

**OVERLAND FLOW**

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## ABSTRACT

Overland flow is defined as a thin sheet flow occurring before surface irregularities cause a gathering of runoff into discrete stream channels. The primary distinguishing characteristic of overland flow is its shallow depth relative to roughness elements. The overland flow is an unsteady free surface flow and the most dynamic part of the response of a watershed due to excess precipitation. Since the floods are mainly due to direct surface runoff resulting from excess precipitation, therefore the accurate estimation of the direct surface runoff will provide the better estimates of the flood peaks and frequencies to the engineers and scientists involved for the design of flood control structures. Linear mathematical models describing stream or river outflow due to storm runoff do not explain several important observed features, such as the change in shape of the discharge hydrograph, the non-linear variation of peak discharge rate with variation of rainfall intensity. Numerical model based upon the shallow water equations or the kinematic wave equations can be used to calculate runoff hydrographs resulting from rainfall on small watershed as it overcomes the deficiencies associated with the linear models. The kinematic wave approximation model has also advantages over the linear model for predicting runoff for ungauged watersheds because the model structures and resistance parameters can be estimated without prior rainfall and runoff records.

The first step in applying these models is to decide upon the model geometry. The simplest way is to represent the catchments by simple geometric elements such as a combination of two planes and channel or linearly converging or diverging sections. The next step is the

solution for the overland flow. Horton and Izzard had solved the mass balance equation for the estimation of overland flow. Later on Behlke, Henderson, Wooding, Morgali, Linsley, Abbott, Brakensick and Woolhiser had applied method of characteristics for solving continuity and momentum equations numerically. Different shapes of input are considered as a separate case, out of those three cases are of our interest: (i) the rising hydrograph for a constant input and initially dry conditions, (ii) the recession from steady outflow conditions after the cessation of input and (iii) the transition from one steady state to another when there are two different constant supply rates in successive intervals of time.

Generally two main problems arise with kinematic flow modelling (i) the physical relevance of the kinematic shocks that result from the mathematics (ii) the adequacy of the numerical scheme with respect to its stability and convergence to the equations that the scheme is intended to represent it. Indeed numerical schemes based upon a direct discretization of the partial differential equations are incapable of predicting and tracking the shocks that are known to occur in the exact solution. Instead, instability or convergence to a different set of equations result. A general review has been made for different numerical schemes utilized by various investigators for modelling the overland flow. The advantages and limitations of these numerical schemes are also described.

## 1.0 INTRODUCTION

Whenever and wherever the rate of precipitation exceeds the infiltration rate at the surface, the excess water begins to accumulate in static surface storage. The capacity of this storage is governed by the extent to which geometrical surface irregularities and surface tension can develop forces to balance the increasing gravitational forces. When the local static storage capacity is exceeded, surface runoff begins as a thin sheet flow. The local gravitational potential gradients are developed due to surface irregularities and due to this gathering, the runoff into discrete stream channels take place. These channels form a tree like network which ensures that the flow immediately below each confluence exceeds that in either of the merging branches. It is clear therefore that in general there exists a whole spectrum of channel geometries and flow types. At one extreme lies the thin sheet flow called overland flow. It is likely to be the primary flow in urban runoff and in surface runoff from very small natural areas having little topographic relief. The next distinctive type is found in the smallest stream channels which gather the overland flow in a continuous fashion along their length of form the lowest order of stream flow. As these smallest streams merge with one another, they form streams of higher order which will have concentrated tributary inflow as well as continuous lateral inflow.

The build up and decay of river flow as a consequence of heavy rainfall is a central problem in design of hydraulic structures and the planning of flood control measures, often the prediction of flood peaks needs to be made with considerable precision.

Most of the standard methods of analysis of surface runoff are

based upon linear models and of these, the unit hydrograph is the most widely used. However, the underlying physical mechanism of runoff is not taken into account and the assumption of linear model appears to be in conflict with known profession of the runoff equation.

Recent application use the equations of motion (particularly with kinematic flow approximation) in describing the hydrodynamics of overland flow. Kinematic cascade models, which consider the outflow from one area as the boundary condition for the next area serve as a generally good physical model for overland flows and are amenable to economical numerical solutions.

Generally two main problems arise with Kinematic flow modelling: (1) the physical relevance of the kinematic shocks that result from the mathematics, as evidenced by intersecting characteristics in the method of characteristics solution and (2) the adequacy of the numerical schemes with respect to its stability and convergence to the equations that the scheme is intended to represent. Indeed, numerical schemes that are based upon a direct discretization of the partial-differential equations are incapable of predicting and tracking shocks that are known to occur in the exact solutions. Instead, instability or convergence to a different set of equations result. Here a general review has also been made for different numerical schemes utilised by various investigators in order to estimate the overland flow and the advantages and limitations of these various numerical schemes.

## 2.0 REVIEW

### 2.1 General

The movement of water in surface or overland flow is another important land surface process. One should consider the interactions between overland flow and infiltration as both the processes occur simultaneously. During overland flow water held in detention storage remains available for infiltration. Surface conditions such as heavy turf or very mild slope reduces the total quantity of runoff and increases the infiltration rate. Short high intensity rainfall bursts are appeared as surface detention storage reducing the maximum outflow rate from overland flow.

Thus, the essential problem of overland flow is to determine the flow off the plane at the downstream end for given physical conditions and a given pattern of lateral inflow along the plane. The continuity equation for this problem can be written as:

$$\frac{\partial Q}{\partial x} + \frac{\partial y}{\partial t} = q(x,t) \quad \dots(1)$$

where

$Q = Q(x,t)$  = rate of overland flow per unit width

$y = y(x,t)$  = depth of overland flow

and  $q = q(x,t)$  = rate of lateral inflow per unit area.

The dynamic equation for two dimensional overland flow can be written as

$$\frac{1}{g} \frac{\partial u}{\partial t} - \frac{\partial y}{\partial x} \frac{u}{g} - \frac{\partial u}{\partial x} = S_o - S_f - \frac{g}{gy} r(x,t) \quad \dots(2)$$

where  $u = u(x, t)$  = velocity of overland flow

$S_o$  = Slope of plane

$S_f$  = friction slope

Though the continuity equation is linear in  $Q$  and  $y$ , the dynamic equation is highly nonlinear. Equations(1) and (2) can be solved numerically by means of a high speed digital computer for any given set of boundary conditions. This approach will be considered in the later part of this review. For the present, however, the simpler approaches to the particular problem are considered and an attempt is made to find a simple mathematical or a simple conceptual model.

The classical problem of overland flow is to solve the particular case where the lateral flow is uniform along the plane and takes the form a unit step function:

$$q(x,t) = U(t) \quad \dots(3)$$

The complete solution of this problem can be divided into several parts:

- (i) There is the steady state problem of determining the equilibrium profile when the outflow at the downstream end of the plane is equal to the inflow over the surface of the plane.
- (ii) There is the problem of determining the rising hydrograph of outflow before equilibrium for the special inflow case represented by equation 3. If the problem were linear one, the solution of this second problem would be sufficient to characterize the response of the system, and the outflow hydrograph for any other inflow pattern could be derived from it. However,



since the problem is inherently non-linear, the principle of superposition can not be applied, and each case of inflow must be treated on its merits.

- (iii) There is the basic problem of determining the recession from the equilibrium condition after the cessation of long continued inflow.
- (iv) Another basic problem is to study the nature of the recession when the inflow ceases before equilibrium is reached (that is before the outflow builds up to a value equal to the inflow).
- (v) The next step is to investigate the effect of an inflow formed by the superposition of two or more step functions. Thus, the fifth basic problem involves consideration of case where there is a sudden increase from one uniform rate of inflow to a second higher rate of uniform inflow.
- (vi) The sixth case considered is that when a uniform rate of inflow is suddenly changed to a second uniform rate of inflow which is smaller than first.

## 2.2 Different Approaches For The Solution of Overland Flow Problem

Two approaches have been considered by various investigators for the solution of overland flow problem.

### 2.2.1 Mass balance approach

In this approach the dynamic equation 2 is replaced by an assumed relationship between outflow and storage. Because this method was first proposed by Horton (1938) for overland flow on natural catchments and subsequently used by Izzard (1946) for paved surfaces, it may be referred to as the Horton-Izzard approach.

Hydrologists noted that when the equilibrium runoff (that is the equilibrium discharge at the downstream end) of a number of experimental plots was plotted against the average surface detention (or total surface detention) at equilibrium on log-log paper, the experimental points fell approximately along a straight line. An exact linear relationship on logarithmic paper would indicate that the equilibrium outflow at the downstream end and the equilibrium storage were connected as follows:

$$Q(L, t_e) = Q_e = a S_e^C \quad \dots(4)$$

where  $Q_e$  was the discharge at the downstream end of the plane under equilibrium conditions,  $S_e$  was the total surface storage at equilibrium conditions and  $a$  and  $C$  were parameters.

In the Horton-Izzard approach to the overland flow problem, the assumption is made that such a power relationship holds not only at equilibrium, but also at any time during the rising hydrograph or during the recession. Using this equation one can write

$$Q(L, t) = Q_L = a S^C \quad \dots(5)$$

where  $Q_L$  is the discharge at the downstream end at any time and  $S$  is the corresponding total storage on the surface of the plane of overland flow. The equation of continuity, equation 2 can be written in the hydrological form as:

$$qL - Q_L = \frac{ds}{dt} \quad \dots(6)$$

which for the above assumption can be written as

$$Q_e - aS^C = \frac{ds}{dt} \quad \dots(7a)$$

$$\text{or} \quad a dt = \frac{ds}{S_e^C - S^C} \quad \dots(7b)$$

The solution of the equation(7) is:

$$t = \frac{1}{aS_e^{C-1}} \int \frac{d(s/S_e)}{1 - (s/S_e)^C} \quad \dots(8a)$$

$$t = \frac{1}{(a)^{1/C} (Q_e)^{(C-1)/C}} \int \frac{d(Q/Q_e)^{1/C}}{1 - (Q/Q_e)} \quad \dots(8b)$$

Equation 8 can be solved for different values of C. Horton (1938) solved the equation of the rising hydrographs due to the step function input for the case of C=2, which he described as 'mixed flow' since the value of C is intermediate between the value of 5/3 for turbulent flow and value of 3 for laminar flow. Horton's solution may be written as:

$$\frac{Q}{Q_e} = \tanh^2 ( t/K_e ) \quad \dots(9a)$$

where

$$K_e = \frac{S_e}{q_e} \quad \dots(9b)$$

Since the system is non linear, the time parameter  $K_e$  will depend on the intensity of inflow. Horton gave an empirical expression for the equilibrium storage per unit width and his equation for the rising hydrograph has been used in the design of airport drainage since that time. Izzard(1944) presented the solution for the case of C=3 (for laminar flow) in the form of a dimensionless rising hydrograph. Izzard used as his time parameter a time to virtual equilibrium, which is exactly twice the time parameter used in equation 9b above.

For recession from equilibrium, the recharge in equation 6 becomes zero and the insertion of the value for  $q$  from equation 5 leads to the solution:

$$\left(\frac{Q}{Q_e}\right)^{(C-1)/C} = 1 - (c-1) t/K_e \quad \dots(10)$$

where  $Q$  is the ordinate of the recession curve and  $t$  is the time elapsed since the cessation of inflow, that is, the time since the start of recession.

If the duration of the inflow(D) is less than the time required to reach virtual equilibrium, we get a partial recession from the value

of the outflow( $Q_D$ ) which has been reached at the end of the inflow having the same shape as for recession from equilibrium except that the recession flow enters the curve defined by equation 10 at the appropriate value of  $Q_e/Q_D$

If there is a change to a new rate of uniform inflow during the rising hydrograph, two cases can occur. If the new rate of inflow is higher than the rate of outflow when the change occur, the same dimension less rising hydrograph can still be used, but since  $q_e$  is equala to the inflow at equilibrium the value of  $q/q_e$  will change as soon as the rate of inflow changes. If the new rate of inflow is less than the outflow when the change occurs, the hydrograph will correspond to the falling curve of the varied flow function.

The conceptual model based on Horton-Izzard solution clearly assumes that the whole system can be lumped together and treated as a single non-linear reservoir whose outflow-storage relationship is given by equation 5. Even though this conceptual model is extremely simple in form, the fact it is nonlinear makes it less easy to handle than some of the apparently complex conceptual models used to simulate linear or linearized systems. Thus the impulse response for such a system no longer characterized the system because the output will also depend on the form and intensity of the input. The solution for a step function input can not be used to obtain the output for a complex pattern of input.

### 2.2.2 Solution of kinematic wave equations

The second simple solution proposed for the overland flow problem is kinematic wave solution. The kinematic wave equation comprise an equation of continuity and an approximation to the momentum equation.

These equations for the plane can be written respectively as

$$\frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} = q(x,t) \quad \dots(11)$$

$$Q = \alpha y^m \quad \dots(12)$$

where

Q = rate of outflow

y = depth of flow

q(x,t) = lateral inflow per unit area varying in time and space.

t = time coordinate

x = space coordinate, and

$\alpha$  and m = Kinematic wave parameters of friction relationship.

Combining equation (11) and (12):

$$\frac{\partial y}{\partial t} + \alpha m y^{m-1} \frac{\partial y}{\partial x} = q(x,t) \quad \dots(13)$$

Equation(13) is a partial differential equation having a single family of characteristics. The solution of equation(13) in conjunction with appropriate initial and boundary conditions, will completely characterise the outflow hydrograph.

The above forms of continuity and momentum equations was first derived by Koulegan in 1945 for overland flow. After an analysis of the magnitude of the terms in the momentum equation, he suggested that the above form of equation which we now call the kinematic wave equation would be appropriate for overland flow. The following two are methods available for solving the kinematic wave equations:

a) Analytical methods

b) Numerical methods.

a) Analytical Methods:

Parsen(1949) used the kinematic approach in describing the rising hydrograph of small runoff experimental plots. In a landmark paper,

Lighthill and Whitham(1955) developed the mathematical theory of kinematic waves including the mathematical phenomenon of kinematic shock. They also suggested the kinematic approach to overland flow modelling. Iwagaki(1955) independently came to the same conclusion for channels with steep slopes. Henderson (1964) obtained analytical solutions for the kinematic wave equation for simple plane and channel geometries. He analysed the problem and developed the equations for the rising hydrograph and falling hydrograph by arguments used on the method of characteristics. Wooding(1965) also obtained analytical solutions by the method of characteristics, firstly, for flow over a plane V shaped catchment under a constant uniformly distributed rainfall of finite duration and secondly for the stream outflow arising from the catchment discharge. This simplified two component model had only four parameters: two dimensionless indices in the power law equations of the motion assumed for catchment and stream flow, a scale of (i) rainfall intensity or(ii) total rainfall or (iii) rainfall duration and a dimensionless parameter which represents a ratio of suitably defined time constants for stream and catchment respectively. Wooding found the catchment stream hydrograph for either stage or discharge rate was a smooth curve having six discontinuities in curvature. He also gave the locations of these discontinuities, and hence the shape of the function depending upon the value of the parameters. He noted that the rising hydrograph depends initially only upon the integral of the catchment out flow. In this region various segments of the curve exhibit power law behaviour, but this is in fact a consequence of assuming power law depth discharge relationships. For a similar reason, the falling part of the curve exhibits the power-law decay.

b) Numerical methods

Since the assumptions leading to analytical solutions are so restrictive that their practical utility is greatly diminished, therefore, numerical or hybrid solutions of the kinematic wave equations were obtained by investigators.

Wooding(1965) obtained the numerical or graphical solutions for the problems where the rainfall varies arbitrarily with time over the catchment. He examined the effects of varying rainfall over the outflow hydrographs. The variation in rainfall was performed by considering (i) total rainfall constant or(ii) constant intensity of rainfall or (iii) constant duration of rainfall. He also investigated the possible modifications due to infiltration at the catchment outflow hydrograph.

Brakensick(1966) used numerical solutions of the kinematic wave equation to describe surface runoff from rural watersheds. Schaakey (1965) used numerical solutions of the continuity and momentum equations to describe runoff from urban watersheds and Margali and Linsley(1965) used a similar approach for rural watersheds. In 1967, Woolhiser and Liggett identified a dimensionless parameter that could be used to determine when the kinematic approximation is adequate. They demonstrated that kinematic approximation is very good for overland flow on a plane surface if the dimensionless parameter ( $K = S_0 L_0 / H_0 F_0^2$ ) is greater than 20. Here  $S_0$  is the slope,  $L_0$  is the slope length,  $H_0$  is the normal depth at the lower end of the plane and  $F_0$  is the Froude number. Morris and Woolhiser(1980) showed that the additional criterion  $S_0 L_0 / H_0 \geq 5$  is also required. From the typical rising hydrograph found by Woolhiser and Liggett, it appears that the shape of the early stage of the rising hydrograph approximates to the kinematic solution, whereas in the later stage it approximates more to the Horton-Izzard solution. This is not

unexpected because in the early stages of the flow  $\frac{dQ}{dx}$  would be relatively small, thus approximating the kinematic solution for which  $\frac{dQ}{dz}$  is zero downstream of the characteristics which starts from the upstream end of the plane at the start of the inflow. In the later stages of the rising hydrograph, the value of  $\frac{dQ}{dx}$  would approach the lateral inflow rate. In this case Horton-Izzard solution based on an empirical relationship, which is a good approximation at equilibrium, might be expected to give better prediction than the kinematic model.

Various numerical schemes which have been frequently utilized to solve the equation(13) are:

- (i) Upstream finite differencing scheme,
- (ii) Crank-Nicholson scheme;
- (iii) Brakensick scheme, and
- (iv) Lax-Wendroff scheme

Equation(13) is a non-linear partial differential equation and for these equations convergence and stability conditions are not yet established, except for some simplified cases. An alternative approach is to linearize the non-linear equations and then perform the analysis. After linearizing the equation(13), the equation becomes:

$$\frac{\partial y}{\partial t} + \alpha m \bar{y}^{m-1} \frac{\partial y}{\partial x} = q(x,t) \quad \dots(14)$$

where  $\bar{y}$  is a constant. Let  $a = \alpha m \bar{y}^{m-1}$ . It will be seen that the analytical treatment of step error remains unaffected (since  $q(x,t)$  is always known) making equation(14) homogeneous. Thus the homogeneous form of equation(14) may be written as:

$$\frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} = 0 \quad \dots(15)$$

The formulae for each schemes to solve the equation(15) are given as :



i) Upstream finite differencing scheme

Equation(15) can be approximated by this scheme as:

$$y_{i,(n+1)} = y_{i,n} - \frac{\Delta t}{\Delta x} \left( \frac{y_{i,n} - y_{(i-1),n}}{2} \right) \quad \dots(16)$$

where  $\Delta x$  and  $\Delta t$  are step lengths in space and time respectively.

ii) Crank-Nicholsen Scheme

This scheme approximates equation(15) as:

$$y_{i,(n+1)} = y_{i,n} - a \frac{\Delta t}{2\Delta x} (y_{i,n} - y_{(i-1),n} + y_{i,(n+1)} - y_{(i-1),(n+1)}) \quad \dots(17)$$

iii) Breakensick scheme

This is four point implicit scheme. Equation(15) can be approximated by this scheme as:

$$\frac{y_{(i+1),(n+1)} - y_{(i+1),n} + y_{i,(n+1)} - y_{i,n}}{2\Delta t} = -\frac{a}{2\Delta x} (y_{(i+1),(n+1)} - y_{i,(n+1)} + y_{(i+1),n} - y_{i,n}) \quad \dots(18)$$

iv) Lax-Wendroff scheme

This scheme is the most popular one. It is a single step second order explicit scheme. Equation(15) can be approximated by this schemes as:

$$y_{i,(n+1)} = y_{i,n} + \frac{\Delta t}{\Delta x} \left[ -\frac{a}{2} (y_{(i+1),n} - y_{(i-1),n}) \right] + \left( \frac{\Delta t}{\Delta x} \right)^2 \frac{a^2}{2} (y_{(i+1),n} - 2y_{i,n} + y_{(i-1),n}) \quad \dots(19)$$

Singh, V.P. (1976) has developed the step error of the above finite difference schemes. He has also shown that for convergent and stable schemes, the production of step error of one scheme may not be same as that of another. Such treatment can be useful in:

- a) determining the accuracy of a method,
- b) estimating a priori the step length to be used in the scheme, and
- c) choosing among the schemes.

Singh(1976) also found that the step error of Lax-Wendroff scheme is the least of all. This partly explains the popularity of this scheme. The Brahensiek 4-point implicit scheme, although unconditionally stable, has the highest step error. This points out that this scheme should not be recommended for use under all circumstances. Thus the important point is that the stability and step error of a numerical scheme must be considered simultaneously. That is, one must decide the step length such that the criteria of stability and possibly minimum error production are simultaneously satisfied. It will be useful to choose a step length that leads to minimum step error but the scheme becomes unstable. Stability and step error are intertwined and must be treated in the light of one another.

### 2.3 Overland Flow Models

The amount, temporal distributions and spatial distribution of the lateral inflow exerts the most significant influences on the runoff hydrograph predicted from a model. Therefore, errors in estimating infiltration are the most serious in watershed response simulation. Infiltration rates are governed by initial water content of the soil, vegetative cover, soil macro porosity and by soil depth. Although the infiltration process and flow in porous media have been studied intensively, but the complexity of the processes and spatial and temporal variability of soil characteristics create barriers that are difficult to overcome. Many overland flow models treat rainfall excess as given and route it over the soil surface. This procedure ignore the interactive nature of the problem and is not compatible with the variable source area concept. Other models(i.e.Rovey, Woolhiser and Smith,1977) include an interactive, one dimensional infiltration model which is effective

at each grid point. If the conditions or infiltration model parameters are not uniform, this model will generate overland flow over only part of the watershed. The area contributing overland flow will change with the rainfall rate. This model will not handle runoff induced by saturation from below. The model presented by Freeze(1972) can, in principle, accommodate all types of runoff generation, but computer time and storage requirements have prevented practical application.

#### 2.4 General Procedure for Modelling Overland Flow

In order to apply the equations describing overland flow to complex watersheds to better understand watershed response or to make predictions the following decisions must be made:

- i) A decision must be made regarding the method of spatial representation of the watershed.
- ii) A decision must be made on the form of the hydraulic resistance law and the infiltration law and several key parameters,
- iii) Finally, the user must select appropriate numerical methods for solving the equations. In this section a review has been done for different methods of spatial representation of watersheds and the different methods for estimating the parameters. Various numerical scheme for solving the equations of overland flow and criteria for selecting an appropriate scheme have already been described in section 2.2.2.

##### 2.4.1 Method of spatial representation of the watersheds

The methods of spatial representation that maintain model flow pattern similar to those in the prototype watershed are:

i) Regular grid method, rectangular and triangular grid

ii) Kinematic cascade

iii) Finite element methods

In 1937, Merrill Bernard divided a watershed into elemental areas 150 metres square. He used a graphical technique to route rainfall excess from the elemental areas to a channel and subsequential to the mouth of the basin. The problem of kinematic shock arose in his calculations and he handled it by essentially the same technique as we use today. He also analyzed the influence of various cropping systems on runoff hydrographs.

Almost after 30 years of Bernards work, Huggin and Monke(1966) used the same grid technique to represent watershed geometry. Rainfall excess was computed for each element and routed to downstream elements by assuming a relation between storage of water within an element and the outflow-an approach that is essentially kinematic.

Kibler and Woolhiser(1970), Harley, Perkins and Eagleson(1970) and Rovey, Woolhiser and Smith(1977) used a network composed of planes and channels to represent watershed geometry. All used kinematic routing, but Harley, Perkins and Eagleson(1970) included an option of the linear response function to the complete equations for flows not dominated by lateral inflow. In his representation all planes(catchments) with the same number had the geometric characteristics. This method has advantages over the square method in that it may require fewer elements and the programming logic and storage requirements are not so severe

A triangular grid representation of a watershed has certain appealing features in that it would confirm more closely to watershed topography. But one may require more difficult programming logic. The other most popular way of representing the watershed is in the form of

rectangular grid.

Recently there have been attempts to use the finite element technique for numerical solutions of the equations describing overland flow. An examination of the results of this work, however, reveals serious problems with continuity errors. Furthermore, the kinematic wave equations seem particularly ill suited to this approach because the analytical solution may contain jump discontinuities at abrupt changes in slope of roughness at element boundaries. Also the advantage of downstream sequential solution of the kinematic equation is lost. Beven and Woolhiser set up the equation for a Galerkin finite element solution to the kinematic wave equations using triangular elements. They found that the solution algorithm gave accurate results if the triangular elements were part of a longer plane but the solution deteriorated when the triangular elements were allowed to conform to a realistic overland flow surface.

The choice of the model watershed configuration is highly subjective. Lane, Woolhiser and Yevjevich(1975) have examined the effects of simplified watershed geometry as represented by a kinematic cascade on the goodness of fit of the response hydrograph. They also noted that the optimum hydraulic resistance parameter depends on the geometric goodness of fit. Singh and Woolhiser(1976) and Singh(1976) investigated the possibility of using a converging section as a simplified representation of natural watersheds.

#### 2.4.2 Estimation of parameters and the form of a hydraulic resistance law

Once the decision regarding the method of spatial representation of a watershed has been made, the user must decide on the form of a hydraulic resistance law and must estimate several key parameters.

The informations regarding slope and slope lengths etc. can be derived from the map. However, the channel characteristics are poorly defined at the map scales normally available. Machmeier and Larson(1968) and Golany and Larson(1971) used stream hydraulic geometry relations to define stream characteristics in numerical studies of watershed response utilising a physically based model. Betson(1979) suggested the use of geomorphic equations relating discharge to cross sectional area or channel width in flood routing for planning studies. Considering natural channel variability and the costs associated with the field measurements, this method appears to have considerable promise as a practical tool.

Resistance to overland flow over natural and manmade surface is influenced by several factors, and is frequently much greater than that encountered in ordinary hydraulic structures. These factors are:

- i) Rates of flow
- ii) Raindrop impact

At low rates of flow, the boundary elements protrude through the free water surface, and at high rates of flow the boundary geometry may change in time and distance because of erosion or bending of vegetation. On non-vegetated surface raindrop exerts a significant retarding effect. The ideal resistance law would include a hydraulically smooth plane or densely vegetated surfaces as special cases of the general law, and would have parameters that could be measured by direct physical means.

- a) Form for the resistance equation

There have been many laboratory and field investigations aimed at finding the best form for the resistance equation and methods for estimating parameters for hydrologically significant surface. The most

common approach has been to assume that the Darcy-Weisbach equation is the appropriate form and then to relate the friction factor  $f$  to hydraulic and geometric variables. The Darcy-Weisbach equation is ,

$$u = \sqrt{\frac{\epsilon g}{f} S_o R} \quad \dots (20)$$

It has been observed that overland flow behaves initially as if it were laminar. However, the turbulence may generate by rain drop impact. As the Reynolds number increases there is a transition from laminar to turbulent flow in the range  $100 < Re < 1000$ . The most frequently cited values for transition range from 300 to 500. Within the laminar flow range the friction factor  $f$  is related to the Reynolds number by the relationship:

$$f = K/R_e \quad \dots (21)$$

For hydraulically smooth surfaces with no raindrop disturbance,  $K=24$  if the Reynolds number is defined as  $R_e = uh/\nu$  where  $\nu$  is the kinematic viscosity. Raindrop impact produces the same effect as an increase in viscosity. However, rather than introducing a pseudo-viscosity term the parameter  $K$  can be approximated by (Woolhiser 1975):

$$K = K_o + Ai^B \quad \dots (22)$$

where  $K_o$  is the parameter without rainfall,  $i$  is intensity of rainfall and  $A$  and  $B$  are parameters. If the surface is hydraulically smooth the resistance effect of raindrops is significant. However, this effect can be neglected for vegetated surfaces.

The parameter  $K$  is related to the characteristics of the surface. Set of values for  $K_o$  and the parameters  $A$  and  $B$  have been reported by Woolhiser(1975) for different surface characteristics. Most of the data, have been used to estimate hydraulic resistance parameters, have been from small plots so bias is undoubtedly present. The small rills present on longer slope would certainly affect hydraulic resistance.

For turbulent flow, the Manning formula has been most commonly used. The tabulated Mannings n in handbooks are suitable for most channels. But for shallow overland flow or flow in grassed waterways, it has been found that the values change substantially with Reynolds number. Kouwen Li and Simons(1980) have developed a method to estimate n or f for vegetated channels where resistance is due to flexible roughness. They include a method to estimate the critical shear velocity above which the flexible roughness becomes prone. This velocity depends upon the number of steps per unit area and the elastic and geometric properties of the vegetation. Wu, Yevjevich and Woolhiser(1978) utilized data from an outdoor experimental watersheds to demonstrate that an equivalent hydraulic resistance can be estimated for watersheds with non-uniform roughness.

b) Infiltration models

Infiltration models which required detailed numerical solution of the partial differential equation of unsaturated flow require too much computer time and soil data for practical use. Approaches based on simplifications of those equation such as the Green and Ampt equations or the model presented by Smith and Parlange(1978) or Morel-seyouse (1978) appear to be the best at the moment.



### 3.0 CONCLUSIONS

Significant advances have been made in developing physically based models through the scientific study of watershed runoff processes. Several different runoff generating mechanisms exist, however, they do not have a significant effect on the mathematical description of unsteady surface runoff processes including overland and open channel flow. The continuity and momentum equation may be simplified to the kinematic wave equation for most overland flow cases. To apply the kinematic wave equations to practical situation one must first decide on the method of spatial representation of watershed and level of geometric details to be presented. Then an appropriate model for infiltration must be selected and linked to the overland flow model. Also, appropriate parameters for hydraulic resistance and porous media characteristics must be estimated. Finally, a stable and consistent numerical scheme may be selected for the solution of kinematic wave equation.

Much progress has been made to describe the spatial variability in watershed characteristics affecting infiltration, depression storage and surface runoff velocities. However, more objective techniques are badly needed to describe this variation in watershed characteristics. One may develop some criteria in order to determine the optimum levels of aggregation of watershed element.

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