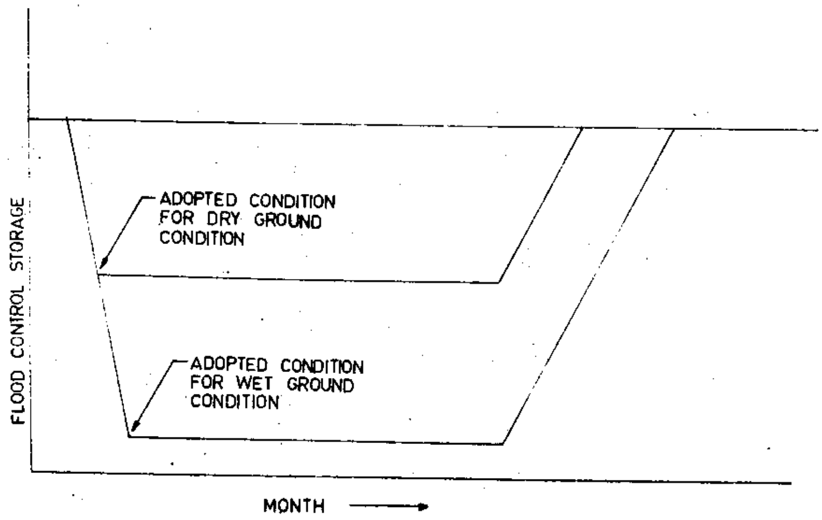

FIG. 5 CONDITIONAL RULE CURVES

3.0 ECONOMICS OF HYDROTHERMAL COMBINATION

The purpose of this discussion is to analyse the economic interaction of Hydropower stations and thermal powerstations. To maintain clarity initially analysis is confined to deterministic environment wherein, the inflows to Hydropower are assumed known and extensions to stochastic situations are discussed. The operation problem considered is of one year duration divided into number of weeks. The objective is to maximize net economic benefits or minimize the cost of generation, subject to constraints of the system. The discussion below which is very clear is due to Read(1982).

3.1 Deterministic Case:

(i) Scheduling of Isolated Thermal Stations:

Optimal scheduling of a purely thermal system involves equating marginal cost of generation from all sources as nearly as possible. So in a lossless system, the mathematical problem can be stated as

$$\text{Min} \sum_{i=1}^{I} C_i(g_i) \qquad \text{(Minimize costs)} \qquad \ldots(6)$$

$$\text{S.T.} \sum_{i=1}^{I} g_i \geq D \qquad \text{(meet load)} \qquad \ldots(7)$$

Where i= 1, 2,...,I is the index of thermal units

g_i = generation of ith unit

C_i = cost of generation of ith unit which is a functions of generation level g_i

D = system load (demand)

The inequality (7) helps to maintain feasibility by allowing excess production should that be necessary.

Incorporating the constraint(7) into the objective function using a lagrange multiplier λ, the problem is written as

$$\text{Min} \quad \Sigma \; (C_i(g_i) - \lambda g_i) + \lambda \; D \qquad \ldots(8)$$

As all C_i are convex functions, this may be solved(for a particular λ) by differentiating with reference to g_i to find g_i (λ) such that

$$\frac{dC_i}{dg_i} = \lambda \text{ for } i = 1,2,\ldots,I \qquad \ldots(9)$$

Equation (9) states that the marginal cost of generation equals its 'price' or value to the system. Therefore, the optimal solution to the problem then corresponds to the energy price $\lambda^*(D)$ for which the generation levels $g_i^*(\lambda)$, $i = 1,2,\ldots I$ just satisfy (7).

In presence of losses (L), the constraint (7) gets modified to

$$\sum_{i=1}^{I} g_i - L\,(g) \geqslant D \qquad \ldots(10)$$

Where L depends on the whole vector of generation levels $g = g_i$; $i = 1,2,\ldots,I$

The corresponding optimality condition is

$$dC_i/dg_i^* = \lambda\,(\,1 - \partial L)/\partial g_i) = \lambda_i \qquad \ldots(11)$$

The situation is complicated by the existence of fixed running costs which introduce non-convexities into the cost functions. It is further complicated by start up costs, constraints on generation rates, ramping rates and minimum down time. These with load continuously varying, mean that units often generate at levels other than their 'most economic'level. However, for purposes of reservoir operation it is assumed that a well defined short term marginal cost λ exists corresponding to any load curve. Further it is assumed that λ is a monotone increasing function of system load levels(This guarantees uniqueness of solutions). The assumption of a single value of λ over a short duration is a simplification.

Mixed Hydrothermal System-with Run of the River Plants

In a mixed hydrothermal system with only run of the river hydroplants, all inflows to hydroplants must be released in the period in which they arrive and the thermal system must then meet the residual load. The system marginal cost λ^* is also the marginal value of hydro generation. When the efficiency of hydrogeneration is accounted for, this marginal value corresponds to 'Natural price of water' denoted by π. It is the marginal value of water in the absence of long term storage capacity. Mathematically, the constraint equation (7) changes to

$$\sum_{i=1}^{I} g_i + \sum_{h=1}^{H} g_h(q_h) \geq D \qquad \ldots (13)$$

ignoring losses and $g(q_h)$ denotes the volume of water released from hydroplant h. In the absence of storage capacity $q_h = F_h$, the inflow to plant h, and the natural price of water for plant k is given by

$$\pi_k(F) = (\lambda^*(D - \sum_{h=1}^{H} g_h(F_h))) \frac{dq_k}{dF_k} \qquad \ldots (14)$$

This is correct if the load is specified and is to be met fully.

Alternatively if we think in terms of social welfare, wherein the prices are determined by marginal costs, the optimality condition is that the marginal cost of generation is equal to marginal benefit which is obtained through demand curve.

Mathematically, the problem is

$$\text{Max } B(D) - \sum_{i=1}^{g} C_i(g_i) \qquad \ldots (15)$$

When constraint is incorporated, the optimality condition, in addition to (9) includes

$$\partial B/\partial D = \lambda = \partial C_i/\partial g_i \qquad i=1,2,\ldots, T \qquad \ldots (16)$$

i.e. marginal benefit from load reduction equals marginal cost of producing power.

Hydrothermal System-Unbounded Reservoir:

The case of unbounded reservoir coupled with a thermal system is somewhat a hypothetical situation, but provides necessary insight into the operation of finite reservoir. If the reservoir never attains either the empty or full levels, then in the absence of release bounds and head effects, the marginal value of releasing water should be same throughout the planning horizon. If this were not so then savings could be achieved by increasing the amount of water released in period in which it is more valuable and decreasing the amount released in periods in which it is less valuable.

Suppose a system is being operated without any long term storage, then the marginal value of water, its natural price π, will vary from period to period. Naturally by building a reservoir, one can buy water from the system when its relative abundance makes its natural price low, storing it in reservoir, then 'selling' it back when its relative scarcity make its natural price high. Of course, since the system actually own the reservoir, all benefits really accrue to the system.

One effect of such trading will make more water available when it is most needed. So lowering the price of water in such periods to λ' less than π (Here λ', the marginal value of releasing water, differs from λ because of the (varying) marginal productivity of release).

Likewise the price will rise above π for periods in which water is relatively abundant. 'Trading' will continue so long as the price in any period exceeds that in any other. An equilibrium situation will be achieved when the prices in all periods are the same. Let ψ denote this equilibrium price. For any value ψ it is possible to define a corresponding storage trajectory since in periods for which π is not equal to ψ, just enough water will be added to or released from storage to bring the marginal value of releasing water to equal ψ. However only one of these trajectories will be feasible since only one will exactly attain the final storage target. The corresponding equilibrium price $\bar{\psi}$ is then the true marginal value of water in the reservoir, a constant through out the year. So the reservoir should be used to even out marginal costs through the year as far as possible.

Mathematically

$$\text{Min} \sum_{t=1}^{T} \sum_{i=1}^{I} C_i(g_i^t) \qquad \ldots (17)$$

such that

$$\sum_{i=1}^{I} g_i^t + g_H^t(q^t) \geq D^t, \qquad t=1,\ldots,T \qquad \ldots(18)$$

$$s^o = S^o \quad \text{initial storage} \qquad \ldots(19)$$

$$s^t = s^{t-1} + F^t - q^t \qquad t - \text{flow balance} \qquad \ldots(20)$$

$$s^T = S^T \qquad \ldots(21)$$

Here $t=1,\ldots,T$ indexes time periods

g_H = generation from the reservoir

S^t = Storage at end of period t

Now the load constraints may be incorporated into the objective using a separate energy price λ^t for each week. The optimality conditions for the thermal systems are exactly as discussed previously(for each week). For the hydrosystem the following sub-problem must be solved

$$\text{Max} \sum_{t=1}^{T} \lambda^{T} g_H{}^{t}(q^t) \qquad \ldots (22)$$

$$\text{S.T.} \sum_{t=1}^{T} q_t = S^o - S^T + \sum_{t=1}^{T} F^t \qquad \ldots (23)$$

Writing down the Lagrangian in terms of the 'marginal water value' the problem becomes

$$\text{Max} \sum_{t=1}^{T} (\lambda^t g_H{}^t(q^t) - \psi q^t) + \psi (S^o - S^T + \sum_{t=1}^{T} F^t) \ldots (24)$$

provided g_h is a concave function, the optimal solution for a particular ψ, and λ, will be $q^{t*}(\lambda, \psi)$ such that

$$\lambda^t \, dg_H{}^t/dq^{t*} = \psi^t \qquad t = 1,\ldots,T \qquad \ldots (25)$$

that is marginal benefit from release = marginal benefit from storage.

Now for a particular set of energy prices(λ) the hydro subproblem(22 and 23) can be solved by finding $\bar{\psi}$ such that $q^{t*}(\lambda, \bar{\psi})$ satisfies (23). Then the energy prices can be adjusted to find $\bar{\lambda}$, for which the load constraints (18) are also met.

The optimality conditions are

$$dC_i/dg_i{}^t = \bar{\lambda}^t \qquad i = 1,2,\ldots,I \qquad \ldots (26)$$
$$t = 1,2,\ldots,I$$

$$\bar{\lambda}^t \, dg_H{}^t/dq^t = \bar{\psi} \qquad t = 1,\ldots,T \qquad \ldots (27)$$

The true marginal water value, $\bar{\psi}$ is the optimal multiplier on the constraint that the final storage target be met. This means that in order to ensure a final storage volume of S^T and so provide adequate water for the next year the system pays a 'price' of ψ per unit of volume. This price represents the marginal value of generation foregone in the current year. $\bar{\psi}$ is frequently referred to as the 'value of water' or the 'water value'.

It should be noted that it is a marginal value.

Once this water value has been determined, short-term scheduling is straight forward. In any period there are two alternative uses for water: releasing it to reduce current thermal generation costs, or storing it for future use. Obviously, water should be released if and only if marginal benefit of doing so exceeds its marginal value if stored for future ψ. So water can be treated as a fuel, which can be 'bought' from the reservoir at the price ψ. Indeed all of the long-term reservoir scheduling models, deterministic or stochastic can be seen as more or less complicated ways of estimating ψ for one or more reservoirs.

When scheduling releases over a one year planning horizon it will generally be necessary to ensure that sufficient water remains at the end of the year for the next years' operation. This may be achieved by specifying a target storage level as above. If the reservoir is in a stable cycle than the initial storage level may be an appropriate target. However, this does not allow any flexibility in balancing the benefits from using water in the current year with the benefits from leaving it in storage for the next year. Of course this balance could be achieved by extending the planning horizon indefinitely. More practically it may prove possible to estimate the marginal value of storing water for the next year as a function of the end of year storage level. Then the optimal storage trajectory will be the one in which the marginal value of using water in the current year equals the marginal value of keeping it for the next year as determined by the final storage level of that trajectory

Mathematically constraint(21) is dropped and the value of final storage $V^T(S^T)$ is added to the objective. The optimality condition (25) then becomes:

$$\lambda^t \frac{dg_H^t}{dq^T} = \frac{\partial V^T}{\partial q^t} = \frac{\partial V^T}{\partial x^T} \bar{\psi} \qquad \ldots (28)$$

In practice higher storage levels may lead to greater productivity of release. So generation is really a function not only of current releases but also of the storage level and hence of all previous releases. If this 'head effect' is significant the previously outlined policy may have to be modified. This is because there is an additional benefit in storing water to increase the productivity of later releases. The impact of this on release decisions will be greatest in early weeks, falling away to zero in the final week. So when this effect is accounted for, the value of storing water, and hence of releasing water, should gradually fall during the planning horizon. So releases will be lower than they otherwise would be in early weeks leading to higher storage levels. In later weeks this will be compensated by releasing more water than would have been released otherwise, so that the final storage target is not exceeded.

Mathematically the optimality condition (27) becomes:

$$\lambda^t \frac{\partial g_H^t}{\partial q^t} = \sigma - \sum_{r=t+1}^{T} \lambda^r \frac{\partial g_H^r}{\partial q^t}$$

$$= \sigma^T + \sum_{r=t+1}^{T} \lambda^r \frac{\partial g_H^t}{\partial s^r} = \psi t' \qquad \ldots (29)$$

Here σ^T is the optimal multiplier on the final storage target constraint, that is the price of ensuring sufficient storage for next year. Obviously this will be lower in this case than it would have been without the head effect, otherwise ψ would rise and total releases would fall. This is because there is now more incentive to store water and so the target plays a less important role. It is even conceivable that the head effect could make it economic to exceed the final target. If the target constraints is an equality (as in this formulation) the multiplier on it(σ^T) will then become negative. This means that there is a cost to the system in keeping storage down to S^T. If (21) is relaxed to become a lower bound on end point storage, σ^T would drop to zero and end point storage would rise above the target. Of course such excess storage will only occur if it is beneficial to the system.

Finally if future benefits are discounted then the marginal water value, rather than being constant, should rise at the discount rate. So the present discounted value of water is the same in all periods. The above analysis naturally corresponds to a zero discount rate.

One Bounded Reservoir Case:

Real reservoirs obviously do have limited capacity and in general it is not economic to construct reservoirs so large that these limits are never attained. So storage bounds do have a significant impact. The impact of lower bounds alone will be considered before upper bounds are introduced. The bounds discussed here need not be constant throughout the year. Head effects will be ignored.

If a high water value is assumed, then water will tend to be held in storage and so the final storage level of the corresponding trajectory will be high. Obviously the analysis of the previous section still applies if there is a single value $\bar{\psi}$ resulting in a trajectory which attains the storage target without emptying the reservoir. But suppose that starting from an initial water value which resulted in the final storage target being exceeded, successively lower values are tried until at ψ_D the trajectory just empties the reservoir in some week t^o but still results in too much water in storage at the end of the year. Such a trajectory is shown in Figure 7 . It may be said to be 'tangential' to the lower constraint set in the sense that it just touches it at t^o then rises above it. Now, if it were not for the fact that the reservoir is already empty at t_D, it is apparent that a lower final storage level could be achieved by lowering ψ further to induce more water to be released throughout the year. But it is evidently impossible to release more water, in total, during the period before t_D.

In fact it will be shown that the first arc of the trajectory corresponding to ψ_D (i.e.,that part of the trajectory up to t_D) is optimal. So ψ_D is the optimal water value throughout that period. Any higher water value would not fully utilize the capacity of the reservoir, since it would never completely empty it. Any lower value could not induce a greater total release in the period upto t_o. It would only resualt in misallocating of releases within that period. Too much would be released early on, then less would be available to be released in the weeks immediately prior to t_o than should be.

Now after t_o the water value must still be lowered until the excess final storage is eliminated. As itis lowered, it is almost certain that the trajectory corresponding to some new value, $\psi_1 < \psi_0$ say,will empty the reservoir at $t_1 > t_0$ but still exceed the final storage target. So, again, the trajectory segment between t_0 and t_1 is optimal and the water value ψ_1 optimal throughout this interval (from t_0 to t_1) .
This process may be continued, finding successively lower marginal water values for successively later intervals until the final storage

target is met.

So, in the absence of upper bounds on storage the optimal marginal value of water will stay constant at $\psi = \psi_0$ until the reservoir first becomes empty. Then, typically, the reservoir should stay empty for some weeks while the water value drops. During this time the optimal trajectory may rise above the empty level in some weeks, during which time the water value will ramain constant. But in no case will the water value rise in any week. Whenever the reservoir is empty for two successive weeks the release must equal the natural inflow, so the optimal water value ψ^t is in fact equal to the natural price π^t. Eventually the water value drops to a value ψ_T which is low enough to avoid excess final storage. Then the optimal trajectory rises above the empty level to finish at the storage target, with ψ_T being the optimal water value over this final trajectory arc.

Obviously if there is only one reservoir, there is no need to pursue the analysis beyond the determination of Ψ_0 since it is only this value which is required to determine an optimal schedule for the first week. The tangency condition discussed above is the basis of the trajectory method introduced.

Mathematically this problem only differs from the previous one in that lower bounds are introduced on the storage level by:

$$s^t \geqslant \underline{s}^t \qquad t=1,\ldots\ldots,T \qquad (30)$$

These constraints can be incorporated into the objective function using a set of multipliers γ . Consideration of the resulting Lagrangian problem shows that the optimality condition(25) remains unchanged except that the marginal water value must be redefined to be:

$$\psi^t = \sigma^T - \sum_{r=t+1}^{T} \gamma^r \qquad t = 1,\ldots\ldots,T \qquad (31)$$

(If the final storage target is actually treated as a lower bound, there is no real need to distinguish between this and other bounds and σ may be dropped).

Now the marginal water value can obviously change from week to week, because of the γ multipliers. But these multipliers are only positive when the constraints are active. So $\bar{\Psi}$ remains constant over unconstrained arcs of the trajectory and drops in each week a lower bound is active. This is because after that week that bound is no longer in the future and the marginal value of water in storage depends only on its future utilisation,not on the past.

The effect of upper bounds is very similar to that of lower bounds, although generally in an opposite sense. They become active when it is impossible for a constant water value trajectory to leave enough water in storage at the end of the year without exceeding them. This means that the ability of the reservoir to assign inflows during the early part of the year to meet loads in later parts of the year is not great enough to even out marginal thermal costs to a constant level. As for lower bounds the optimal initial trajectory arc is the first arc of the trajectory which just fills the reservoir at some time t_0(i.e.,it is tangential to the upper bound). The corresponding water value ψ_0,will be valid up until that time. Any extra water available in that interval could only be used in that interval and so its value does not depend on any later week. After that time the water value will rise, following the natural price function while the reservoir stays(approximately) full. When the water value is high enough so that the trajectory arc corresponding to it just touches the next storage target, the storage level will fell away from the upper bound again, the water value remaining constant until the next target is attained. Here the next target may be the final storage target in the final week. But it will be the empty (full) level at the next time at which the reservoir becomes empty (full) if it is impossible to find a constant water value trajectory from the full level to the final target without emptying (filling) the reservoir in between. The initial storage level for each arc will be either S^o or the bound on which the last arc terminated.

So in general the optimal storage trajectory will contain a series of arcs over which the water value remains constant. Between these will be periods when the reservoir is full and the water value rising and periods when it is empty and the water value falling. Figure(7) shows a typical optimal trajectory while figure(8) shows the corresponding water values.

Mathematically, there are now upper bounds as well as lower bounds on s^t. So Ψ^t is again redefined to be:

$$\Psi^t = \sigma^T + \sum_{r=t+1}^{T} (\gamma^r - \delta^r) \quad t = 1,\ldots\ldots,T \qquad \ldots (32)$$

Hence δ^t is the multiplier on the upper storage bound in week t. It may be seen from equation (32) that the marginal water value will rise while the reservoir is full (i.e. while δ^t is positive).

It may be noted that the existence of such positive multipliers implies some economic incentive to expand the storage capacity of the reservoir so as to allow more water to be transferred from periods where its value is comparatively low to those where it is higher. That is δ is the price the system should be prepared to pay for more storage capacity in week t. Also a discrepancy between initial and final water values may indicate a poor choice of storage target. The marginal value of water at the beginning of two successive years should not be substantially different,unless there are special circumstances.

Of course only the water value for the first arc of this trajectory is required. Note that if it should prove necessary to spill water in some week the marginal value of water will fall to zero throughout the trajectory arc preceding that week, resulting in maximum discharge, if not spill, throughout that arc. Extra water available during that time could not be utilized and so is worth nothing.

Conceptually, while the reservoir is approaching its full level the manager's primary concern is to release enough water to avoid spill. So the marginal water value is low. Mathematically this is due to the δ multipliers which set a 'price 'on violating the full constraint, and so spilling water, which is high enough to ensure that this avoided if possible. As soon as the reservoir becomes full, however, his concern will be with avoiding the next constraint, generally the empty level. Mathematically this'concern' is expressed by the γ multipliers which set a price on running out of water. So the value of water quickly rises to conserve storage. The reservoir tends to stay full for some weeks. After this water is released much more continuously until the reservoir actually becomes empty. Then concern switches back avoidinq spill, or excess storage, and water values fall once more.

Many Reservoirs:

The case of many unbounded reservoirs which are not in the same river system is identical to that of one unbounded reservoir. There will be a single marginal water value for all reservoirs at all times, apart from differences caused by differences in release productivity.

With many bounded reservoirs, however, this will not be the case. Water value will vary from reservoir to reservoir depending on the interactions between each reservoir's inflows and storage bounds. But each reservoir will behave in exactly the same manner as the single reservoir discussed above. Furthermore, the marginal cost of producing power from each reservoir in any period will be equated as nearly as possible to λ^t. This fact is the basis of the decomposition algorithm. So the reservoirs behave just like thermal units with fuel costs given by their water values.

If plant efficiency were a constant independent of the rate of release, this would mean that those reservoirs with ψ less than or equal to λ^t would generate at their full output, while the remainder generated at minimum levels. If,however, the efficiency of generation varies as a function of the release rate, several reservoirs with differing water values may be generating at intermediate levels to produce at the same marginal cost. Realistically this will usually be the case because the value of generation is not in fact constant at λ^t during the week, being higher in peak and lower in off-peak hours. So if water in a reservoir has been assigned a high value then it may only be used during the peak. At the same time another reservoir with a lower water value may release throughout the week. Also there may be several hydro plants downstream of one reservoir so that the marginal productivity of release will fall off as each plant reaches its maximum output level. So for any λ^t there may be many reservoirs with differing water values for which the optimal release is intermediate between minimum and maximum levels.

If reservoirs are in the same river system the same principles apply. But the marginal value of releasing water from one reservoir is then equal to the marginal value of water in the next reservoir downstream,plus the marginal benefit from generation from the intervening stations.

In the multi-reservoir case it is still true that only the water value for the first arc of each reservoir's trajectory is required. But unless all reservoirs come into constraint simultaneously, a model covering the entire planning horizon will be required to estimate those values. This is because the first arc of one reservoir's trajectory may interact with the second arc of another's. This in turn may interact with the third arc of another's and so on throughout the planning horizon.

Many Sub-Systems:

Finally, if the reservoirs are in different sub-systems a further complication may occur. Limited transmission capacity between the sub-systems may mean that marginal costs can not be equalled throughout the system. In this case marginal costs are equalized, when losses are accounted for, if the transmission links are not at their maximum limits.

Suppose that two adjacent sub-systems were not linked and so had different equilibrium prices in each period. Then if a hypothetical enterpreneur build a transmission link between them he could buy power from whichever region had the lower marginal cost and sell it to the region with higher marginal cost. This would make the marginal cost in the cheaper region rise while that in the more expensive region fall. Such trading would continue until the difference between the two region's prices exactly equalled the marginal losses incurred by the transmission link or until the capacity of the link was reached. Then ,when power is already being transmitted at the maximum rate from one sub-system to another, the marginal cost in the receiving sub-system may have to remain above that in the sending one in order to meet load levels there. Technically this discrepancy may be accounted for by a multiplier on the transmission capacity constraint. The existence of such multipliers indicates an economic incentive to expand the capacity of the link. It is the marginal value to the system of increasing transmission capacity in that period.

Mathematically, if the losses incurred by the link(L)are a convex function of the transfer through it(e) between the bounds $\underline{E}$ and $\bar{E}$, the optimal transfer from sub-system n to sub-systgem m is e^{*}_{nm} where:

$$e_{nm}^{*} = \mathrm{MIN}(\mathrm{Max}(\bar{e}_{nm}, \bar{E}_{nm})\ \underline{E}_{nm}) \qquad \ldots (33)$$

$$\lambda_m = \frac{dL}{de_{nm}} = \lambda_m - \lambda_n \qquad \ldots (34)$$

(i.e.marginal lost value= marginal gain from transfer).

Stochastic Analysis:

It has been pointed out earlier that the certainty of future flows is the most critical and difficult problem as compared to uncertainty in any other related matters. Because of this uncertainty, the release decisions may lead to crossing of bounds prescribed(i.e. may not satisfy the constraints specified) and thereby produce greater variations in thermal generation levels than that would be with perfect knowledge of the future inflows.

As compared to the deterministic analysis, the basic difference in stochastic analysis is the uncertainty of future inflows. This small difference creates a very large computational problem in reservoir scheduling problem as it is necessary to account for all possible inflows at any time and in the near future of operating horizon, and further the data necessary to evaluate all such possibilities is not available. Under these conditions, a real stochastic analysis may be impossible as of today,however, some simplified and approximate solutions are possible which might be closer to the desired optimum solutions.

by ignoring uncertainty, the policy derived become unrealistic. In addition to uncertainty of inflows, there is uncertainty of the model, the impact of which on the model's recommendation is not recognized so far.

The sequential decision process must be based on the information available at hand. Many models suggested deviate from this ideal situation. Such approximations are expected to introduce a systematic bias into the models conclusions.

In the case of runoff the river plants, the decision on the release of water at any time has no link with the releases at any other time. The releases correspond to the inflow. As such there is absolutely no difference in the deterministic or stochastic analysis in this case.

Unbounded Reservoir:

Consider a single reservoir with upper and lower bounds, but assume that the reservoir is so large that it will always be within the bounds. As is the deterministic case, the optimal first period decision depends only on the expected marginal value of water stored at the end of first period. Furthermore the marginal value of water stored in the multiplier on the final storage target constraint. But the basic difference from the deterministic case is:

(i) The non-anticipative nature of decision process must be modelled.

(ii) The concepts of feasibility and optimality must be examined.

(iii) The true magnitude of the stochastic problem is to be recognized.

Non-Anticipative decision is one in which the decision at any time is based only on the information available at that time. This is the basic difference from deterministic case wherein decisions at any time is based also on all future inflows which are assumed known. The non-anticipative decision on release R at any time as a function of inflow Q upto that time can be written as:

$$R(Q) = (R^1(Q^1), R^2(Q^2), R^3(Q^3) \ldots . R^T(Q^T)) \qquad \ldots(35)$$

where

$$R^t = R^1, R^2, \ldots \ldots, R^+ \qquad \ldots(36)$$

$$Q^t = Q^1, Q^2, \ldots \ldots, Q^+ \qquad \ldots(37)$$

The distribution of Q may be quite general but most naturally thought of as the set of all historically observed inflows.

For every sequence of Q, there exists a sequence of R. Together these sequences define a decision strategy. Then the question arises that for a strategy to be non-anticipative,is it necessary to consider all conceivable inflows from zero to infinity. This is not necessary; the analysis may exclude sequences of zero probability or the analysis can be further curtailed through chance constraints.

In practice it is recognised that decisions are made and modified continuously in response to inflows and other contingencies as they occur. But this cannot be modelled exactly. In the models proposed one can see a difference between'inflow-first-decision next' and 'decision first-inflow next' concepts. The later one is more realistic. Though in a continuous process ,not much difference appears in these two cases, the results are different from these two types of analysis but the earlier type is easy to model as the final storage is known with certainty once the decision is made.

Optimality:

In a deterministic case, optimal decision sequence is one which maximizes net benefits.

In a stochastic environment, there must be one such sequence for every possible inflow sequence. So it is more sensible to talk of a decision strategy(also referred to as decision rule, decision policy, or recourse function).

The strategy for release in each period is derived in response to any inflow sequence in a way which maximizes expected benefits. The solution has to satisfy the constraints of the problem for all sequences of inflow, and also the non-anticipative nature of decision.

Suppose two inflow sequences, same up to a particular point in time t, and diverging thereafter. The same decision must apply to both the sequence upto and including t.

Variability of final weeks inflow must be absorbed in the last week. If releases are constrained it may well prove impossible to find a common penultimate storage level from which the target can be reached for both sequences.

A decision strategy can be non-anticipatively feasible unless at stage it is possible to find for each future inflow sequence, a feasible non-anticipative release sequence from then on. The release decisions at any time must be based on the information available upto that time and the strategy should be such that the decisions should remain feasible for all future inflows. The solution with this property is said to possess complete recourse. This means that the decisions maker is never caught in a position, unable to proçeed without violating constraints. This is the most desirable feature and can be achieved by increasing the flexibility of model and modification of constraints through:

(i) penalty functions

(ii) Storage target can be replaced with and end of the year storage benefit function.

(iii) The targets may be relaxed to become a simple lower bound on final storage or a target storage range.

(iv) It may be necessary to introduce induced constraints to ensure complete recourse.

These are projection backward in time of the final constraints.

These manipulations are necessary to maintain feasibility and they are similar to the thinking of multiple zones discussed earlier.

Often ensuring feasibility is a problem. So one can think of violating some of the constraints(rather than rigidly satisfying the constraints) and specify that the probability that it is violated not exceeding a certain level. These types of constraints are referred to as chance constraints. The computation of problem of a stochastic programming is very large despite the fact that all flows with zero probability need not be considered. The model uncertainty with regard to the input based on historical flows may lead to suboptimal results. The data required to evolve a proper model of inflow is usually not available.

With these major difference brought out between deterministic and stochastic analysis it is clear that the present stochastic analysis is only approximations to the real stochastic problem in terms of inflow seqeuences considered, the non-anticipativity of decision process and the optimality of solution.

Modelling Approximations:

Ideally the reservoir operation model must be a stochastic one and must be of non-anticipative type. However, models incorporating all aspects including real time, stochastic and non-anticipative character is not in sight. Some sort of approximations and simplification in representative of the physical situation is an accepted fact. In practice even the models nearest to actual situation physical and environmental one are only a compromize between reality, accuracy and computational efficiency. Even under such circumstances, evaluation of the impact of operation on benefits of the simplifications and approximations will be most useful and serves as a guide to the operator to adjust model operation strategies suitably while implementing them in real life situation. However, this assessment of impact is not very easy and not much of attention is paid in this direction. It may be possible to some extent to estimate the impacts through simulation, but then there could be bias, introduced through modelling of stochastic flows.

Despite the deficiency, the modelling approach will be most useful if:

(i) The impacts and biases introduced due to simplifications and assessment could be estimated which might help in introducing to model compensating approximation, and that the model could provide an upper and lower bounds on the solutions which can provide a confidence measure and also indicate the maximum benefits that could be obtained from more accurate models.

The model structure is an important thing in actual modelling. Nested model with detailed model for the immediate future decision with somewhat approximate model for future long term operation is most desired. In stochastic models the effect of the system representation may be to reinforce or reduce the effect of the assumptions made about future management strategies.

The definition of the objective function in the model is most crucial. If the model objective and the actual social attitudes within which the actual system is to operate do not have at least similarity of purpose, the model results are totally useless so far as the actual system operation is concerned.

The adaptive ability is the most desirable feature of the model. The model, must use the latest information available . So the implementation of models results will only be for the first period and it is assumed that the model is rerun and decisions taken at every subsequent time.

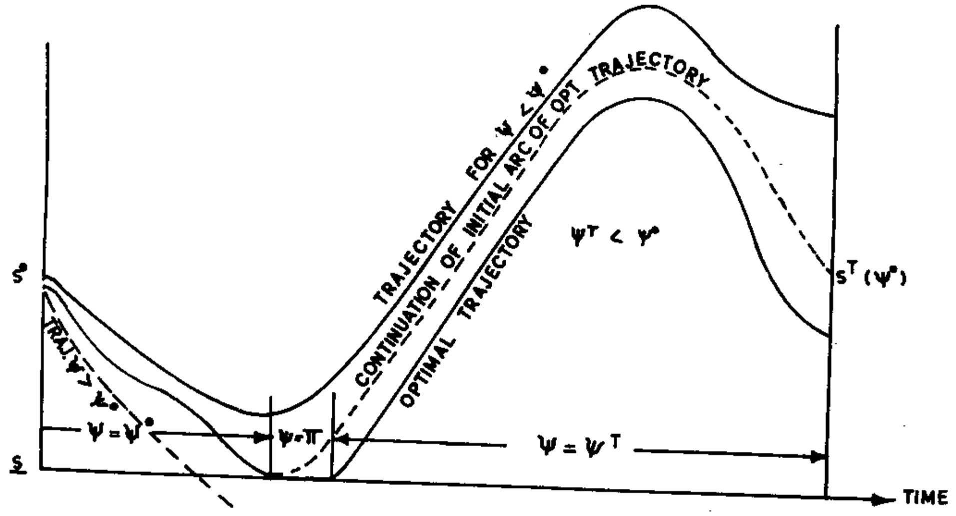

FIG 6 - OPTIMAL STORAGE TRAJECTORY WITH LOWER BOUNDS

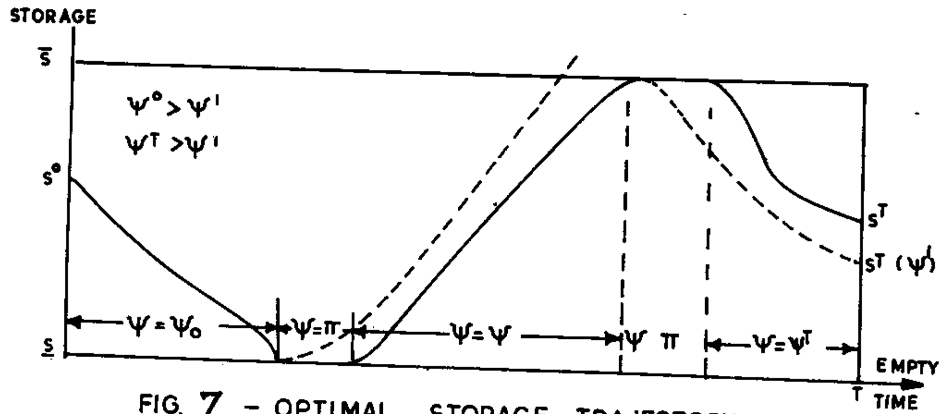

FIG. 7 - OPTIMAL STORAGE TRAJECTORY

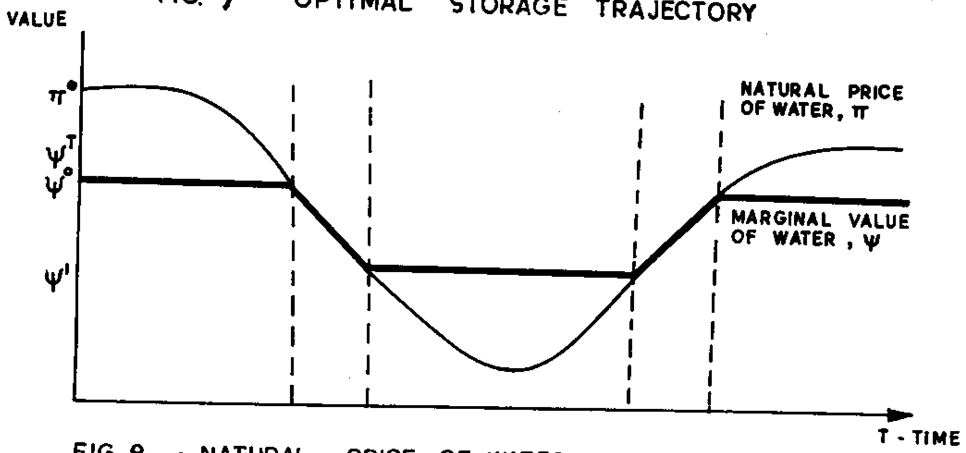

FIG. 8 - NATURAL PRICE OF WATER VALUES FOR OPTIMAL TRAJECTORY

4.0 MODELLING APPROACH

The reservoir management problem will never be solved in its full generality, so separate models must be applied carefully, with due account taken of interactions between those aspects addressed by the model and other relevent aspects of the problem. Further a hierarchy of models are required with proper nesting to study the operation of online, mid term and long term operation problems. It is the practice in general that although the model may recommend a full years schedule of releases, the release decisions recommended for the first period(week or month) will be the only one ever actually implemented. So many models, particularly those with model uncertainty, give special attention to this first release decision. In general the following comments are appropriate.

(1) The goal of complete automated system remain unattainable for the foreseeable future.

(2) The ultimate goal of all such studies is to improve system operation, directly or indirectly. It is not possible to experiment with real systems, and more so with capacity expansions.

(3) Models can improve reservoir management by providing or confirming, insight into the nature of the problem. The result of the model may not be directly implemented but will be used as one of many inputs into the decision making process(this is because the experience over years of research management may not be possible to incorporate into the model). The model studies may bring into focus certain aspects of reservoir operation. So it serves to improve the manager's ability to manage system wisely.

A model which is not itself implemented but which provides a robust management rules is a good investment. The various models and the studies reported based on these models can be classified into three general categories:

(i) Optimizing models for single reservoirs.

(ii) Optimizing models for multi-reservoir systems.

(iii) Simulation models

Optimizing models for single reservoirs:

The first study reported by Little(1955) for optimal operation of a single reservoir is an adaptations of inventory control theory. In this dynamic programming was used for obtaining an optimal reservoir regulating plan within one year or less with the objective of minimizing thermal power generation cost. The stochastic nature of inflow into the reservoir is considered in terms of its probabilities. Following this, Manne (1960), who applied linear programming to inventory problems and extended it to reservoir management problems. The worthwhile contribution was to derive release policies in terms of the storage and average inflow. Thomas and Watermeyer (1962) approached the problem in a different way, though they also used linear programming. The inflows in any time period were considered as independent random events neglecting any possible serial correlations. The objective function was linear in terms of releases, shortfall and excesses defined with reference to specified targets. Dietrich and Loucks(1967), Gablinger and Loucks(1970) used the same approach in principle and carried out more detailed investigations. Parallel to these studies based on linear programming, studies were also reported based on Dynamic programming for example Bather(1962), Buras(1963) for conjunctive operation of a reservoir and an aquifer. Falkson(1961) used a combined LP and DP approach and called it 'Policy iteration approach'. All these models are classed as explicit stochastic models wherein the inflow into tne reservoir is expressed in terms of its probability distribution and is used in deriving the release policy.

Contemporary developments are the approaches referred to as implicit stochastic approach. The analysis reported are essentially in the planning context with deterministic optimization and stream flow generation techniques to take care of the stochasticity of inflows. In this category falls the work of Hall(1964), Hau and Buras(1961). In 1966 Young used the same approach to derive a policy through regression analysis. The independent variables in the regression analysis were the storage and inflow during the previous period. The implicit approach is quite appealing as it is easy to conceptualize, but the main question as to the independent variables to be used in the regression, analysis for defining the policy remains unanswered. Further the validity and the possible improvement in gains by implementing such derived policies in practice is still to be established. But still implicit methods are much better compared to explicit methods as errors induced in discretization and use of incorrect probability distributions in describing streamflow discharges are not properly qualified and assessed.

Multiple Reservoirs:

Multireservoir operation problem is quite complex in terms of problem formulation and solution procedures. The problem is computationally intractable in the case of explicit stochastic models. This is because the multireservoirs offer a lot of flexibility in operation with varying site characteristics and multiple use requirements at different sites and interaction among reservoirs being sometimes hydrological as in the case of series configuration of reservoirs. The best examples of stochastic optimization to multireservoir operation is due to Schweig and Cole(1968) who applied Dynamic Programming to a two reservoir system, Gablinger(1971) and Houck and Cohon(1978).

Various general approaches to multireservoir operation through implicit approaches have been suggested. The earliest work is by Hall and Roefs(1966) who studied the optimal operation of a three reservoir system in northern California. Here again they used dynamic programming with 6 years of specific hydrologic sequence.

A major break through in this is due to Parikh(1966) who used the decomposition concept. He studied the individual reservoirs by dynamic programming and these subproblems were integrated through a master LP programme. Parikh called this procedure as'Linear Dynamic Decomposition Programme'. Parikh used his model for analyzing two test problems a two reservoir system for 24 months of hydrology and a four reservoir system for 36 months hydrology. In both the cases the solution came close to the optimal relatively quickly. However, a substantial number of iterations was conducted before finally reaching the optimum solution. Although the computational effort was substantial, it was not prohibitive and thus demonstrated the potential of the method for further development and application. Further this concept was used and extended in reservoir operation studied by Buras(1965), Hau' et al(1969) Roefs and Bodin(1970). The main improvements in these studies were realistic representation of systems and introduction of nonlinear benefit functions.

A parallel development was the concept of linear decision rules introduced by Revelle, Jores and Kirby (1960). This was an adaptation of similar rule suggested by Charnes et al (1958) for determining refinery rates for heating oils to meet stochastic weather dependent demands. The linear decision rule that was suggested is quite simple, and the relationship is

$$r_t = S_t - b_t \qquad \ldots(38)$$

Where r_t is the release at time t and S_t is the storage at time t and b_t is the decision variable derived through the model maximizing or minimizing the stated objective. This decision rule is very convenient as the reservoir operation problem can be written as a linear programming problem.

Because of simplicity and mathematical tractability, linear decision rules have been improved and studied by various researchers despite the controversy about its validity for adoption in practical operations of reservoirs. ReVelle and Kirby(1971),Joeres, Liebman and ReVelle (1971), Nayak and Arora(1971,1974), Eastman and ReVelle(1973) and Leclerc and Marks(1973) have modified and extended and/or applied this to reservoir management problems. However, Eisel(1972), Sobel(1974) and Loucks and Dorfman(1975) have all questioned the utility of linear decision rules for reservoir management. It has been established that the linear decision rules produce conservative results and it is suggested that this type of linear decisions rule in its original form or improved versions might be useful only for screening purposes and is not itself satisfactory for deriving optimal operating policies for a single or multiple reservoirs.

As a part of Texas Water Plan studies ,Evenson and Moseley(1970) developed an operation model called allocation model based on the Ford Fulkersons(1961,1962) 'Out-of-Kilter' algorithm. Ford- Fulkersons algorithm is an efficient algorithm to solve linear programming problems of special type of maximizing flow through a capacitated network. Many water resources problems(both planning and operation) can easily be represented as a network problem and solved through this algorithm. The algorithm is very efficient in terms of computational requirements.

Simulation Approach:

Simulation in essence is to duplicate the system behaviour under given hydrologic and other input data.Thus the approach is not useful for deriving any operation policy, but helps in evaluating any policy. Quite a lot of effort has been diverted by many to build detailed and generalized simulation models, and there is an extreme view that simulation is the only approach best suited for analyzing complex water resources systems. This however does not mean that the so called generalized programmes of various agencies can blindly be applied universally. The three generalized simulation models which are accessible through published reports and manuals are:

1. HEC-3
2. SIMLYD-2
3. Acres Model

All these are based on the zoning concept of reservoir management. The SIMYLD-2 and Acres model have built in it the optimization process through 'Out-of-Kilter' algorithm.

The application of these simulation models to practical problems are available in Beard(1967), U.S.Army Corps of Engineers(1971), Beard(1975), Frederich and Beard(1972), Texas Water Development Board(1972), Acres Consulting Services Limited(1973a and 1973b), Sigvaldason (1976),Sigvaldason,Bradford and Granz(1975), Sigvaldason, Ellis and Allen(1975) Tedro, Liu, Halton and Hiney,(1971). Of these, Acres model has been claimed to be in use in day to day reservoir operation of Trent River system in Ontario. Some of these models permits evaluation of reservoir policy options in terms of system reliability, resilience and vulnerability. Reliability is a measure of how often a failure, however defined, occurs. Resilience is a measure of how quickly the system recovers from failure and vulnerability is a measure of the magnitude of consequences of failure, should failure occur. It must be noted that despite lot of improvements in methodology and understanding of the system behaviour the 'improved operation' is still a collective perception and interpretation.

Objective Function

The objective function is crucial choice in mathematical optimization model for reservoir operation studies. The benefit is to be expressed in one or more of the following variables in any reservoir operation problem: (i) Reservoir content, (ii) Vacant storage space and flood absorption capacity, (iii) Amount of water released at any time, (iv) Rate of release, (v) Rate of depletion of reservoir level, (vi) Anticipated discharge, (vii) Downstream channel capacity, (viii) Demand for various outputs like irrigation power, recreation etc., (ix) Reliability, stability and robustness. Some of the objective functions used by various authors are:

(i) $\text{Max } F\ (PW_j\ R_j + Pe_j\ Ep_j + Pn_j\ En_j)$

where Pw_j = water price

R_j = Release in period j

Pe_j = Peak energy price

Ep_j = peak energy

Pn_j = Non peak energy price

En_j = Non peak energy

(ii) $\text{Max} \sum_{t=1}^{36} Cp_t P_t + CU_t . U_t + CY_t\ Y_{3t}$

This is the study for three reservoirs in series

CP_t = value of peak energy

CU_t = Value of Off-peak energy

Cr_t = Value of release

p_t = peak energy produced at time t

U_t = Off peak energy produced at time t

r_{3t} = Release from downstream reservoir at time t.

(iii) $\text{Max AFE} = \text{Max}\left(\text{Min}\left(\frac{OE_n}{OPH_n} - A_{OPH}\right)\right)$

AFE - Annual firm on peak energy contract

OE_n - On peak energy production in month n

OPH_n - Number of on peak hours during month n

AOPH- Annual number of on-peak hours availability energy year

(iv) $\text{Max} \sum_{i=1}^{24} W_i \, P'_i(S_i S_i)$

where W_i - is the weighting factor for the outputs of period i.

$P'_i(D_i, S_i)$ - power generated in the period i

D_i -Plant release at time i

S_i - Storage at the beginning of ith time

(v) $\text{Min.}F(S,Q) = D + W \sum_{j=1}^{J} (D_j - D)^2$

where $D_j = L_j - P_j$

$$D = \frac{1}{T} \sum_{j=1}^{J} D_j T_j$$

$$T = \sum_{j=1}^{J} T_j$$

L_j is the system load in the period J

W is a suitable weighting factor

P_j is system power generation in period j

S= storage

Q= discharge

(vi) Min $\sum_K (C_i^k R_i^k + C_i' R_i'^k)$

R_i and R_i' are the firm and non-firm releases in period i for reservoir K,

C_i and C_i' are functions of energy rate and average storage at any time i and are known.

(vii) Min $f(x) = \sum_{k=1}^{P} W_k(Z_k(X))$

$Z_k(X)$ - Value of index K of operation efficiency with decision variable X.

p- the total number of indices.

W_k - Weight assigned to index K.

The various indices used are

(a) energy shortage index

(b) Downstream discharge shortage index

(c) Number of times salt-water barrier is installed in the period of analysis

(d) Number of times the salt water barrier fails (washed out) in the period of analysis.

(e) Average annual energy shortage

(f) Average annual downstream discharge shortage

(g) Average monthly conservation pool elevation fluctuation

(h) Average annual energy

(i) Number of times the conservation pool is emptied.

(j) Number of times the downstream discharge shortage occurs.

(viii) Min $Z = \sum_{t=1}^{T} Loss\ (R_t)$

where $Loss(R_t) = A[\ \exp\ (R_t/R\ UP) - \exp\ (1)\ ]$

$R_t \geq RUP$

$$\text{Loss } R_t = 0 \quad RLOW \le R_t \le RUP$$

$$\text{Loss } R_t = B[\exp(-R_t/RLOW) - \exp(-1)]$$

$$R_t \le RLOW$$

where A and B are constants depending on the price of water and how extensive the property damage is (known)

R_t - Release during period t.

RUP- Upper limit of the Safe range(known)

RLOW- Lower limit of safe range(known)

(ix) $\quad \text{Max} \sum_{t=1}^{T} \text{(benefit)}$

Benefit = Benefit 1 + Benefit 2 + Benefit 3 + Benefit 4

Benefit 1 - firm energy benefit

Benefit 2- Dump energy benefit

Benefit 3- Firm water benefit

Benefit 4- Dump water benefit.

Rosenthal (1980)provided a comparative bibliography of reservoir Management models. The models listing however excludes those primarily dealing with capacity expansion and other system design features and those incorporating water quality, temperature and multiple objectives. Despite this deficiency the review is very useful and instructive for researchers.

The comparative study considers the problem features such as reservoir network topology(single or multiple reservoirs), multiple time periods(which is the most essential feature), the stochastic or deterministic character describing the inflow and the benefits (separable or non separable).

(i) Reservoir network topology

An x under the heading 'multireservoir' means the model allows for general network topology. A blank indicates that the model is of single reservoir. Additional footnotes indicate the series and parallel configurations of the multiple reservoir.

(ii) Multiple time periods:

This feature is shared by almost all the models.

(iii) Stochastic inflows:

The non anticipative stochastic control aspect is the most desirable feature of any reservoir scheduling model. It means that the operating policy derived through the model must have the property that, in each period, the release must depend on exactly the information available to the manager at the time of decision. This ideal is very difficult to attain. So approximations to this are employed. The classification is one of dealing with stochastic inflows or deterministic hydrology.

(iv) Non-separable benefit:

A function is called separable if it can be expressed as a sum of single variable functions. If f is twice differentiable and it has all its off diagonal elements zero in the Hessian matrix $\partial^2 f/\partial x_j \partial x_k; j \neq k$) then it is separable. In physical terms it explains the non existence of economic interaction between two reservoirs for any time period. In multiple reservoirs, especially with serial structure, it is difficult to visualize the absence of such interactions. Even in the case of independent multiple reservoir in integrated operation when integrated with other systems(such as Thermal System), the release at any reservoir depends on concurrent releases from all other reservoirs. This point stresses the need for consideration of the non separability of the benefits. However, non-separability of the objective function in some cases can be removed by explicit representation of the hydrothermal interactions in constraints. The price paid for this modification is the loss of exploitable network structure in the constraints(except in the case when reservoirs are in parallel).

COMPARATIVE DISPLAY OF EXISTING RESERVOIR MANAGEMENT MODELS

Author and Year				Techniques
Trott and Yeh, 1973	x	x	x	Dynamic programming successive approximation
Gagnon et al., 1974	x	x	x	Penalty functions, reduced gradient method
T.V.A., 1974	x	x	x	Discrete differential dynamic programming
T.V.A., 1974, Giles and Wunderlich, 1981	x	x	x	Dynamic programming Successive approximation
T.V.A., 1976	x	x	x	Reduced gradient
Hanscom et al., 1980	x	x	x	Reduced gradient method, control theory
Divi et al., 1979	a	x	x	Penalty function minimization with conjugate gradients and Powell's restart method, cubic splines.
Rosenthal, 1981	x	x	x	Nonlinear network flows
Gilbert and Shane, 1982; Shane and Gilber 1982	x	x	x	Linear programming with prepriority constraints; separable programming
Bodin and Roefs, 1971	x	x	x	Separable programming Dantzig Wolfe decomposition
Joeras, Liebman and ReVelle, 1971	b	x	x	Chance constrained linear programming, simulation
Jacoby and Loucks, 1972	x	x	x	Dynamic programming simulation
Driscoll, 1974	x	x	x	Network flows
Takeuchi and Moreau, 1974	x	x	x	Linear &dynamic programming
McKerchar, 1975	c	x	x	Deterministic dynamic programming multivariate stream flow simulation

a. Reservoirs in series.
b. Single reservoir with pumped water backup
c. Two reservoirs in series.

Author and Year				Techniques
Sobel,1975	x	x	x	Dynamic programming with structured policies.
Casti,1976	x	x	x	Dynamic programming;network flows
Pinter,1976	x	x	x	Stochastic programming
Sigvaldason,1976	x	x	x	Out-of-kilter algorithm,simulation
Toebes,Rukvichai and Lin,1976	x	x	x d	Time series analysis;simulation
Electricite de France, (See Read,1979)	x	x	x	Trajectory method
Peters,Chu and Jamshidi, 1977	x	x	x	Discrete differential dynamic programming with successive approximations
North,et al,1977	x	x	x	Discrete differential dynamic programming;regression analysis
Houck and Cohon,1978	x	x	x	Sequential linear programming; Markov chain
Divi,et al.,1978	x	x	x	Simulation;penalty function minimization;regression analysis
Arunkumar & Chon,1978	x	x	x	Dynamic programming with heurestic structured policies
Yeh,Becker & Chu, 1978	x	x	x	Linear programming and incremental dynamic programming with successive approximations
Kindler,et al.	x	x	x	Out-of-kilter algorithm;simulation
Gal,1979	x	x	x	Dynamic programming
Read,1979	x	x	x	Lagrangean duality;decomposition
Yeh,Becker,Chu,1979	x	x	x	Linear programming and incremental dynamic programming with successive approximations
Turgeon,1980	x	x	x	Dynamic programming
Diacon,Seteano,and Popa,1981	a	x	x	Linear programming & pattern search
Turgeon,1981-2	a	x	x	Decomposition

d. Model is descriptive,hence it has no objective function.

Author and Year			Techniques
Little, 1955	x	x	Dynamic programming
Young, 1967	x	x	Dynamic programming; simulation
Loucks, 1968	x	x	Markov decision chain
ReVelle, et al., 1969	x	x	Linear decision rule
Loucks and Falkson, 1970	x	x	Comparison of dynamic, linear and Markov
ReVelle and Kirby, 1970	x	x	Linear decision rule
Butcher, 1971	x	x	Markov decision chain
Davis and Pronovost, 1972	x	x	Dynamic programming
Russell, 1972	x	x	Dynamic programming
Lane, 1973	x	x	Chance constrained programming
Torabi and Mobasheri, 1973	x	x	Markov decision chain
Croley, 1974-1	x	x	Dynamic programming
Croley, 1974-2	x	x	Stochastic dynamic programming
Guitron and Roefs, 1974	x	x	Markov decision chair(modified value iteration)
Mawer and Thorn, 1974	x	x	Markov decision chain
Su and Deininger, 1974	x	x	Markov decision chain
Askew, 1975	x	x	Chance-constrained dynamic programming.
Chow, et al, 1975	x	x	Dynamic programming
Gundelach and ReVelle, 1975	x	x	Linear decision rule
Loucks and Dorfman, 1975	x	x	Linear decision rule
Roefs and Guitron, 1975	x	x	Markov decision chair
ReVelle and Gundelach, 1975	x	x	Linear decision rule
Arunkumar, 1975	x	x	Successive approximations
Swedish State Power Board, (See Daellenbach and Read, 1976)	x	x	Dynamic programming

Author and Year				Techniques
Heidari et al,1971	x	x		Discrete differential dynamic programming
Chang and Toehes ,1972	x	x		Comparison of reservoir operating policies by simulation
Fults and Hancock,1972	x	x		State increment dynamic programming
Kerr,1972	x	x		Linear,dynamic,and out-of-kilter programming
Windsor and Chow,1972	x	x		Separable model with integer variables,no solution given
Liu and Tedrow,1973	x	x	g	Dynamic programming;pattern search
Tauxe et al.,1973	x	x		Multi-state incremental dynamic programming
Becker and Yeh,1974	x	x		Linear and dynamic programming
Chow and Cortes-Rivera,1974	x	x		Discrete differential dynamic programming
Jensen et al.,1974	x	x		Generalized network flows
Mejia et al,1974	x	x		Linear programming,forecasting
Meredith,1975	x	x		Discrete differential,dynamic programming
Becker et al,1976	x	x		Linear and dynamic programming
Fults et al.,1976	x	x		Dynamic programming
Nopmongcol & Askew,1976	x	x		Multi-level incremental dynamic programming
Yeh, 1976	x	x		Linear and dynamic programming
Boshier and Lermit,	h	x		Out-of-kilter algorithm
Collins,1977	x	x		Dynamic programming
Jamshidi,1977	x	x		Discrete differential dynamic programming
T.V.A.,1977	x	x		Linear Programming
Tauxe and Mays,1977	x	x		Dynamic programming

g. Nonseparable function in model is approximated by separable function in solution procedures.

h. Reservoirs in parallel

Author and Year			Techniques
Arun Kumar,1977	x	x	Nonhomogenous Markov chair
Croley and Rao,1977	x	x	Multi-objective trade-off analysis
Rossman,1977	x	x	Dynamic programming;Lagrangean duality
Bogle and O'Sullivan,1979	x	x	Dynamic programming;release rules
Boshier & Read,1979	x	x	Stochastic economic modelling
	x	x	Deterministic economic modoelling
	x	x	Trajectory method
Bhaskar and Whitlatch,1980	x	x	Dynamic programming and regression analysis
Soares,Lyra, and Tawares, 1980	x	x	Decomposition
Sniedovich,1980	x	x	Dynamic programming with a (non-separable) variance constraint
Joeres,Seus, and Engelmann, 1981	x	x	Linear decision rule
Manning and Gallagher,1982	x	x	Extension of Hotelling's rule
Mannes,1955	x	x	Linear programming
Meier and Beightler,1967	x	x	Decomposition
Schweig and Cole,1968	e	x	Dynamic programming
Hall et al.,1969	x	x	Dynamic programming
Larson and Keckler,1969	x	x	Discrete differential dynamic programming
Evenson and Mosely,1970	x	x	Out-of-Kilter algorithm
Fitch et al.,1970	x	x	Dynamic programming
Lee and Waziruddin,1970	f	x	Gradient projection and conjugate gradient methods
Roefs and Bodin,1970	x	x	Separable programming with Dantzig-Wolfe decomposition
Drobny,1971	x	x	Linear Programming

e. Two servoirs.

f.Reservoirs are in series and there are no exogenous inflows except at first reservoir

Author and Year				Techniques
Murray and Yakowitz,1978	x	x		Constrained differential dynamic programming
Singh,1978	x	x		Simulation
Toebes and Rukvichai,1978	x	x		Simulation
Daellenbach,1979	x	x		Dynamic programming;price directive decomposition
Turgeon,1981-1	x	x		Progressive optimality
Opricovic & Djordjivic, 1976	x	x		Three level dynamic programming
Dagli and Miles,1980	x	i	x	Adaptive Planning
Manning,1981	x		x	Monte Carlo techniques
Hall and Roefs,1966		x		Dynamic programming
Hall et al. 1968		x		Dynamic programming
Erickson,1969		x		Dynamic programming,pattern search
Harboe,et al,1970		x		Dynamic programming
Fronza, et al,1977		x		Game theory
Chu and Yeh,1978		x		Lagrangean duality and gradient projection
Laufer & Morel-Seytoux,1979		x		Two-phase indirect solution of Kuhn-Tucker conditions
Tauxe,et al.,1979		x		Multi-objective dynamic programming
Tauxe,et al.,1980		x		Multi-objective dynamic programming
Guitron,1981		x		Dynamic programming

i. Multiple time periods are handled independently with the release for period t given as a forecast for period t+1.

5.0 CONCLUSIONS:

Multireservoir operation under stochastic environment is a very complex problem with no complete solution in sight. The procedures available at present can generally be classified as:

Heuristic procedures

Optimization models

Simulation models

The major stumbling blocks are unmanageable non-linearities, inseparable objective functions and stochasticity of inflows and the resulting computational incapabilities. Each one of these need a detailed study.

The operation problem should be incorporated right at the planning stage as unrealistic operational policies used in planning stage may introduce constraints during operation stage resulting in inflexibilities and lesser benefits.

Since all the reservoirs are planned with some sort of yield models, a critical review of such models and their behaviour under stochastic conditions is very much desired and based on such studies suitable guidelines are to be provided for preparing of rule curves and guide lines. This is an immediate task because this procedure is easily understood by the persons incharge of real world situations and the procedure continue to dominate until such time the new aproaches are well established, demonstrated for actual applications and understood and accepted by field people.

Regarding optimization models, the models worth is judged interms of objective function. There is a need to quantify objectives as applicable to reservoirs and their relevance to various situations and expressing these objectives in terms of the reservoir parameters and the information at hand. Majority of the models referred in the Bibliography are concerned with the methodology for solution and optimization scheme. In terms of the objective function, they are unrealistic and so not directly useful for adaptations.

A hard look at the computational schemes on optimization and their simplications for use in reservoir operation studies is desirable.

Forecasting models play a vital role in real time operation of reservoirs. Development of operational policies, use of the policies so derived through forecasted inflows and adapting the policies(Adaptive aspects) are the important aspects which are still in the developing stages.

There are quite a number of multireservoir simulation programmes documented and circulated. It is doubtful whether one can simply borrow those programmes and try to cut short the effort required in developing quantitative analysis capabilities. It is just like the saying'Having a screw driver and looking for a screw it can turn'. This leads to lot of wastage of manhours and computer time with no useful purpose. There is a necessity to direct efforts to build our own technological capabilities in this direction.

REFERENCES

1. Acres Consulting Services Limited,(1973),"The Trend Basin: Plan of Operation", Rept. to Department of Indian and Northern Affairs.

2. Acres Consulting Services Limited (1973),"The Trend Basin:Analysis of the System", Rept. to Dept. of Indian and Northern Affairs.

3. Acres Consulting Services Limited (1973),"Study of the Operation of the Rideau-Cataraqui System, Niagara Falls, Ontario, Canada.

4. Arun Kumar,S.(1975), "Characterisation of Optimal Operating Policies for Finite Dams," Journal of Mathematical Analysis and Applications, Vol.49, No.2,pp.267-274.

5. Arun Kumar,S.(1977),"Optimal Regulation Policies for a Multipurpose Reservoir with Seasonal Input and Return Function", Journal of Optimization Theory and Application, Vol.21, 319-328.

6. Arun Kumar, and K Chon,(1974),"On Optimal Regulation Policies for Certain Multi-Reservoir Systems", Operations Research, Vol.10,1107-1112.

7. Askew,A.J.,(1975), "Chance-Constrained Dynamic Programming and the Optimization of Water Resources Systems", Water Resources Research, Vol.10, 1099-1106, 1974; "Comment", M.Sniedovich and D.Davis and "Reply",Water Resources Research", Vol.II,1937-9.

8. Bather, J.A.,(1962),"Optimal Regulation Policies for Finite Dams", J.Soc.Ind.Appl.Math.,Vol.10, 395-423.

9. Beard, L.R.(1967),"Functional Evaluation of a Water Resources System", Intnl.Conf. on Water for Peace,Washington,D.C.

10. Beard,L.R.,(1975),"Models for Optimizing the multipurpose operation of a Reservoir System," Proc. of Symposium and Workshops on the Application of Mathematical Models in Hydrology and Water Resources Systems, Bratislava, Czechoslovakia.

11. Beard,L.R.(1976),"Flood Control by Reservoirs, Vol.7 of Hydrologic Engineering Methods for Water Resources Development, U.S.Army Corps of Engineers, Hydrologic Engineering Center, Davis, California.

12. Beard,L.R.,et.al.,(1977)"Reservoir System Analysis for Conservation", Vol.9 of Hydrologic Engineering Methods for Water Resources Development, U.S.Army Corps of Engineers, Hydrologic Engineering Center, Davis, California.

13 Becker,L.,and W.W.G.Yeh,(1974),"Optimization of Real Time Operation of a Multiple-Reservoir System", Water Resources Research,Vol.10, 1107-1112.

14. Becker,L., W.W.G.Yeh, De Fults and D.Sparks,(1976),"Operations Models for the Central Valley Project," Journal of Water Resources Planning and Management Division,ASCE, Vol.102,No.WRI,101-115.

15. Bhaskar, Nageshwar,R. and Elbert E.Whitlatch, Jr.,(1980),"Derivation of Monthly Reservoir Release Policies", Water Resources Research, Vol.16, No.6, Pages 987-993.

16. Bodin, L.D., and T.G.Roefs (1971),"A Decomposition Approaoch to Non-linear Programs as Applied to Reservoir Systems", Networks, Vol.1,-pp.59-73.

17. Bogle,M.G.V., and M.J.O'Sullivan(1979),"Stochastic Optimization of a Water Supply System," Water Resources Research, Vol.15, 778-786.

18. Boshier,J.F., and R.J.Lermit,(1973),"A Network Flow Formulation for Optimum Reservoir Management of the New Zealand Power Generating System", New Zealand Operations Research,Vol.5, No.2, 85-100.

19. Boshier,J.F.,and E.G.Read,(1979),"A Stochastic Single Reservoir Model for Optimal Operation of the New Zealand Power System,"Proceedings of Energy Modelling Symposium.

20. Boshier,J.F. and E.G.Read(1980),"Stochastic Single Reservoir Models for Long Term Scheduling of a Hydrothermal Power System," New Zealand Ministry of Energy Planning Division, Internal Report.

21. Boston,W.T.,(1978),"Computer Applications Used in Operational Planning and Real Time Economic Control of Hydro Power", Paper presented at ASCE Workshop on Reservoir Systems Operations, Boulder, Colorado.

22. Buras, N.(1963),"Conjunctive Operation of Dams and Aquifers,"ASCE, J.of the Hydraulics Div.,Vol.8P, No. HY6, pp.111.

23. Buras,N.(1965),"The Optimization of Large Scale Water Resources Systems,Part I , Conceptual Framework," University of California, 1-34.

24. Butcher,W.S.(1971),"Stochastic Dynamic Programming for Optimum Reservoir Operation",Water Resources Bulletin, Vol.7, 155-123.

25. Calhoun,W.F.,Benefied,E.F., and Contractor,D.N.(1980),"An Interactive Simulation of Pumped Storage Reservoir Systems," Water Resources Bulletin, Vol.16,No.1, p.63-68.

26. Casti,J.(1976),"Algorithms for the Stochastic Inflow-Nonlinear Objective Water Reservoir control Problem" in Szollosi-Nagy,op. cit., 129-136.

27. Chang, T.P., and G.H.Toebes(1972) ,"Operating Policies for the Upper Wabash Surface Water System," Technical Report No.31,Purdue University Water Resources Research Center.

28. Charnes, A.,W.W.Cooper and G.H.Symonds(1958),"Cost Horizons and Certainty Equivalents: An Approach to Stochastic Programming of Heating Oil", Mgmt.Sci.4,3,235-263.

29. Chow,V.T., and G.Cortes-Revera(1974),"Applications of DDDP in Water Resources Planning," Research Report No.78, University of Illinois,-Water Resources Center, Urbana.

30. Chow,V.T.,D.H.Kim,D.R.Maidment and T.A.Ula,(1975),"A Scheme for Stochastic State Variable Water Resources Systems Optimization", Research Report No.105, Water Resources Centre, University of Illinois at Urbana-Champaign.

31. Chow,V.T.,D.R.Maidment, and G.W.Tauxe,(1975),"Computer Time and Memory Requirements for DP and DDDP in Water Resources Systems Analysis", Water Resources Research, Vol.11,620-628.

32. Chu,W.S., and W.W.G.Yeh(1978),"A Nonlinear Programing Algorithm for Real Time Hourly Reservoir Operations," Water Resources Bulletin, No.14, 1048-1063.

33. Collins,M.A.(1973),"Implementation of an Optimization Model for Operation of a Metropolitan Reservoir System," Water Resources Bulletin, Vol.13. 57-70.

34. Coomes,R.T.(1979),"Development of a Comprehensive Multi-Purpose Reservoir Regulation Plan for the Arkansas River Basin System", paper presented at ASCE Water Systems Speciality Conference, Houston, Texas.

35. Croley,T.E.,(1974-a),"Sequential Deterministic Optimization in Reservoir Operation", Journal of Hydraulics Division,ASCE,Vol.100,No.HY3, 443-459.

36. Croley,T.E.,(1974b),"Sequential Stochastic Optimization for Reservoir System", Journal of Hydraulics Division, ASCE, Vol.100, HY1, 201-219.

37. Croley, T.E.,and K.N.R.Rao (1977),"Multipurpose Reservoir Operation Using Stochastic Trade-Off Analyses", in Stochastic Processes in Water Resources Engineering, L.Gottschalk et.al.(eds.), 483-513.

38. Daellenbach, H.G.,(1979), "Schedualing Hydro Power Systems by D.P. Using Decomposition by Prices", Proceedings of the Operations Research Society of New Zealand.

39. Daellenbach, H.G., and E.G.Read (1976),"Survey on Optimization for the Long Term Scheduling of Hydro-Thermal Power Systems", Economics Department, University of Canterbury, Christchurch, New Zealand.

40. Dagli, C.H. and J.F.Miles,(1980)," Determining Operating Policies for a Water Resources System", Journal of Hydrology, Vol.47,No.3/4, pp.297-306.

41. Davis,R.E., and R.Pronovost,(1972),"Two Stochastic Dynamic Programming Procedures for Long Term Reservoir Management," IEEE PES Summer Meeting, San Francisco, California.

42. Dietrich,G.N. and Loucks, D.P.,(1967)," A Stochastic Model for Operating a Multipurpose Reservoir, Proc. Third Annual American Water Resources Conf. 92-104, San Francisco, California.

43. Divi,R.,D.K.Sen, R.Thompstone,A.Vallee and T.E.Unny,(1978)," On Evaluation of Operating Rules for a Multi-Reservoir Hydro-Electric Systems", TIMS/ORSA National Meeting, New York.

44. Divi ,R.,H.Patrung, A.Vallee, M.Vidyasagar, S.R.K.Dutta, and D.K.Sen (1979),"Nonlinear Programming Applied to the Optimal Operation of a Hydro-Electric Power System", Tenth International Symposium of the Mathematical Programming Society, Montreal

45. Driscoll,W.D.,(1974),"Multireservoir Operating Policies Considering Uncertainty", in Jensen,Baumik, Driscoll, op.cit.

46. Drobny, N.L.,(1971)," Linear Programming Application in Water Resources", Water Resources Bulletin, Vol.7, 1180-1193.

47. Durbin,E.P., and Korenke, D.M.,(1967),"The Out-Of-Kilter Algorithm: A Primer, RM472-R, Rand Corporation, Santa Monica, California,p.22.

48. Eastman,J. and ReVelle, C.S.,(1973),"The Linear Decision Rule in Reservoir Management and Design 3, Direct Capacity Determination and Intraseason Constraints," Water Resources Res.,Vol.9, No.1. 29-42.

49. Eisel,L.M.,(1972),"Chance Constrained Reservoir Model , Water Resour. Res. Vol.8, No.2 , 338-347.

50. Erdle, W.P.,(1979),"Lake Ontario Regulation," Water Spectrum, Vol.11, No.2, 17-25.

51 Erickson,L.E.,(1969)," A Nonlinear Model of a Water Reservoir System with Multiple Uses and Its Optimization by Combined Use of Dynamic Programming and Pattern Search Techniques," Water Resources Bulletin, Vol.5, 18-36.

52. Evenson, D.E., and Moseley, J.C.,(1970),"Simulation /Optimization Techniques for Multi-Basin Water Resource Planning, Water Resources Bulletin, Vol.76, No.5, 725-736.

53. Falkson,M.,(1961),"An Application of Howard's Policy-Iteration Method for the Solution of Sequential Decision Processes to the Problem of Determination the Optimal Operation Procedure for a Water Resources System, Seminar paper, Harvard Water Program, Cambridge, Massachusetts.

54. Fedorov,B.G.,(1975)," Principles of Optimization of Reservoir Utilization Regime," Translated from Russian, Water Resources by Consultants Bureau.

55. Fitch,W.N., P.H.King and , G.K.Young,Jr.,(1970),"The Optimization of the Operation of Multi-Purpose Water Resource System", Water Resources Bulletin, Vol.6, 498-518.

56. Ford,L.R., and Fulkerson, D.R.,(1962),"Flow in Networks," Princeton Univ. Press., Princeton, p.194.

57. Frederich, A.J., and L.R.Beard,(1972),"System Simulation for Integrated Use of Hydro-Electric and Thermal Power Generation", Tech. Paper 33, Hydrologic Engineering Center, U.S.Army Corps of Engineers, Davis.

58. Fronza, G.,A.Karlin and S.Rinaldi,(1977),"Reservoir Operation Under Conflicting Objectives", Water Resources Research, Vol.13, 296-302.

59. Fulkerson, D.R.,(1961),"An Out-of-Kilter Method for Minimal Cost Flow Problem", JSIAM,Vol.9.No.1.

60. Fults,D.M., L.Hancock, G.Logan,(1976),"A Practical Monthly Optimum Operations Model", Journal of the Water Resources Planning and Management Division, Proceedings of A.S.C.E.,Vol.102, No.WR1.

61. Fults, D.M.,L.F.Hancock,(1972),"Optimum Operations Model for Shasta Trinity System", Journal of Hydraulics Division, ASCE,Vol.98, No. HYO,1497-1514.

62. Green,G.G.,(1979),"Regulation of the Columbia Basin Reservoir System for Hydro Power and Other Purposes", Paper presented at ASCE Workshop on Reservoir Systems Operations, Boulder, Colorado.

63. Gagnon,C.R.,R.H.Hicks, S.L.S.Jacoby and J.S.Kowalik,(1974)," A Non-linear Programming Approaoch to a Very Large Hydro-Electric Systems Optimization", Mathematical Programming,Vol.6, 28-41.

64. Gal,S.,(1979),"Optimal Management of a Multireservoir Water Supply System", Water Resources Research, Vol.15, 737-749.

65. Gilbert,Kenneth and Richard M.Shane,(1982)," TVA Hydro Scheduling Model: Theoretical Aspects", Journal of the Water Resources Planning and Management Division,ASCE,Vol.108, p.21-36.

66. Giles, James E., and Walter O.Wunderlich,(1981),"Weekly Multipurpose Planning Model for TVA Reservoir System", Journal of the Water Resources Planning and Management Division,ASCE,Vol.107, No.WR2, pp.495-511.

67. Green,G.G.(1979),"Regulation of the Columbia Basin Reservoir System for Hydro-Power and Other Purposes", Paper presented at ASCE Workshop on Reservoir System Operations, Boulder, Colorado.

68. Guitron,R.A.,(1974),"Improved Stochastic Dynamic Programming for Optimal Reservoir Operation Based on the Asymptotic Convergence of Benefit Differences", M.S.Thesis in Hydrology and Water Resources, Arizona University.

69. Guitron,R.A.(1981),"Hydro-Electrical Model for Optimal Operation of a Single Multi-Purpose Reservoir", Journal of Hydrology, Vol.51, No.1/4, p.67-73.

70. Gundelach, J., and C.ReVelle,(1975),"Linear Decision Rule in Reservoir Management and Design 5: A General Algorithm", Water Resources Research, Vol.11, 204-207.

71. Hall, W.A.,(1964),"Optimal Design of a Multi-Purpose Reservoir", ASCE Journal of the Hydraulics Div.,Vol.90, No.HY1, p.141.

72. Hall, W.A. and Buras, N.(1961),"Dynamic Programming Approach to Water Resources Development", J.Geophysical Res.,66, p.517.

73. Hall, W.A. and Roefs, T.G.,(1966),"Hydropower Project Output Optimization", ASCE, J.of the Power Divi.Vol.92, No.PO1,p.67.

74. Hall, W.A.,W.S.Butcher, and A.Esogbue,(1968),"Optimization of the Operation of Multiple-Purpose Reservoir by Dynamic Programming", Water Resources Research, Vol.4, 471-477.

75. Hall, W.A.,Tauxe, G.W. and Yeh,W.W.G.,"An Alternative Procedure for the Optimization of Operations for Planning with Multiple River Multiple Purpose Systems", Water Resources Res.5,6, 1367-1372.

76. Hanscom, M.,L.Lafond,L.Lasdon, and G.Pronovost (1980),"Modelling and Resolution of the Medium Term Energy Generation Planning Problem for the Hydro-Quebec System", Management Science, Vol.26, 659-668.

77. Harboe, R.C.,F.Mobasheri, W.W.G.Yeh (1970),"Optimal Policy for Reservoir Operation", Journal of the Hydraulics Division, ASCE, Vol.96, No.HY9, 2297-2308.

78. Hashimoto, T.,(1979),"Robustness, Resilience, Reliability and Vulnerability Criteria for Water Resources Planning, unpub. manuscript, International Institute for Applied Systems Analysis, Laxenburg, Austria.

79. Heidari, M.,Chow,V.T. and Meredith, D.D.,(1971),"Water Resources Systems Analysis by Discrete Differential Dynamic Programming", Civil Engineering Studies, Hydraulic Engineering Series No.24, University of Illinois, Urbana, pp.118.

80. Heidari, M., Chow,V.T., Kokotovic, P.V., and D.D.Meredith (1941), "Discrete Differential Dynamic Programming Approach to Water Resources Systems Optimization, Water Resources Research, 7, 273-282.

81. Houck, M.H., and Cohon, J.L.(1978),"Sequential Explicitly Stochastic Linear Programming Models: A Proposed Method for Design and Management of Multipurpose Reservoir Systems, Water Resources Res., Vol. 14, No.2

82. Joeres,E.G.,Liebman, J.C. and ReVelle, C.S.(1941),"Operating Rules for Joint Operation of Raw Water Sources, Water Resources Research Vol.7, No.1.

83. Jones, I.J.(1979)," Hydro System Seasonal Regulation Program(A Hydro Simulator)", Paper presented at ASCE Workshop on Reservoir System Operations, Boulder, Colorado.

84. Kerr, J.A.,(1972),"Multireservoir Analysis Techniques in Water Quantity Studies", Water Resources Bulletin,Vol.8, 871-880.

85. Kindler, J.,et.al,(1979),"Operation of Multiple Reservoir Systems: A Case Study of the Upper Vistuala System," in IIASA, op.cit.

86. Knop,H.(Ed),(1974),The Tennessee Valley Authority Experience, International Institute for Applied Systems Analysis, CP-76-2, Vol.1, Laxemburg, Austria.

87. Lane, M.,(1973),"Conditional Chance-Constrained Model for Reservoir Control", Water Resources Research, Vol.9 937-948.

88. Larson, R.E., and W.G.Keckler,(1969),"Applications of Dynamic Programming to the Control of Water Resources System ", Automatica, Vol.5, 15-26.

89. Laufer, F.and H.J.Morel-Seytoux,(1979),"Optimal Weekly Releases from a Seasonal Reservoir 1. Deterministic Future", Water Resources Research, Vol.15, 383-398.

90. Leclere, G. and Marks, D.H.,(1973),"Determination of the Discharge Policy for Existing Reservoir Networks Under Differing Objectives, Water Resources, Res. Vol.9, No.5, 1155-1165.

91. Lee, E.S.and S.Waziruddin,(1970),"Applying the Gradient Projection and Conjugate Gradient to the Optimum Operation of Reservoirs", Water Resources Bulletin, Vol.6,713-724.

92. Little, J.D.C.,(1955),"The Use of Storage Water in a Hydro-Electric System, Oper.Res.,Vol.3, p.197.

93, Liu, C.S. and A.C.Tedrow,(1973),"Multilake River System Operation Rules", Journal of Hydraulics Division,ASCE,Vol.99, No.HY9, 1369-1381.

94. Loucks, D.P.,(1968),"Computer Models for Reservoir Regulation, Proc. ASCE, J. of the Sanitary Eng.Div.,Vol.94,No.SA4, 657-669.

95. Loucks, D.P.,(1970),"Some Comments on Linear Decision Rules and Chance Constraints," Water Resources Research, Vol.6,No.2, 668-671.

96. Loucks, D.P.,(1976),"Surface Water Quantity Management, Chapter 5 in: Biswas, A.K.(Ed.), Systems Approach for Water Management (McGraw -Hill), New York, 156-218.

97. Loucks, D.P., and L.M. Falkson, .,"A Comparison of Some Dynamic Linear and Policy Iteration Methods for Reservoir Operation, Water Resources.Bull., AWRA Jnl.,Vol.6, 384-400.

98. Loucks,D.P. and P.J.Dorfman,(1975),"A Comparison of Some Linear Decision Rules in Chance Constraint Models for Reservoir Planning and Operation, Water Resources Research, Vol.11, No.6, 772-782.

99. Loucks, D.P., and P.J.Dorfman,(1975),"An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation," Water Resources Research,Vol.11, No.6, 777-782.

100. Loucks, D.P. and O.T.Sigvaldason,(1978),"Multiple Reservoir Operation in North America", in IIASA, op.cit.

101. McKerchar, A.I.,(1975)," Optimal Monthly Operation of Interconnected Hydro-Electric Power Storages", Journal of Hydrology, Vol.25, 137-158.

102. Maidment, D.R.,(1978),"Water Resources Control Through Systems Analysis", Water Supply and Management, Vol.2, 243-251.

103. Manne,A.S.,(1960),"Linear Programming and Sequential Decisions, Mgmt.,Sci,6, p.259.

104. Manne,A.S.,(1962),"Product-Mix Alternatives: Flood Control, Electric Power and Irrigation", Intl.Econ.Rev.,3,p.30.

105. Manning,G.B.,(1981),"Practical Use of the Stage and Larson Method", New Zealand Electricity Internal Report.

106. Manning, R. and D.Gallagher,(1982),"Optimal Water Pricing and Storage: The Effect of Discounting", Water Resources Research, Vol.18, No.1,65-70.

107. Mannos, M.,(1955),"Application of Linear Programming to Efficiency in Operation for a System of Dams",Econometrica, Vol.33, 335-336.

108. Marks, D.H.,(1974),"Models in Water Resources", in a Guide to Models in Government Planning and Operations(eds.S.Gass and R.Siskin), Environmental Protection Agency.

109. Mawer, P.A.,and D.Thorn,(1974),"Improved Dynamic Programming Procedures and Their Practical Application to Water Resources Systems", Water Resources Research, Vol.10, 183-190.

110. Meier,W.L., and C.S.Beightler,(1967),"An Optimization Method for Branching Multistage Water Resource Systems", Water Resources Research, Vol.3, 645-652.

111. Mejia, J.M., P.Egli, and A.LeClerc,(1974),"Evaluating Multireservoir Operating Rules", Water Resources Research, Vol.10, No.6, 1090-1098.

112. Meredith, D.D.,(1975),"Optimal Operation of Multiple Reservoir System", Journal Hydraulic Division.ASCE ,Vol.101, No.HY2, 299-312.

113. Meyer, H.W., and D.L.Sparks,(1978),"Measuring Results in Strategy Model Applications for Central Valley Project" Paper presented at ORSA/TIMS National Meeting, Los Angeles, California.

114. Murray, D.M., and S.Yakowitz,(1978),"Constrained Differential Dynamic Programming and Its Application to Multireservoir Control",

Deptt. of Systems and Industrial Engineering, University of Arizona.

115. Nayak, S.C., and S.R.Arora,(1971)," Optimal Capacities for a Multi-reservoir System Using the Linear Decision Rule" Water Resources Research, Vol.7, No.3, 485-498.

116. Nayak, S.C., and S.R.Arora, (1974),"Linear Decision Rule: A Note on Control Volume Being Constant", Water Resources Research, Vol.10, No.4, 637-642.

117. New York State Department of Environmental Conservation, (1977) "Proposed Alternative Releases from New York Reservoir in the Upper Delaware River Basin", Albany, New York (March 1974),plus revision of Appendix C.

118. Nopmongool, P.,and A.Askew,(1976),"Multilevel Incremental Dynamic Programming", Water Resources Research, Vol.12, No.6, 1291-1297.

119. North, H.A., T.E.Unny, R.Thompstone, D.K.Sen,and R.J.Selver,(1977)," Long Term Optimum Operating Policies for a Multi-Reservoir Hydro-System", TIMS/ORSA National Meeting, San Francisco.

120. Opricovic, S., and B.Djordjivic,(1976),"Optimal Long-Term Control of a multi-purpose Reservoir with Indirect Users", Water Resources Research, Vol.12, No.6, 1286-1290.

121. Orlob,G.T.,(1977),"Mathematical Modelling of Surface Water Impoundments",Vol.I, Rept. to USDI, OWRT, Res. Mgmt. Assoc. Inc.Lafayette, California.

122. Parikh, S.C.,(1966),"Linear Dynamic Decomposition Programming of Optimal Long-Range Operation of a Multiple Multipurpose Reservoir System, Operations Research Center, University of California, Berkeley.

123. Peters, R.J.,K.C.Chu, and M.Jamshidi,(1977),"Optimal Operation of a Water Resources System by Stochastic Programming", IBM Thomas J.Watson Research Center.

124. Pinter, J.,(1976),"A Stochastic Programming Model Applied to Water Resources Management", Computing Center for Universities, Budapest, Hungary, Abstract in Selected Water Resources Abstracts, Vol.9, No.21, 24.

125. Read,E.B.,(1978),"A Deterministic Multi-Reservoir Model for Optimum Operation of the New Zealand Power System", Proceedings of Energy Modelling Symposium.

126. Read, E.G.,(1979),"Stochastic Long-Term Scheduling Models for a Power System", Proceedings of the Operations Research Society of New Zealand, 41-52.

127. Read, E.G.,(1979),"Optimal Operation of Power Systems", Ph.D.Dissertation, University of Canterbury, Christchurch, New Zealand, 408 pp.

128. ReVelle, C.,and J.Gundelach,(1975) ," Linear Decision Rule in Reservoir Management and Design 4, A Rule that Minimizes Variance", Water Resources Research, Vol.11, No.2,197-203.

129. ReVelle, C.,E.Joeres and W.Kirby (1969)," The Linear Decision Rule in Reservoir Management and Design I, Development of the Stochastic Model", Water Resources Research, Vol.5, 767-777.

130. ReVelle, C.,and Kiroby,W.,(1970),"Linear Decision Rule in Reservoir Management and Design, 2, Performance Optimization", Water Resources Research, Vol.6, No.4, 1033-1044.

131. Riley, John G. and Charles R.Scherer,(1979),"Optimal Water Pricing and Storage with Cyclical Supply and Demand", Water Resources Research, Vol.15, No.2, 233-239.

132. Roefs, T.G.,(1968),Reservoir Management: The State of the Art, IBM Science Center Rept. 320-2508.

133. Roefs, T.G. and Bodin, L.D.(1970),"Multireservoir Operation Studies", Water Resources Research, Vol.6, No.2, 410-420.

134. Riley, John G. and R.A.Guitron,(1975),"Stochastic Reservoir Models: Relative Computational Effort," Water Resources Research, Vol.11, 801-804.

135. Rogers, P.,(1979),"On the Choice of the 'Appropriate Model' for Water Resources Planning and Management",Water Resources Research, Vol.14, 1003-1010.

136. Rosenthal, R.E.,(1980),"The Status of Optimization Models and Non Separable Benefits", Report No.75, Tennessee Water Resources Research Center, The University of Tennessee, Knoxville.

137. Rosenthal, R.E.,(1981),"A Nonlinear Network Flow Algorithm for Maximization of Benefits in a Hydroelectric Power System", Operations Research, Vol.29,763-786.

138. Rossman, L.A.,(1977),"Reliability-Constrained Dynamic Programming and Randomized Release Rules in Reservoir Management," Water Resources Research, Vol.13, 247-255.

139. Russell, C.B.,"An Optimal Policy for Operating a Multi-Purpose Reservoir"Operations Research, Vol.20, No.6.

140. Schultz, M.A.(1978),"Experience with Optimizer Techniques for Regulation of the Columbia River System", Paper presented at ASCE Workshop on Reservoir Systems Operations, Boulder Colorado.

141. Schweig, Z. and J.A.Cole,(1968),"Optimum Control of Linked Reservoirs", Water Resources Research, Vol.4, No.3, 479-497.

142. Sen, D.K.,R.Thompstone, M.Kermani, and R.Divi,(1977),"An Explicit Stochastic Optimization Approach to Develop Operating Policies for a Multi-Reservoir Hydroelectric System", ORSA/TIMS National Meeting, Atlanta.

143. Shane,Richard and Kenneth C.Cilbert,(1982)," TVA Hydro Scheduling Model: Practical Aspects", Journal of the Water Resources Planning and Management Division, ASCE, Vol.108, p.1-19.

144. Shelton R.A.(1979),"Management of the TVA Reservoir", Paper presented at ASCE Workshop on Reservoir Systems Operations, Boulder, Colorado.

145. Sigvaldason, O.T.,(1976),"A Simulation Model for Operating a Multi-Purpose Multireservoir System", Water Resources Res.,Vol.12, No.2, 263-278.

146. Sigvaldason, O.T.,(1976),"Models for Scheduling Releases from Multi-Reservoir Systems: A State-of-the-Art Review, Environmental Aspects of Irrigation and Drainage, 331-348, ASCE, New York.

147. Sigvaldason,O.T.,(1978),"Models to Aid the Planning and Management of Multi-Purpose Multireservoir Systems," Paper presented at Intnl. Symp. on Logistics and Benefits of Using Mathematical Models of Hydrologic and Water Resources Systems, Pisa, Italy.

148. Sigvaldason, O.T. and Bennett, W.D.,(1978),"Late Winter-Spring Forecasting and Operation of Multipurpose Multireservoir Systems in Canada," Paper presented at Third Intnl.Water Congress, Mexico City.

149. Sigvaldason, O.T., Ellis, R.,Bradford, W.E. and Granz, H.,(1975), "The Trent River System: An Improved Operational Strategy", Proc. of Second World Congress on Water Resources, New Delhi, India.

150. Sigvaldason, O.T.,Ellis, R. and Allen, R.B.,(1975),"Early Experience in Using a Simulation Model for Operating the Trent River System", Proc. of Second Canadian Hydrotechnical Conf., Burlington, Ontario.

151. Singh, K.P.,(1978),"Optimal Operation of Shelbyville and Carlyle Lakes" Water Resources Bulletin, Vol.14, 1201-1219.

152. Sniedovich, Moshe,(1980),"A Variance-Constrained Reservoir Control Problem", Water Resources Research, Vol.16, No.2, p.271-274.

153. Soares, S.,C.Lyra, H.Tavares,(1980),"Optimal Generation Scheduling of Hydrothermal Power Systems," IEEE Transactions on Power Apparatus and Systems, Vol.PAS-99, No.3, pp.1107-1118.

154. Sobel, M.J.,(1974),"Reservoir Management Models", Yale Univ.,New Haven, Connecticut, 29p.

155. Su,S.Y. and R.A.Deininger,(1974),"Modelling the Regulation of Lake Supervior Under Uncertainty of Future Water Supplies," Water Resources Research, Vol.10, 11-25.

156. Szollosi-Nagy, A(Editor),(1976),"Workshop on the Vistala and Tisza River Basins", International Institute for Applied Systems Analysis.

157. Takeuchi,K., and D.H.Moreau,(1974)," Optimal Control of Multi-Unit Interbasin Water Resources Systems", Water Resources Research, Vol.10, 407-414.

158. Tauxe, G.W., R.R.Inman, D.M.Mades,(1979),"Multiobjective Dynamic Programming with Application to a Reservoir," Water Resources Research, Vol.15, No.6, p.1403-1408.

159. Tauxe, G.W.,R.R.Inman, D.M.Mades,(1978),"Multiobjective Dynamic Programming: A Classic Problem Redressed," Water Resources Research, Vol.15, No.6, p.1398-1402.

160. Tauxe, G.W.,D.M.Mades, and R.R.Inman,(1980),"Multiple Objectives in Reservoir Operation", Journal of the Water Resources Planning and Management Division, ASCE, Vol.106, p.225-238.

161. Tauxe, G.W., and L.W.Mays,(1977),"Application Expedients in Dynamic Programming to Water Resources Systems," ORSA/TIMS National Meeting, Atlanta.

162. Tauxe, G.W., W.A.Hall , and W.W.G.Yeh,(1973),"Joint Operation of a Linked Reservoir System with Multi-State Incremental Dynamic Programming",American Geophysical Union Transactions, Vol.54, 266-277.

163. Tedrow, A.C.,Lfu,C.S.,Halton, D.B. and Hiney, R.A.,(1971),"The Use of Systems Analysis in the Development of Water Resources Management Plans for New York State, N.Y.State Dept. of Environmental Conservation Rept. to U.S.Dept. of Interior, Office of Water Resources Res.,Grant No.14-01-001-1586.

164. Tennessee Valley Authority Water Resources Management Methods Staff, (1974a),"Development of a Comprehensive TVA Water Resources Management Program".

165. Tennessee Valley Authority Water Resources Management Methods,(1974b), "Evaluation of Weekly Hydropower Benefits".

166. Tennessee Valley Authority Water Resources Management Methods, (1976), "Weekly Release Scheduling by REDGRAD,"Report B-27, Knoxville.

167. Tennessee Valley Authority Water Resources Management Methods, (1977),"Short-Term Scheduling by Linear Programming (MPSX)", Note B-33.

168. Tennessee Valley Authority,(1974)," Development of a Comprehensive TVA Water Resources Management Program, Project Status, July 1974, Knoxville, Tennessee.

169. Texas Water Development Board,(1970),"System Simulation for Management of a Total Water Resources, Report 118.

170. Texas Water Development Board,(1972),Economic Optimization and Simulation Techniques for Management of Regional Water Resources Systems, River Basin Simulation Modoel, SIMYLD II Program Description.

171. Thomas, H.A.,Jr. and Watermeyer, P.,(1962),"Mathematical Models: A Stochastic Sequential Approach" in" Design of Water Resources Systems", Chapter 14,Maass .,A., et.al.(Eds) (Harvard Univ. Press, Cambridge, Massachusetts, 540-561.

172. Toebes, G.H., and C.Rukvichai,(1978),"Reservoir System Operating Policy: Case Study," Journal of the Water Resources Planning and Management Division, ASCE, Vol.104, No.WR.1 175-191.

173. Toebes, G.H., C.Rukvichai, and Y.S.Lin,(1976),"Operating Policy Simulation for a Reservoir System," Technical Report No.80, Purdue University Water Resources Research Center.

174. Torbi, M., and F.Mobasheri,(1973),"A Stochastic Dynamic Programming Model for the Optimum Operation of Multi-Purpose Reservoir", Water Resources Bulletin, Vol.9, 1089-1099.

175. Trott, W.J. and W.W-G.Yeh,(1973),"Optimization of Multiple Reservoir System", Journal of Hydraulics Division, ASCE, Vol.99, No.HY10, 1865-1884.

176. Turgeon,A.,(1981),"A Decomposition Method for the Long-Term Scheduling of Reservoirs in Series," Water Resources Research, Vol.17, No.6, pp.1565-1770.

177. Turgeon,A.,(1980),"Optimal Operation of Multireservoir Power Systems with Stochastic Inflows," Water Resources Research, Vol.16, No.2, p.275-283.

178. Turgeon, A.,(1981),"Optimal Short-Term Hydro Scheduling from the Principle of Progressive Optimality", Water Resources Research, Vol.17, No.3, p.481-486.

179. U.S.Army Corps of Engineers,(1971),HEC-3: Reservoir System Analysis Genralized Computer Program Description, The Hydrologic Engineering Center, Davis, California.

180. Watermeyer, P.and Thomas, H.A.,Jr.,(1962),"Queuing Theory and Reservoir Design", The Annals of the Computation Laboratory of Harvard University, Vol.31, Harvard Univ.Press, Cambridge, Massachusetts.

181. Windsor, J.S., and V.T.Chow,(1972),"Multireservoir Optimization Model," Journal of Hydraulics Division, ASCE, Vol.98, 1827-1845.

182. Wunderlich, W.D.,(1979),"Planned Enhancement of Water Management Methods for the TVA Reservoir System", presented at ASCE National Workshop on Reservoir Systems Operation, Boulder, Colorado.

183. Wunderlich, W.D.,(1979),"Application of System Analysis Techniques to Water Management in the Tennessee Valley Authority Reservoir System", presented to International Association for Hydraulic Research.

184. Wunderlich,W.D.,"Overview of Water Management Methods for the TVA-Operated Reservoir Systems", in IIASA, op.cit.

185. Yeh, W.W-G,(1976),"Optimization of Real Time Daily Operation of a Multiple Reservoir System", Report No.,UCLA-ENG -7828, Department of Engineering Systems, UCLA.

186. Yeh,W.W-G.,et.al.,(1978),"Optimization of Real Time Hourly Operation of a Complex Multiple Purpose Reservoir System, Rept. to USDI, OWRT,Sch. of Eng. and Apl.Phys. University of California at Los Angeles.

187. Yevjevich, V.,(1979),"Overview of Research on Operation of Multiple Reservoir Systems(Colorado State University Activities) in IIASA, op.cit.

188. Yevjevich, V.,L.Becker, and W.S.Chu,"Optimization of Real-Time Hourly Operations of a Complex Multiple Purpose Reservoir System", Engineering Systems Department, UCLA.

189. Yoganarasimhan,G.N.,(1976),"A Study of Operation of Rihand Reservoir", Ph.D.Thesis, B.I.T.S.,Pilani.

190. Young, G.K.,(1966), Techniques for Finding Reservoir Operating Rules,Ph.D.Thesis, Harvard University, Cambridge, Massachusetts.

191. Young, G.K.,(1967),"Finding Reservoir Operating Rules", ASCE, J. of the Hydraulics Div.Vol.93, No.HY6, 297-231.