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HYDROLOGIC FLOOD ROUTING INCLUDING DATA REQUIREMENTS

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CONTENTS

List of Symbols

a_o, a_1, a_2	-	parameters used in the Kulandaiswamy's storage equation
B	-	The Channel width
b_o, b_1	-	parameters used in the Kulandaiswamy's storage equation
C	-	the wave celerity
C_1, C_1, C_2	-	coefficients used in the conventional Muskingum method
C'_o, C'_1, C'_2	-	Coefficients used in the modified Muskingum method; also the Coefficients used in the more refined scheme
D	-	the diffusion coefficient
g	-	acceleration due to gravity
I(t) or I	-	inflow discharge
I_o	-	initial steady flow
I_1, I_2	-	inflow at the end of the successive routing time interval
K	-	Storage Coefficient
m	-	exponent of the nonlinear storage equation
n	-	number of linear reservoirs in series
Q(t) or Q	-	Outflow discharge
Q_o	-	the reference discharge;also the initial steady outflow
Q_1, Q_2	-	outflow at the end of successive routing time interval
q_o	-	reference discharge/unit Width

r	-	ratio of routing time interval to travel time
$S(t)$ or S	-	Storage in the reach
S_o	-	the bed slope
T	-	travel time of the average discharge through the reach
T_r	-	period of rise of inflow hydrograph
t	-	notation denoting time
U'_1	-	The first moment of IUH about origin
U_2, U_3	-	the second and third moments of IUH about the first moment
u_o	-	the Velocity corresponding to the reference discharge, q_o
$u(o,t)$	-	the IUH
V	-	average velocity at a channel cross section
x	-	the distance from upstream end of the reach
y_o	-	the depth corresponding to the reference discharge, q_o
T	-	lag time of the outflow
θ	-	Weighting parameter used in the Muskingum method
θ_k	-	Weighting parameter of the Koussi difference scheme
Δt	-	routing time interval
Δx	-	length of the single reach
δ	-	Dirac-delta function

ABSTRACT

Flood routing is used to simulate flood wave movement through river reaches and reservoirs. There are two types of flood routing methods used in practice viz.,hydraulic flood routing and hydrologic flood routing. Hydrologic flood routing uses the continuity equation in lumped form and some relationship between discharge and storage within the reach. This report reviews, the various hydrologic flood routing methods available in literature including the two parameter models which incorporate the Muskingum method, lag and route method, Kalinin-Milyukov method, diffusion analogy method, modified Puls method, and working R and D method; the three parameter models which incorporate diffusion added with lag model, multiple Muskingum method and three parameter gamma distribution etc. The linearized St.Venant's equations model applied to wide rectangular channels, the multiple linearization flood routing model, and the simple non-linear model are also reviewed. A broad discussion on the relationship between some of the hydrologic flood routing methods and the basic St.Venant's equations describing the one dimensional flow in channel is made. The advantages of the hydrologic flood routing methods over the hydraulic flood routing methods are described. The limitations of the hydrologic flood routing methods for their inability to take into account the back water effects and discontinuities in the water surface such as jumps or bores are considered. The data requirements of the hydrologic flood routing methods are indicated.

1.0 INTRODUCTION

A great many different methods and procedures for solving flood routing problems have been described in engineering literature. In general, those methods that attempt a strict mathematical treatment of the many complex factors affecting flood wave movement are not easily adaptable to the practical solution of problems of routing floods, as they demand on high computer resources as well as quantity and quality of input data. In order to keep the amount of computation within practical limits and to conform to limits ordinarily imposed by the type and amount of basic data available, it is generally necessary to use approximate flood routing methods that either ignore some of the factors affecting flood wave movoement or are based on simplifying assumptions in regard to such factors. Approximate methods produce results at considerably less expense but are limited in generality and accuracy(U.S. Army Corps of Engineers,1960).

Methods of flood routing are broadly classified as empirical, hydraulic and hydrological (Fread,1981). Empirical methods generally were developed from intuitive processes rather than from mathematical formulation of the problem. In practice, the methods have been applied to long reaches. Some of the methods found useful in practice are the successive average lag method, progressive lag method and the method based on gauge relations(Linsley et al.,1949). Empirical methods are limited in application with sufficient observations of inflows and outflows to calibrate the essential coefficients. The classification as hydraulic or hydrological depends on whether or not the St.Venant's equations are employed in the derivation of the model structure(Whitehead et al.,1979). However,this classification is more artificial as the so called hydrological methods can also be derived from the St. Venant's equations(Price, 1973; Weinmann and Laurenson,1979). The Kalinin-Milyukov method (Kalinin and Milyukov, 1957) is a particular example of such a case. Leaving aside the classification of flood routing method for the present, it can be seen that regardless of the method chosen, however, the estimation of model parameters is almost always accomplished in practice by fitting the predicted flows to noisy observations and consequently on application to actual rivers, the hydraulic method may be no better than the simpler hydrological method (Young,1977). The weight of empirical evidence also tends to substantiate the claim that simple routing methods are adequate for most purposes, particularly in the planning stages of flood control and multi-purpose projects, and in real time forecasting.(Price, 1975 ; Keefer, 1976; Weinmann and Laurenson,1979). Keefer (1976) compared the solutions of various linear hydrological flood routing models with that of St.Venant's solution obtained using implicit method. He found linear hydrological models are more cost effective from the consideration of computer resources and data needs. He found for many practical purposes the application of linear models is justified. In general, the more complex the model, the higher the price in terms of data and computer time. This is particularly true of accurate finite difference models, which may require frequent cross sectional and roughness information along a stream reach. Stability requirements often require tight time steps and thus high computer cost. Finite difference models, for many cases, represent a form of overkill, in that predicted output is required only at one or two points. To get this prediction it may require data from two or several hundred points. Thus for many problems, a simple hydrologic model which treat the stream reach as a lumped system may be of advantage. Such type of hydrologic models are abundant in literature. Muskingum method introduced by McCarthy(1938) and modified by many researchers, lag and route method(Meyer,1941), the diffusion analogy method (Hyami,1951), Kalinin-Milyukov method (Kalinin and Milyukov,1957),the complete linearized model(Harley,1967),multiple linearization from routing method (Keefer and McQuivey,1974),the simple non-linear model (Mein et al.,1974) etc. are some specific examples of hydrological models. The diffusion analogy model can be solved either by lumped linear system approach(Hyami,1951; Harley 1967, Dooge, 1973) or by numerical method approach(Thomas and Wormleaton,1970). The former approach in which the reach being studied as a single unit can be regarded as a hydrological approach and the latter approach where the cross section information is required at short intervals may be regarded as a hydraulic approach. Therefore it may be considered, at this point, that the hydrologic routing methods, in general,treat the channel or river reach as a single unit or a series of single units, and as such information within this unit can not be obtained. In this review note no attempt is made to review the flood routing methods based on empirical methods or that of so called hydraulic methods. Only the literature on hydrologic flood routing methods have been reviewed.

The hydrologic method of routing can be broadly classified as 1) storage routing method,and 2) complete linearized method and its simplifications. Although there are some methods like that of HYMO flood routing method introduced by Williams (1975) which do not fall in the above two categories and yet considered as hydrologic methods. The storage routing method may deal with linear,quasilinear and nonlinear flood routing problems. In linear routing the parameters of the model are kept constant throughout the routing operation. Examples are the conventional Muskingum flood routing method, lag and route method, Kalinin-Milyukov method etc. In quasi-linear routing some or all the parameters of the model change from one time step to another. The variable parameter Muskingum-Cunge method (Ponce and Yevjevich, 1978), the variable lag and route method introduced by Quick and Pipes(1975) are the specific examples of such category. The method proposed by Mein et al. (1974) and Rockwood (1958) form the class of nonlinear storage routing method.

All the above categories of hydrologic flood routing methods have been reviewed in this note with reference to the aspects of mathematical development, data requirement, advantages and limitations and comparison among themselves. The note ends with the remarks for further work to be carried out on the hydrologic flood routing area.

2.0 REVIEW

2.1 Storage Routing Models

All the storage routing models are based on the continuity equation in lumped form which can be written for a channel reach as:

$$I(t) - Q(t) = \frac{dS(t)}{dt} \qquad \ldots (1)$$

where, I(t) and Q(t) are inflow and outflow respectively, and S(t) the storage in the reach under study at time t. Since there are two unknowns viz.Q(t) and S(t) and only one equation, the solution for Q(t) can not be obtained. In order to eliminate one of the unknowns, expression for storage S(t) in terms of I(t) and Q(t) or Q(t) is used. The storage equation may be linear or nonlinear in form. The following are the commonly used forms of storage equations in flood routing:

$$S(t) = K\,Q(t) \qquad \ldots (2)$$

$$S(t) = KQ(t+\tau) \qquad \ldots (3)$$

$$S(t) = K[\theta I(t) + (1-\theta)Q(t)] \qquad \ldots (4)$$

$$S(t) = a_o Q(t) + a_1 \frac{dQ(t)}{dt} + a_2 \frac{d^2Q(t)}{dt^2} \qquad \ldots (5)$$

$$S(t) = a_o Q(t) + a_1 \frac{dQ(t)}{dt} + b_o I(t) \qquad \ldots (6)$$

$$S(t) = a_o Q(t) + a_1 \frac{dQ(t)}{dt} + b_o I(t) + b_1 \frac{dI(t)}{dt} \qquad \ldots (7)$$

$$S(t) = K[Q(t)]^m \qquad \ldots (8)$$

For the sake of brevity the time functions attached with the notations for inflow, outflow and storage would be dropped here afterwards except for the case of equation(3) which represents the storage of the lag and route model. Equation(2) represents the storage of a Single Linear Reservoir(SLR) model proposed by Zoch(1934). Using a series of n-SLRs Nash (1957) conceptualised the catchment behaviour for a unit impulse input and derived the Instantaneous Unit Hydrograph (IUH) for the catchment. Dooge(1973) pointed out the same can also be used for modelling the flood flows in a river reach. Equation(3) forms the basis of the lag and route model proposed by Meyer (1941). It relates the outflow of time $(t + \tau)$ to storage at time t. The term τ represents the response delay time or the time taken for the leading edge of the flood wave to reach the outflow section. Equation(4) forms the basis of the classical Muskingum flood routing method proposed by McCarthy(1938). Equations (5) - (7) were studied by Kulandaiswamy et al.(1967) as particular cases of general linear storage routing model applied to route floods in channels and river reaches. Equation(8) represents the nonlinear relationship between storage and discharge and it has been employed by Rockwood (1958) and Mein et al.(1974) for channel routing. The work based on each of the above equations have been reviewed.

2.1.1 Single linear reservoir(SLR) model and its variations

(a) The SLR model

The storage equation for SLR Model is given as:

$$S = KQ \qquad \ldots(2)$$

combining equation (1) and (2) and solving for Q, for the Dirac-delta input gives the response as:

$$U(o, t) = \frac{1}{K} e^{-t/K} \qquad \ldots(9)$$

where,

u(o, t) = the IUH

K = the storage constant of SLR

The form of the impulse response suggests that it can represent only the attenuation effect of the flood wave.

(b) Nash model

To overcome the deficiency of SLR Model, Harley(1967) and Dooge (1973) suggested the use of n-SLRs in series to partially simulate the translation behaviour of the flood wave in a channel reach. The form of IUH of such a model is given as:

$$u(o,t) = \frac{1}{K\,\Gamma n}\left(\frac{t}{K}\right)^{n-1} e^{-t/K} \qquad \ldots \quad (10)$$

in which,

n = the number of SLRs, in series.

Equation (10) represents Nash model (1957). The parameters n and K can be determined from the inflow and outflow hydrographs using the moments theorem(Nash,1958).

(c) Kalinin-Milyukov model

This model proposed by Kalinin and Milyukov (1957) and widely used in USSR is a physically based n-linear reservoirs model. It is physically based because the length of the reach which can be modeled by a SLR is given in terms of the channel and flow characteristics of the reach. They computed the length of the reach assuming that there is a single value relationship between the downstream discharge and the stage at the middle of the SLR reach. The characteristic unit reach length is given as:

$$\Delta x = \frac{Q_o}{S_o BC} \qquad \ldots (11)$$

in which, Q_o, the reference discharge about which the unsteady behaviour of the flood flow is linearised; So, the bed slope; B, the channel width, and C, the wave celerity. The lag time of the characteristic unit reach length is given as:

$$K = \frac{\Delta x}{C} \qquad \ldots \quad (12)$$

where, Δx is the reach length under consideration. Using the characteristic unit reach length, the number of reservoirs required for routing floods in a given reach can be computed. Miller and Cunge(1975) explain the procedure of this method for use in practice.

It can be seen now, that the black box modelling of flood flow in a river reach using Nash model given by equation (10) has physical basis through Kalinin-Milyukov model. Whereas in the case of Nash model the parameters n and K are estimated by the method of moments(Nash,1958) using the past recorded inflow and outflow information, they are estimated in the case of Kalinin-Milyukov model based on the physical considerations. Thus the Kalinin-Milyukov model enables one to compute the parameters of the model for those floods which are greater in magnitude than the past observed floods. However, both the Nash model and Kalinin-Milyukov model treat the entire flow at a particular section including the flood plain flow,as a single channel flow.

Data requirements:

In the case of Nash model the required data for its application to flood routing problem are only the inflow, outflow and the parameters n and K. But in the case of Kalinin-Milyukov model besides the inflow information, one requires the steady state stage-discharge relationship at the end of the characteristic reach length. The characteristic reach length itself is determined from the channel and flow characteristics, and with the change in channel and flow characteristics from flood to flood the characteristic length also changes. The characteristic reach length determined by equation(11) assumes a rectangular cross section for the entire reach length. However due to natural river being non-rectangular, and due to sediment deposition and erosion problem, the application of such a method is only approximately correct. Nevertheless

Kalinin-Milyukov method is used in USSR as one of the expedient method to solve the flood forecasting problem.

2.1.2 Lag and route models

The simple lag and route model is a two parameter model based on the following input-storage-output relationship:

$$I - Q = \frac{dS}{dt} \qquad \ldots (1)$$

$$S = KQ\,(t + \tau) \qquad \ldots (3)$$

in which, τ is the travel time of the leading edge of the flood wave to arrive at the outlet section and K, the storage constant of the linear reservoir. The concept of lag and route model was based on intuition (Meyer, 1941) and thus it has been considered for a long time as an empirical model(Harley, 1967; Dooge, 1973). This model attempts to duplicate the complex action of a channel by a simple combination of a linear channel and a linear reservoir in series. The impulse response function of this particular system is as follows:

$$u\,(o,t) = \frac{1}{K}\, e^{-(t-\tau)/K} \quad \text{for } t > \tau$$

$$u\,(o,t) = 0 \quad \text{for } t < \tau \qquad \ldots (13)$$

A similar unit response approach for routing through a SLR was reported by Saur (1973). Saur assumes a translation hydrograph for routing through a SLR to arrive at the IUH of the channel reach, similar as in the case of catchment routing using Clark's model(Clark,1945). He states there is no physical reasoning for assuming a translation hydrograph as inflow to the SLR of a channel reach. Having obtained the IUH of the channel reach it is translated to outlet of the reach by the leading edge travel time of the flood wave. Saur varies the leading edge travel time depending on the average antecedent flow existing prior to the flow to be routed and thus, accounting for the non-linearity in translation time. Consequently, the leading edge travel time changes during a routing period as antecedent flow changes. When travel time is shortened as a resualt of this process, stacking of response hydrographs will occur. This results in a steepening of the rising limb. Conversely, when travel time is lengthened, there will be separation between response hydrographs. Quick and Pipes(1975) have proposed a nonlinear lag and route model which is based on the physical characteristics of the channel and flow characteristics. Their model does not exhibit the defect of stacking and separation as experienced in the case Saur's (1973) model. However Keefer et al.(1976) criticised the approaoch of Quick and Pipes as it has been developed based on daily flows only, and the method itself is derived from Hyami's theory (1951) which could have been applied directly incorporating the non-linear behaviour of flood wave. They also showed by example that Quick and Pipes method causes inconsistency in the mass balance of inflow and outflow. Recently Perumal (1984) has given physical justification for the lag and route method based on a consideration which is different from that of Quick and Pipes (1975). He showed that the parameters K and T of equation (13) can be estimated using the channel and flow characteristics. The parameter T has been kept constant and the parameter K can be either constant or varying. Perumal(1984) demonstrated his approach using two hypothetical problems and found that the solution obtained is comparable with that of St.Venant's equations solutions.

Instead of using one SLR in the lag and route method, Harley(1967) proposed a n-multiple SLR in series along with a single linear reservoir. Such a model has the flexibility of applying it to long river reaches time depending on the average antecedent flow existing prior to the flow to be routed and thus, accounting for the non-linearity in translation time. Consequently, the leading edge travel time changes during a routing period as antecedent flow changes. When travel time is shortened as a resualt of this process, stacking of response hydrographs will occur. This results in a steepening of the rising limb. Conversely, when travel time is lengthened, there will be separation between response hydrographs. Quick and Pipes(1975) have proposed a nonlinear lag and route model which is based on the physical characteristics of the channel and flow characteristics. Their model does not exhibit the defect of stacking and separation as experienced in the case Saur's (1973) model. However Keefer et al.(1976) criticised the approaoch of Quick and Pipes as it has been developed based on daily flows only, and the method itself is derived from Hyami's theory (1951) which could have been applied directly incorporating the non-linear behaviour of flood wave. They also showed by example that Quick and Pipes method causes inconsistency in the mass balance of inflow and outflow. Recently Perumal (1984) has given physical justification for the lag and route method based on a consideration which is different from that of Quick and Pipes (1975). He showed that the parameters K and τ of equation (13) can be estimated using the channel and flow characteristics. The parameter τ has been kept constant and the parameter K can be either constant or varying. Perumal(1984) demonstrated his approach using two hypothetical problems and found that the solution obtained is comparable with that of St.Venant's equations solutions.

Instead of using one SLR in the lag and route method, Harley(1967) proposed a n-multiple SLR in series along with a single linear reservoir. Such a model has the flexibility of applying it to long river reaches and its IUH form is given as:

$$u(o,t) = \frac{1}{K\,\Gamma n} \left(\frac{t - \tau}{K}\right)^{n-1} e^{-(t-\tau)/K} \quad \text{for} \quad t > \tau \qquad ...(14)$$

Equation (14) reveals that the model is a combination of n-linear reservoirs in series alongwith a linear channel whose characteristics is to translate the flow to downstream end of the reach without any modification. Equation(14) represents the three parameter Gamma distribution. The parameters n, τ and K of this model can be estimated using moments theorem.

Data requirements:

The method can be used either as empirical or as physically based. If it is empirical as suggested by Dooge (1973) and Harley(1967), then the data required for calibration are the inflow and outflow with no lateral inflow contribution. Using the moments theorem, the parameters τ and K or the parameters n, τ and K are estimated. These parameters are used to find the outflow knowing only inflow. The physically based lag and route model as suggested by Quick and Pipes(1975) requires the inflow hydrograph, the steady state stage-discharge relationship at the upstream gauging station as well as at the downstream gauging station in addition to the channel slope, cross section and roughness information. The lag and route model suggested by Perumal(1984) requires the inflow hydrograph, the steady state stage-discharge relationship at the outlet point and the storage-discharge relationship of the channel reach.

2.1.3 Conventional Muskingum method and its variations

(a) The conventional Muskingum method

The Muskingum method is the widely used **method** for routing floods in rivers and channels. McCarthy(1938) introduced this method as a storage routing method, in connection with studies of the Muskingum Conservancy District Flood Control Project. The method employs the lumped continuity equation given as:

$$I - Q = \frac{dS}{dt} \qquad \ldots(1)$$

and the storage equation given as:

$$S = K [\theta \ I + (1 - \theta) Q] \qquad \ldots(4)$$

in which θ is the weighting parameter and all the other notations being as defined earlier. The conventional or classical Muskingum routing equation is obtained by expressing equation(1) and (4) in the finite difference form and their simplification leads to:

$$Q_2 = C_o I_2 + C_1 I_1 + C_2 Q_1 \qquad \ldots(15)$$

in which, Q_1,Q_2 and I_1,I_2 respectively are the outflow and inflow at the beginning and end of the routing interval, Δ t.

The coefficients C_o, C_1 and C_2 are expressed as:

$$C_o = \frac{-K \ \theta + \Delta t/2}{K(1 - \theta) + \Delta t/2} \qquad \ldots(16)$$

$$C_1 = \frac{K \ \theta + \Delta t/2}{K(1- \theta) + \Delta t/2} \qquad \ldots(17)$$

$$C_2 = \frac{K(1-\theta) - \Delta t/2}{K(1-\theta) + \Delta t/2} \qquad \ldots(18)$$

The sum of C_o, C_1 and C_2 are equal to 1.0. Since I_1, I_2 and Q_1 are known for every time increment, routing is accomplished by solving equation(15) recursively. The Muskingum solution can also be obtained using electronic analogue formulated based on the basic equation(1) and (4). Ramanamurthy (1965) has formulated such an electronic analogue model for the reach between Baramul and Karmudi on the river Mahanadi.

From the numerical solution of the Muskingum flood routing method, it is generally recognised in the standard text books(Viessman et al., 1977) that for some particular values of the Muskingum parameters K and θ (when Δt= K and θ = 0.5) the outflow hydrograph is the same as the inflow hydrograph, but lagged in time by K. Substituting these particular values of K= Δt and θ = 0.5 in equation (16),(17) and (18) leads to the solution of equation (15) as:

$$Q_2' = I_1 \qquad \ldots(19)$$

However this is a misleading conclusion since routing of the inflow hydrograph by these constants does not reproduce the outflow hydrograph adequately. (Hopkins, Jr.,1956; Nash, 1959; Singh and McCann,(1980). The error lies in violating the condition that Δt must be small relative to K. Where this condition is relaxed, as in the case of modified Muskingum method, it can be shown that equation(19) is not obtained.

(b) The modified Muskingum method

Because of the condition that Δt should be small relative to K in the conventional Muskingum method, Nash(1959) linearised the inflow over the time interval Δt and solved the basic equations (1) and (4) using linear system approach. Nash's solution is given as:

$$Q_2 = C_o' I_2 + C_1' I_1 + C_2' Q_1 \qquad \ldots(20)$$

in which

$$C_o' = 1 - (1-c)\, K/\Delta t \qquad \ldots(21)$$

$$C_1' = (1-c)\, K/\Delta t - c \qquad \ldots(22)$$

$$C_2' = c = \exp(\Delta t/K(1-\theta)) \qquad \ldots(23)$$

Here the Δt is restricted only from the consideration of adequately describing the inflow and outflow, and not with reference to K as in the conventional Muskingum method. Substituting K = Δt and θ= 0.5 in equation (20) does not show pure translatory characteristics.

(c) Further studies on the Muskingum method

Kulandaiswamy(1966) studied the translatory characteristics of the Muskingum method and for this purpose he derived the following exact solution of the Muskingum method using the initial condition

$$Q_o = I_o \text{ at } t = 0;$$

$$Q(t) = -\frac{\theta}{1-\theta} I(t) + \frac{e^{-t/k(1-\theta)}}{K(1-\theta)^2} \int_o^t I(t) e^{t/K(1-\theta)} dt$$

$$+ \frac{I_o}{(1-\theta)} e^{-t/K(1-\theta)} \qquad \ldots(24)$$

The same solution was also independently derived by Diskin (1967). Kulandaiswamy pointed out that when $\Delta t = K$ and $\theta = 0.5$, equation(24) does not lead to translatory solution. However he pointed that the Muskingum solution is approximately translatory if the third and higher order derivatives of the inflow is negligible. In his controversial paper Gill(1979a) stressed that the Muskingum solution is purely translatory under the condition laid out by Kulandaiswamy (1966). As a response to this conclusion Singh and McCann(1980) proved unequivocally that the pure translatory solution is a myth. Strupczewski and Kundzewicz (1980), and Kundzewicz and Strupczewski(1982) showed by systems approach that for $\theta = 0.5$, the Muskingum solution is approximatly translatory when compared with the other value of θ. Based on these works, one can conclude that the Muskingum solution does not exhibit the pure translatory characteristics.

One of the disturbing fact of the Muskingum method is the formation of negative outflow in the beginning of the solution. Nash(1959) recognised this defect and recommended the use of lag and route method especially for steep rising rivers, wherein this defect is prominent. However, Nash pointed out that the solution with the negative outflow is mathematically correct.

Venetis (1969) explicitly brought out this defect in the form of IUH of the Muskingum reach as:

$$u(o,t) = -\frac{\theta}{(1-\theta)}\delta(0_{+}) + \frac{1}{K(1-\theta)^2}e^{-t/K(1-\theta)} \quad .....(25)$$

in which, δ is the Dirac-delta function. The development of negative outflow has even led to the suggestion of rejecting the Muskingum method for the field use other than for mere academic interest (Meehan,1978; Meehan and Wiggins,1979). Gill(1979a,1979b) argued that the negative outflow development is due to the adoption of wrong initial conditions and suggested a new set of initial conditions as:

$$Q(\tau) = I(0) \quad \text{when} \quad \tau > 0 \quad ....(26)$$

and quoted that Q is undefined for $t < \tau$. Gill's (1979a) initial conditions as proposed above created a lot of controversies in the literature of Muskingum method, (Singh and McCann,1980; Strupczewski and Kundzewicz,1980,1981) and it was emphasised that the use of initial condition $Q(o) = I(o)$ at $t=0$, for solving the Muskingum equation is correct. Singh and McCann(1980) argued that when Q(t) is undefined for $t<\tau$, then it leads to inconsistency in the mass balance of the Muskingum method and therefore such an initial condition is not suitable. In a reply to the discussion of Strupczewski and Kundzewicz(1980), Gill(1980) changed his previously stated initial condition and modified it as $Q(t) = I_o$ for $t<\tau$.

His argument in favour of the modified initia condition is quite physically based. But he did not explain the impact of that inflow which enters the reach between time o and τ on the outflow. This made the researchers not to heed much attention to his concept. In a recent paper Gill(1981) has convincingly clarified his concept on the modified initial conditions. It is worthwhile to mention here that the concept proposed by Gill(1980) had been employed much earlier by Pitman and Midgley(1966). However, they accounted the travel time of the leading edge of the wave as a time varying parameter. An indirect support to the concept of Gill's modified initial conditions has been given by comer et al.(1981) when they suggested the modified ATTKIN routing model. The basic concept behind the ATTKIN Model is that the proper unsteady flow models describing the passage of flood hydrograph, through a reach must atleast be a combination of both kinematic characteristics for translation and storage characteristics for attenuation. While the full dynamic equations simultaneously account for both effects in a distributed manner, the hydrologic models should account them in a lumped fashion. Without explicitly stating Gill's modified initial condition, recently Perumal(1984) has demonstrated while developing a physically based flood routing method, that the Gill's suggestion for alleviating the defect in the Muskingum method is valid and its use does not lead to violation of mass balance of the Muskingum solution. In this context one should remember that a mathematical model should imitate the physical process for which it is intended and vice versa is not possible in a natural system.

Parameter estimation:

The determination of parameters values is more important to the development of a good routing procedure than is the choice of the model structure. The effectiveness of the Muskingum method depends on the accuracy with which to estimate its parameters. Most hydrology text books(Linsley et al., 1949; Viessman et al., 1977) present the graphical technique for estimating the parameters K and θ using observed inflow and outflow data with no lateral flow. Although the graphical method is generally satisfactory, it certainly is not the most convenient method to work with. Harley(1967) and Dooge(1973) estimated these parameters using the method of moments. Gill (1978) proposed the least squares method to estimate the Muskingum parameters. Stephenson

(1979) proposed a direct optimization method for parameter estimation. Singh and McCann(1980), and Singh and Choudhury(1980) studied five different methods of estimating the parameters of the Muskingum method. These methods include (1) graphical meothod (2) least squaresmethod (3) method of moments,(4) method of cumulants and (5) direct optimisation method. However the inclusion of method of moments and method of cumulants as separate methods of parameter estimation is not correct, as the method of cumulants has to be reduced to method of moments before computing the parameters and thus they would give the same parameter values (Dooge, 1973), Singh and McCann concluded that these methods of parameter estimation are comparable and one method does not have a particular advantage over the other. Laurenson(1959) investigated the parameter estimation of the Muskingum method for long river reaches and found that the linear storage relationship given by equation(4) is no more valid. He suggested that long reach has to be sub-divided into smaller reaches, such that the travel time K of the small reach is less than half the time of rise of the inflow hydrograph.

(d) Range of parameters

The parameter K signifies the average travel time of the flood wave between upstream and downstream section of the reach (Dooge, 1973). The value of K for practical purposes may range between Δt, the time interval used for routing the flood, and half the time of rise of the inflow hydrograph as suggested by Laurenson(1959). The weighting parameter θ ranges between 0 and 0.5. The lower limit of θ corresponds to linear reservoir case and the upper limit corresponds to the non-attenuation of the inflow hydrograph. Using Fourier transform analysis, Strupczewski and Kundzewicz(1980) demonstrated that $\theta > 0.5$ leads to inflow hydrograph amplification which is not a desired result in the flood routing.

(e) The working R and D method

This method uses the same storage discharge relationship as that of Muskingum method, but the parameters K and θ can be varied in time. This method uses the concept of virtual working discharge, which represents a steady flow that would produce storage equal to that produced by the actual inflow I and outflow Q, during the passage of flood. The working value meothod is considered more advantageous than the Muskingum method, if an independent variable such as tributary inflow or controlled discharge through a gated low dam, is involved. The relationship between the storage in a reach under consideration and the corresponding virtual discharge can be established from the field measurements. In deciding the working discharge, the parameter θ plays a crucial role as it accounts for the watersurface slope indirectly. However, it seems, other than U.S.Army Corps of Engineers(1960) no other organisation has been found to use this method for flood routing. No description is given by the U.S.Army Corps of Engineers in deciding the value of θ for the purpose of estimating the virtual dischare versus storage relationship.

(f) Flood stage forecasting

Chander et al.(1975) have used the combination of linear reservoir represented by Muskingum storage equation and linear channel to forecast the flood stages at Delhi bridge on river Yamuna, using the stage information at Kalanaur which is located 195 km.upstream of Delhi bridge. Note the earlier described models based on the Muskingum storage equation employ the discharge as variable, to forecast discharge at the required point. Lack of discharge information but availability of stage information during flood season has led Chander et al.(1975) to develop such a model. They formulated this technique using 'the indirect linear approximation of the different segments of the stage-discharge relationship between upstream and downstream gauging stations. Since stage is not a absolute variable unlike discharge, it is quite difficult to take into account the lateral inflow contribution in modifying the downstream stages.

2.1.4 Muskingum-Cunge method and its variations

The predictive accuracy of the Muskingum method hinges upon the correct determination of the parameters K and θ These parameters are functions of the flood characteristics, i.e.,the hydrograph travel time and attenuation rate, which are usually determined by the various methods described earlier. However, since the flood characteristics are likely to vary from one flood to another, it would be rash to assume that the parameters determined from one set of flood records could be used to predict the behaviour of an altogether different flood. This, in effect, limits the predictive capability of the method to floods similar to that used in the calibration, and any attempt at extrapolation is unwarrented.

The use of constant parameters in the classical Muskingum method is tantamount to an assumption of linearity, and this is in contradiction with the quasilinear property of flood waves. Nevertheless, the classical Muskingum method remains an expedient way of obtaining approximate answers for a number of practical problems.

(a) The Muskingum-Cunge method

The Muskingum-Cunge method (Cunge, 1969) considerably enhances the predictive capability of the Muskingum method, while remaining within the same computational framework. Cunge related the parameters of the method K and θ with the channel and flow characteristics using the analogy between the finite differenceapproximation of the conventional Muskingum difference scheme derived using the Kinematic wave theory concept, and the linear convection diffusion equation or the diffusion analogy model. Cunge's approach essentially converts the conventional Muskingum difference scheme being hydrological in theory into a method based on hydraulic principle. Cunge's relationship between model parameters, and channel and flow characteristics are given as:

$$K = \Delta x / C \qquad \text{...(27)}$$

$$\theta = \frac{1}{2} \left(1 - \frac{Q_o}{S_o B C \Delta x} \right) \qquad \text{...(28)}$$

in which Q_o the reference discharge about which the unsteady flow is linearized; S_o, the bed slope; C, the wave celerity, and Δx the channel reach length.

Muskingum-Cunge method employs equation(15) with the coefficients C_o, C_1, and C_2, given by equations (16), (17) and (18) respectively, being computed using the parameters estimated through equations (27) and (28). Following Cunge's study a number of papers and reports have been added to the literature of Muskingum method, based on the principle of diffusion analogy (Miller and Cunge, 1975; NERC 1975; Koussis,1976,1978,1980,1983;Weinmann, 1977; Ponce and Yevjevich,1978; Ponce et. al.,1978,1979; Ponce,1979;Ponce and Theurer,1982; Weinmann and Laurenson,1979; Cameron,1980; Smith,1980; Jones,1981; Chang et al,1983; Ferrick et al 1984). Ponce and Yevjevich(1979) improved the Muskingum-Cunge method by varying the parameter K and θ in time and space, and showed that the use of variable parameters computed based on an iterative four point approach simulates the flood flow accurately. Cameron(1980) developed a Kalman filter application based Muskingum-Cunge method for real time application to flood forecasting. Cunge's work(1969) and its derivatives basically try to mimick the physical diffusion present in the flood routing using the numerical diffusion. However, researchers try to misuse the same to their convenience. Ponce(1980) attempted to use the numerical diffusion concept to reason out the diffusion present in the n-linear cascade flood routing model whose IUH form is given in equation(10). Strupczewski and Kundzewicz(1981) questioned the necessity of introducing numerical diffusion effect to simulate the diffusion effect of n-linear cascade model as the model itself is a fair approximation of the diffusion analogy.

(b) Muskingum-Koussis method

Koussis advocated (Koussis,1976,1978,1980,1983) an alternate finite difference scheme of Muskingum method, obtained through the space averaged continuity and linear storage weighted discharge equations, after linearising the inflow hydrograph over the routing time interval. He named the scheme as more refined scheme(1978) and related the parameters K and θ with the channel and flow characteristics in a similar manner as Cunge had done for the conventional scheme. The more refined scheme was also derived by Nash(1959). The scheme reads as:

$$Q_2 = C'_o\ I_2 + C'_1\ I_1 + C'_2\ Q_1 \qquad ...(29)$$

The coefficients C'_o, C'_1 and C'_2 remain same as that of equations (21)-(23). The weighting factor of the Koussis Scheme is given as:

$$\theta_k = 1+r/\ln((1+\lambda-r)/(1+\lambda+r) \qquad ...(30)$$

$$,r = \Delta t/K \qquad ...(31)$$

and

$$\lambda = Q_o / (S_o B C \Delta x) \qquad ...(32)$$

However in both the schemes the travel time K is given as:

$$K = \Delta x / C \qquad ...(27)$$

Koussis(1980) noted that the numerical experimentation could not conclusively prove the superiority of his scheme over the Cunge's scheme and noted that more refined scheme, however, is less prone to the well known dip at the beginning of the outflow hydrograph. Perumal(1984) showed by theoretical analysis that both the schemes yield the same coefficients of the routing scheme He reasoned that both the schemes, by appropriate selection of their respective weighting factors afford a second order, approximation of the same physically based convection-diffusion equation.

(c) Weighting parameter θ

Cunge(1969) limits the value of θ to 0.5 at the upper end from the numerical stability point of view and to zero at the lower end. However, Perumal (1984) has shown using discrete impulse response analysis that there is no numerical instability even if $\theta > 0.5$, but points out that the value of θ has to be limited to 0.5 to avoid amplification down the channel. The numerical values of θ_k of the Koussis method can be greater than 0.5 without affecting the stability of the solution and at the same time maintaining the attenuation of the flood wave down the channel(Chang et al; 1983). Note that in contrast to the weighting factor of the conventional scheme (Cunge's scheme), the Koussis θ_k depends on the time step Δt. However, as pointed out by Perumal (1984), the coefficients of both the schemes are exactly the same in terms of the physical and flow characteristics of the channel reach.

(d) Time and spatial resolution

The problem of negative outflow exists, whether one is dealing with the conventional Muskingum method or with that of diffusion analogy based Muskingum method. U.S.Army Corps of Engineers(1960) suggest that the routing time interval Δt should be greater than $2K\theta$ in order to avoid negative value of C_o which causes negative outflow in the beginning. Weinmann and Laurenson (1979) suggest that for practical purposes, these negative outflows are small enough and sufficiently short lived to be ignored if the following inequality is satisfied:

$$\frac{\Delta x}{C} \leqslant \frac{T_r}{2\theta} \qquad ...(33)$$

in which, T_r = period of rise of inflow hydrograph. Chang et al(1983) arrived at the lower limiting condition on Δt based on the Koussis scheme, which eliminates the dip of the outflow hydrograph using the implicit equation:

$$t^* = \Delta t = K(1-\theta) \ln[1 - \Delta t/K)]^{-1} \qquad ...(34)$$

Kundzewicz(1984), and Perumal and Seth(1984) emphasized that by taking a minimum value of Δt, the problem of negative outflow or reduced outflow in the beginning of the outflow hydrograph is not really solved and it is simply skipped.

Ponce and Theurer (1982) have suggested, based on their experience with the diffusion analogy method, that the upper limit of the spatial resolution Δx can be limited to:

$$\Delta x < \left(\frac{q_o \Delta t}{S_o \varepsilon} \right)^{1/2} \qquad \ldots(35)$$

in which q_o being the reference discharge per unit width and $\varepsilon = 0.25$. They found that there is no theoretical limit on Δx. However, Jones(1983) argued that the accuracy criterion proposed by Ponce and Theurer is not correct due to the unusual way the finite difference scheme is constructed to match the convection-diffusion equation in which the amount of diffusion is assumed to be small. He emphasized, therefore, that it is not possible to use consistency or convergence of the finite differencescheme as accuracy criterion in this case. He pointed out based on his earlier studies (Jones,1981) on the Muskingum-Cunge method, that the time resolution should be based on the following criterion:

$$\frac{T_r}{5} > \Delta t > 1.43 \left(\frac{q_o}{2 S_o C} \right) \qquad \ldots (36)$$

and the space resolution being equal to the channel length.

(e) Reference discharge

The reference discharge used in the diffusion analogy model for determining β and θ is based on the representative value of discharge both for the length of reach considered and for the range of discharges encountered. Price (1973) used the average peak discharge of inflow and outflow as a representative value.

(f) Diffusion analogy model corrected for dynamic effects

The Muskingum-Cunge method and the Koussis method described earlier, use the kinematic wave speed parameter C for determining the travel time of flood in a given river reach. This parameter C is a function of depth only as this is viewed through a kinematic wave model and this implies a single valued rating curve. A number of attempts have been made to make these models more general by allowing for a looped rating curve,rather than a single valued rating curve characteristic of the kinematic wave model in determining the travel speed parameter. Koussis(1976) has used "Jones formula" Henderson,1966) to obtain the travel speed of the flood wave based on the loop rating curve. The Jones formula used for establishing loop rating curve at a particular section in a river is given as:

$$Q = Q_n \left(1 + \frac{1}{BC^2 S_o} \frac{\partial Q}{\partial t} \right)^{1/2} \qquad ...(37)$$

in which, Q_n corresponds to the normal flow at a section.

The advantage of this model is that it not only takes care of the variation in C,but the effect of loop rating curve on C is also taken into account in a more rational manner than the method proposed by Ponce and Yevjevich(1978).

Data requirement for the Muskingum based models:

The application of conventional Muskingum method to flood routing problem requires the inflow and outflow information for determining the parameters K and θ . The parameter K and θ can be determined by any of the methods described earlier. The storage within the reach required for graphical or least squares estimation of parameters K and θ can either be obtained from recorded inflow and outflow hydrographs or from the topographic information within the reach. The correct determination of K and θ for a given flood and for the given reach depends on the amount of lateral inflow present in the outflow. Therefore, information regarding the lateral inflow is necessary both in the cases of calibration of the method and in the case of application to a given flood. The required data for the application of the conventional Muskingum method are the inflow,& the parameters K & θ The required data for the application of Cunge method or the Koussis method are the inflow, the average channel cross section for the reach under consideration, the bed slope. So; the channel reach length, Δx; the representative discharge, Q_o and the corresponding wave celerity determined from the steady stage-discharge relationship of the upstream site.

2.1.8 Kulandaiswamy's general storage equation based model

Kulandaiswamy et.al(1967) studied some particular forms of the general storage equation (Kulandaiswamy,1964) for use in the flood routing problem. The particular forms of the storage equations studied are:

$$S = a_o Q \qquad ...(38)$$

$$S = a_o Q + a_1 \frac{dQ}{dt} + a_2 \frac{d^2Q}{dt^2} \qquad ...(39)$$

$$S = a_o Q + a \frac{dQ}{dt} + b_o I \qquad ...(40)$$

$$S = a_o Q + a_1 \frac{dQ}{dt} + b_o I + b_1 \frac{dI}{dt} \qquad ...(41)$$

Each of the above equations was studied in conjunction with the space averaged continuity equation for its suitability to describe the flood movement, using the data of three floods. They found that the form of storage equation given by equation(40) gave satisfactory results., In all the three floods studied, 'a' and 'b' coefficients were determined by the method of least squares. The coefficients a_o, a_1 and b_o have been assumed to be constants for a given flood. The data requirements for the application of this model to flood routing problems remain same as that required in the case of conventional Muskingum method.

2.1.6. Nonlinear storage routing model

This model is based on the space averaged continuity equation given by equation (1) as:

$$I - Q = \frac{dS}{dt} \qquad \ldots(1)$$

and the non-linear storage equation given by equation(8) as:

$$S = KQ^{m} \qquad \ldots(8)$$

Equation(1), written in finite difference form for routing reach of length Δx and a time interval, Δt may be combined with equation(8) to yield:

$$\frac{Q_2}{2} + \frac{KQ_2^{m}}{\Delta t} = \frac{I_2 + I_1 - Q_1}{2} + \frac{KQ_1^{m}}{\Delta t} \qquad \ldots(42)$$

where, the parameters K, m and the terms on the right hand side are known. This equation can be solved for Q_2 by an iterative procedure such as the Newton-Raphson procedure. The details of the model solution procedures and methods of parameter evaluation are described by Laurenson et al.(1975). Rockwood (1958) has also used the same form of the model described by equations (1) and (8), but with a different solution procedure. Napiorkowski and O'Kane (1984) have presented a non-linear lumped conceptual model which is composed of a cascade of equal nonlinear storage elements preceded by an element of pure delay, and this model depends on four parameters only.

2.1.7 Modified Puls method for flood routing through a river reach

The Hydrologic Engineering Center uses in their HEC-1 'flood hydrograph package' (HEC, 1981), the modified Puls method as one of the method for routing floods in river reaches. Routing in river using the modified Puls method is same as reservoir routing, except for the method of developing storage outflow curves. The method consists of repetitive solution of the space averaged continuity equation and is based on the assumption that outflow at the end of the river reach considered is a unique function of storage within that river reach. The storage values are based on water surface profiles determined from: (1) steady flow profiles; (2)observed profiles; (3) normal depth calculations and (4) inflow and outflow hydrographs to make storage calculations. This methods falls under the category of nonlinear storage routing method, as the constructed storage-discharge curve can be expressed as power function. Using the developed storage discharge relationship and the space averaged continuity equation, the following recursive equation required for modified Puls method of routing is obtained.

$$\left(\frac{S_2}{\Delta t} + \frac{Q_2}{2} \right) = \left(\frac{S_1}{\Delta t} + \frac{Q_1}{2} \right) - Q_1 + \frac{(I_1 + I_2)}{2} \qquad \ldots(43)$$

Where, S_1 and S_2 are storages at the beginning and end of the time step, Δt. Note that the storage within the reach is computed based on the assumption of steady flow situation for the given outflow discharge, which does not really hold good for long river reaches during the passage of flood. Therefore in 'HEC-1 flood hydrograph package', the modified Puls method is used generally by dividing the longer reaches into many subreaches.

Data requirements:

The nonlinear storage routing method proposed by Mein et al.(1974) requires the data on storage characteristics of the reach. The parameters F and m which characterise the nonlinear storage within reach can be obtained either by fitting the computed storage arrived using the topographic information of the reach or from the inflow and outflow hydrographs of the past observed floods. Adopting a constant K and m values imply the use of nonlinear storage relationship but still following the principles of steady flow. The application of the method to field problem requires the inflow hydrograph and K and m parameters. The application of modified Puls method requires the inflow hydrograph and the reach oriented storage-discharge relationship.

2.2 Complete Linearized Model and its Simplifications

2.2.1 Complete linearized model

Harley (1967) obtained a general linear solution to the flood routing problem by solving the linearized equation of motion described by the St.Venant's equation, for a semi-infinite uniform open channel subject to a Direc-delta function input at the upstream end. The linearization of St.Venent's equations about a reference discharge q_o, applied to a unit width of channel carrying a unit discharge leads to the following linear form:

$$(gy_o - u_o^2)\ \frac{\partial^2 q}{\partial x} - 2u_o\ \frac{\partial^2 q}{\partial x\ \partial t} - \frac{\partial^2 q}{\partial t^2} = 3gS_o \frac{\partial q}{\partial x} + \frac{2gS_o}{u_o} \frac{\partial q}{\partial t} \quad \ldots(44$$

in which,

q = the disocharge/unit width of the channel

u_o = the velocity corresponding to the reference discharge, q_o

Y_o = the depth corresponding to the reference discharge, q_o

g = acceleration due to gravity

x = the distance from upstream end of the reach.

Harley obtained the following unit response function due to the Dirac-delta function as input at the upstream end of the reach:

$$u(x,t) = e^{-px}\delta\left(t - \frac{x}{C_1}\right) + h\,(x/C_1 - x/C_2)\; e^{sx-rt}\; I\,(2hm)/m$$

where ...(45)

$$C_1 = u_o + \sqrt{gy_o}$$

$$C_2 = u_o - \sqrt{gy_o}$$

$$F = u_o / \sqrt{gy_o}$$

$$p = S_o\,(2-F)/(2Y_o\,(F^2 + F)$$

$$r = S_o\,u_o\,(2 + F^2)/\,2Y_oF^2$$

$$s = S_o / 2y_o$$

$$h = S_o\,u_o\,\sqrt{(4 - F^2)\,(1 - F^2)}/\,4Y_oF^2$$

$$m = \sqrt{(t - x/C_1)(t - x/C_2)}$$

and I( ) is a first order Bessel function of the first kind and δ is the Dirac-delta function.

This model's accuracy is very much dependent on the reference discharge magnitude. The moments of the linear channel response are related with the channel and flow characteristics of the complete linearized solution modoel (Harley, 1967) as:

$$U'_1 = x/C \quad \text{... (46)}$$

$$U_2 = \frac{2}{3}\,(1-F^2/4)(Y_o / S_ox)(x/C)^2 \quad \text{...(47)}$$

$$U_3 = 4/3\,(1-F^2/4)\,\left(1+\frac{F^2}{2}\right)(Y_o/S_ox)^2\,(x/1.5\,u_o)^3 \quad \text{...(48)}$$

in which, U'_1 is the first moment about the origin and U_2 and U_3 are the second and third moments respectively abut the mean; C, the wave celerity obtained using Chezy's coefficient. As this is a three parameter model the first three moments are sufficient to estimate the parameters of the model.

2.2.2 The diffusion analogy model

By neglecting the last two terms on the left hand side, equation(44) can be reduced from hyperbolic to parabolic equation of the following form:

$$\left(gy_o - \frac{u_o^2}{4}\right) \frac{\partial^2 q}{\partial x^2} = 3gS_o \frac{\partial q}{\partial x} + \frac{2gs_o}{u_o} \frac{\partial q}{\partial t} \qquad \ldots(49)$$

Rearrangement of eoquation(48) to the form of convection diffusion equation proposed by Hyami (1951) reduces to:

$$\frac{\partial q}{\partial t} + \frac{3}{2} u_o \frac{\partial q}{\partial x} = \frac{q_o}{2S_o} (1-F^2/4) \frac{\partial^2 q}{\partial x^2} \qquad \ldots(50)$$

The wave celerity is given as:

$$C = 1.5\, u_o \qquad \ldots(51)$$

and the hylraulic diffusivity is given as:

$$D = \frac{q_o}{2S_o} (1-F^2/4) \qquad \ldots(52)$$

When the Froude number is less than one-half, it may be neglected and D is expressed as:

$$D = \frac{q_o}{2S_o} \qquad \ldots(53)$$

Equation (50) with D expressed as in equation(53) may be traced back to Hyami (1951). The linear channel response of equation (50) at distance x, is given for a Dirac-delta input as (Harley, 1967):

$$U(o,t) = \frac{x}{\sqrt{4\pi Dt^3}} \exp\left(-(x-ct)^2/4Dt\right) \qquad \ldots(54)$$

By convoluting the unit discharge at x=o with equation(54), the response of channel at any x and t may be determined. As long as q(t) does not vary greatly from q_o, the reference discharge, or atleast in a range in which C and D do not vary greatly with discharge, equations(54) is a satisfactory routing model.

The relationship between the first two moments of the impulse response function and the channel and flow properties are given as (Harley, 1967):

$$U_1' = x/C \qquad ...(55)$$

$$U_2 = 2Dx/C^3 \qquad ...(56)$$

Using the above two equations, the parameters C and D can be determined from the known input and output. The parameters C and D of the model are related with the channel and flow characteristics as indicated by equations(51) and (52) based on the assumption of wide rectangular channel. While applying this model for flood routing in natural channels, equations (55) and (56) can be used to find the equivalent wide rectangular channel characteristics for the given reach with past flood data. The equivalent parameters arrived in such a manner can be used for routing floods in the same reach of the natural channel for those input which were not used for calibration of the model.

2.2.3 Multiple linearization model

Keefer and McQuivey (1974) realized that the complete linearized model and the diffusion analogy model, linearizes the flow behaviour in a channel about a single discharge which forces all the blocks of discharges of a hydrograph to travel at a single velocity. They argued that the stream channels behave very nearly as linear systems over small ranges of discharge and so they divided a single block of discharge, say over the routing interval Δt, in different ranges, in such a manner that there is a unique unit response for each range of flow. These unit responses are convoluted with the discharges according to the range they belong to, and then these convoluted discharges are summed up to give the outflow hydrograph. Keefer and McQuivey's aproach can be used with any of the methods of generating response functions such as complete linearized model, diffusion analogy model etc. Keefer and McQuivey(1974) evaluated the performance of their multiple linearization model for four river reaches in U.S.using the response functions of the complete linearized model as well as that of diffusion analogy model. They found that, multiple linearization model performs much better than a single linearization model. They also found that complete linearized model proved difficult to apply to actual data when compared with the diffusion analogy model. They noted, although the diffusion analogy model is a particular case of complete linearized model, the mathematical elegance of the complete linearized model is last when converting from instantaneous to hourly or daily response functions.

Eventhough the method proposed by Keefer and McQuivey yield acceptable results from the practical consideration, their approach of dividing a single block of discharge into different ragnes different ranges for the purpose of linearization seems to be artificial. Instead if they had adopted a single response function for a single block of discharge, but varying with different ranges of single discharge, then the nonlinearities would have been taken realistically.

Data requirments:

One of the data requirements other than inflow, stage discharge relationship and channel characteristics, for the application of complete linearized models or its derivatives like diffusion analogy model, multiple linearization model, is the information regarding the reference discharge about which the nonlinear flow phenomena is linearized. As these model parameters are related with the wide rectangular channel flow characteristics, it would be appropriate to calibrate the model parameters in order to establish the equivalent wide rectangular channel properties using the given past inflow and outflow of the natural river. The number of range of discharges about which the multiple linearization is carried out in the Keefer and McQuivey model(1974) is decided by the user's judgement. Adopting too many ranges makes the procedure cumbersome and too less number of ranges affect the accuracy of the model.

2.2.4 Comparison of two parameter models

Dooge(1973) compared the performance of the two parameter models like diffusion analaogy model, Muskingum model, lag and route model, and Kalinin-Milyukov model with reference to the solution of the complete linearized model. He found the diffusion analogy model and the Kalinin-Milyukov model predict discharges which are graphically indistinguishable from the complete linear solution. The lag and route method predicts the travel time to a fair degree of accuracy, but underestimates the degree of attenuation. The Muskingum meothod is seen to predict negative ordinates for a considerable period and a higher peak discharge with a 50% small time to peak. Dooge found that for the short channel lengths the Muskingum method performs as satisfactorily as the other methods and found for long reaches the method fails as noted by Laurenson (1959).

2.2.5 Comparison of three parameter models

The complete linearized model is a three parameter model. If expressed in dimensionless form, the dimensionless discharge can be formulated as a function of dimensionless time parameter, a dimensionless length parameter, and the Froude number. It may appear pointless to attempt to simulate the three parameter complete linear solution by another three parameter which, at best, will be an approximation to it. However, the complete linear solution is complex in form and relatively difficult to compute. If it can be approximated with a sufficient degree of accuracy by another three parameter system which is easier to comprehend and easier to compute , then the simulation may be more convenient than the use of the original mathematical solution.

With the above consideration Dooge(1973) studied the performance of three parameter models like diffusion plus lag, multiple Muskingum method, and three parameter gamma distribution model with that of complete linearized solution. The diffusion plus lag model is a combination of diffusion analogy model and linear channel which simply translates the hydrograph in time. Dooge found that a change in the length of the channel considered does not result in any change in the value of convective velocity, C or the hydraulic diffusivity, D, but the third parameter,the lag, varies in order to maintain the optimum solution and is directly proportional to the length of channel. In the case of three parameter gamma model, the reservoir lag time, K remains constant as in the two parameter Kalinin-Milyukov model, but both the number of reaches,n and lag of the linear channel , τ vary directly with the length to maintain similarity with the complete linear solution. Dooge found that both diffusion plus lag model and three parameter gamma distribution model are well able to simulate the complete linearized solution. In the case of the multiple Muskingum model, the values of K and θ are independent of the reach length and the complete linearised solution is matched by using a number of Muskingum reaches which is proportional to the length.

2.3 Physically Based Hydrologic Flood Routing Models

Many of the hydrologic flood routing models discussed above are semi-empirical in nature. The semi-empiricism arises, because the parameters involved in the mathematical formulation of the flood movement phenomena by various models are estimated from the past flood data. Application of such models for input whose magnitude is greater than the recorded input, which was used for calibration of the model, is not warrented. However, some of the models described above are physically based, in the sense that the parameters of the models are related with the measurable channel and flow characteristics. Such models can be used with confidence, for inflow which had not been recorded in the past or used for calibration of the model. Models like Kalinin-Milyukov method, Muskingum-Cunge method, complete linearized model and diffusion analogy model fall under this category. The Muskingum-Cunge method, although relates the parameters of the model with flow and channel characteristics, their derivation is based on mimicking the numerical diffusion with the physical diffusion. The other three models are based on approximations of the St.Venean's equations which govern the one dimensional flow in channel and river reaches. On the same lines, recently Perumal(1984) has related the parameters of the lag and route model with the channel and flow characteristics.

An entirely different approach has been proposed for routing floods in channels by Williams (1975) which forms a component of the HYMO Watershed model. He has proposed a Variable Travel Time (VTT) flood routing method which takes into account the variation in travel time of the discharge with stage and water surface slope. Williams has shown that by accounting the variation in water surface slope, the VTT method yields results that are comparable to the accurate results obtained with an implicit solution of the unsteady flow equations of continuity and motion. The basic equations employed in this method are the space averaged continuity equation and the travel time equation.

$$T = 2S/(I+Q) \qquad ...(57)$$

in which, S, the storage within the reach and T, the travel time through the reach. By expressing equation(57) in terms of normal flows at both ends of the reach and the corresponding depths, Williams was able to get the solution of outflow using iterative procedure. He found the method is well suitable for inbank as well as flows within flood plain, and demonstrated the applicability of the methoa to channel reaches of Brushy creek in U.S.A.

Data requirements:

The data requirements for the physically based flood routing models except that of HYMO flood routing method were specified earlier. The data required for the HYMO flood routing are the steady stage discharge relationships and the cross sections information at both ends of the reach, and the inflow hydrograph.

2.4 Wave Speed- Discharge Relationships

Most of the physically based models employ either Chezy's or Manning's formula to estimate the wave speed required to determine the average travel time of the flood wave movement from upstream to downstream of the reach under consideration. For example, Kalinin-Milyukov method, Muskingum-Cunge method,diffusion analogy method, and the physically based lag and route method (Perumal,1984) determine the travel time as:

$$K = \Delta x / C \qquad \ldots(12)$$

In general, travel time varies nonlinearly with discharge. Since single valued stage(and therefore storage) versus discharge relations are assumed in most of the hydrologic models, this propagation speed is called the kinematic wave speed. Wave speed-discharge investigations carried out by Wong and Laurenson(1983a, 1983b,1984) on six Australion river reaches, which include in bank as well as flood plain flow, show consistent variations of wave speed with discharge and show that the wave speed first increases rapidly with discharge, then decreases sharply and finally increases at a slower rate. The establishment of such empirical relationships between wave speed and discharges are immensely useful for the application of physically based hydrologic flood routing models to field problems.

2.5 Accounting Lateral Inflow

One of the factors which affects the accuracy of flood forecasting is the magnitude of lateral inflows to the channel. If lateral inflows to the channel are ignored in routing the flows from upstream to downstream of the reach, then the computed flows at downstream point would be smaller than the observed flows. These errors propagate as floods are routed down the stream. Consequently, estimating lateral flows would improve the routing accuracy. Estimation of lateral inflows has not received much attention. Infact the development of different flood routing techniques has been investigated more than the lateral inflow estimation. Nevertheless attempts have been made to estimate the lateral inflow in a meaningful manner. In 'HEC-1 flood hydrograph package' [illegible] the lateral inflow is estimated based on the supplied pattern of lateral inflow hydrograph and the volume difference between outflow and inflow hydrograph of the given reach. The form of pattern hydrograph is usually of the shape of inflow hydrograph or outflow hydrograph depending on where the major lateral inflow contribution is added to the main channel. However during the forecasting stage it is not possible to supply the pattern hydrograph based on the outflow hydrograph, A similar approaoch has been adopted by Ramanamurthy(1965) in modelling some river reaches of Godavari using the Muskingum analog model. To determine the lateral inflow contribution in a more rational manner Mimikov and Rao(1976) have proposed a model in which the lateral inflowe volume of each storm is distributed over time equal to storm duration in a manner similar to the time distribution of rainfall of the storm within the intervening catchment of the reach. They have applied their model for daily flow and daily storm and therefore, the application of such a concept to estimate lateral inflow based on hourly data is questionable due its dynamic nature than that of daily data. Slocum and Dandekar(1975) have estimated the lateral inflow hydrograph by subtracting the outflow hydrograph estimated using different K and θ values of the Muskingum method, from the observed outflow hydrograph. The lateral inflow hydrograph which does not contain even a single negative ordinate has been considered as the correct one. Using Muskingum-Cunge model, the Price (1973) has taken into account the lateral inflow assuming a uniform contribution along the channel reach. However, such assumptions are more valid for the urban storm sewer system rather than to natural river system. In all these approaches the influence of addition of lateral inflow on the wave speed characteristics of the flood has not been considered.

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3.0 REMARKS

The hydrologic approach to flood routing is based on the consideration that in a large number of practical cases, the inertia terms of the equations of motion play an exceedingly small role. Such methods have advantages and disadvantages from the practical consideration. Miller and Cunge (1975) have listed the advantages and disadvantages in detail. Some of them are listed below:

3.1 Advantages

(1) Hydrologic methods may provide answers in much less time than solution procedures based on the complete equations.

(2) The channel geometry need not be described in detail.

(3) Computation cost is less; programming for solution is simpler.

(4) Hydrologic flood routing models probably can be more easily integrated with rainfall-runoff models.

(5) The uses of the results from mathematical modelling often do not require the accuracy provided by the complete model.

3.2 Disadvantages

(1) Hydrologic methods do not have the accuracy of a solution procedure based on the complete equations. Probably a hydrologic routing may give sufficiently accurate results for a particular application, but there is often considerable doubt as to how accurate the results are for any application.

(2) Considerable amount of past data, especially of inflow and outflow is required for reliable estimation of the parameters involved

in the model.

(3) Backwater effect can not be accounted for by hydrologic methods as they are single characteristic passing only the upstream disturbance to downstream.

(4) The impact of lateral inflow on the parameters of the hydrologic models can not be taken into account due to longer length of reach usually considered.

(5) Solutions of simplified equations may lack desired generality.

Although current interest in the field of flood routing seems to be in methods utilizing numerical solutions of the comlete equations of continuity and momentum, hydrologic methods are still useful, and may be preferable in some circumstances. The limitations of each technique must be throughly understood so that an intelligent choice of method to be used may be made.

3.3 Choice of Methods

The final choice of the best method of flood routing ultimately depends on the factors such as accuracy, the type and availability of data, the available computational facilities, the computational costs, the extent of flood wave information desired, and the familiarity of the user with a given model(Fread,1981). Taking for granted that the modeler is well versed with all the techniques and there is no dearth for required data and computational facility, the impartant factors which influence the appropriate flood routing method are:

(1) The objective of the flood routing exercise, and

(2) The nature of available data.

If flood routing is to be done on a real time basis as part of a flood forecasting scheme, then what is required is a simple model in which the flood forecast at the downstream end of the channel reach can be updated at hourly or lesser intervals. If, on the other hand, the flood routing is to be carried out as part of the design of a large scale hydraulic engineering project, then it may be more appropriate to use a more accurate method because of the potential savings may result from the analysis (Dooge, 1980). If more flow data are available than the channel characteristic information like cross sectional area, wetted perimeter, roughless informaotion etc.,then the hydrologic method is best suited. Even when there is less flow information, but more channel characteristic information are available, one can still prefer the hydrologic methods which are physically based, for computing the parameter of the method using the channel characteristics.

Having decided to adopt the hydrologic method one is faced with problem of selecting the best among them. Perhaps the conclusions arrived by Dooge(1980) in this aspect may be worth considering. Based on the comparative study of various two parameter models with reference to complete linearized solution of the St.Venant's equation Dooge(1980) indicating that the diffusion analogy model appear to give better simulation than any other two parameter model studied. Accordingly there seems no reason to prefer either the Muskingum method or the Kalinin-Milyukov method. In a similar manner he also concluded that among the three parameter models studied, the Kalinin-Milyukov method with pure lag added performs better. However, he has restricted his studies to physically based linear models and the performance of the simple non-linear models such as proposed by Mein et al. (1974), Multiple linearized model(Keefer and McQuivey, 1974), HYMO flood routing model, Muskingum-Cunge model(Cunge, 1969) and Muskingum-Koussis model corrected for dynamic effects has not yet been studied with reference to complete linearized solution model. NERC(1975) has recommended the use of Muskingum-Cunge method over the diffusion analogy model as the former can take into account the lateral inflow explicitly. However, it should be remembered that the Dooge's (1980) recommendations on the choice of 2 parameter and 3 parameter models are based on the use of hypothetical data like sinusoidal input applied to a wide rectangular channel reach. He has used the chezy's resistance equation in his studies, and accordingly for wide rectangular channels the wave celerity is of the magnitude if 1.5 V, where V is the average mean velocity in a cross section. Keefer and McQuivey(1974) pointed out while applying their multiple linearization flood routing model to the reaches of Catawba river in U.S.A., that the wave celerity was 4V rather than 1.5 V meant for wide rectangular channels. Therefore Dooge's(1980) conclusions should not be taken as the ultimate rule, but only as a suggestion for a better method to begin with.

3.4 Further Research Needs

Future work on the hydrologic flood routing methods need to be focussed on improving the existing methods to suit the practical needs rather than to attempt to develop new methods. More attention should be paid in studying the physically based flood routing models for their field application, so that the parameters of the models are evaluated from the known characteristics of the channel reach and the flow characteristics. Attempt should be made to find method to vary the parameters in time depending on the flow magnitude and the conveyance of the channel. Initial attempts have been made by Perumal(1984) in this direction to estimate the varying travel time of the lag and route model. Perhaps the sotudies conducted by Wong and Laurenson (1983a,b;

1984) may be of much use in this direction, for modelling the flood flow including the flood plain flow. Although their study is site specific, no doubt, they throw more light on the possibility of adopting same procedure for rivers in any other part of the world. Little attention has been paid for modelling the lateral inflow in a river reach. Although the backwater effect created by the bulk addition of lateral inflow, such as from a tributary to the main channel, can not be modelled by a hydrologic meothod, its impact on the wave speed of the main channel flow can be studied. Such a study would be of immense use for real time forecasting. Studies in the direction of finding the suitability of applying the hydrologic methods to alluvial rivers should be made. This would be useful in real time forecasting of flood s in rivers like Ganga and Yamuna where the channel cross sections get modified, during the progress of flood wave, due to sediment transportation.

The hydrologic flood routing methods are most suitable for use in forecasting in Indian rivers due to the lack of cross sectional information and the gauging stations being located far apart. The magnitude of flood problem in Indian rivers, especially the Himalayan rivers, is increasing day by day due to increased upland activities such as deforestation, changing land use, water resources development works etc. Rather than continuing the use of empirical methods, such as the gauge to gauge relationship, peak to peak travel time relationship for forecasting floods, it is better to adopt simple, but rational methods of flood routing like hydrologic flood routing for flood forecasting. For this purpose the problems of lateral inflow and the sediment transporation have to be studied in detail, as indicated earlier. Literature survey indicates that there are few studies in our country on flood routing problems in river reaches. It is high time a systematic study of flood routing problems in each of the flood prone rivers in our country is carried out using hydrologic flood routing methods for the purpose of forecasting floods.

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