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HYDRAULIC ROUTING TECHNIQUES

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ABSTRACT

Routing of flood in open channel is one of the unsteady flow problem of importance to engineers. Flood routing is a computational procedure aiming at tracing of a flood wave incident and known at the upstream location. The objective of flood routing is to find the maximum elevation reached and corresponding time, the maximum volume rate of flow, for use in the design of spillways, bridges, culverts, channel sections, etc., and total volume of flow for design and operation of storage facilities for flood control, irrigation and water supply. Engineers are mainly interested in finding the two field quantities which are stage and discharge(or velocity). In the case of hydraulic routing continuity equation and momentum equation are used. These are frequently referred to as St.Venant equations. These are as follows:

Continuity:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

Momentum:

$$\frac{\partial Q}{\partial t} + \frac{Q}{A} \frac{\partial Q}{\partial x} + Ag\ (S_f - S_o + \frac{\partial y}{\partial x}) = 0$$

where, A is area of the flow(m^2), Q is discharge (m^3/sec), g is acceleration due to gravity(m/sec^2) S_f is energy slope, S_o is bed slope and Y is depth of flow.

The above partial differential equations along with initial conditions and appropriate boundary conditions are solved using numerical methods. The finite difference models and the finite element models are commonly used. The review includes a discussion on

the above in addition to the method of characteristics. The review also includes uncertainties, in natural flood flow, like flood plain and channel interaction, alluvial bed deformation etc.

1.0 INTRODUCTION

Routing of flood in an open channel is one of the unsteady flow problems of interest to engineers. Flood routing is a computational procedure aiming at tracing the temporal variation and special history of a flood wave incident and known at the upstream location.

1.1 Importance in Water Resource Development

Floods are natural events affecting mankind. Man's response to this problem is interdisciplinary because of industrial development and urbanization man chooses to live in flood prone areas(flood plains). Hence one is interested in the extent of inundation of flood plain etc. Irrigation development and improvement of water transport, require one to understand the movement of floods. This is achieved through methods of flood routing.

1.2 Physical aspects of flow/flood phenomena in canals and natural rivers(including flood plain system):

The main river flowing to the sea is fed by numerous tributaries and also by small gullies through which water trickles from rain or from other sources like subsurface etc. Larger floods normally overflows the banks and used to occupy the flood plains. The existence of wide flood plains will have a favourable influence on attenuation of flood waves, because of the large storage capacity for flood water. In almost all the rivers, the bed on which water moves will get altered. For certain rivers an assumption of rigid bed is appropriate for flood routing. For rivers in alluvial plains the deformation of bed produces different forms which cause different resistance of flow. In addition the **rivers** in alluvial plains transport sediments also. They cause problems of erosion or siltation. The hydraulic connection between stream and aquifer also modify the flood to a greater extent in alluvial river reaches. Sometime simplified models are used. Table 1 gives comparison of simplified & complete **models**.

1.3 Basic Equations:

Flow in a river is generally changing with time. It also varies spatially. It may form a three dimensional. But for many engineering purposes it is enough to study rivers in one dimensional model especially for flood routing, with exception of cases where flood plains are also involved. This review is mainly concerned with one dimensional problems.

Water movement in a river is described by continuity and momentum equations(2,6) as follows:

Continuity equation:
$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad \ldots(1)$$

Momentum:
$$\frac{\partial Q}{\partial t} + \frac{Q}{A} \frac{\partial Q}{\partial x} + Ag(S_f - S_o + \frac{\partial y}{\partial x}) = 0 \qquad \ldots(2)$$

where:

A is the wetted area of cross section of flow(m^2)

Q is the discharge(m^3/S)

g is the acceleration due to gravity (m/s^2)

S_f is energy slope

S_o is bed slope

y is the depth of flow

x is the general flow direction

Since the area(A) is a unique function of the depth(y) these two equation essentially contain two unknown y and Q, provided the energy

Table 1 - Comparison of Simplified Vs.Complete Models

No.	Simplified Model(Hydrologic Routing)	Complete Model(Hydraulic Routing)
	<u>Advantages</u>	
1.	Computationally easy	Relatively difficult
2.	Many problems can be solved even without recourse to a digital computer	A computing machine is very essential.
3.	Provides answer in much less time	Takes time
4.	Computational cost is less	More
	<u>Disadvantages</u>	
5.	Normally do not have necessary accuracy	Desired accuracy can be obtained depending upon the uncertainties involved
6.	Large amount of past flood records are needed	Model calibration requires only few.
7.	Tributory flows and other controls can not be modeled, in general.	Can be modeled.
8.	Accuracy of the results very much depend on the length of reach.	The reach length is normally discretised.
9.	All acceleration terms are excluded.	Included.
10.	Does not require data on cross-section etc.	Needs data on cross-sections, Mannings 'n' at all discretised points.

slope (s_f) is known. Hence before going to any solution procedure evaluation of (S_f) is to be made.

1.4 Objective and Scope

In majority of the cases, in India the flood problems are causes by over bank flows. These problems are associated with aggredation or degradation, especially in northern parts of this country. The National Commission on floods has examined various aspects related to floods and discussed them in Volume I of its report (1980). The table 2 provides the regionwise flood problems in India ~~as~~ extracted from the above report. Floods passing the alluvial river reach are modified due to the bank storage effects.

In this review note, the following are briefly dealt with:

1. Complete equations and boundary conditions

2. Numerical methods

a) Finite Difference

b) Finite Element

c) Method of Characteristics

3) Determination of energy slope

4) Bank storage effects and

5) Flood plain effects.

Table 2 - Regionwise Flood Problems in India

No.	Name of the Region	States covered	Problems are due to	Notable events
1.	Brahmaputra region	Assam West Bengal North eastern states and union territories.	1. Hill sides are friable 2. Practice of shifting cultivation(Jhooming) 3. Frequent earthquakes 4. Land slides.	a) The earthquake in the river bed by 3.0 m near Dibrugarh. b) Considerable sediment in the flow. c) In Sikkim certain amount of bank erosion is also noticed.
2.	Ganga region	Uttar Pradesh Bihar West Bengal Haryana Himachal Pradesh Rajasthan Madhya Pradesh Union Territory Delhi	1. The north tributaries rising in the hills brings in lot of sediment. 2. Over spills of banks. 3. Breach of embankments 4. Changing course 5. Bank erosion 6. Drainage congestions	a) In 1978 a breach of river Yamuna affected Delhi b) Recent trend of heavy rainfall in Rajasthan cause heavy flooding. c) Drainage problems are noticed in Haryana and north-eastern districts of U.P.
3.	North West Region	Jammu & Kashmir Himachal Pradesh Punjab Haryana Rajasthan	1. Jhelum causes the main problem of flooding the Srinagar. 2. The chenab and the Ravi cause erosion in Jammu. 3. Water logging is noticed in Haryana and Punjab 4. Hill torrents and flash floods bring extensive silt over agricultural lands.	a) Large storage have been constructed on Beas and Sutlej rivers.

Contd.

No.	Name of the region	States covered	Problems are due to	Notable events.
4.	Central India and Deccan region	Madhya Pradesh Orissa Gujarat Maharashtra Karnataka Tamil Nadu Kerala and Andhra Pradesh	1. Intermingling of flood water from one river and the other resulting drainage congestion. 2. In delta areas of this region, occasional coincidence with high tides cause substantial damage	

2.0 REVIEW

In solving the unsteady problem of flood routing, it is usual to go in for one dimensional approach. The principal limitation of this approach is that the flow is assumed to vary in one direction i.e.,along the river. In most of the practical situations (Chen 1973), this approach is sufficient to model flood flows. When large area of flood plain is inundated, quasi one dimensional models are used as adopted by Dass and Simon(1976).

2.1 Complete equations and boundary conditions:

The one dimensional differential equations of gradually varied unsteady flow in natural alluvial channels can be derived based on the following assumptions:

1. The channel is sufficiently straight and uniform in the reach.
2. The velocity is uniformly distributed over the cross section.
3. Hydrostatic pressure prevails at every point in the channel.
4. Water Surface slope is small.

Continuity of water:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \qquad ...(3)$$

Momentum equation:

$$\frac{\partial Q}{\partial t} + \frac{Q}{A} \frac{\partial Q}{\partial x} + Ag \left(\frac{\partial y}{\partial x} + S_f - S_o\right) \qquad ...(4)$$

where,

Q is discharge in m^3/S

A is area of flow in m^2

g is acceleration due to gravity m/sec^2

y is the depth of flow in (m)

S_f is energy slope

S_o is bed slope.

In the case of alluvial rivers the continuity of sediment is accounted using the following equation.

$$\frac{\partial P A_d}{\partial t} + \frac{\partial AC}{\partial t} + \frac{\partial Q_T}{\partial x} = 0 \qquad ...(5)$$

where;

P is the volume of sediment per unit volume of bed layer

A_d is the volume of sediment per unit length of channel(m^2)

C is the mean concentration of sediment in the flow

Q_T is the total sediment discharge(m^3/S)

A is the area of flow in (m^2)

(a) Supplementary equations

In the above equations (3,4) Q,y are basic unknown. In equation 5 A_d is an additional unknown. The other variables must be expressed as a function of these three variables to obtain a solution. Physical properties of the prototype found in the equations are treated as follows:

1. The geometric properties of cross-sections are expressed as a function of stage from the known channel geometry.
2. The mean bed slope S_o is known from initial bed elevation.
3. The energy slope is treated separately.
4. The sediment discharge can be computed as suggested by Shen(1971), Garde(1960)

(b) Boundary conditions:

There are two kinds of boundary conditions for one dimensional (Chen,1977) routing models:

(i) exterior boundary conditions, and

(ii) interior boundary conditions

(i) Exterior boundary conditions:

If the flow is sub-critical the routing solution is possible only when one condition is specified at the downstream end of the reach. If the flow is supercritical the condition at the downstream condition is redundant. For this type of flow the conditions can not influence the upstream situation and the boundary condition should be specified at the upstream boundary.

There are three different ways of specifying this boundary condition as follows:

a) the stage hydrograph

b)the discharge hydrograph

c) the rating curve, and

in case of sediment routing following additional boundary condition may be used

d) the sediment discharge hydrograph or

e) the bed elevation

If there is a disturbance (according to R.K.Price,1974) such as from a triburary or from a tide, downstream of the boundary it is preferable to define either the discharge or the stage. If there is no disturbance affecting the downstream of the boundary and if a rating curve is available the latter can serve as the boundary condition. If no such rating curve is available an approximate boundary condition can be obtained by extrapolation and used. R.K.Price-(1974) justified the approximate boundary condition, saying that for most of the river flows the Froude's number is small, which ensures that any disturbance generated by this boundary condition, do not propogate upstream appreciably. Price,R.K.(1974) discussed the number of boundary conditions and described them to be dependent upon the method of solution. But it is to be understood that the number of boundary conditions are independent of the method of solution(Cunge,1975) whether it is numerical or analytical.

(ii) Interior boundary conditions:

They are usually concerned with continuity of discharge or water level or energy balance. The following are some of the examples.

1. At abruptly varying river cross-section as shown in Figure 1. Two compatibility equations are to be satisfied.

a) $Q_1 = Q_2$

b) $Y_2 + V_2^2/2g + \Delta H = Y_1 + V_1^2/2g$

where, Δ H is the expansion or contraction loss. In case of sediments, the third compatibility should be satisfied.

c) $G_1 = G_2$

In energy loss and kinetic energy head can be neglected the condition(b) would become

$$y_2 = y_1$$

2. At tributory confluence as shown in figure 2.

a) $Q_3 = Q_1 + Q_2$

b) $h_1 + V_1^2/2g = h_3 + V_3^2/2g + h_{f1-3}$, or

$h_1 = h_3$

c) $h_2 + V_2^2/2g = h_3 + v_3^2\ /2g + h_{f2-3}$, or

$h_2 = h_3$

d) Sediment continuity is also to be satisfied.

3. Flow through other control structures

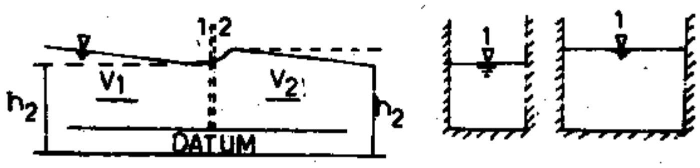

FIGURE 1 -- CROSS SECTION VARIATION BETWEEN TWO REACHES

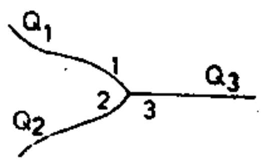

FIGURE 2 - JUNCTION OF TWO RIVERS

a) $Q_1 = Q_2$

b) Discharge formula incase of weir etc.

c) Sediment continuity equation.

2.2 Numerical Methods:

The partial differential equations are solved by numerical methods. With the development of digital computers, the sophisticated numerical methods have come up. There are a variety of numerical techniques each claiming certain specific advantages in terms of convergence, stability, accuracy and efficiency. Subramanya, K(1984) classifies the numerical methods used in flood routing as follows:

I. Direct Methods (FDM)

a) implicit

b) explicit

II Method of characteristic(MOC)

a) characteristic nodes

b) rectangular grid

III Finite element method

In case of method of characteristics he further classifies them into implicit and explicit methods.

Numerical solution approximates the continuous (defined at every point) partial differential equations with a set of discrete equations in time and space. The region of interest is usually divided into discrete nodes and the nodes will be used to make discrete equations for each sub region and time step. These equations are assembled to form a system of algebraic equations to be solved for each time step.

Numerical considerations:

In applying numerical methods, three characteristics are important; 1) accuracy, 2) efficiency and 3) stability. Accuracy deals with how well the discretized solution approximates the solution to the continuous problem it represents. Efficiency is a measure of how much computational work and computer resources are required to obtain a solution. Stability addresses the question of whether or not a solution is possible at all. The above definitions are, oversimplifications for practical purposes. Although numerical analysis provides the above information on alternative numerical methods, it does not tell which one is optimum for particular (Mercer,J.W.1981) application. Mercer,J.W.(1981) concluded that no particular combination of numerical technique and matrix solution procedure is best for all applications.

The numerical techniques commonly used in flood routing as used different researchers are briefly reviewed below.

2.2.1 Method of characteristics

In the method of characteristics (MOC) the St.Venant equations are converted into a set of two pairs of ordinary differential equations(in characteristics forms) and then solved by finite difference methods. Four computational schemes of calculations are commonly made in MOC. While applying finite difference method, an implicit or an explicit method in combination either to a rectangular grid or irregular characteristic grid are used. The Courant condition required for stability($\Delta t \leq |(\Delta x/v + c|)$ is automatically satisfied in MOC.

The method of characteristics was proposed for graphical integration of the shallow water equations by Massan(1905).This method assumes a constant wave property(f(h,Q) along the wave path defined by the following

$$\frac{dx}{dt} = C \qquad ...(7)$$

where, C is the wave velocity. This velocity can be found by

$$C = \frac{Q}{A} \pm \sqrt{gA/B} \qquad ...(8)$$

where, Q is the discharge in m^3/sec,

A is the area of cross section of flow in m^2

B is the top width of flowing water in m.

Introducing this in the momentum equation and continuity equation the following can be obtained.

$$\frac{dQ}{dt} + B\left(C - \frac{2Q}{A}\right)\frac{dh}{dt} - Ag(S_o + S_f) = 0 \qquad ...(9)$$

A computational mesh can be built up from the characteristics as shown in figure 3. If x and t for two points 2 and 3 are known along with the values of depth and discharge the new point 1 can be found using approximations of $\frac{dx}{dt} = C$.

$$x_1 - x_2 = C_2^+(t_1 - t_2) \qquad ...(10)$$

$$x_1 - x_3 = C_3^-(t_1 - t_3) \qquad ...(11)$$

where $C^{\pm} = V \pm \sqrt{gA/B}$

From these x and t, can be solved. Infact the velocity of propagation at the right hand sides of the equations can be described more accurately by the mean values between 1,2, and 1,3 respectively. In this case the system becomes implicit. The computation will become more accurate. Writing the equation 9 in a different form:

$$(Q_1 - Q_2)/(t_1-t_2) + B_2(C_2 - 2Q_2/A_2)\ (h_1-h_2)/(t_1-t_2) - Ag(S_o+S_{f2}) = 0$$

$$...(12)$$

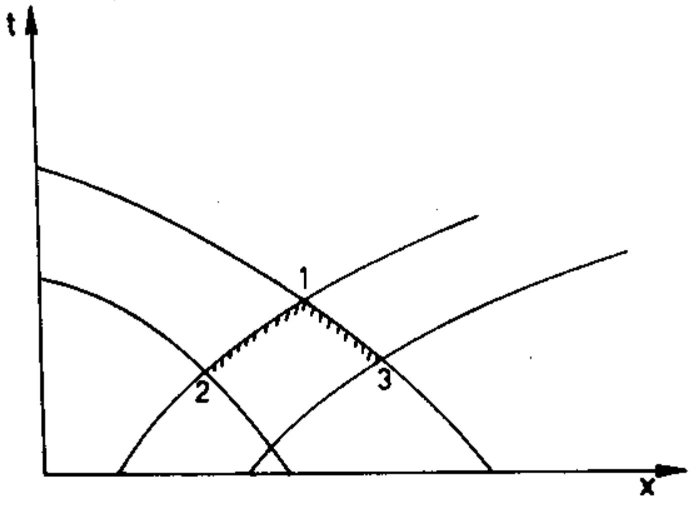

FIGURE 3 - IRREGULAR CHARACTERISTIC GRID

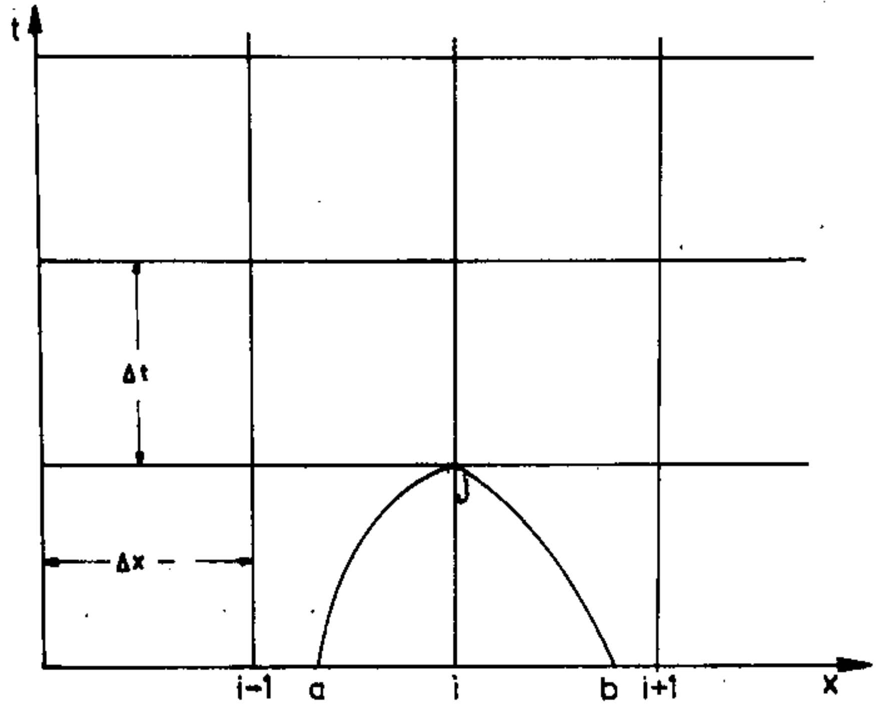

FIGURE 4 - REGULAR CHARACTERISTIC GRID

$(Q_1 - Q_3)/(t_1 - t_3) + B_3(\bar{C}_3 - 2Q_3/A_3)(h_1 - h_3)/(t_1 - t_2) - Ag(S_o + S_{f2}) = 0$

from which Q_1 and h_1 can be solved. ... (13)

In this way, starting from the initial conditions, a new layer of points in the x-t diagram can be determined which in turn is the basis for another layer. Repeating this process, the full region of interest and the full time interval under consideration can be covered by computational points. It can be seen that in the case of boundaries the scheme has to be modified, since only one characteristic is available. At the boundary, x-coordinate will be known and a boundary condition either on Q or h will also be known. This allows a solution at the boundary. More on boundary conditions are explained by Liggett,J.A.et al(1975).

Explicit schemes were used by Liggett and Woolhiser(1967) Streeter and Wylie(1967) John Ellis (1970). Ellis(1970) has applied this method to a channel of varying cross section which has also been explained in this review. Amein(1966) used an implicit scheme.

Ellis(1970) used the following form of equations of flow:

Continuity:

$$\frac{u\,\partial d}{\partial x} + d\,\frac{\partial u}{\partial x} + \frac{\partial d}{\partial t} = -\frac{u}{b}\,d\,\frac{db}{dx} \quad \ldots(14)$$

Momentum:

$$\frac{\partial U}{\partial t} + \frac{u\partial u}{\partial x} + g\left(\frac{\partial d}{\partial x} + \frac{\partial z}{\partial x} + S_f\right) = 0 \quad \ldots(15)$$

where, u is mean velocity in m/sec

d is mean depth (A/b) in m

b is mean width in m

In addition to these equations the following were also used:

$$\frac{\partial U}{\partial x}\,dx + \frac{\partial U}{\partial t}\,dt = du \quad \ldots(16)$$

$$\frac{\partial d}{\partial x}\,dx + \frac{\partial d}{\partial t}\,dt = dd \quad \ldots$$

From these equations Ellis(1970) obtained the following characteristic equations

$$du + (g/d)^{1/2}\, dd + g(dz/dx)\, dt + g s_f dt + u(gd)^{1/2}(db/b)(dt/dx) = 0 \qquad \ldots(18)$$

and

$$du - (g/d)^{1/2} dd + g(dz/dx)dt + g\; S_f dt - u(gd)^{1/2}(db/b)(dt/dx) = 0 \qquad \ldots(19)$$

Integrating along(1-2) and (3-2) as shown in figure 3 the following are written:

$$[u + 2(gd)^{3/2}]_1^2 + e_{12} + f_{12} + W_{12} = 0 \qquad \ldots(20)$$

$$[u - 2(gd)^{3/2}]_3^2 + e_{32} + f_{32} + w_{32} = 0 \qquad \ldots(21)$$

where

$$e = \int g(dz/dx)dt$$

$$f = \int gS_f dt$$

$$w = \pm \int \{u(gd)^{1/2}/(u \pm (gd)^{1/2})\} \frac{db}{b}$$

The integral 'e' is concerned with changes in bed elevation along the length of the channel. The integral 'f' is evaluated using the Mannings equation wherein the hydraulic radius is replaced by the mean depth. The third integral 'w' involves change of channel breadth along the length of the channel. Ellis(1970) used the following expression for evaluating W between the limit a,b.

$$W = [\; u - \frac{2u^2}{(gd_b)^{1/2} \mp (gda)^{1/2}} \;] \ln [\; \frac{d_b}{d_a} \;]$$

where u is the mean velocity,

$$u = \frac{1}{x_a - x_b} \int_a^b \frac{Q}{bd}\, dx$$

and Q,b,d are assumed to be linear function of x.

The characteristic equations are now written as

$$u_2 + 2(gd_2)^{1/2} = u_1 + 2(gd_1)^{1/2} + e_{12} + f_{12} + w_{12} = IA$$

$$u_2 - 2(gd_2)^{1/2} = u_3 - 2(gd_3)^{1/2} + e_{32} + f_{32} + w_{32} = IB$$

$$u_2 = \frac{IA + IB}{2} \quad ; \quad d_2 = \frac{IA - IB}{16g} \qquad ....(22)$$

Conditions at 1 and 3 are known but due to the unknown conditions at point 2 some initial estimates have to be made in order to evaluate 'e''f' and 'w'. Therefore, an iterative progress has to be employed to solve the unknown at point 2.

Ellis(1970) applied the above scheme in studying flows in Rhu Narrow at Clyde Sea area(Scotland). The grid is very similar to that of Hartree, who first developed this method (Fox 1962).

In the figure 4 the conditions at node i are assumed to exist at j and using these values the characteristic velocities $u_i \pm (gd_i)^{1/2}$ are calculated. From these celerity and the time step Δt, the intercepts a and b of the positive and negative characteristic on the line t_o are found. Interpolations between nodes i-1 and i and between i and i+1 give conditions at a and b respectively. The relationship between section spacing, the wave celerity and the time increment is such that the intercepts a and b always fall within the adjacent nodes. From conditions at i,a and b, values IA and IB are calculated as explained earlier. An iterative method is used till desired accuracy is achieved. It has been said that the computing time required for a 16 hr.tide simulation was 1 hr.

It can be found from Ellis(1970) work and others that the method of characteristic is accurate. This method generally and automatically makes a closer mesh in area of rapid change and sparse mesh in other regions of the domain. This is a favourable spacing in computation. But often practical problems need the grid as used by Ellis(1970). In either case this method is associated with a main disadvantage that it requires river cross-sectional data and roughness character etc.at intermediate sections in between the sections at which they are known. Since, these data will be known only at fixed locations and not at intermediate sections, which needs interpolation, this method did not gain importance, in the field problems.

However, Wylie,E.B.,(1980) developed an alternate formulation where interpolation are made on time line rather than space line as has been done in Ellis method.

2.2.2 Implicit Modelling

Stoker and his colleagues (1953) first anticipated the need for an implicit method for flood routing. Later Isacson (1954) developed further and applied them to river problems. In implicit method, two or more unknowns at the upper time level are related to one another. Since implicit methods do not require short time steps for solution, they are preferred.

Among the various implicit schemes used in open channel flows, the four point scheme of Amien(1970),Abbot(1967) are mentionable. In a comparative study, Price,R.K.(1974) has concluded that implicit scheme of Amien to be the most efficient one. Amien(1975) presented a modified version of his earlier work.

Amien's four point scheme:

Amien(1975) considered a non uniform rectangular grid on x-tplane as shown in figure 5. The partial derivatives at a point,

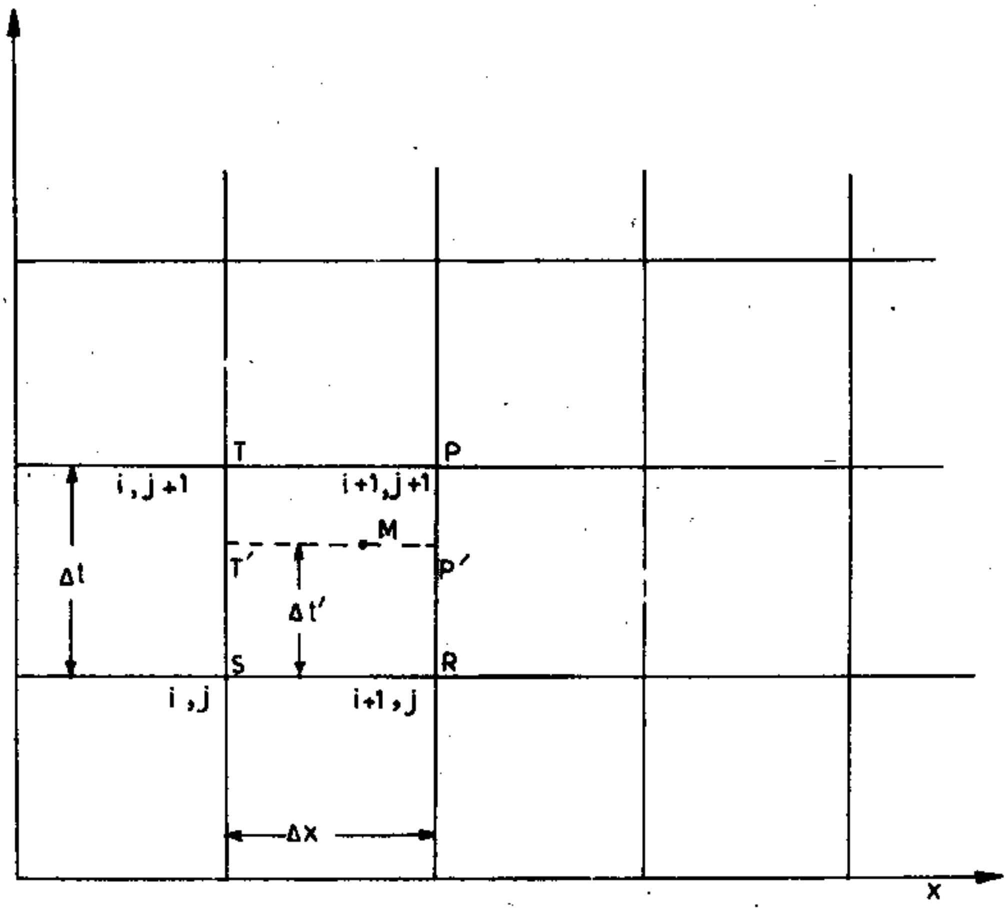

FIGURE 5 - SPACE TIME GRID

M, of a function of Q,y,etc.,with respect to x,t are expressed as:

$$\frac{\partial \alpha}{\partial t}(m) = \left[\frac{\alpha(P) + \alpha(T)}{2} - \frac{\alpha(R) + \alpha(S)}{2} \right] \frac{1}{\Delta t} \quad \ldots(23)$$

where, α is a variable

$$\frac{\partial \alpha(m)}{\partial x} = \frac{\alpha(P') - \alpha\ (T')}{\Delta x}$$

$$= \frac{1}{\Delta x} \{ (1-\theta)\ [\alpha\ (R) - \alpha\ (s)] + \theta\ [\alpha\ (P) - \alpha\ (T)]\}$$

in which; $\theta = \Delta t'/\Delta t$; is a weighting factor. ...(24)

The weighting factor, has been found to be important, in the stability of the numerical methods. When $\theta = 0$ the schedme reduces to explicit, when $\theta = .5$ the scheme produces an implicit centered difference scheme known as box scheme. The box scheme is accurate and stable for slowly varying flows. However it produces numerical oscillations under certain transient conditions. They do not occur for $0.5 < \theta < 1.0$. Amien says for $\theta = 1$ the method can take flow problems ranging from abrupt to slowly varied. But Cunge(1976) recommends a value of less than unity.

Applying the above approximations with $\theta = 1$ the continuity and momentum equations 1 and 2 can be written as follows:

a) Continuity equation

$$\frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x} + \frac{A_{(i+\frac{1}{2})}^{j+1} - A_{(i+\frac{1}{2})}^{j}}{\Delta t} - q_{i+\frac{1}{2}}^{j+1} = 0 \quad \ldots(25)$$

in which:

$$A\ (i + \frac{1}{2}) = \frac{1}{x_{i+1} - x_i} \int_{x_i}^{x_{i+1}} A(x)dx \quad \ldots(26)$$

The value of $A_{(i + 1/2)}$ depends on the shape of the channel and how the cross sectional area varies with distance. A practical approximation is mentioned in section:

b) **Momentum equation.**

$$\frac{1}{A^{j+1}_{(i+\frac{1}{2})}} \frac{Q^{j+1}_{i+\frac{1}{2}} - Q^{j}_{i+\frac{1}{2}}}{t^{j+1} - t^{j}} +$$

$$\frac{1}{A^{j+1}_{i+\frac{1}{2}}} \left[\frac{(Q^{j+1}_{i+1})^2}{A^{j+1}_{i+1}} - \frac{(Q^{j+1}_{i})^2}{A^{j+1}_{i}} \right] \frac{1}{x_{i+1}-x_i}$$

$$+ \frac{g}{x_{i+1}-x_i} \left[(y^{j+1}_{i+1} + z_{i+1}) - (y^{j+1}_{i} + z_i) \right] + g\, S^{j+1}_{f_{i+\frac{1}{2}}} = 0 \qquad \ldots(27)$$

in which

$$Q^{j+1}_{i+\frac{1}{2}} = \frac{1}{x_{i+1} - x_i} \int_{x_i}^{x_{i+1}} Q(x,t)\, dx$$

and

$$S^{j+1}_{fi+\frac{1}{2}} = \frac{1}{x_{i+1} - x_i} \int_{x_i}^{x_{i+1}} S_f(x,t)\, dx$$

Amien assumed a linear variation in Q, S_f, A and y with distance x between the grid points.

The finite difference representation of the two equations are modified with the above assumption and written as follows:

$$Q^{j+1}_{i+1} - Q^{j+1}_{i} + (\Delta x_i / \Delta t_j)(1/2)[(A^{j+1}_{i+1} + A^{j+1}_{i}) - (A^{j}_{i+1} - A^{j}_{i})] - q(x,t)\, \Delta x_i = 0 \qquad \ldots(28)$$

$$(1/2)(Q^{j+1}_{i} + Q^{j+1}_{i+1} - Q^{j}_{i} - Q^{j}_{i+1}) + [(Q^{j+1}_{i+1})^2 / A^{j+1}_{i+1} -$$

$$(Q^{j+1}_{i})^2 / A^{j+1}_{i}] (\Delta t_j / x_i) + (g/2)(\Delta t_j / \Delta x_i)$$

$$(H^{j+1}_{i+1} - H^{j+1}_{i})(A^{j+1}_{i+1} - A^{j+1}_{i}) + (g/4)(S^{j+1}_{fi} + S^{j+1}_{f_{i+1}})(\Delta t_j)$$

$$(A^{j+1}_{i+1} - A^{j+1}_{i}) = 0 \qquad \ldots(29)$$

All variables with superscript j are known with J+1 are unknowns. In the above two equations, there are 4 independent unknowns. These unknowns are the values of the discharge and stage at grid points (i,j+1) and (i+1,j+1). The distance increment Δx_i and the time increments Δt_i, need not be constant. The equations (28), (29) constitute a system of two non linear algebraic equations in four unknowns. But they are not sufficient to solve for the unknowns. It may be noted that if there are N points, 2(N-1) equations can be written in 2N unknowns. Two additional equations are needed to determine all the unknowns and they are provided by the boundary conditions. The following may form the necessary boundary condition:

1. Stage or discharge as a function of time at the upstream end.
2. Stage or discharge as a function of time or stage-discharge relationship.

The generalized Newton iteration method is recommended by Amien for the solution of the system of algebraic equations. Amien also applied the methodology to various field problems and showed that implicit scheme requires much less computer time, in view of large time steps used. Another advantage of the implicit method is that the time step can be selected in accordance with the physical requirements of the problem rather than those of numerical stability.

Price's R.K.(1974) conclusion on four numerical methods has been mentioned in the beginning of this section on implicit modeling. It is to be noted that his comparison is based on modeling of monoclinal wave with Chezy form of equation for energy slope(S_f). He compared the following methods: 1) Leap Frog explicit method, 2) Two step Lax-Wendroff explicit method 3) Four point implicit method of Amein and 4) Fixed mesh characteristic method. He further discussed that if a rating curve is available at the downstream section of the river and if the flood wave speed is sufficiently smaller than the courant speed($\Delta x/\Delta t$) then the implicit method is definitely faster than other methods for a similar accuracy. This advantage is particularly useful in the case of floods in rivers with small bottom slope. In case of steep slopes the speed of the wave may be comparable with the courant speed, there is little choice between the methods. The speed of the peak of the flood wave, which is over the bank, can be considerably lesser than the speed of a flood peak which is just with in the banks of the same river. This variation in the speed of the peak has a direct bearing on the accuracy of the implicit method. Price,R.K.(1974) suggested that the accuracy of the numerical solution for a large flood, which inundates the flood plains can be maximised by a choice of the time step appropriate to the speed for the peak of the flood just overlaps the banks of the river channel. Finally, it is to be noted that the dependence of a variable at one mesh point on the values of variables at all other mesh points on the same and the previous time levels, can not be explained on physical basis.

2.2.3 The Finite Element Methods(FEM):

The ability to model curve boundaries accurately and to represent non linear material properties easily make this method a powerful technique to solve many engineering problems. Among various methods in FEM Galerkin technique is popular in solving complete St.Venant equations. In this method algebraic approximation to the variables appearing in complete equations produce an expression for the residual error. Then by using certain mathematical criterion each method forces the residual to zero. This criterion is different for different methods. In this process of making the residual vanish, the method produces an algebraic system of equations solving this system yields the values of unknown variable at the required points in time and space. Fread, D.L.(1981) commented that the mathematical basis for finite element solution schemes is not as easily understood as finite difference approach. The use of finite element method to route floods in channels and natural streams is presented by Cooley and Moin(1976).

A variance of conventional FEM method was proposed by Nikolaos D Katopodes(1984). He concluded that the cost of computation using above approach is comparable to that of an implicit finite difference. Scheme using same number of grid points. Table 3 brings out the relative merits and demerits of finite difference and finite element methods.

2.3 Energy Slope

The dynamics of the river flow is much influenced by the energy slope (S_f) caused by the frictional resistance produced by bed and banks. Prediction of this still pose a problem to engineers.

There are many models which are using uniform flow equations like the Manning's equation, the Chezy's equations etc. For example HEC 2, HEC6, HEC2SR use the Manning's equation.

$$Q = \frac{1}{n} AR^{2/3} S^{1/2} \qquad ...(30)$$

where R is hydraulics radius and others are as defined earlier.

Einstein and Barbarossa(1952) recommended the use of Mannings-Strickler equation

$$U/U_* = 7.66(Y/K_s)^{1/6} \qquad ...(31)$$

where, $U_* = (gRS_f)^{1/2}$ and is known as shear velocity and K_s is sand

Table - 3 Comparison of Finite Difference Methods(FDM) and Finite Element Method(FEM)

No.	FDM	FEM
1.	A difference approach	An integral approach
2.	Approximation by grid points	Approximation is by finite element
3.	Non linear variation between grid points are not possible	Non linear variation adds no complexity.
4.	Curved boundaries require larger grid points	Curved boundaries can be represented with few curved elements.
5.	Grid formation is simple	Needs greater care to discretize
6.	Unit cost of computation (Per node per time step) is independent of the overall size of the problem (Weare,T.J.1976)	Unit cost increases with size of the problem.
7.	Matrix techniques are not a must	Matrix techniques are generally required.
8.	Accuracy is comparatively less (in few cases)	High order accuracy can be achieved.
9.	Hand computations are possible	Computer is a must
10.	Problem formulation and subsequent computer programs are relatively simple	Both formulation and computer program development are formidable task.

grain diameter taken as bed sediment size such that 65% of the materials are finer.

Both the equations(30,31) can be used with confidence in the case of rigid bed channels and also for alluvial rivers with flat bed.

However, when undulations are present in the bed these equations can not give any reasonable prediction of energy slope. The Manning's n is found (Garde, et al 1977) to be a function of discharge Q. Krishnappan,B.G.(1985) derived the following form of the equation based on works of Kishi and Kuroki(1974).

$$S_f = Const(R/D)^M (V/gR)^N \qquad ...(32)$$

where, V is the average velocity, M, N are constants and other terms are defined as earlier. He has also brought number of existing formulae to the above form which are shown in table 4.

In the model MOBED developed at the Hydraulic Division of National Water Research Institute at Burlington, Ontario, Canada the generalized equation (32)for energy slope is used. This permits easy updating of the analysis without any structural modification and enables the users to improve predictive ability as new relationships are developed for energy slope.

Boundary Conditions:

The equations 1 and 2 are solved for the following boundary conditions.

Initial condition:

A(x,o) Should be known for all

Q(x,o) x under consideration when t=o

Upstream condition:

A(o,t) should be known for all

Table 4 - Existing friction formulae in the form of Equation 32.

FORMULA NAME	TYPE OF BED	CONST.	M	N
Engelund	Dune	$1.86 \times 10^4(\gamma_s/\gamma)$	-1/5	1.8
	Antidune	$(0.12)^6(\gamma/\gamma_s)^{5/3}$	-7/3	-3.0
Garde and Ranga Raju	Ripples and dunes	0.098	-1/3	1.0
	Transition	0.028	-1/3	1.0
	Antidune	0.028	-1/3	1.0
Kishi and Kuroki	Dune I	0.0052	1.0	3.0
	Dune II	0.013	0.0	1.0
	Transition I	$0.018(\gamma_s/\gamma)^{6/7}$	-3/7	1/7
	Flat bed	0.021	-1/3	1.0
	Anti dunes	$0.0021(\gamma_s/\gamma)$	1/5	3.0
Griffth	Gravel bed	0.026	-0.34	0.66
Manning	Rigid bed	$n^2 g D^{1/3}$	-1/3	1
Chezy	Rigid bed	g/c^2	0	1

Note : C-is Chezy's coefficient;D is sediment size; γ_s, γ-specific gravity of sediment and water respectively.

$Q(o,t)$ Time steps $t > o$ at $x = o$

Downstream condition:

$A(L,t)$ Should be known for all time steps t o at x=L

When L is the total length of the river for which routing is done and t is time. Almost all the modelers use these boundary conditions. However a simplification is usually made by using the water depth $y(x,t)$ instead of $A(x,t)$. This means that an assumption of uniform width throughout the depth of the cross sections is made.

2.4 Bank Storage Effects:

The flood problems in India are concentrated in places where hydraulic connections between stream and aquifer are actively present. In the case of rivers in the Ganga region where alluvial river lengths are large the bank storage can modify the flood magnitude.

Pinder G.F. and Sauer,S.P. have made a numerical simulation of flood wave modification due to bank storage effects. The partial differential equations used by them are:

$$y\frac{\partial v}{\partial x} + v\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = (q_1 + q_o)/b \quad \ldots(33)$$

$$v\frac{\partial v}{\partial x} + g\frac{\partial y}{\partial x} + v(q_1 + q_o)/by + \frac{\partial v}{\partial t} = g(S_o - S_f) \quad \ldots(34)$$

$$T_x\frac{\partial^2 h}{\partial x^2} + T_y\frac{\partial^2 h}{\partial y^2} = S\frac{\partial h}{\partial t} + q_o/(b+2y) + W(x,y,t) \quad \ldots(35)$$

$$q_o/(b + 2y) = -k_p(y + z_o - h)/\Delta z \quad \ldots(36)$$

where:

Note: Bank storage is caused temporarily during the passage of flood and consequent raise in ground water level near a channel because of inflow from the stream.

y - the depth of flow(L)

v - velocity of flow (L/T)

q_l - flow into the channel per unit length of wetted perimeter (L^2/T)

q_o - lateral flow per unit length over the channel banks and from tributaries(L^2/T)

b - is width (L)

g is acceleration due to gravity

S_o - bed slope

S_f - energy slope

T - transmissivity (L^2/T)

h - hydraulic head L

S - Storage coefficient (dimension less)

$W(x,y,t)$ - Vertical discharge from the aquifer per unit area(L/T)

= 0 if there is no source or sink.

K_p - hydraulic conductivity

The following boundary conditions are also applied

$$\frac{\partial h}{\partial x}\bigg|_{(o,y,t)} = 0$$

$$\frac{\partial h}{\partial x}\bigg|_{(l,y,t)} = 0$$

$$\frac{\partial h}{\partial y}\bigg|_{(x,o,t)} = 0 \qquad \frac{\partial h}{\partial y}\bigg|_{(x,d,t)} = 0$$

together with an initial condition

$$h\,(x,y,o) = f(x,y)$$

Pinder,et al. used finite difference method to solve these equations. They found that bank storage attenuates a flood wave and this modification of the wave may be considerable in a long alluvial reach. The length of the channel reach and the hydraulic conductivity of the flood plain aquifer has a considerable influence on the modification of a flood wave by bank storage.

Hall and Moench(1972) have given analytical equations that would give the hydraulic gradient at the interface between stream and aquifer for an unit change in stream stage. Making use of these expressions and convolution techniques Larry F.Land developed a model for streamflow routing with losses due to bank storage. The model is being used in U.S.Geological Survey. Routing is carried out by a simplified model known as diffusion analogy.

2.5 River and Delta

A delta is usually formed by sediment deposition as a river enters into a reservoir, a lake or a massive water body. It is necessary to determine the variation of the flow pattern during a delta formation, since it affects the hydraulic parameters, such as flow velocity, energy gradient.

Chang and Hill(1977) use minimum stream power concept in hydraulic routing of a flood in river delta stream. It is postulated that during delta formation the delta stream width varies in such a way that the total stream power of the river flow is minimum. However, physical constraints are to be taken into account. In other words,a the delta stream adopts a width that represents the most efficient pattern of river flow with the minimum rate of work done to overcome flow resistance. Yang(1976) has also used this concept. But he used minimum unit stream power and hence different from Yang's who used total power.

Basic equations used in hydraulic routing by Chang are :

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \qquad \ldots(37)$$

$$g\,\frac{\partial H}{\partial x} \qquad \frac{\partial Q}{\partial t} \qquad \frac{1}{A}\,\frac{\partial(Q^2/A^2)}{\partial x} + gS - \frac{Q}{A^2}\,q = 0 \qquad \ldots(38)$$

$$(1-\lambda)\;\frac{\partial A_b}{\partial t} + \frac{\partial Q_s}{\partial x} - q_s = 0 \qquad \ldots(39)$$

where,

Q is flow discharge(m^3/s);

A is area of flow (m^2);

H is watersurface elevation(m);

S is the energy slope computed using Manning's equation;

A_b is the cross sectional area of bed above an arbitrary frame;

Q_s is the volumetric sediment rate; and

q_s is lateral inflow of sediment per unit length.

The sediment rate Q_s needs to be evaluated using proper equation.

The non linear partial differential equations will be written as difference equations at a number of discrete points in the space time domain. Usually the cross-section along the river reach are taken as the discrete points. Using initial and boundary conditions, the solutions are obtained.

In the finite difference scheme used by Chang and Hill(1977) the partial derivative at point 'B' of a function with respect to x, t can be written as

$$\frac{\partial \Phi}{\partial x} = \frac{1}{\Delta x_i}\left(\Phi^{j+1}_{i+1} \quad \Phi^{j+1}_{i} \right) \qquad \ldots(40)$$

and

$$\frac{\partial \Phi}{\partial t} = \frac{1}{\Delta t_j}\left(\Phi^{j+1}_{(i+\frac{1}{2})} - \Phi^{j}_{(i+\frac{1}{2})} \right) \qquad \ldots(41)$$

in which

$$\Phi_{(i + 1/2)} = \frac{1}{\Delta x_i} \int_0^{x_i} \Phi(x)\, dx \qquad \ldots(42)$$

and i,j represent space and time. For practical purposes the equation 42 can be simplified as

$$\Phi_{(i+1/2)} = (\Phi_i + \Phi_{i+1})/2 \qquad \ldots(43)$$

The primary equations 37,38 are written in the finite difference form as follows:

$$F_i = \theta(Q_{i+1}^{j+1} - Q_i^{j+1}) + (1/2)(A_{i+1}^{j+1} + A_i^{j+1} - A_{i+1}^{j} - A_i^{j}) - q(x,t)\,\Delta t_j = 0 \qquad \ldots(44)$$

$$\theta = \Delta t_j / \Delta x_i \qquad \ldots(45)$$

$$\begin{aligned} G_i = \; & (1/2)(Q_i^{j+1} + Q_{i+1}^{j+1} - Q_i^{j} - Q_{i+1}^{j} \\ & + \theta[(Q_{i+1}^{j+1})^2 / A_{i+1}^{j+1} - (Q_i^{j+1})^2 / A_i^{j+1}] \\ & + (g/2)\,\theta(H_{i+1}^{j+1} - H_i^{j+1}]\,[A_{i+1}^{j+1} + A_i^{j+1}] \\ & + (g/2)\, S_{i+1/2}^{j+1}(A_{i+1}^{j+1} + A_i^{j+1})\,\Delta t_j \\ & - [(Q_{i+1}^{j+1} + Q_i^{j+1})/(A_{i+1}^{j+1} + A_i^{j+1})]\, q(x,t)\Delta t_j = 0 \end{aligned} \qquad \ldots(46)$$

In the above two equations, all variables at time j are known and at j+1 they are unknown. There are N-1 points in the space domain where similar equations can be written. Therefore there would be 2(N-1) equations containing 2N unknowns in Q,H. Other variables like A and S are dependent upon Q and H. Two additional equations are obtained from boundary conditions at the upstream and downstream ends of the stream. At the upstream boundary, the discharge or stage may be given as a function of time, and resulting the following additional equation:

$$G_o = Q_1^{j+1} - Q'(t^{j+1}) = 0 \qquad \ldots(47)$$

or

$$G_o = H_1^{j+1} - H'(t^{j+1}) = 0$$

in which $Q'(t^{j+1})$ and $H'(t^{t+1})$ are the discharge and stage at the upstream end at time t^{j+1}. At the down stream end if the stage is known as a function of time, the boundary condition is:

$$F_N = H_N^{j+1} - H''(t^{j+1}) = 0 \qquad \ldots(48)$$

If the stage and discharge relationship is known, then

$$F_N = H_N^{j+1} - H''' (Q_N^{j+1}) = 0$$

is imposed as an additional equation. The generalized iteration method of Newton is used for solution. This method achieves the solution by successive solution of linear system of above equations 44-48.

The sediment bed area correction ΔA_b for a time increment Δt is found using sediment continuity equation 39 as follows:

$$\Delta A_b = - (\Delta t/(1-\lambda)) [(\partial Qs/\partial x) - q_s] \qquad \ldots(49)$$

At a section i the term for lateral sediment inflow is written as

$$q_{si} = (1/2)\ (q_{si} + q_{si}^{j+1})$$

A backward difference in space and centered difference in time is used to represent $(\partial Qs/\partial x)$

Thus $$\frac{\partial Q_s}{\partial x} = \frac{1}{\Delta x_{(i-1)}} \left[\frac{Q_{si}^{j} + Q_{si}^{j+1}}{2} - \frac{Q_{Si-1}^{j} + Q_{Si-1}^{j+1}}{2} \right] \qquad \ldots(50)$$

The total stream power of the river is

$$\epsilon = \int_0^L \gamma QS\, d \qquad \ldots(51)$$

where, ϵ is streampower or energy loss per unit time;

L is the total length of the river;

γ is the specific weight of water and sediment mixture;

Q is the discharge; and

S is the energy slope.

This stream power, ϵ is computed at each of the time steps. The width at which stream power is minimum is found by trial and error procedure. A new delta radius is obtained according to:

$$\Delta X_{N-1}^{j+1} = \Delta x_{N-1}^{j} + \frac{Q_{SN}^{j} + Q_{SN}^{j+1}}{A_{d}^{j} + A_{d}^{j+1}} \Delta t \qquad \ldots(52)$$

The computer program FLUVIAL 5 is based on above methods developed by Chang and Hill(1977). They have given step by step procedures for computations.

2.6 Flood Plain

Complications occur in routing with flood of flood plains. Extensive flood plains will generally show a flow pattern which is essentially two dimensional(in a horizontal plane). This situation is complicated by the occurance of field irregularities and obstructions like bridges etc. Grijsen and Vrengdenhil,(1976) have modeled Rhab Plains in Morocco. Implicit finite difference technique has been used. Inundation maps were reproduced very satisfactorily.

Radojkovic(1976) used two separate equations defining flow in the channel and plains. A similar study has been made by Purushotam Das and D B Simon (1976). They have divided the flow according to the conveyance of the channel portion and other divided portions of the sections. This is a multi-stream flow approach and can be called as quasy two dimensional model. Dass et al(1976) included sediment transport also in this model. D.L.Fread(1976) studied unsteady flow in a natural river which meanders through flood plains. He has distinguished flood plain length from channel length. Flood peak attenuation and travel time were found to increase as flood plain roughness and width increases and as channel sinosity decreases.

Attenuation increases and travel time decreases as the flood plain flow increases except at low flood plain flows.

2.7 Data requirements:

1. The primary data needed for hydraulic routing is the geometric coordinates of the cross-sections. More the number of cross-sections better would be the physical description in the model subjected to the limitation of cross-sections should adequately describe the flood plains on either side

2. For rivers in plains the characteristics of river bed materials are necessary. The size distribution of bed materials are to enable the estimation of friction factor.

3. Upstream hydrograph the hydrographs at tributaries alongwith their locations.

These are general requirements however different programme may need different additional data.

3.0 CONCLUSIONS

Routing models for water in rivers and to some extent in alluvial river have been reviewed. Although all the methods present in the literatures are not included, some useful methods that have been tested and verified are reviewed. They are one dimensional models. In general, a one dimensional model is adequate to study large river problems when extensive flood plains are not involved.

The complete equations are sufficiently correct and only the supplementary equations require considerable attention and further research.

The method of characteristics have been applied to field problems. Since this method uses a finite difference grid, it requires the same amount of effort as FDM. Complex curved boundaries produce difficulties in both the methods.

Finite difference approximation are relatively easy to apply. There are several applications of this method. Amein's (1970) implicit finite differences method is reported to be the best.

The finite element method is found to have been used by few modeler in routing. These methods have admirable quality of modeling curved boundaries.

No particular numerical method is best for all applications. For any given problem the choice of the method depends on the processes being modeled, and the accuracy desired. However, as Fread,D.L. pointed out that the personal preference(based on his familiarity) of the model developer is determining factor in selecting the method.

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