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RESERVOIR CAPACITY COMPUTATION

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ABSTRACT

The present report deals with computation of the storage capacity required in a reservoir. In the beginning definition of a number of terms used in this connection is given. In all three methods which are used for the purpose of computing the required storage capacity of a reservoir are given. These are mass curve method, method based on linear programming and the simulation method. A detailed list of data required for this computation is also given. The recommendations of IS code have been given in appendix which also contains description of the Linear Decision Rule which is a formulation based on linear programming.

1.0 INTRODUCTION

A reservoir is constructed to change the temporal and spatial availability of water of a stream. Since the natural flow in a stream varies in quantity with time and it seldom follows the demand pattern, it is essential to store the water when the availability is more than the requirements and release it from storage when the situation reverses. In the streams where the flows have high variability, this strategy is particularly beneficial as both the deficiency and glut in water availability are harmful. Another added benefit is obtained by generating hydro-electric power using the head developed due to piling up of water in a reservoir.

On a particular stream, a number of sites may be available where the construction of reservoir can take place. Once a particular site has been selected, the next important decision to be taken is about the capacity of the reservoir to be constructed. The aim of this manual is to describe some of the methods which are frequently used for this purpose. However, before going for this description, the terms which are used in this connection are being described.

1.1 The Reservoir System

The terms which are used in connection of a reservoir system are explained here.

1.1.1 Storage terms

The amount of storage available in a reservoir can be divided in several zones. It may be mentioned here that this division is more important from operational point of view. Most commonly the division is among following five sub-zones:

a) Dead Storage Zone

This is the bottom most zone in a reservoir. Generally it is meant to cater for the sediment entering in the reservoir or to provide recreation facilities. Usually all the outlets are located above this zone. The withdrawls from this zone, if any at all, are made only in extreme dry conditions.

b) Buffer Zone

This storage space is located just above the dead storage zone. As the name implies, this zone is a buffer between the conservation and dead storage zones and the releases from this zone are made in dry situations to cater for essential requirements only. Sometimes, dead storage and buffer storage are together called as inactive storage.

c) Conservation Zone

Water is stored in this zone to cater for various conservation requirements like irrigation, water supply and hydropower generation etc. This zone, normally accounts for most of the storage space available in a reservoir.

d) Flood Control Zone or Surcharge Zone

This storage zone is used to moderate floods impinging the reservoir. Depending upon the flood volume and downstream release constraints, water is stored in this zone to attenuate a flood peak.

After the flood peak has passed, this zone is emptied as soon as possible to prepare for subsequent flood events.

e) Spill Zone or Induced Surcharge Storage

This topmost zone is located above the flood control zone and is occupied only in the extreme flood events. Releases from this zone are trade-offs between the downstream flood damage and structural safety of the dam and thus are near the maximum spillway capacity.

All these zone are shown in figure 1. The entire reservoir storage which lies above the inactive storage is called live or active storge.

f) Within-Year Storage

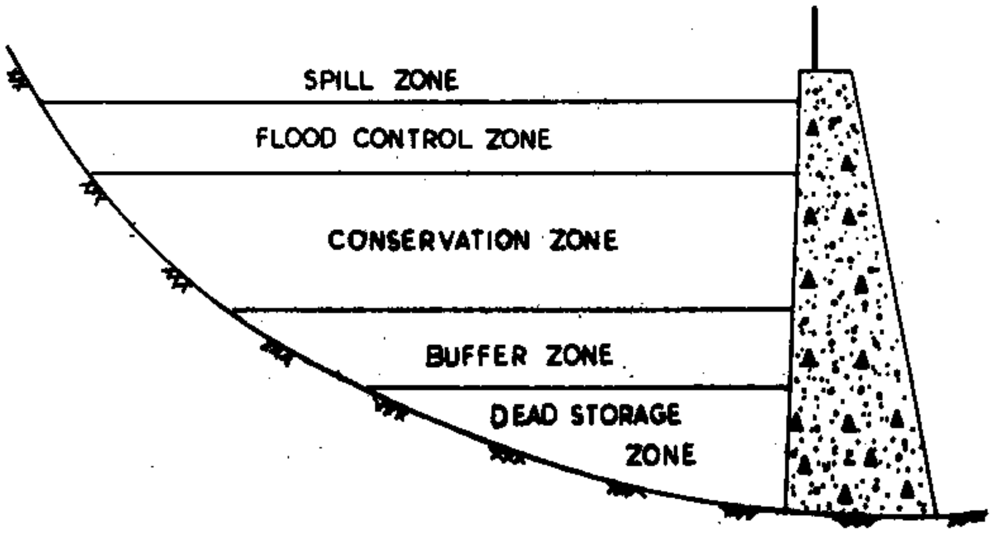
This term is used to denote the storage of a reservoir which is constructed to provide water for a small period say of the order of few months. This type of reservoir may spill and run dry several times in a year.

g) Carryover Storage

In many cases, the entire water stored in a reservoir is not used up in a year and some water is carried over to the next year. This amount is called carry over storage.

h) Conceptual Storages

Three terms related to conceptual storage are particularly useful in computation of reservoir capacity. As defined by McMahon and Mein (1978), a finite storage is the storage of a reservoir which can spill as well as run dry. A semi finite storage is the storage which has no lower bound, i.e., it is a bottomless reservoir which can spill but never run dry. It is contrasted from an infinite storage which has no upper bound - a topless reservoir which can become empty



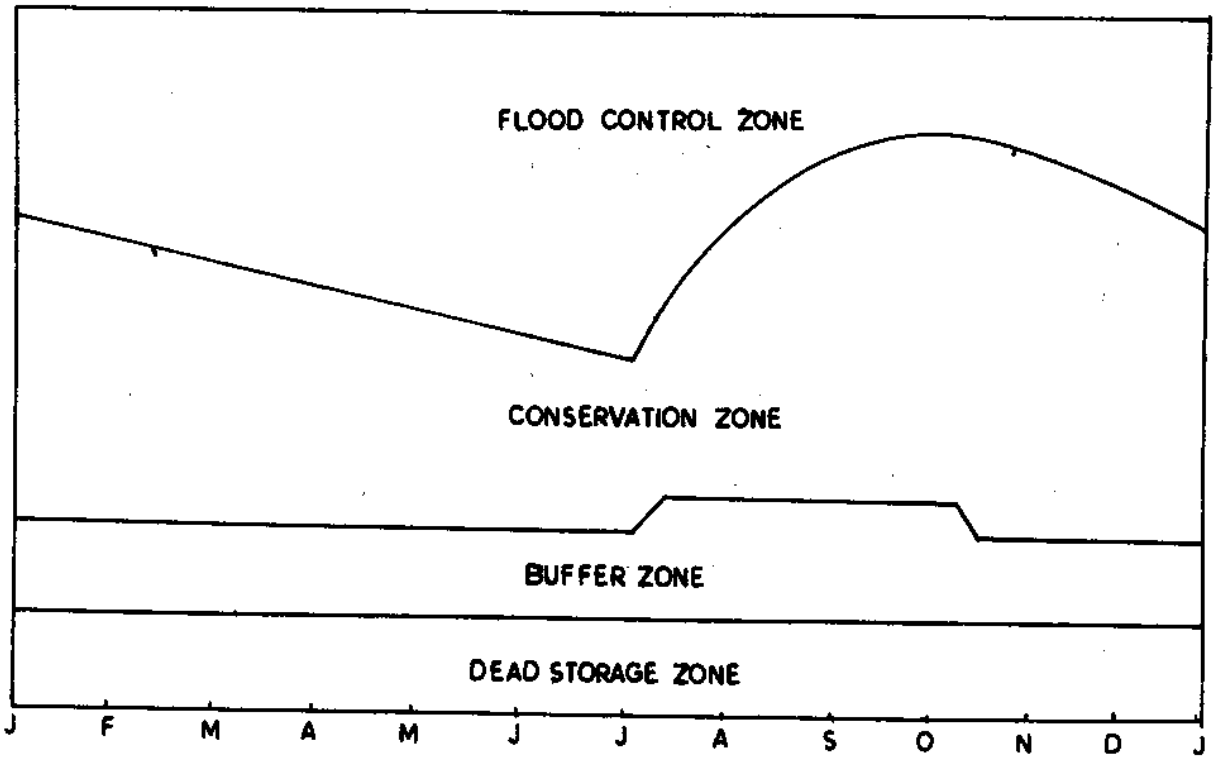


FIGURE 1 - SCHEMATIC REPRESENTATION OF VARIOUS RESERVOIR ZONES

but never spill.

i) Normal Conservation Level (NCL)

This is the highest level of reservoir at which water is intended to be held for various uses, other than flood control. Genérally it refers to the top of conservation zone.

j) Full Reservoir Level (FRL)

This is the highest level of the reservoir at which water is intended to be held for various uses, including part or total of the flood storage without allowing any passage of water through the spillway.

k) Maximum Water Level (MWL)

It is the highest level to which the reservoir water will rise while passing the design flood with the spillway facilities in full operation. This level refers to the top of spill zone.

l) Minimum Draw Down Level (MDDL)

This level corresponds to the reservoir level below which no withdrawl is permissible except by evaportion or seepage losses etc.

1.1.2 Terms connected with design for flood control

a) Maximum Probable Flood (MPF)

It is the flood which may be expected from the most severe combination of critical meteorologic and hydrologic conditions that are reasonably possible in the region and is computed by using the maximum probable storm which is an estimate of the physical upper limit to the storm rainfall over the basin.

b) Standard Project Flood (SPF)

It is the flood that may be expected from the most severe combination of meteorologic and hydrologic conditions considered reasonably characteristic of the region. It is computed from the standard project storm rainfall reasonably capable of occurring over the basin in question and may be taken as the largest storm which has occurred in the region of the basin during the period of weather record.

c) Design Flood

This is the flood adopted for spillway design purposes. It may be the maximum probable flood, standard project flood, or a flood corresponding to some desired frequency of occurrence depending upon the standard of security that should be provided against failure.

1.1.3 Terms related with outflow

a) Release

Releases or draft is the amount of controlled outflow from a reservoir during a given time interval to satisfy the downstream demands.

b) Yield

For the reservoirs serving for irrigation, or water supply, or municipal or industrial areas, the amount of water released for these purpose is called the reservoir yield. For the reservoirs where the stored water is used to generate hydroelectric power, yield is defined as the amount of power delivered during a time interval.

c) Firm Yield

Firm water yield from a reservoir is defined as the maximum quantity of water that can be guaranteed to be delivered 100% of time each year according to some prescribed monthly distribution. Firm power yield of a reservoir can also be described in a similar manner.

1.1.4 Performance terms

a) Reliability

Reliability of a system is described by the probability α that the system is in the satisfactory state. Hence

$$\alpha = \text{Prob}\ \{ x_t \in S \} \qquad \ldots(1)$$

where x_t is the state of the system at time t and S is the domain of admissible states. Further, risk, which is the probability of failure is one minus reliability.

b) Annual Reliability

Defining a failure as occurrence of an outflow ..ss than demand, annual reliability is the probability that no failure will occur within a year. Mathematically

$$R_a = (n-m)/n \qquad \ldots(2)$$

where m is the number of failure years in the total number of n years.

c) Time reliability

Time reliability is the portion of the total operation time during which the demand was fully satisfied.

$$R_t = \frac{1}{T} \sum_{y(\Delta t) \geq q} \Delta t \qquad \ldots(3)$$

where T is the period of operation, q is the target and y is the release.

d) Volume Reliability

This is the actually delivered portion of the total volume of demand during the period T. Hence

$$R_v = 1 - \int_{y<q} (q-y)\ dt \ / \int_0^T q\ dt \qquad ...(4$$

In case of constant target demand, the relation among annual reliability, time reliability and volume reliability is

$$R_a \leq R_t \leq R_v \qquad ...(5)$$

The reason of this behaviour of these indices is that most failure years contain periods of nonfailure operation and that during most failure periods, the release is not completely curtailed.

e) Resiliency

Resiliency describes how quickly the system recovers from failure once the failure has taken place. Hashimoto (1982) defines resiliency as the inverse of expected value of the length of time a system's output remains unsatisfactory after a failure.

f) Vulnerability

Vulnerability refers to the likely magnitude of a failure if it occurs. Here emphasis is placed on how severe the failure is but not on how long it persists.

1.2 Techniques for Reservoir Capacity Computation

Depending upon the type of data and the computational techniques used, reservoir capacity computation procedures are classified into following four categories:

i) Critical period techniques,

ii) Stochastic techniques,

iii) Simulation techniques, and

iv) Optimization techniques.

Among these techniques, those based upon critical period concepts are the earliest techniques. One such method, known as 'Mass Curve Technique' was the first rational method proposed to compute the required storage capacity of a reservoir.

With the advent of computer, the techniques, which beneficially use its computational capabilities are increasingly being used. Among the category optimization techniques, those based on Linear Programming (LP) have been found to be particularly suitable.

For the purpose of this report, among the several available techniques, only three are being discussed here. First, the mass curve technique is being discussed in detail. Mass curve technique is basically a graphical technique. The technique is very simple and very useful also to conceptually understand the idea of storage. Manual computations can be easily performed while using this technique. It can be very easily programmed. The second technique, essentially base on mathematical programming is a computer based technique. With the wider availability of computers, the use of this technique is also growing. Among the available optimization techniques, linear programming is most popular. This technique is applicable when the objective function and constraints are linear in nature. Thus, to consider the evaporation losses and hydroelectric power generation, linearization is necessary which may be conveniently done using the technique of piecewise linearization.

Many times, it is suggested that the results obtained from other methods should be checked using the simulation technique.

In the present report, the mass curve method, the method using linear programming and the simulation technique are being discussed in detail. The recommendations of Indian Standard in connection with reservoir capacity computation are given in appendix 1.

2.0 DATA REQUIREMENTS

Following data is in general needed for determination of required storage capacity of a reservoir.

a) Runoff Data

Sufficiently long runoff data series at the site of the reservoir must be available. If this series is not available from the past records, the same can be developed using precipitation data or using the data at nearby sites.

b) Elevation-Area-Capacity Table

This is constructed by measuring area enclosed between two contours and then multiplying the average area by the contour interval. The map used should be an accurate one and the contour interval should be sufficiently small.

c) Evaporation data at the Site

If it is not available, data for a nearby site which is hydro-meteorologically similar to the proposed site may be considered. Sometimes evaporation losses are neglected in the preliminary computations

d) Time series of demands for the reservoir alongwith target demand values. Target demands, say for a water supply reservoir may be computed by using projected population and water use per capita.

e) Reliability of meeting various demands. By default, 75% dependability is assumed for irrigation, 90% for hydroelectric power and 100% for water supply. These norms are specified by Government of India.

f) Rate of sediment inflow into the reservoir.

g) Height vs cost of the construction of dam.

3.0 METHODS FOR RESERVOIR CAPACITY COMPUTATION

In this section, three methods for computing the required reservoir capacity are being discussed. The first, mass curve technique, is essentially a graphical method based on critical period concept. The second method which uses a popular optimization technique, linear programming ,is a computer based method. The third method is simulation which can also be used to further modify and test the results of first two methods.

The critical period is defined as the duration in which an initially full reservoir depletes and passing through various states (without spilling), empties.In the methods based on critical period concept a sequence of streamflows containing a critical period is passed through an initially full reservoir in presence of specified demands. The reservoir capacity is obtained by finding the maximum difference between cumulative inflows and cumulative releases.

3.1 Mass Curve Technique

If we define a function X(t) as

$$X(t) = \int_{t_o}^{t_f} X(\tau)\ d\tau \qquad \text{...(6)}$$

then the graph of X(t) versus time is called the mass curve. The mass curve technique, proposed by Ripple in 1883 to determine storage capacity of a reservoir, is a graphical integration technique. A similar method was introduced in Europe and was known as 'The Stretched -Thread Rule'. In fact the method originally proposed by Ripple was a residual Mass curve method in which inflow minus long term mean flow is plotted intead of inflow. The idea behind this exercise was to obtain higher accuracy (using graphs).

Let x be the series of inflows to the reservoir and q be the outflow or draft series and Z be defined as

$$Z_t = \int_0^t (x-q)\, d\tau$$

$$= \int_0^t x\, d\tau - \int_0^t q d\tau$$

$$\text{or} \quad Z_t = X_t - Q_t \qquad \ldots(7)$$

The plot of Z_t with respect to time represents storage fluctuations in an unconstrained reservoir subject to inflow x and outlow q. This graph can be used to find the smallest size of the reservoir required to supply draft series Q_t throughout the critical period without failure. Here it is assumed that the reservoir does not spill during the critical period or it is a topless reservoir. It may be mentioned that Ripple asumed reservoir to be empty at the start of critical period. Klemes (1979) pointed out that this assumption is more realistic than the one assumed in one other variation of mass curve technique where the reservoir is assumed full in the beginning. This is particularly true for large reservoirs.

Using the mass curve technique graphically, the computations are performed as given in following steps:

i) First the mass curve of cumulative inflow relative to draft is plotted. A typical of such graphs is shown in figure 2.

ii) Now horizontal lines from the troughs of this mass curve are drawn.

iii) The required reservoir capacity is given by the maximum intercept between the mass curve and the horizontal lines. This will be the size of smallest reservoir which will be able to supply desired draft without failure during the time period under consideration. Sometimes, it is suggested to plot cumulative inflow curve and then plot cumulative draft curve and then measure the intercept after drawing tangent from the mass curve. But by plotting cumulative inflow less draft the procedure is made very simple and variable demands can be easily taken care of.

Many times, it is more convenient to express release as a ratio of mean inflow and this ratio is called the degree of regulation or release ratio. Similarly, the storage capacity can also be expressed as a ratio of mean annual inflow and is called storage ratio or storage coefficient.

The mass curve technique , although very simple and straight forward, has few shortcomings. One drawback pointed out many times is the implicit assumption that the storage which would have been adequate in past will also be adequate in future. Although this is not clearly true, the error caused is not really serious particularly if sufficiently long flow series has been considered. Secondly this problem will arise in any other method since true future is not known.

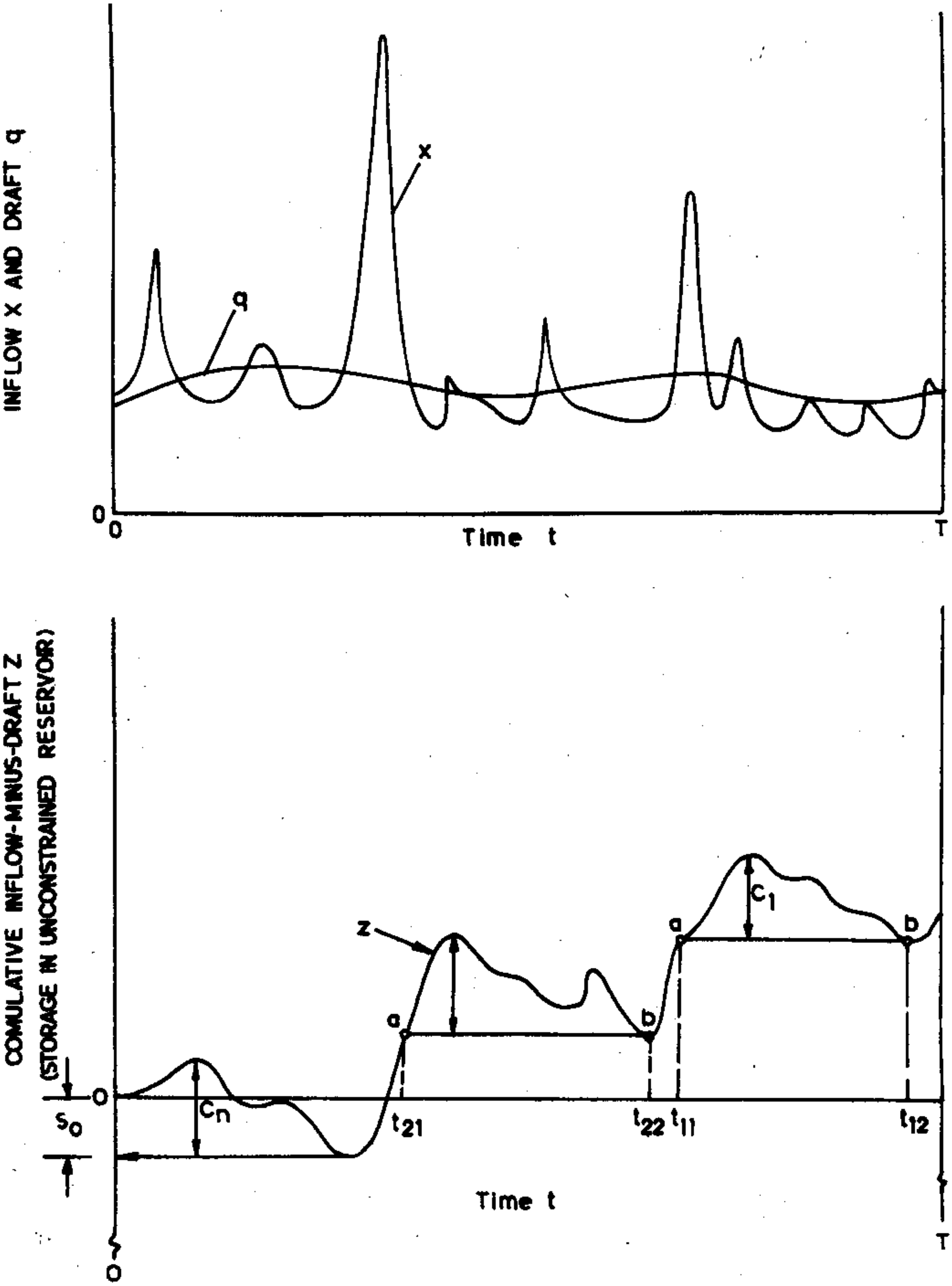


FIGURE 2 - ILLUSTRATION OF MASS CURVE TECHNIQUE

Some methods try to address this problem by explicitly considering the stochasticity of the inflows.

Another criticism which is in general true for all critical period techniques is the circularity of the definition of critical period. By definition, the critical periods depends upon the operating policy of the reservoir upto that period, its capacity and demand level. These are essentially the factors which our analysis aims to find out.

One more drawback of the mass curve is that explicit economic analysis can not be done in this technique. The storage size can not be related to the economic life of the project. Further, it can not be computed for a particular level of reliability.

According to Klemes(1978),the streamflow regulation problems have two physical objectives:

a) Regulation based on a firm value of target release which is either constant or periodic function of time, and

b) Regulation aimed at the greatest possible equalization of reservoir outflow.

The second aim is what is known as maximum economic effect.He showed that the currently applied LP and DP formulations of the performance optimization problem for a single reservoir lead towards above objectives only. It was proved by him that this can be better carried out by using mass curve technique.The computation time required is significantly less and the accuracy far better.He concluded that for the important special case of convex loss function,both the dynamic and linear programming formulations of optimum reservoir operation as developed over the past decade still have a long way to go to match mass curve technique in terms of exactness, accuracy,as well as computational efficiency.

Klemes(1981) derived a general equation relating reservoir storage, reliability and releases. This has been called ' Regime

Function '. It has been shown that the mass curve is a special case of regime function when the reliability of the system is unity.

A typical plot of regime function is shown in figure 3.

Let there be a storage reservoir of capacity C with target release q and some reliability characteristic R. The variables are related using the regime function as

$$\phi = \phi\ (C,q,R) \quad \text{...(8)}$$

Any two of the three variables can be chosen to form appropriate functional form with the third variable.

$$C = C(q,R) \quad \text{...(9a)}$$

$$q = q(C,R) \quad \text{...(9b)}$$

$$R = R(C,q) \quad \text{...(9c)}$$

$$C > 0$$

$$q > 0$$

$$0 < R < 100\%$$

The stochastic storage theory can be used to directly solve equation (9c). Equations (9a) and (9b) can be solved only for the special case of a finite deterministic streamflow series and R=100% in the mass curve method.

The regime function can be solved for inflow series of any detail, i.e.,annual, monthly, weekly etc. The required storage capacity will, however, generally decrease with increase in time interval between the successive values, keeping R and q constant. This is because of greater smoothing of the series when the averaging is performed over larger intervals. The elimination of peaks is obtained by storage action and the average operation eliminates fluctuations

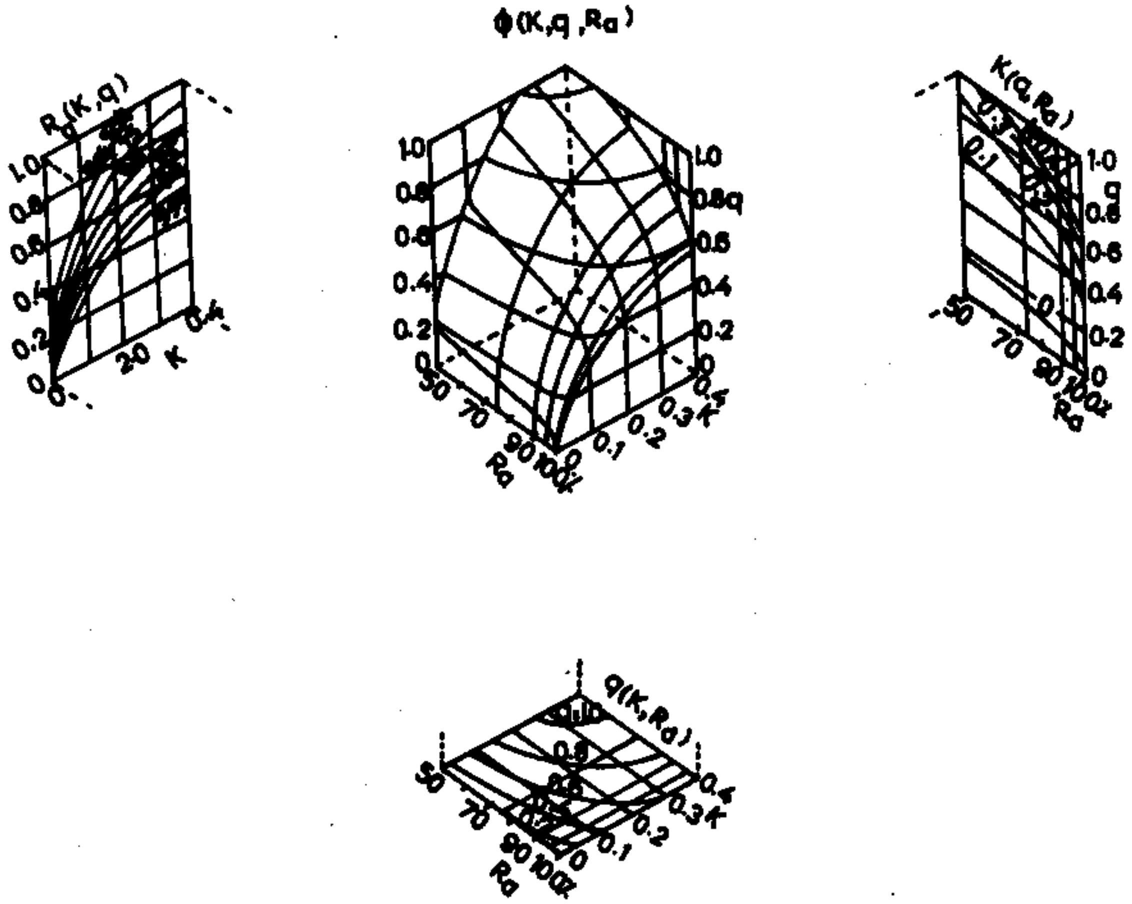


FIGURE 3 - REGULATION REGIME FUNCTION(AFTER KLEMES (1981))

within that period. In practice the storage capacity based on the mean annual flows is called long term storage. The difference between the long term storage capacity and the storage obtained using sub-annual flows is called seasonal or within year storage.

3.2 Fixing the size of dead storage

Dead storage is provided in a reservoir to serve two purposes. The river, during its course to the reservoir, picks up sizeable amount of sediment and carries it along either as suspended load or bed load. Upon entering a reservoir, the velocity of flow becomes virtually zero and hence its carrying capacity is lost. So the sediment settles down and it keeps on accumulating as the time passes on. On account of this accumulation,the effective storage capacity of the reservoir and hence its reliability goes on reducing with time.

Many times, the water released from the reservoir is passed through turbines of power plants located downstream of the dam to generate hydroelectric power. For efficient working of turbines, it is necessary that head variation must be within a specified range and a minimum head must always be available.

These two considerations necessitate the provision of dead storage in the reservoir.

To compute the amount of sediment inflow expected in the reservoir, average sediment yield of the catchment is determined. This is then used to compute total sediment load expected during the economic life of the project. The storage actually provided is governed by the greater of the two factors discussed above.

3.3. Storage requirement for flood control

The requirement of storage for flood control is in conflict with the requirements for conservation needs. The conservation requirements, like water supply and hydropower generation require the storage space to be full while the flood control aspect requires the availability of empty storage space.

From the point of view of analysis, the demands for water supply and hydroelectric power are relatively deterministic in nature while the demand for flood control storage is completely stochastic. Further, the time period for analysis is usually of the order of one month for the conservation purposes while for flood control purposes, it is of the order of few hours.

There are several ways in which this storage requirement can be computed. For example, it can be done in optimization framework involving optimal operation of reservoir or in a simulation framework by studying the consequence of adopting a particular combination of variables involved and then modifying them to suit the needs.

Alternatively, the storage requirement can be estimated by using the design flood hydrograph. An initial reservoir level is assumed at which this flood hydrograph impinges the reservoir. The maximum level attained by the reservoir is computed by routing the hydrograph through the reservoir. The maximum height of the dam is obtained after adding the free board to this level. To begin with, top of conservation pool is a good choice for initial storage for computations. But in case it is required to have the reservoir full after the flood season is over, the water level in the reservoir is likely to be above the conservation pool in the later part of flood season and choice of a higher initial storage for computation will be more appropriate.

One important variable in this analysis is spillway-its length, crest level and whether it is gated or not. The reservoirs which cater for more than one purpose almost always have gated spillways. The first two factors determine the flow that can be passed through the spillway. In case the spillway crest is at low elevation, the reservoir release can be controlled from the early stages of arrival of flood(when the water level is low). This gives rise to additional flexibiliy in the flood control operation of the reservoir.

Economic analysis of benefit from the moderation of flood and the cost of providing additional storage space to achieve higher protection is also quite essential. The upper bound of storage will be for a capacity where the marginal benefit equals marginal cost, provided it is physically feasible to have this must storage.

3.4 Optimization techniques for reservoir capacity computation

The advent of computer and the development of optimization techniques has led to the use of both of these to solve the problems of reservoir capacity computation. Amongst various available optimization techniques, Linear Programming(LP) and Dynamic Programming(DP) are two techniques which have been used extensively. Here, only a LP based formulation is being discussed. Before this, the theory of linear programming is being discussed very briefly. The problem formulation is essentially same in case of dynamic programming.

3.4.1 Linear programming

Linear programming (LP) is a technique to solve an optimization problem in which the objective function and constraints are linear in nature. The procedure that is used for this purpose is called the Simplex Method. In this method,first of all,a solution of the given problem is obtained without worrying about its optimality,i.e.,it may be optimal or it may not be optimal. Only restriction is that this solution must be feasible,i.e.it should not violate any constraint. This solution is called basic feasible solution. The process of obtaining a feasible solution of the problem is called the First Phase of Simplex method. If,as the result of first phase, a feasible solution to the problem can not be obtained then the problem is termed infeasible and the computations are terminated.

If a feasible solution is obtained after the first phase then the second phase of Simplex Method is taken up. This phase starts with the feasible solution obtained in the first phase. The aim in this phase is to achieve maximum possible improvement to the solution of the first phase. The solution obtained in the first phase is tested for optimality. If optimality condition is satisfied the procedure is terminated after printing out the results. If optimality condition is not satisfied then a new basic solution is found which is also feasible and at the same time, leads to improvement in objective function. In a big problem , there may be a large number of possible basic feasible solutions and the computation of all of them and testing for optimal among them may not be computationally feasible. In the Simplex method, systematic search is made in the feasible domain. In case of LP, the optimal solution must lie on a corner point of the feasible region and hence Simplex moves from one corner point to other until an optimal solution is found.

Nowadays standard LP packages are widely available on major computer systems. Typical input to these programs consits of number of variables, number of constraints, coefficients in objective function and constraints etc. Output from the program includes values of variables in basic solution and optimal value of objective function. Further computational details such as intermediate results etc.can be printed out depending upon requirements.

3.5 Determination of Reservoir Capacity Using LP

Let us consider a situation in which a reservoir is to be constructed at a particular site. Monthly inflow data for past n months is available. The projected demand of water during a critical year is known alongwith its fractions for each month. The losses from the reservoir are neglected for the time being. The problem is to find out the minimum capacity of reservoir which will supply the required quantity of water without failure. Let X be the annual water demand from the reservoir & α_i, i=1,2,...12 be its fractions for different months. Hence the demand in a particular month will be α_i X. Let I_i be the inflow to the reservoir during the i^{th} month and R_i be the water actually released from the reservoir.

Representing by S_i the storage content of the reservoir at the beginning of month i, the continuity equation can be written as:

$$S_i + I_i - R_i = S_{i+1} \qquad ...(10)$$

This equation has to be satisfied for each of the n months and hence we shall have n such equations which will be constraints in the formulation. The value of S_1 is given as input.

It is also required that the amount of water actually released from the reservoir must be more than or equal to the amount demanded. This can be mathematically expressed as

$$R_i \geqslant \alpha_i X \qquad ...(11)$$

Since this condition also must hold for each month, there will be n such constraints also.

If the capacity of the required reservoir is C then in any month, from physical point-of-view, the storage content of the reservoir must be equal to or less than this value. Hence

$$C \geq S_i \qquad i=1,\ldots\ldots n \qquad \ldots(12)$$

$$\text{or} \quad C - S_i \geq 0 \; ; \; i=1,2,\ldots\ldots n \qquad \ldots(13)$$

Moreover, the storage S_i capacity C and release R_i can take only positive values.

Now the entire problem of minimizing the required capacity can be expressed in the linear programming framework as:

Min C

Subject to

$$R_i \geq \alpha_i \; X \quad i = 1,\ldots\ldots n$$

$$S_i + I_i - R_i = S_{i+1} \; i=1\ldots\ldots n$$

$$C - S_i \geq 0 \quad i=1,\ldots\ldots n \qquad \ldots(14)$$

$$C, S_i, R_i \geq 0 \quad i=1,\ldots\ldots n$$

The above problem is quite easy to solve particularly due to availability of standard package programs. If the number of months for which the data is available is 240(20 years) then there will be 720 constraints in total. Though the computational time requirement for a problem of this size will be well within the acceptable limits, preparation of data for a problem of this size is quite tedious. Moreover, a sample of 20 years is not long enough to carry out such an analysis for which it is preferred to have bigger sample size, say 30-40 years or so. In that case, the problem will be really very big in size. The special structure of the problem can be advantageously used to apply the decomposition principle.

Several ways have been suggested to reduce the problem size.

One of them is to select a critical year and then carry out the analysis for that year only. But this is not very correct as the effect of carryover storage is not considered in this variation.

Another technique uses stochastic programming concepts. In this technique, instead of using the inflow values directly, appropriate probability distributions are fitted to inflow series for each month separately. Now a particular reliability level is chosen and that particular value of inflow is adopted which is exceeded with this particular reliability. Thus the continuity equation is rewritten as follows

$$S_i + I_i\ (P) - R_i = S_{i+1} \qquad ...(15)$$

Where $I_i(P)$ is the value of inflow which is exceeded in month i with probability P. There will be twelve such constraints, one for each month. Thus the complete formulation can now be presented as:

$$\text{Min } C$$

Subject to

$$R_i \geqslant \alpha_i X \qquad i= 1,2......12$$

$$S_i + I_i(P) - R_i = S_{i+1} \qquad i=1,.....12 \qquad ...(16)$$

$$C - S_i \geqslant 0 \qquad i=1,2....12$$

$$C, S_i, R_i \geqslant 0 \qquad i=1,2,...12$$

In the above formulation, the number of constraints has again been reduced to 48 and the stochasticity of the inflows has been explicitly considered. Further, one cay vary the level of reliability and see its effects on the reservoir capacity. However, there is one drawback in the above formulation. Implicitly it has been assumed that all the inflows are perfectly correlated since the constraints are interactive. Or, in other words, it has been assumed that inflow with probability P will occur in each month of the critical year. In practice this is not true and this assumption leads to conservative results. However, there is lot of controversy about this aspect and some investigators report that the error is negligible.

As pointed out earlier, the computation of evaporation losses and hydrological power generation poses a small problem in linear programming formulation. This is because of highly non-linear nature of elevation-area-capacity curve. For rough and preliminary computations this curve may be assumed linear but this will give rise to unwanted distortions which may be quite significant depending upon the nonlinearity of elevation-area-capacity curve. The refined way of tackling this problem is the piecewise linearization technique. In this technique this curve is subdivided into a number of segments such that the curve can be reasonably approximated by a line in a particular segment. Naturally,more is the number of such segments, higher will be the accuracy of the results. Thus the correct choice is made depending upon the degree of accuracy required and the availability of the computing facilities.

Apart from the simple formulation given above, the linear programming has been widely used in the Linear Decision Rule(LDR) frameworks. This has been described in appendix 2.

3.6 The method of Simulation

Simulation is essentially a search procedure. It is one of the most widely used techniques to solve a large variety of problems associated with the design and operation of a water resources systems. The reason is that this approach can be realistically and conveniently used to examine and evaluate the performance of a set of alternative options available.

The response or behaviour of a particular alternative to a given set of inputs can be studied by developing a model of that alternative. This model is subjected to inputs and its performance is monitored. The models can be of several types like physical models, mathematical models etc. It can be very easily appreciated that physical modelling is not a very good alternative for studying a water resources system. With the development and wide availability of digital computers, mathematical modelling has become very attractive for this purpose. A mathematical model, as developed for a digital computer consists of a series of mathematical instructions and relations which represent the inherent characteristics, behaviour and response of a particular system. This sequence of instructions, when carried out using the input data, simulates the behaviour of the prototype.

The attractiveness of the simulation model lies in the fact that using them, it is very easy to answer the question: what if. These models can be easily developed and used. Their output is very easy to understand. The simulation models can be simple as well as realistic.

A simulation programme can be easily developed and used to determine the minimum storage capacity of a reservoir which is required to satisfy given demands. How this can be done is explained here with the help of a simple example.

Let there be a given site identified for the construction of a reservoir. The reservoir has to cater for irrigation for a nearby area and the target demand of water for different months is given. The elevation-area-capacity table for the site is available. A sufficiently long series of streamflows at the site is available. Further it is required that the reliability of the reservoir should be at least 75 percent. An efficient procedure of binary search can be used in the present case. In this method, first the upper bound and lower bound on the capacity of the reservoir are determined.The lower bound can be taken to be zero or the dead storage and the upper bound can be determined from physical factors such as water availability etc. A trial value for the reservoir capacity is selected which lies at the center of the feasible region(i.e.at the mean of upper bound and lower bound). Now,starting with a suitable value of initial storage content, the reservoir is operated using the streamflow data. The effect of this initial storage value will not be very significant if the inflow series for a sufficiently long period, say 30-40 years is being used. During any time period, the release is made equal to the demand if that much water is available in the storage. Otherwise whatever can be made available is released and the reservoir is said to have failed in that period. The evaporation losses can be easily considered here if the information about the depth of evaporation is available. In this way, the reservoir is operated for the entire period of record. Now the number of periods during which the reservoir has failed is counted and the reliability is computed. If this reliability is less than the desired value, it means that the capacity of the reservoir must be increased. In this case the present capacity is adopted as the lower bound for next iteration. The feasible region below this lower bound is discarded and the trial value for the next iteration is chosen midway the upper bound and new lower bound. If,however, the reliability comes out to be higher than the required limit, the size of the reservoir is bigger than what it should have been and hence the region between the current value and upper bound is discarded for further examination. The present capacity value becomes the new upper bound. Again the trial value for the next iteration is chosen as mean of new upper bound and old lower bound.

The computations are repeatedly performed in this manner and they are terminated when the required number of iterations is over. This method converges quite rapidly as the feasible region is halved every time. It may be seen that in this method, generation of hydro-electric power can also be easily considered.

4.0 CONCLUSION

Three methods of reservoir capacity computation have been presented in this manual. The first, mass curve method is a graphical technique. This is useful when hand computations are to be performed. The second technique is based on linear programming and is advocated when a computer is being used to perform computations. As the standard linear programming packages are easily and widely available nowadays, this technique becomes quite handy. The user has just to prepare the input data.

The third technique is the method of simulation which can form the alternative to above two techniques. Many times, it is insisted that the results obtained from the other methods should always be tested by simulation.

APPENDIX -1

A.1.1 IS Guidelines for Fixing Reservoir Capacity

The determination of reservoir capacity has been divided in three aspects according to the recommendations of Indian Standard IS: 5477-1969. These are being described here.

A.1.2 Dead Storage

This storage space is provided to cater for the sediment entering into the reservoir along with the streamflow. As the water is sufficiently still in the reservoir, the carrying capacity is lost and the sediment settles down. By earmarking a zone as dead storage, it is ensured that the live storage will work at the full efficiency and also a minimum required head will be available for power plants.

For fixing dead storage,it is very essential to determine sediment yield of the catchment. This can be done by either sedimentation surveys of the reservoir with similar catchment characteristics or by sediment load measurements of the stream. In the first method, the sediment yield is determined by measuring the accumulated sediment in a reservoir for a known period, by using precise measuring devices. The difference between the present reservoir capacity and the capacity just after completion of construction gives the sediment yield. In the second method suspended load and bed load measurements are taken along with the discharge measurements and the yield is determined using them.

IS:5477(part II)-1969 recommends following methods for determination of sediment distribution in a reservoir for design purposes:

a) Empirical area reduction method .

b) Area increment method,

c) Moody's method to find new zero elevation.

A.1.3 Live Storage

Live storage is provided in a reservoir to store excess water during high flows for use during low flows. The live storage is the useful storage between the full reservoir level and the minimum draw down level(in case of power projects) and dead storage(in case of irrigation projects).

According to IS:5477(Part III)-1969, the following data should be used to fix live storage capacity :

a) Streamflow data for a sufficiently long period ,

b) Losses, such as evaporation and seepage and recharge(during the depletion),

c) The contemplated irrigation, power or water supply demand, and

d) Storage-capacity curve at the site.

In case streamflow records are not available at the required site,these at the nearby sites must be used to generate the data for the reservoir site. Similarly, short records must be suitably extended.

Evaporation losses are computed using depth of evaporation multiplied by average water spread area. Unless adequate data are available, no allowance be made for seepage and recharge.

The storage provided for irrigation projects must be able to supply for the demands with 75% dependability; 90% dependability for hydroelectric power and 100% dependability for water supply has been suggested. It has been recommended that mass curve technique should be used to determine required storage capacity.

A.1.4

One of the important aspects to be considered in the design of a dam is safety consideration during the extreme floods. The Indian Standards recommend the assumption that the reservoir would be filled to the full reservoir level at the beginning of spillway design flood. This assumption is made to consider improper operation of regulation mechanisms as a result of incorrect flood predictions and mechanical problems in their operation. The methods suggested for estimation of the design flood are broadly classified into:

a) Application of suitable factor of safety to maximum observed flood or maximum historical flood,

b) Empirical flood formulae,

c) Envelope curves,

d) Frequency Analysis, and

e) Rational method of derivation of design flood from storm studies and application of unit hydrograph principle.

The maximum water level of reservoir is obtained by routing the design flood through the reservoir and spillway. It has been recommended to use continuity equation or Sorensen's Method. Step-by-step method for doing the computations has been discussed in IS:5477(Part IV)-1971.

The standard also recommends that the following governing factors should be considered while determining the storage capacity of a reservoir.

a) Long-term precipitation records for the catchment,

b) Long term runoff data at or near the reservoir site,

c) Sediment yield into the reservoir from the catchment, and

d) Area and capacity curves.

APPENDIX 2

A.2.1 The Linear Decision Rule

Linear Decision Rule(LDR), in which the release from a reservoir is expressed as a linear function of storage, was originally proposed by Re Velle et al(1969) in following form

$$X_t = S_{t-1} - b_t \qquad \ldots(17)$$

Where X_t is the release during period t, S_{t-1} is the storage at the end of the period (t-1) and b_t is a decision parameter chosen to optimize some performance criterion. After its introduction, this rule was criticised by some investigators and extended and modified by others. In the above equation the release is completely dependent upon the previous end of period storage. A general form of the above rule was proposed by Re Velle and Gundelach(1975) in which the release was expressed as a function of inflows during previous periods. The modified form expressed by them was:

$$X_t = S_{t-1} + \alpha_t R_t - \beta_{t-1} R_{t-1} - \gamma r_{t-2} R_{t-2} \ldots\ldots + b_t \qquad \ldots(18)$$

Here R_t is the inflow during period t, α_t β_{t-1} and γ_{t-2} etc. are the coefficients to be determined. Thus the release increases with increase in value of inflows in the present period and decreases with increase in the inflows in the previous periods. Further when α_t, β_{t-1} and γ_{t-2} etc.are zero, the rule is analogous to the original rule (equation 17).

Now a model can be constructed to solve the problem of reservoir capacity computation. As the objective function and constraints are linear functions, any standard LP package can be used to solve the problem. The problem can be presented in a deterministic as well as chance constrained formulation. It is assumed here that we are dealing with monthly inflows.

A.2.2 Deterministic LDR

Here, it is required to find the minimum value of reservoir capacity which can meet the specified performance targets and give twelve LDR parameters, one for each month. The objective function is

$$\text{Min } C \qquad \ldots(19)$$

where C is the reservoir capacity to be determined. This objective function is subjected to the following constraints.

For flood control operations, it is required that a specified amount of empty storage space is available in the reservoir. Depending upon the flood moderation envisaged, this requirement will vary from month to month. Hence

$$C - S_t \geq V_t \qquad t=1,2,\ldots n \qquad \ldots(20)$$

where V_t is the free board required at the end of t^{th} month.

If m is the number of years for which data is available then n=12m . Writing continuity equation for period(t-1)·

$$S_{t-1} = S_{t-2} + R_{t-1} - X_{t-1} \qquad \ldots(21)$$

Substituting X_{t-1} from equation(17) for period (t-1)

$$S_{t-1} = (1-\alpha_{t-1})\, R_{t-1} + \beta_{t-2}\, R_{t-2} + \gamma_{t-3}\, R_{t-3} + \ldots b_{t-1} \qquad \ldots(22)$$

or

$$S_{t-1} = J_{t-1} - b_{t-1} \qquad \ldots(23)$$

where $J_{t-1} = (1-\alpha_{t-1})\, R_{t-1} + \beta_{t-2}\, R_{t-2} + \gamma_{t-3}\, R_{t-3}$ ...(24)

substituting from eq.(22) into eq.(18)

$$X_t = \alpha_t\, R_t + (1-\alpha_{t-1} - \beta_{t-1})\, R_{t-1} + (\beta_{t-2} - \gamma_{t-2}) R_{t-2} \qquad \ldots(25)$$

$$+ \ldots\ldots - b_{t-1} + b_t$$

or

$$X_t = K_t - b_{t-1} + b_t \qquad \ldots(26)$$

where $K_t = \alpha_t R_t + (1-\alpha_{t-1} - \beta_{t-1}) R_{t-1} + (\beta_{t-2} - \gamma_{t-2}) R_{t-2} + \ldots$ ...(27)

Now using equation(24), eq.(20) can be written as

$$C - J_t + b_t \geq V_t \qquad t=1.....n \qquad ...(28)$$

or $$C+b_t \geq J_t+V_t \qquad t=1.....n$$

The next constraint specifies that the storage at the end of any period t must be greater than the minimum(dead) storage C_{min}:

$$S_t \geq C_{min} \qquad t=1.....n$$

or $$J_t-b_t \geq C_{min} \qquad t=1....n$$

or $$b_t \leq J_t-C_{min} \qquad t=1.....n \qquad ...(29)$$

Further, the release in any period must be greater than a minimum mandatory release q_t for that month:

$$X_t \geq q_t \qquad t=1.....n \qquad ...(30)$$

Substituting X_t from equation (26)

$$K_t - b_{t-1} + b_t \geq q_t \qquad t=1.....n$$

or $$b_t - b_{t-1} \geq q_t-K_t \qquad t=1....n \qquad ...(31)$$

It can be seen here that the number of constraints is quite large for even a moderate problem. If inflow data for 10 years is available which is not appropriate sample length, each constraint will occur 120 times. However, in this case, each constraint in the same form occurs 10 times (for each month) except for a different stipulation on the right hand side. It is clear from hydrology that of each appearance(10 in this case) of each constraint(for a particular month) one must be more binding than others. Only this most binding constraint must be retained.

Thus eq(28) can be written as

$$C+b_t \geq \text{Max } (J_{t+12}) + V_t \qquad t=1.....12 \qquad ...(32)$$

Similarly eq.(30) can be written as

$$b_t \leq (J_{t+12m}) - C_{min} \quad t=1.....12 \qquad ...(33)$$

Equation (31) can be rewritten as

$$b_t - b_{t-1} \geq q_t - \min_m (K_{t+12m}) \quad t=2....12 \qquad ...(34)$$

$$b_1 - b_{12} \geq q_1 - \min_m (K_{12+12m})$$

This completes the linear programming formulation of the problem in the linear decision rule framework. The objective function is given by equation(19) and the constraints by equation(32) to equation equation (34). The size of the problem is relatively small and it can be quickly solved using any standard LP package.

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