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RIGOROUS DERIVATION OF HOOGHOUDT'S EQUATION FOR DRAINAGE SPACING

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PREFACE

Waterlogging can be caused by excess soil moisture due to periodic flooding, overflow by runoff, over irrigation, seepage, artesian water and impeded subsurface drainage. These conditions affects the growth and yield of crops and in course of time, turns the land saline or alkaline and ultimately render it unfit for cultivation. The valleys of Tigris and the Euphrates, which were once very fertile, were rendered barren because of this malady. Usually the cause of waterlogging in Indian subcontinent is limited to impeded drainage, over irrigation and inadequate drainage facilities. This is a very paradoxical situation where on one hand water being a scarce resource, is required to be conserved and its availability maintained through measures for maximising retention and minimising losses, and on the other hand indiscriminate use of water of limited availability results into waterlogging. The most effective answer to waterlogging is a properly designed drainage system. In view of the above, the present study has been carried out for the rigorous derivation of Hooghoudt's equation for drainage spacing.

This report entitled "Rigorous Derivation of Hooghoudt Equation for the Drainage Spacing" is the part of the research activities of 'Drainage Division' of the Institute. This study has been carried out by Dr. Vivekanand Singh, Scientist 'B' and Dr. G. C. Mishra, Scientist 'F'

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Director

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ABSTRACT

Hooghoudt's drain spacing formula is based on an implied assumption that in the radial flow zone there is no spatial variation in the flux and hence no vertical recharge in this zone. The head loss in the radial flow zone has been accounted presuming that half of the total vertical recharge within the drains enters at the outer boundary of the radial flow zone. In the present report, a drain spacing formula has been derived rigorously considering the variation in radial flux and the area above the drain level in the radial flow zone. The Hooghoudt's formula computes maximum water table height very close to that computed by the present rigorous method. For a ratio of high recharge rate to hydraulic conductivity of drainage layer, R/K =0.5, and a ratio of close drain spacing to thickness of soil layer below the drains, L/D=4, the divergence between the two sets of results is 8%. For normal recharge rate, shallow soil layer below the drains and usual drain spacing, the error involved in computation of the maximum water table height by Hooghoudt's formula is less than 1 %. The water table height near the drains computed using the equivalent depth proposed by Hooghoudt differs considerably from that computed by the rigorous method.

1.0 INTRODUCTION

An agricultural land is said to be waterlogged when the soil pores in the crop root zone gets completely saturated with water. This is usually caused by a rise of the subsoil water table. When the soil in the crop root zone becomes saturated, the plant roots are denied normal circulation of air, the level of oxygen declines and that of carbon-dioxide increases, as organic matter decomposes with the saturated results in wilting and ultimately in the death of the plants. An artificial subsurface drainage is provided in agricultural lands of inadequate natural drainage to guarantee suitable condition for plant growth.

Drainage is the removal of excess water, by artificial means, from the soil or from the land surface, its objective being to make the land more suitable for use by man. In agriculture, the prime requirement for successfully irrigated agriculture is the development and maintenance of a soil root zone in which the moisture, oxygen and salt balance is favourable for plant growth. Plant requires both moisture and oxygen to live. The soil moisture deficiency is abated through irrigation and the oxygen deficiency is overcome by drainage. Its aims are to increase production, to sustain yields, or to reduce production costs - all helping the farming enterprise to maximise its net profit.

In an arid region, efficient use of land and water resources is to a great extent dependent upon the control of salinisation problem. The long history of irrigation has recorded severe deterioration of land resources due to salinisation and waterlogging. It is well known phenomenon that, when an area is irrigated excessively over an extended period of time, the groundwater level rises. When the water table reaches a height, which is within the capillary lift of the soil, the soil moisture is brought to the surface where it evaporates. Salts, which were originally present in the irrigation water or which were dissolved in the rising groundwater, get concentrated on the land surface by the so called 'tea kettle effect'. This causes soil salinity and some times alkalinity, which are harmful to plant growth. When a saline water table rises and remains in the root zone longer than about 48 hours, resulting in an abnormally high saline moisture condition, agricultural production is usually seriously affected and this heavy concentration of salts renders the soil infertile. In modern times, the rate of salinisation and land destruction has been greatly accelerated, especially in areas irrigated with plentiful, low cost water, which contains dissolved salts.

The most effective answer to waterlogging is a properly designed drainage system. The drainage system of an area is the reverse of the irrigation system. Just as the main canal takes off from the river, branches off into distributories and minors and finally ends in field channels supplying water to individual fields, in reverse order, the drainage is collected into main drains, and then discharged off in bigger stream.

In the nineteenth century the French hydrologists, Darcy and Dupuit, were the first to formulate the basic equations for ground water flow through porous media and to apply them to flow to wells. At the beginning of the twentieth century, Rothe applied these equations to groundwater flow to drains, and he was to derive the first drainage formula. But it was Hooghoudt who, in the thirties, gave the real stimulus to a rational analysis of the drainage problem, by studying it in the context of the plant-soil-water system. Since then, great contributions towards a further refinement of this rational analysis have been made by scientists all over the world: childs in England, Donnam, Luthin, and Kirkham in the United States, and Ernst and Wesseling in the Netherlands (Wesseling, 1983). But when drainage theories are applied in practice, we still face a number of limitations. These limitations are a consequence of the wide variability we encounter in nature when dealing with soils and plants. We are faced with such questions as: how to characterise a soil profile consisting of a large number of different layers changing in position and magnitude from one place to another; how to measure the physical soil 'constants'; how to formulate the agronomic requirements in respect to excess water. All these factors contribute to an inevitable inaccuracy, which we have to accept when working in drainage. Therefore, the statement made by Clyde Houston in 1961 is still valid: "Although excellent progress has been made in recent years in developing drainage criteria and investigational tools, it still takes good judgement, local experience, and trial and error – along with a thorough understanding of the basic principles – to design a successful drainage system".

Subsurface drainage problems can be solved either by horizontal subsurface drains or by pumping ground water by tube wells. The design of the subsurface drainage is mainly depends on the geo-hydrological condition of the area, crops cultivated in the command area and the drainage coefficient. Tube well drainage, under some conditions is an effective method of lowering a high water table and reducing salinity hazard in an irrigated area. Before a tube well drainage scheme is installed, careful engineering investigations should be carried out to evaluate the feasibility of drainage.

Most of the formulae used for designing drain spacing for containing water table below root zone are based on the Dupuit-Forchheimer assumptions (Kirkham, 1966; Moody, 1966; VanSchilfgaarde, 1970; Skaggs, 1978; Sakkas and Antonopoulos, 1981; Wesseling, 1983 and Mishra, et al. 1996). Miles and Kitmintto (1989) recently derived a simple drain spacing formula based on the potential theory. Consequently they are to be considered as approximate solutions only. Such approximate solutions, however, are generally accepted as having such a high degree of accuracy that their application in practice is justified (Wesseling, 1983). Though the flow to the drain remains in an unsteady state condition the drain spacing is generally computed using solution of steady state flow. The spacing of fully penetrating vertical walled parallel ditches to contain the water level below the root zone, when prolonged recharge is taking place at uniform rate, is computed using the Donnan's equation (Wesseling, 1983). If the ditches do not reach the impervious floor, the flow lines are not parallel and horizontal but converge towards the drain. In this case the flow system can not be simplified to a flow field with parallel and horizontal streamlines without introducing large errors. The radial flow causes a lengthening of the flow lines. This lengthening causes a more than proportional loss of hydraulic head since the flow velocity in the vicinity of the drains is larger than else where in the flow region. Consequently, the elevation of the water table will be higher, when vertically walled ditches are replaced by pipe drain, the drain level remaining the same. Hooghoudt has derived a simple rational formula relating water level height with spacing of horizontal tile drains, radius of the drain pipe, depth to a horizontal impervious layer below the drains, uniform recharge rate and hydraulic conductivity of the soil. Hooghoudt identified radial flow zones up to a distance of $D/\sqrt{2}$ from each drain and a central horizontal flow zone. Sakkas (1975) has given generalised nomographic solution of Hooghoudt's equation. Hooghoudt's assumptions for the derivation of the drainage spacing are as follows (Luthin, 1970):

1. Soil is homogeneous and of hydraulic conductivity K;
2. The hydraulic gradient at any point is equal to the slope of the water table above the point;
3. Darcy's law is valid for flow of water through soils;

4. An impermeable layer underlies the drain at a depth D;
5. Rain is falling or irrigation water is applied at a rate of R;
6. Recharge in the radial flow zone is neglected;
7. Water depth above the drain level in the radial flow zone is neglected;
8. Developed the concept of equivalent depth.

According to Hooghoudt's assumptions, in the radial flow zone, there is no spatial variation in the flow, which implies no vertical recharge in this zone. Besides the area of flow in the radial flow zone above the drain level has been neglected. In Hooghoudt's method, the head loss in the radial flow zone has been computed assuming that half of the total vertical recharge between the drains enters the radial flow zone at $D/\sqrt{2}$. All these assumptions have led to the derivation of the simple Hooghoudt's formula, which is widely used. In the present study, a drain spacing formula has been derived considering radial variation in flow and the area of radial flow above drain level and accuracy of Hooghoudt's drain spacing formula has been verified.

In this report, an attempt has been made to derive rigorously the equation for drainage spacing by considering parallel tile drains of radius, r_o, which are laid at a height D above a horizontal impervious layer. The soil is homogeneous and isotropic. The flow to the drains due to a constant recharge rate, R, has reached a steady state condition. The pipe drains run half filled. Hooghoudt's assumption that the flow is radial upto a distance of $D/\sqrt{2}$ from the drains is valid. In the central zone, $L - D\sqrt{2}$, Dupuit Forchheimer assumptions are applicable. The objective of the present study is to derive a rigorous expression for the maximum height of water above the drain level.

2.0 DRAINAGE SYSTEM

Drainage system can be classified as surface and subsurface drainage systems. Though the main objective in both the systems is same, i.e. to provide a soil moisture regime conducive to better plant growth, but the method to achieve this is different. Surface drainage system removes water before it has entered the soil. Provision of surface drainage system results in an increase in the surface runoff by which the amount of water going into storage in the soil is reduced. Where as subsurface drainage system removes water after it has entered the soil. Subsurface drainage aims to increase the rate at which water can be drained from the soil so as to lower the water table for increasing the depth of unsaturated soil above the water table. In many areas, both surface and subsurface drainage may be required. Subsurface drainage is accomplished by a system of open ditches and buried tube drains into which water seeps by gravity. Water collected in drains is conveyed to a suitable outlet. Subsurface drainage can also be accomplished by pumping from wells to lower the water table. Water source often governs the type of drainage to be installed. If excess water were due to precipitation the remedial measure would probably be better surface drainage; if due to canal seepage, an interception drain may be indicated; and if due to artesian pressure pumped wells may provide the most practical remedy.

2.1 Surface Drainage System

The natural development of stream systems provides surface drainage for most sloping land. Areas that need artificial surface drainage are either nearly level or depressions. Surface drainage is the orderly removal of excess water from the surface of land through improved natural channels or constructed ditches and through shaping of the land surface. Surface drainage systems, when properly planned, eliminate ponding, prevent prolonged saturation and accelerate flow to an outlet without siltation or erosion of soil. The basic surface drainage systems are (Luthin, 1970):

1. The random;
2. The parallel;
3. The cross slope or diversion system.

2.2 Subsurface Drainage System

Any drain or well, which is installed to control or lower the high water table in an area, is considered to be an element of subsurface drainage system. The high water table may be caused due to percolation from precipitation, seepage from canals and surface water bodies located at higher elevation, irrigation water, leaching water and leakage from artesian aquifer. In arid and semi arid areas a minor portion of excess water comes from precipitation. The major source of excess water is precipitation. The major sources of excess water in irrigated areas are percolation losses from the irrigation and leaching water applied and seepage losses from irrigation canals. In humid, arid and semi arid areas, the sources of excess water may be ground water moving through shallow aquifers and emerging as springs or ground water under artesian pressure. When the total quantity of water introduced into the subsurface in an area from the various sources exceeds the total quantity disposed of through natural drainage processes the water table will rise. It is then necessary to install artificial drains to remove the surplus water to maintain the water table at some predetermined level, which is not harmful to crops. From a functional point of view, subsurface drainage is classified in two groups, Relief Drainage and Interception Drainage. Relief drainage is used to lower a high water table which is generally flat or of very low gradient. Interception drainage is to intercept, reduce the flow and lower the flow line of the water in the problem area. In planning a subsurface drainage system, the site conditions should be evaluated to decide whether to use relief or interception drainage.

2.2.1 Relief Drainage

In relief drainage system one can distinguish three categories of drains: field laterals, collectors and main drains. The field lateral, collectors and main drain can be either buried pipes or open ditches or a combination of both. Relief drainage systems are classified into four general types:

1. Parallel
2. Herringbone
4. Double main, and
5. Random

2.2.2 Interception drainage

The first step towards solving an agricultural drainage problem is to identify the cause. Sometimes what appears to be a major drainage problem can be solved by a simple remedy once the cause is recognized. An example of this is water logging which is caused by water coming in from outside the wet area. This problem can be abated by a single drain, which intercepts the flow. On sloping land it is worth trying the effect of an interception drain before putting in a relief drainage system. If a permeable layer, sandwiched between layers of less permeable material outcrops causing a seep that may affect a considerable area below the out crop, the remedial measure would be installation on intercepting drains.

3.0 DERIVATION OF THE EQUATION

3.1 Dissipation of Head in the Radial Flow Zone:

The problem analysed is presented in Fig. 1, which shows a homogeneous soil of known hydraulic conductivity with an impermeable stratum lying under it. Soil is assumed to be drain by a series of parallel drains. It is assumed that rain is falling at a constant rate on the soil surface. In order to simplify the mathematical analysis, it is assumed that the hydraulic gradient at any point is equal to the slope of the water table above that point. This assumption is known as the Dupuit Forchheimer (D-F) assumption. The D-F assumptions imply that water flows horizontally because all the equipotentials are vertical planes. But it especially incorrect nears the drains, where the paths are quite curved. However, where the slope of the water table is relatively flat the D-F assumptions are nearly valid. Hooghoudt has assumed that near the drain, flow lines are curved i.e. flow is in radial direction up to a distance of $D/\sqrt{2}$ from the drain. Where D is the height above the impervious floor of the water level in the drains i. e. thickness of aquifer below drain level as shown in Fig. 1.

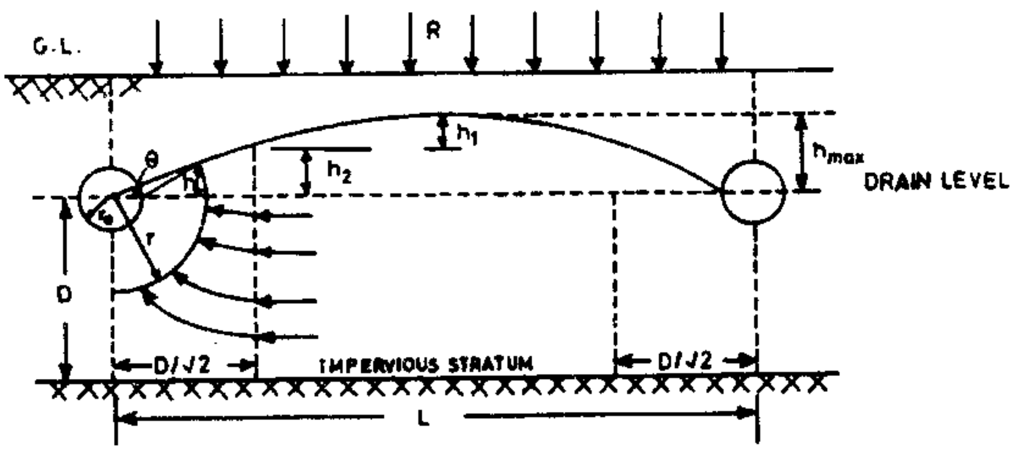

Fig. 1: Steady flow to pipe drains showing horizontal and radial flow zones

The radial flow lines are considered up to a radial distance of $D/\sqrt{2}$ from each drain as shown in Fig. 1. In this case, equation for radial flow line are derived by applying the Darcy's law, the radial flux, q_r, at a distance r from the centre of the drain is given by:

$$q_r = -K\,A\frac{dh}{dr} \tag{1}$$

in which, K is the hydraulic conductivity (m/day); A is the area through which radial flow takes place (m^2); h is the hydraulic head (m) and r is the radial distance from the center of the pipe (m). The datum for the hydraulic head is the center of the drainage pipe. The area of flow, $A = \frac{\pi}{2}r + r\theta$, which is approximated as:

$$A = \frac{\pi}{2}r + h\,, \tag{2}$$

The flux at radial distance r is:

$$q_r = -\left(\frac{L}{2} - r\right)R \tag{3}$$

where, L is the spacing of the pipes and R is the recharge rate.

Incorporating Eqs. (2) and (3) in (1)

$$\frac{dh}{dr} = \frac{\left(-\frac{R}{K}r + \frac{LR}{2K}\right)}{\left(\frac{\pi}{2}r + h\right)} \tag{4}$$

The boundary condition to be satisfied is: at $r = r_0$, $h = 0$. The unknown hydraulic head, h_2, that prevails at the common boundary of the radial flow and horizontal flow zones is to be determined. The differential equation is a non-linear equation of the first order and is solved by standard method.

For solving Eq. (4), assume r = x, h = y, Eq. (4) becomes

$$\frac{dy}{dx} = \frac{\left(-\frac{R}{K}x + \frac{LR}{2K}\right)}{\left(\frac{\pi}{2}x + y\right)} \tag{5}$$

$$\frac{dy}{dx} = \frac{(-ax + b)}{(cx + y)} \tag{6}$$

in which, a = R/K, b = LR/(2K), and c = π/2. Substituting x = X + e and y = Y + f in above equation, where e and f are constants:

$$\frac{dY}{dX} = \frac{(-aX - ae + b)}{(cX + ce + Y + f)} \tag{7}$$

Let the constants e and f be such that -a e + b = 0 and c e + f = 0 from which e = L/2 and f = -π L/4. Hence,

$$\frac{dY}{dX} = \frac{-aX}{cX + Y} \tag{8}$$

Substituting Y = U X in Eq. (8)

$$U + X\frac{dU}{dX} = \frac{(-aX)}{(cX + UX)} = \frac{-a}{c+U} \tag{9}$$

Rearranging Eq. (9) yields

$$\int_{r_o - \frac{L}{2}}^{r - \frac{L}{2}} \left(\frac{dX}{-X}\right) = \int_{\frac{\pi L}{4\left(r_o - \frac{L}{2}\right)}}^{\frac{h + \frac{\pi L}{4}}{\left(r - \frac{L}{2}\right)}} \left[\frac{(c+U)dU}{(U^2 + cU + a)}\right] \tag{10}$$

After integrating and applying the lower and upper limits, i.e. at $r = r_o$ (ro is the radius of drainage pipe), $h = h_o = 0$, and $X = r - \frac{L}{2}$, $U = \frac{h + \frac{\pi L}{4}}{r - \frac{L}{2}}$. Equation (10) yields

$$\log\left(\frac{L}{2} - r_o\right) - \log\left(\frac{L}{2} - r\right) = \frac{1}{2}\log\left(\left(\frac{h + \frac{\pi L}{4}}{r - \frac{L}{2}}\right)^2 + \frac{\pi}{2}\left(\frac{h + \frac{\pi L}{4}}{r - \frac{L}{2}}\right) + \frac{R}{K}\right)$$

$$- \frac{1}{2}\log\left[\left(\frac{\pi L}{4(r_o - \frac{L}{2})}\right)^2 + \frac{\pi}{2}\left(\frac{\pi L}{4(r_o - \frac{L}{2})}\right) + \frac{R}{K}\right] \tag{11}$$

$$+ \frac{\pi}{4}\int_{\frac{\pi L}{4\left(r_o - \frac{L}{2}\right)}}^{\frac{h + \frac{\pi L}{4}}{r - \frac{L}{2}}} \frac{dU}{\left(U + \frac{\pi}{4}\right)^2 - \left(\sqrt{\left(\frac{\pi^2}{16} - \frac{R}{K}\right)}\right)^2}$$

Now, there are 3 cases for which the last term in right hand side of Eq. (11) can be integrated.

These cases are as follows:

<u>Case I</u> $\left(\frac{\pi^2}{16} - \frac{R}{K}\right) > 0, \Rightarrow \frac{\pi^2}{16} > \frac{R}{K} \Rightarrow R < \frac{\pi^2}{16}K$ (12)

<u>Case II</u> $$\left(\frac{\pi^2}{16}-\frac{R}{K}\right)=0, \quad \Rightarrow \frac{\pi^2}{16}=\frac{R}{K} \quad \Rightarrow R=\frac{\pi^2}{16}K \tag{13}$$

<u>Case III</u> $$\left(\frac{\pi^2}{16}-\frac{R}{K}\right)<0, \quad \Rightarrow \frac{\pi^2}{16}<\frac{R}{K} \quad \Rightarrow R>\frac{\pi^2}{16}K \tag{14}$$

Last term in the right hand side of Equation (11) can be integrated for all the three cases.

For case I

$$\left(\frac{\pi^2}{16}-\frac{R}{K}\right)>0, \quad \Rightarrow \frac{\pi^2}{16}>\frac{R}{K} \quad \Rightarrow R<\frac{\pi^2}{16}K \tag{15}$$

Assume

$$I_1=\frac{\pi}{4}\int_{-\frac{\pi L}{4(r_o-\frac{L}{2})}}^{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}}\frac{dU}{\left(U+\frac{\pi}{4}\right)^2-\left(\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}\right)^2} \tag{16}$$

If $$\left(U+\frac{\pi}{4}\right)>\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)} \quad \Rightarrow\left(U^2+\frac{\pi}{2}U+\frac{R}{K}\right)>0 \tag{17}$$

For Case I, after integration Eq. (11) yields

$$I_1=\frac{\pi}{8\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}}\log\left\{\frac{U^2+\frac{\pi}{2}U+\frac{R}{K}}{\left(\left(U+\frac{\pi}{4}\right)+\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}\right)^2}\right\}\Bigg|_{\frac{\pi L}{4(r_o-\frac{L}{2})}}^{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}} \tag{18}$$

Finally Eq. (11) becomes

$$\log\left(\frac{\frac{L}{2}-r_o}{\frac{L}{2}-r}\right)=\frac{1}{2}\log\left[\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)+\frac{R}{K}\right]-\frac{1}{2}\log\left[\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}\right]$$
$$+\frac{\pi}{8\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}}\left[\log\left\{\frac{\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)+\frac{R}{K}}{\left(\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)+\frac{\pi}{4}+\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}\right)^2}\right\}-\log\left\{\frac{\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}}{\left(\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{\pi}{4}+\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}\right)^2}\right\}\right] \quad (19)$$

By putting the values of

$$r=\frac{D}{\sqrt{2}},\quad h=h_2\quad \Rightarrow U=\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}} \quad (20)$$

where. D is the thickness of the aquifer below drain level.

Final equation for Case I will be

$$\log\left(\frac{\frac{L}{2}-r_o}{\frac{L}{2}-\frac{D}{\sqrt{2}}}\right)=\frac{1}{2}\log\left[\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)+\frac{R}{K}\right]-\frac{1}{2}\log\left[\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}\right]$$
$$+\frac{\pi}{8\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}}\left[\log\left\{\frac{\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)+\frac{R}{K}}{\left(\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)+\frac{\pi}{4}+\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}\right)^2}\right\}-\log\left\{\frac{\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}}{\left(\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{\pi}{4}+\sqrt{\left(\frac{\pi^2}{16}-\frac{R}{K}\right)}\right)^2}\right\}\right] \quad (21)$$

<u>For Case II</u>

$$I_1=\frac{\pi}{4}\int_{\frac{\pi L}{4(r_o-\frac{L}{2})}}^{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}}\frac{dU}{\left(U+\frac{\pi}{4}\right)^2} \quad (22)$$

Integration of I_1 will be

$$I_1=-\frac{\pi}{4}\frac{1}{\left(U+\frac{\pi}{4}\right)}\Bigg|_{\frac{\pi L}{4(r_o-\frac{L}{2})}}^{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}} \quad (23)$$

Equation (11). becomes

$$\log\left(\frac{\frac{L}{2}-r_o}{\frac{L}{2}-r}\right)=\frac{1}{2}\log\left[\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)+\frac{R}{K}\right]-\frac{1}{2}\log\left\{\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}\right\}$$
$$-\frac{\pi}{4}\left\{\frac{1}{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}+\frac{\pi}{4}}\right\}+\frac{\pi}{4}\left\{\frac{1}{\frac{\pi L}{4(r_o-\frac{L}{2})}+\frac{\pi}{4}}\right\}$$

(24)

For Case II, the final equation by putting the values r and h from Eq. (20), to Eq. (24) will becomes:

$$\log\left(\frac{\frac{L}{2}-r_o}{\frac{L}{2}-\frac{D}{\sqrt{2}}}\right)=\frac{1}{2}\log\left[\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)+\frac{R}{K}\right]-\frac{1}{2}\log\left\{\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}\right\}$$
$$-\frac{\pi}{4}\left\{\frac{1}{\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}+\frac{\pi}{4}}\right\}+\frac{\pi}{4}\left\{\frac{1}{\frac{\pi L}{4(r_o-\frac{L}{2})}+\frac{\pi}{4}}\right\}$$

(25)

<u>Case III</u>

$$I_1=\frac{\pi}{4}\int_{\frac{\pi L}{4(r_o-\frac{L}{2})}}^{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}}\frac{dU}{(U+\frac{\pi}{4})^2+\left(\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}\right)^2} \tag{26}$$

After integration Eq. (26) becomes

$$I_1=\frac{\pi}{4\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\tan^{-1}\left\{\frac{(U+\frac{\pi}{4})}{\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\right\}\Bigg|_{\frac{\pi L}{4(r_o-\frac{L}{2})}}^{\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}} \tag{27}$$

$$\log\left(\frac{\frac{L}{2}-r_o}{\frac{L}{2}-r}\right)=\frac{1}{2}\log\left[\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)+\frac{R}{K}\right]-\frac{1}{2}\log\left[\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}\right]$$
$$+\frac{\pi}{4\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\left[\tan^{-1}\left\{\frac{\left(\frac{h+\frac{\pi L}{4}}{r-\frac{L}{2}}\right)+\frac{\pi}{4}}{\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\right\}-\tan^{-1}\left\{\frac{\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{\pi}{4}}{\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\right\}\right] \quad (28)$$

By putting the values of r and h from Eq. (20) to Eq. (28)

$$\log\left(\frac{\frac{L}{2}-r_o}{\frac{L}{2}-\frac{D}{\sqrt{2}}}\right)=\frac{1}{2}\log\left[\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)^2+\frac{\pi}{2}\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)+\frac{R}{K}\right]-\frac{1}{2}\log\left[\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)^2+\frac{\pi}{2}\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{R}{K}\right]$$
$$+\frac{\pi}{4\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\left[\tan^{-1}\left\{\frac{\left(\frac{h_2+\frac{\pi L}{4}}{\frac{D}{\sqrt{2}}-\frac{L}{2}}\right)+\frac{\pi}{4}}{\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\right\}-\tan^{-1}\left\{\frac{\left(\frac{\pi L}{4(r_o-\frac{L}{2})}\right)+\frac{\pi}{4}}{\sqrt{\left(\frac{R}{K}-\frac{\pi^2}{16}\right)}}\right\}\right] \quad (29)$$

Equations (21), (25) and (29) are the final equations for all three cases of radial flow respectively. These equations are non-linear in h_2 and L and have been solved numerically by Newton's Raphson technique. Hence, for a given values of drainage spacing, L, the values of h_2, can be evaluated and for the radial distances.

3.2 Dissipation of Head in Horizontal Flow Zone

The horizontal flow lines are considered over a length of L- D√2 between the two parallel drains as shown in Fig 2. The governing differential equation in the horizontal flow zone, L- D√2, (Fig. 2), in which D-F conditions are satisfied, is:

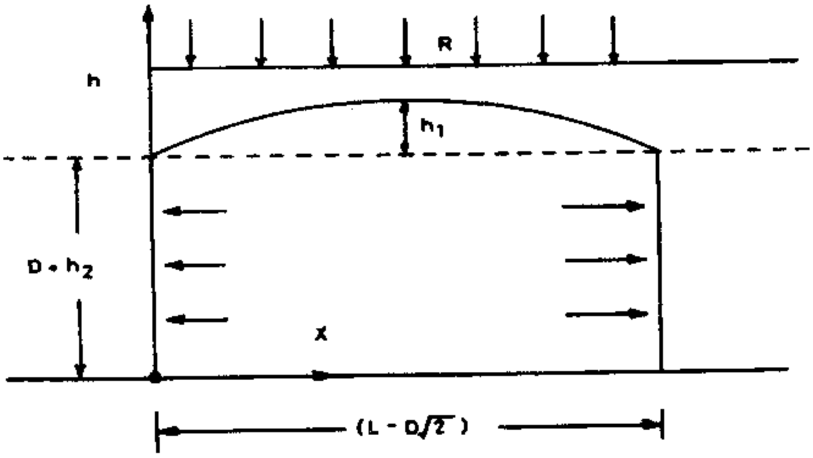

Fig. 2 Problem definition of the horizontal flow

$$\frac{dq_x}{dx} = R \tag{30}$$

$$\frac{d}{dx}\left(-Kh\frac{dh}{dx}\right) = R \tag{31}$$

$$\frac{d^2h^2}{dx^2} = -\frac{2R}{K} \tag{32}$$

Equation (32) after integration yields

$$h^2 = -\frac{2R}{K}\frac{x^2}{2} + Ax + B \tag{33}$$

in Eq. (33), A and B are constants. The hydraulic head, h, is required to be measured from the datum at the impervious boundary. The boundary conditions required to be satisfied are given below. Satisfying these conditions the head can be is found.

Boundary Conditions

1. At x=0, $h = h_2 + D$ and at $x = L - D\sqrt{2}$; $h = h_2 + D$ (34)

$$h^2 = -\frac{R}{K}x^2 + \frac{R}{K}\left(L - D\sqrt{2}\right)x + \left(h_2 + D\right)^2 \tag{35}$$

2. At $x = (L - D\sqrt{2})/2$; $h = D + h_2 + h_1$ (36)

$$\left(D + h_2 + h_1\right)^2 = -\frac{R}{K}\left(\frac{L - D\sqrt{2}}{2}\right)^2 + \frac{2R}{K}\left(\frac{L - D\sqrt{2}}{2}\right)^2 + \left(h_2 + D\right)^2 \tag{37}$$

$$h_1 = -\left(D + h_2\right) + \sqrt{\left(D + h_2\right)^2 + \frac{R}{4K}\left(L - D\sqrt{2}\right)^2} \tag{38}$$

The maximum water table height above drain level, h_{max} is given by:

$$h_{max} = h_1 + h_2 \tag{39}$$

After knowing the values of h_2 for given values of L the values of h_1 can be evaluated by using Eq. (38) for horizontal flow case. Equation (39) is the final equation for the calculation of maximum depth of water table in the middle of the two drains.

4.0 RESULTS AND DISCUSSION

4.1 Worked out Example

The following data reported by ILRI, Vol. II, 1983: have been used for the worked out example:

Hydraulic conductivity of the soil, K, = 0.8 m/day;

Radius of the drain pipe, r_o, = 0.1 m;

Depth to impervious layer below drain level, D, = 5.0 m

Average recharge rate, R, = 0.002 m/day.

For h_{max} = 0.6 m, the drain spacing, L, has been found to be 87 m.

Using the rigorous method, for L = 87 m, the maximum head, h_{max} is found to be 0.591 m.

The head dissipated in the radial flow zone, h_2 , is non-linearly related to all the parameters and the equation which predicts h_2 is an implicit one. Depending upon the range of R/K one of the three derived solutions is applicable. The solution should be consistent in the neighbourhood of $R/K = \pi^2/16 = 0.61685$, which is the neighbour point of all three solutions. To check the consistency the input data are r_o = 0.1 m and D = 5.0 m and the values of R/K are 0.615, 0.61685 and 0.6175. The values of drainage spacing are varying from 10 m to 200 m Results obtained for R/K = 0.615, 0.61685 and 0.6175 are presented in Table 1 for different drain spacing. It can be seen clearly form Table 1 that the results of all three cases are more or less same but there are slight changes in the results as there are slight change in the input values. Thus, the results from all three cases are consistent

4.2 Comparison of the results from both the methods

Results of rigorously derived equation have been compared with the results of Hooghoudt's equation. For this purpose, the input data considered for comparison are as follows (Wesseling, 1983): radius of the drain pipes = 0.1 m; drain pipes are placed at a depth of 1.8 m below the soil surface; impermeable stratum is at 6.8 m below the soil surface i.e. D = 6.8-1.8 = 5.0 m; hydraulic conductivity of the soil = 0.8 m/day; and the average recharge of the drainage system

= 0.002 m/day; h_{max} = 0.6 m. Maximum water table heights above drain level, h_{max} for different drain spacing have been computed for the above set of data using rigorous method and are shown in Fig. 3. The value of h_{max}, is also computed using Hooghoudt's method, for the above case and is presented in Fig. 3 for the purpose of comparison. Both the methods compute all most the same value of h_{max}.

Table 1 : Values of h_{max} for different R/K = 0.615, 0.61685, and 0.6175

Spacing (m)	H_{max} (m) at different R/K			Spacing (m)	H_{max} (m)		
	R/K 0.61500	R/K 0.61685	R/K 0.61750		R/K 0.61500	R/K 0.61685	R/K 0.61750
10	2.623	2.628	2.630	110	39.295	39.359	39.382
20	5.738	5.749	5.753	120	43.158	43.228	43.253
30	9.160	9.177	9.183	130	47.028	47.104	47.131
40	12.750	12.773	12.781	140	50.904	50.986	51.015
50	16.435	16.463	16.474	150	54.785	54.873	54.903
60	20.178	20.213	20.225	160	58.669	58.763	58.796
70	23.960	24.001	24.015	170	62.557	62.657	62.692
80	27.769	27.816	27.833	180	66.448	66.554	66.591
90	31.598	31.651	31.669	190	70.341	70.453	70.492
100	35.441	35.499	35.52	200	74.237	74.355	74.396

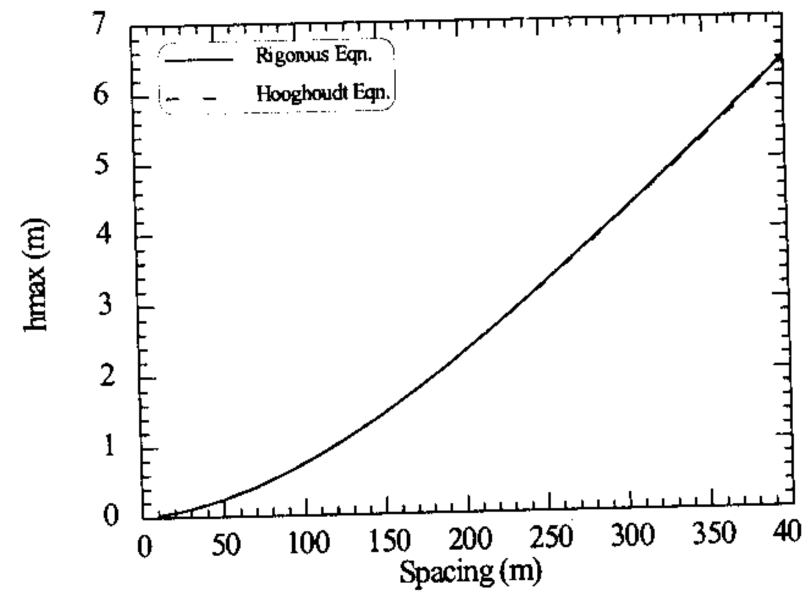

Fig. 3: Comparison of h_{max} Computed using Rigorous Method and Hooghoudt Method

The water surface profiles computed by the rigorous method and by Hooghoudt's method using equivalent depth are presented in Fig. 4. The difference between the two profiles is not significant in the D-F zone but is considerable near the drain. This shows that the Hooghoudt's method is not suitable for the computation of the water surface profile, but it gives the maximum water table height at the middle of the two parallel drains. Though the values of h_{max} computed by Hooghoudt's method is slightly more then rigorous method, which is safe for the purpose of subsurface design.

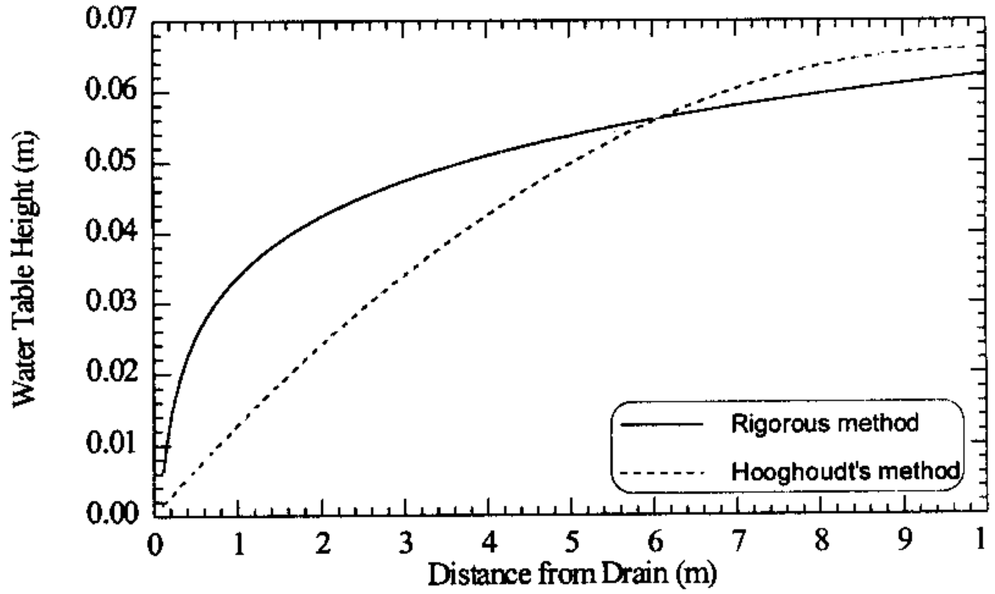

Fig. 4: Water Surface Profile Computed Using Rigorous and Hooghoudt's Methods for R/K=0.25 and L/D = 4

The maximum head, h_{max}, computed by Hooghoudt's method is likely to differ from that by the rigorous method for small value of L/D and large value of R/K. The two methods are compared for low value of L/D and high value of R/K in Table 2. For L/D =4 and R/K = 0.5, the error involved in predicting h_{max} by Hooghoudt's method is 8.4 %. For L/D = 20 and R/K =0.1 the error is 0.7 %. Thus, for normal recharge rate and shallow draining layer simple Hooghoudt's method can be conveniently used for finding the drain spacing.

Table 2: Comparison of h_{max}/D computed by Rigorous Method (RM) and by Hooghoudt Method (HM) for different R/K (r_o/D= 0.01)

L/D	h_{max}/D							
	R/K=0.0025		R/K= 0.01		R/K=0.1		R/K=0.5	
	R M	H M	R M	H M	R M	H M	R M	H M
4	0.012	0.013	0.046	0.050	0.325	0.362	1.005	1.090
6	0.022	0.023	0.081	0.087	0.543	0.585	1.611	1.697
8	0.034	0.035	0.124	0.130	0.786	0.828	2.251	2.330
10	0.048	0.049	0.172	0.179	1.046	1.086	2.910	2.980
12	0.064	0.066	0.227	0.234	1.318	1.355	3.581	3.641
14	0.083	0.084	0.287	0.294	1.599	1.631	4.259	4.311
16	0.103	0.104	0.351	0.358	1.886	1.914	4.944	4.987
18	0.125	0.126	0.419	0.426	2.178	2.202	5.632	5.668
20	0.149	0.150	0.491	0.496	2.474	2.493	6.323	6.353
22	0.174	0.175	0.565	0.570	2.772	2.788	7.016	7.041
24	0.200	0.202	0.642	0.646	3.073	3.085	7.711	7.731
26	0.229	0.230	0.721	0.724	3.376	3.385	8.408	8.423
28	0.258	0.259	0.802	0.805	3.680	3.686	9.106	9.117
30	0.289	0.290	0.885	0.886	3.986	3.989	9.805	9.813
32	0.321	0.321	0.970	0.970	4.293	4.293	10.504	10.509
34	0.353	0.354	1.055	1.054	4.600	4.598	11.205	11.207
36	0.387	0.388	1.142	1.140	4.909	4.904	11.906	11.905
38	0.422	0.422	1.230	1.227	5.218	5.212	12.607	12.605
40	0.459	0.458	1.318	1.315	5.527	5.519	13.309	13.305
42	0.495	0.494	1.408	1.403	5.837	5.828	14.011	14.005
44	0.531	0.531	1.498	1.493	6.148	6.137	14.714	14.706
46	0.570	0.568	1.589	1.583	6.459	6.447	15.417	15.408
48	0.608	0.606	1.681	1.674	6.770	6.757	16.120	16.109
50	0.648	0.645	1.773	1.765	7.082	7.067	16.824	16.812
52	0.688	0.685	1.866	1.857	7.394	7.378	17.527	17.514
54	0.728	0.724	1.958	1.949	7.706	7.689	18.231	18.217
56	0.769	0.765	2.052	2.042	8.019	8.001	18.935	18.92
58	0.809	0.806	2.146	2.135	8.331	8.313	19.639	19.624
60	0.851	0.847	2.240	2.228	8.644	8.625	20.344	20.327
62	0.892	0.888	2.335	2.322	8.957	8.937	21.048	21.031
64	0.935	0.930	2.429	2.416	9.271	9.250	21.753	21.735
66	0.979	0.973	2.524	2.511	9.584	9.562	22.458	22.439
68	1.021	1.015	2.620	2.606	9.897	9.875	23.163	23.144
70	1.065	1.058	2.715	2.701	10.211	10.188	23.868	23.848
72	1.109	1.102	2.811	2.796	10.525	10.502	24.573	24.553
74	1.154	1.145	2.907	2.891	10.839	10.815	25.278	25.257
76	1.196	1.189	3.003	2.987	11.153	11.128	25.983	25.962
78	1.239	1.233	3.099	3.083	11.467	11.442	26.689	26.667
80	1.286	1.277	3.196	3.179	11.781	11.756	27.394	27.372

4.3 Application of Hooghoudt Equations

The values of h_{max}/D with the values of L/D for different values of R/K ratios are computed and presented in Figs. 5 (a) and 5 (b). In this case the values of $r_o = 0.1$ m and D = 5 m. The results of computed h_{max} and drain spacing are divided by the value of D to non-dimensionalised the factors. The values are plotted for different values of R/K = 0.0025, 0.025, 0.25, 0.61685 and 1.0, which covers all the three limits defined above. From these figures the values of the ratios of drain spacing and depth of impermeable stratum can be taken for the given values of h_{max} /D and vice versa for a particular values of R/K ratios for the purpose of the drainage design.

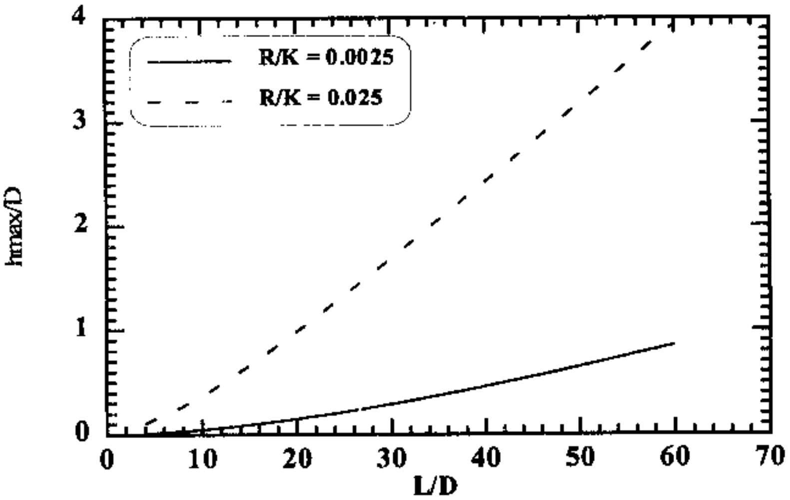

Fig. 5 (a): The values of h_{max}/D with L/D for R/K = 0.0025 and 0.025.

These values are also presented in Table 3. From this table the values of L/D ratios can be taken directly for the corresponding values of h_{max}/D for a particular values of R/K. The main aim to present the results in tabular form is the ease to read the corresponding correct values from the table. Fig. 5 (a) presents the h_{max}/D values for R/K = 0.0025 and 0.025, where as Fig 5 (b) presents the values for R/K = 0.25, 0.61685 and 1.0.

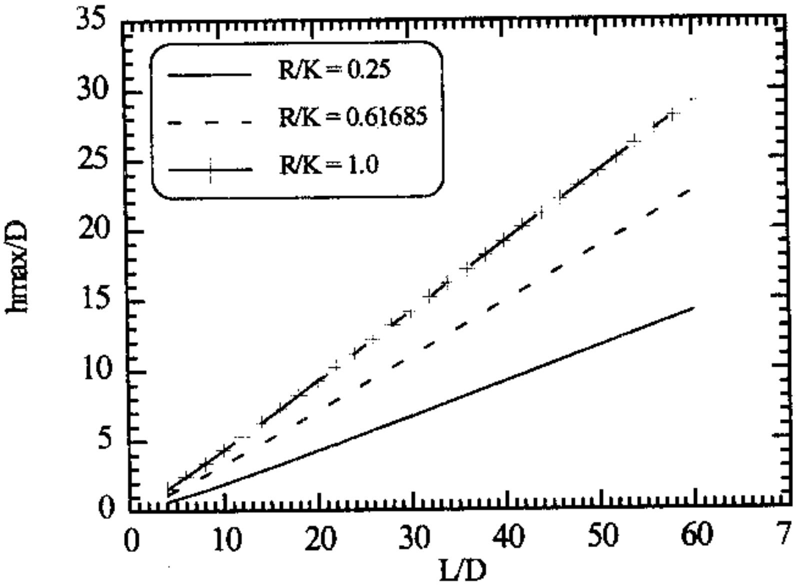

Fig. 5 (b): The values of h_{max}/D with L/D for R/K = 0.25, 0.61685 and 1.0

Table 3: Values of h_{max}/D for different values of L/D and R/K

Sl. No.	L/D	h_{max}/D				
		R/K=0.0025	R/K=0.025	R/K=0.25	R/K=0.61685	R/K=1.0
1	4	0.012	0.105	0.632	1.150	1.554
2	6	0.022	0.182	1.030	1.835	2.456
3	8	0.034	0.272	1.460	2.555	3.392
4	10	0.048	0.373	1.908	3.293	4.347
5	12	0.064	0.483	2.368	4.043	5.313
6	14	0.083	0.600	2.837	4.800	6.287
7	16	0.103	0.724	3.312	5.563	7.266
8	18	0.125	0.852	3.791	6.330	8.248
9	20	0.149	0.984	4.273	7.100	9.233
10	22	0.174	1.119	4.758	7.872	10.221
11	24	0.200	1.257	5.245	8.646	11.210
12	26	0.229	1.398	5.734	9.421	12.200
13	28	0.258	1.540	6.224	10.197	13.191
14	30	0.289	1.684	6.714	10.975	14.184
15	32	0.321	1.829	7.206	11.753	15.177
16	34	0.353	1.975	7.699	12.531	16.170
17	36	0.387	2.123	8.192	13.311	17.165
18	38	0.422	2.271	8.686	14.091	18.159
19	40	0.459	2.420	9.180	14.871	19.154

Sl. No.	L/D	h_{max}/D				
		R/K=0.0025	R/K=0.025	R/K=0.25	R/K=0.61685	R/K=1.0
20	42	0.495	2.570	9.675	15.652	20.150
21	44	0.531	2.720	10.170	16.433	21.146
22	46	0.570	2.872	10.666	17.214	22.142
23	48	0.608	3.023	11.161	17.996	23.138
24	50	0.648	3.175	11.658	18.777	24.135
25	52	0.688	3.327	12.154	19.559	25.132
26	54	0.728	3.480	12.650	20.342	26.128
27	56	0.769	3.633	13.147	21.124	27.126
28	58	0.809	3.786	13.644	21.907	28.123
29	60	0.851	3.940	14.141	22.689	29.120

4.4 Equivalent Depth Theory

Hooghoudt's method is based on the concept of equivalent depth theory, which is more practical to have a formula similar to the equation describing flow to fully penetrating ditches. Equivalent depth for the different values of R/K computed from rigorous method are also presented for different values of drainage spacing, L and depth to impervious level, D. Corresponding to the computed h_{max} by the rigorous method for known values of L, D, r_o and R/K, the equivalent depth, d, is found using Donnan's equation, as

$$d = \frac{R}{K}\frac{L^2}{8h_{max}} - \frac{h_{max}}{2} \tag{40}$$

and is presented in Table 4, 5, 6, 7 and 8 to facilitate the design of subsurface pipe drains. Drainage spacing considered are from 5 to 250 m, where as values of D are from 0.5 to 20 m. Table 4 presents the values of equivalent depths for the R/K ratio equal to 0.0025. Similarly Table 5, 6, 7 and 8 presents the values of equivalent depth for R/K equal to 0.025, 0.25, 0.61685 and 1.0 respectively. These equivalent depths can be used for computing h_{max} for a given value of R/K, L and equivalent depth. Equation for computing h_{max} using equivalent is as follows

$$h_{max} = -d + \sqrt{d^2 + \frac{R}{K}\frac{L^2}{4}} \tag{41}$$

Table 4: Values of the equivalent depth of rigorous method
(r_o = 0.1 m; R/K = 0.0025)

Sl. No.	D (m)	L, Drainage Spacing (m)									
		5	10	15	20	25	30	35	40	45	50
1	0.50	0.48	0.49	0.49	0.49	0.50	0.50	0.50	0.50	0.50	0.50
2	0.75	0.63	0.68	0.70	0.71	0.72	0.72	0.72	0.73	0.73	0.73
3	1.00	0.72	0.83	0.88	0.90	0.92	0.93	0.94	0.94	0.95	0.95
4	1.25	0.78	0.95	1.02	1.07	1.10	1.12	1.13	1.14	1.15	1.16
5	1.50	0.81	1.04	1.15	1.21	1.26	1.29	1.32	1.33	1.35	1.36
6	1.75	0.84	1.10	1.25	1.34	1.40	1.45	1.48	1.51	1.53	1.55
7	2.00	0.85	1.16	1.34	1.45	1.53	1.59	1.64	1.67	1.70	1.72
8	2.25	0.86	1.20	1.41	1.54	1.64	1.72	1.78	1.82	1.86	1.89
9	2.50	0.86	1.23	1.47	1.63	1.74	1.83	1.91	1.96	2.00	2.04
10	2.75	0.87	1.26	1.52	1.70	1.83	1.94	2.02	2.09	2.14	2.19
11	3.00	0.87	1.28	1.56	1.76	1.91	2.03	2.13	2.20	2.27	2.33
12	3.25	-	-	1.59	1.81	1.98	2.12	2.23	2.31	2.39	2.45
13	3.50	-	-	1.62	1.86	2.04	2.19	2.32	2.41	2.50	2.57
14	3.75	-	-	1.65	1.90	2.10	2.26	2.40	2.51	2.60	2.68
15	4.00	-	-	1.67	1.93	2.15	2.33	2.47	2.59	2.69	2.79
16	4.25	-	-	1.68	1.96	2.19	2.38	2.54	2.67	2.78	2.88
17	4.50	-	-	1.70	1.99	2.23	2.43	2.60	2.74	2.86	2.97
18	4.75	-	-	1.71	2.02	2.27	2.48	2.66	2.81	2.94	3.06
19	5.00	-	-	1.72	2.04	2.30	2.52	2.71	2.87	3.01	3.14
20	5.25	-	-	1.73	2.05	2.33	2.56	2.76	2.93	3.08	3.21
21	5.50	-	-	1.74	2.07	2.35	2.60	2.81	2.98	3.14	3.28
22	5.75	-	-	1.75	2.09	2.38	2.63	2.85	3.03	3.20	3.35
23	6.00	-	-	1.75	2.10	2.40	2.66	2.89	3.08	3.25	3.41
24	6.25	-	-	1.76	2.11	2.41	2.68	2.92	3.12	3.30	3.47
25	6.50	-	-	1.76	2.12	2.43	2.71	2.95	3.16	3.35	3.52
26	6.75	-	-	1.76	2.13	2.45	2.73	2.98	3.20	3.39	3.57
27	7.00	-	-	1.77	2.14	2.46	2.75	3.01	3.23	3.43	3.62
28	7.25	-	-	1.77	2.14	2.47	2.77	3.03	3.26	3.47	3.67
29	7.50	-	-	1.77	2.15	2.48	2.79	3.06	3.29	3.51	3.71
30	7.75	-	-	1.77	2.15	2.49	2.80	3.08	3.32	3.54	3.75
31	8.00	-	-	1.77	2.16	2.50	2.82	3.10	3.35	3.57	3.79
32	8.25	-	-	1.78	2.16	2.51	2.83	3.12	3.37	3.60	3.82
33	8.50	-	-	1.78	2.17	2.52	2.84	3.14	3.39	3.63	3.86
34	8.75	-	-	1.78	2.17	2.53	2.85	3.15	3.41	3.66	3.89
35	9.00	-	-	1.78	2.17	2.53	2.87	3.17	3.43	3.68	3.92
36	9.25	-	-	1.78	2.17	2.54	2.87	3.18	3.45	3.70	3.95
37	9.50	-	-	1.78	2.18	2.54	2.88	3.19	3.47	3.73	3.97
38	9.75	-	-	1.78	2.18	2.55	2.89	3.21	3.49	3.75	4.00
39	10.00	-	-	1.78	2.18	2.55	2.90	3.22	3.50	3.77	4.02

Table 4 Cont.

D (m)	L, Drainage Spacing (m)										
	50	60	70	80	100	120	140	160	180	200	250
0.5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
1.0	0.95	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.97	0.97	0.98
1.5	1.36	1.38	1.39	1.40	1.41	1.42	1.43	1.43	1.44	1.44	1.44
2.0	1.72	1.76	1.78	1.80	1.83	1.85	1.86	1.87	1.88	1.88	1.90
2.5	2.04	2.10	2.14	2.18	2.22	2.26	2.28	2.30	2.32	2.32	2.34
3.0	2.33	2.41	2.47	2.52	2.59	2.65	2.69	2.71	2.73	2.74	2.78
3.5	2.57	2.68	2.77	2.84	2.94	3.02	3.07	3.10	3.14	3.15	3.20
4.0	2.79	2.93	3.03	3.13	3.26	3.36	3.43	3.48	3.53	3.55	3.62
4.5	2.97	3.15	3.28	3.39	3.56	3.69	3.78	3.84	3.90	3.93	4.02
5.0	3.14	3.34	3.50	3.64	3.84	4.00	4.10	4.19	4.26	4.30	4.41
5.5	3.28	3.52	3.70	3.86	4.10	4.28	4.42	4.52	4.60	4.66	4.79
6.0	3.41	3.67	3.88	4.06	4.34	4.56	4.71	4.83	4.93	5.00	5.16
6.5	3.52	3.81	4.04	4.25	4.56	4.81	4.99	5.13	5.25	5.33	5.52
7.0	3.62	3.94	4.19	4.42	4.77	5.05	5.26	5.42	5.56	5.65	5.86
7.5	3.71	4.05	4.32	4.57	4.96	5.28	5.51	5.69	5.85	5.95	6.20
8.0	3.79	4.15	4.44	4.72	5.14	5.49	5.74	5.95	6.13	6.24	6.53
8.5	3.86	4.24	4.56	4.85	5.30	5.69	5.97	6.19	6.39	6.53	6.84
9.0	3.92	4.32	4.66	4.97	5.46	5.87	6.18	6.43	6.65	6.80	7.15
9.5	3.97	4.40	4.75	5.08	5.61	6.05	6.38	6.65	6.89	7.06	7.45
10.0	4.02	4.47	4.83	5.18	5.74	6.22	6.58	6.87	7.13	7.31	7.74
10.5	4.07	4.53	4.91	5.28	5.87	6.37	6.76	7.07	7.35	7.55	8.02
11.0	4.11	4.58	4.98	5.37	5.98	6.52	6.93	7.27	7.57	7.78	8.29
11.5	4.14	4.63	5.05	5.45	6.10	6.66	7.09	7.45	7.78	8.00	8.55
12.0	4.17	4.68	5.11	5.52	6.20	6.79	7.25	7.63	7.97	8.22	8.80
12.5	4.20	4.72	5.16	5.59	6.29	6.91	7.40	7.80	8.16	8.42	9.05
13.0	4.23	4.76	5.21	5.66	6.39	7.03	7.54	7.96	8.34	8.62	9.29
13.5	4.25	4.79	5.26	5.72	6.47	7.14	7.67	8.12	8.52	8.81	9.52
14.0	4.27	4.82	5.30	5.77	6.55	7.24	7.80	8.26	8.69	9.00	9.74
14.5	4.29	4.85	5.34	5.82	6.62	7.34	7.92	8.40	8.85	9.17	9.96
15.0	4.31	4.88	5.38	5.87	6.69	7.44	8.03	8.54	9.00	9.34	10.17
15.5	4.32	4.90	5.42	5.92	6.76	7.52	8.14	8.67	9.15	9.51	10.37
16.0	4.34	4.93	5.45	5.96	6.82	7.61	8.25	8.79	9.29	9.67	10.57
16.5	4.35	4.95	5.48	6.00	6.88	7.69	8.34	8.91	9.43	9.82	10.76
17.0	4.36	4.97	5.50	6.03	6.94	7.76	8.44	9.02	9.56	9.96	10.94
17.5	4.37	4.98	5.53	6.07	6.99	7.83	8.53	9.13	9.68	10.11	11.12
18.0	4.38	5.00	5.55	6.10	7.04	7.90	8.62	9.23	9.81	10.24	11.29
18.5	4.39	5.01	5.57	6.13	7.08	7.97	8.70	9.33	9.92	10.37	11.46
19.0	4.40	5.03	5.59	6.15	7.13	8.03	8.78	9.43	10.03	10.50	11.63
19.5	4.41	5.04	5.61	6.18	7.17	8.08	8.85	9.52	10.14	10.62	11.79
20.0	4.41	5.05	5.63	6.20	7.21	8.14	8.92	9.61	10.25	10.74	11.94

Table 5: Values of the equivalent depth of rigorous method
(r_o = 0.1 m; R/K = 0.025)

D	L, Spacing of the Drainage (m)									
	5	10	15	20	25	30	35	40	45	50
0.50	0.48	0.49	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.75	0.63	0.68	0.69	0.70	0.71	0.71	0.72	0.72	0.72	0.72
1.00	0.72	0.83	0.87	0.89	0.90	0.91	0.92	0.93	0.93	0.93
1.25	0.78	0.94	1.02	1.06	1.08	1.10	1.11	1.12	1.13	1.14
1.50	0.82	1.04	1.14	1.21	1.25	1.27	1.29	1.31	1.32	1.33
1.75	0.84	1.11	1.25	1.34	1.39	1.43	1.46	1.49	1.51	1.52
2.00	0.86	1.17	1.34	1.45	1.53	1.58	1.62	1.65	1.68	1.70
2.25	0.87	1.21	1.42	1.56	1.65	1.72	1.77	1.81	1.84	1.87
2.50	0.87	1.25	1.49	1.65	1.76	1.84	1.91	1.96	2.00	2.03
2.75	0.88	1.28	1.54	1.73	1.86	1.96	2.03	2.09	2.14	2.18
3.00	0.88	1.30	1.59	1.80	1.95	2.06	2.15	2.22	2.28	2.33
3.25	-	-	1.63	1.86	2.03	2.16	2.26	2.34	2.41	2.47
3.50	-	-	1.67	1.91	2.10	2.25	2.36	2.46	2.53	2.60
3.75	-	-	1.70	1.96	2.17	2.33	2.46	2.56	2.65	2.72
4.00	-	-	1.72	2.00	2.22	2.40	2.54	2.66	2.76	2.84
4.25	-	-	1.74	2.04	2.28	2.47	2.63	2.75	2.86	2.95
4.50	-	-	1.76	2.07	2.33	2.53	2.70	2.84	2.96	3.06
4.75	-	-	1.77	2.10	2.37	2.59	2.77	2.92	3.05	3.16
5.00	-	-	1.79	2.13	2.41	2.64	2.83	3.00	3.14	3.26
5.25	-	-	1.80	2.15	2.44	2.69	2.89	3.07	3.22	3.35
5.50	-	-	1.81	2.17	2.48	2.73	2.95	3.14	3.30	3.43
5.75	-	-	1.82	2.19	2.50	2.77	3.00	3.20	3.37	3.51
6.00	-	-	1.82	2.21	2.53	2.81	3.05	3.26	3.44	3.59
6.25	-	-	1.83	2.22	2.56	2.84	3.09	3.31	3.50	3.66
6.50	-	-	1.83	2.23	2.58	2.88	3.14	3.36	3.56	3.73
6.75	-	-	1.84	2.24	2.60	2.91	3.17	3.41	3.62	3.80
7.00	-	--	1.84	2.25	2.61	2.93	3.21	3.45	3.67	3.86
7.25	-	-	1.85	2.26	2.63	2.96	3.24	3.50	3.72	3.92
7.50	-	-	1.85	2.27	2.65	2.98	3.27	3.54	3.77	3.98
7.75	-	-	1.85	2.28	2.66	3.00	3.30	3.57	3.81	4.03
8.00	-	-	1.85	2.28	2.67	3.02	3.33	3.61	3.86	4.08
8.25	-	-	1.85	2.29	2.68	3.04	3.36	3.64	3.90	4.13
8.50	-	-	1.85	2.29	2.69	3.05	3.38	3.67	3.93	4.17
8.75	-	-	1.86	2.30	2.70	3.07	3.40	3.70	3.97	4.22
9.00	-	-	1.86	2.30	2.71	3.08	3.42	3.73	4.00	4.26
9.25	-	-	1.86	2.31	2.72	3.10	3.44	3.75	4.04	4.29
9.50	-	-	1.86	2.31	2.72	3.11	3.46	3.78	4.07	4.33
9.75	-	-	1.86	2.31	2.73	3.12	3.47	3.80	4.10	4.37
10.00	-	-	1.86	2.31	2.74	3.13	3.49	3.82	4.12	4.40

Table 5 Cont.

D\L	50	60	70	80	100	120	140	160	180	200	250
0.5	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51
1.0	0.93	0.94	0.95	0.95	0.96	0.96	0.96	0.97	0.97	0.97	0.97
1.5	1.33	1.35	1.36	1.37	1.39	1.40	1.40	1.41	1.42	1.42	1.43
2.0	1.70	1.73	1.75	1.77	1.80	1.82	1.83	1.84	1.85	1.86	1.87
2.5	2.03	2.08	2.12	2.15	2.19	2.22	2.24	2.26	2.27	2.29	2.31
3.0	2.33	2.40	2.46	2.50	2.56	2.61	2.64	2.67	2.69	2.70	2.74
3.5	2.60	2.70	2.78	2.83	2.92	2.98	3.03	3.06	3.09	3.11	3.16
4.0	2.84	2.97	3.07	3.15	3.26	3.34	3.40	3.44	3.48	3.51	3.57
4.5	3.06	3.22	3.35	3.44	3.58	3.68	3.76	3.82	3.86	3.90	3.97
5.0	3.26	3.45	3.60	3.72	3.89	4.02	4.11	4.18	4.23	4.28	4.37
5.5	3.43	3.66	3.83	3.97	4.18	4.33	4.44	4.53	4.59	4.65	4.75
6.0	3.59	3.85	4.05	4.22	4.46	4.64	4.76	4.87	4.95	5.01	5.13
6.5	3.73	4.02	4.25	4.44	4.72	4.93	5.08	5.19	5.29	5.36	5.51
7.0	3.86	4.18	4.44	4.65	4.97	5.20	5.38	5.51	5.62	5.71	5.87
7.5	3.98	4.33	4.61	4.85	5.21	5.47	5.67	5.82	5.94	6.04	6.23
8.0	4.08	4.46	4.77	5.03	5.43	5.72	5.94	6.12	6.26	6.37	6.59
8.5	4.17	4.58	4.92	5.20	5.64	5.97	6.21	6.41	6.56	6.69	6.93
9.0	4.26	4.69	5.06	5.36	5.84	6.20	6.47	6.69	6.86	7.00	7.27
9.5	4.33	4.79	5.18	5.51	6.03	6.42	6.72	6.96	7.15	7.30	7.60
10.0	4.40	4.89	5.30	5.65	6.21	6.63	6.96	7.22	7.43	7.60	7.93
10.5	4.46	4.97	5.41	5.78	6.38	6.83	7.19	7.47	7.70	7.89	8.24
11.0	4.52	5.05	5.51	5.90	6.54	7.03	7.41	7.71	7.96	8.17	8.56
11.5	4.57	5.12	5.60	6.02	6.69	7.21	7.62	7.95	8.22	8.44	8.86
12.0	4.61	5.19	5.69	6.12	6.83	7.39	7.82	8.18	8.47	8.70	9.16
12.5	4.66	5.25	5.77	6.22	6.97	7.55	8.02	8.40	8.71	8.96	9.45
13.0	4.69	5.31	5.84	6.31	7.10	7.72	8.21	8.61	8.94	9.21	9.74
13.5	4.73	5.36	5.91	6.40	7.22	7.87	8.39	8.81	9.16	9.46	10.02
14.0	4.76	5.40	5.98	6.48	7.34	8.02	8.56	9.01	9.38	9.69	10.29
14.5	4.79	5.45	6.04	6.56	7.44	8.15	8.73	9.20	9.60	9.93	10.56
15.0	4.81	5.49	6.09	6.63	7.55	8.29	8.89	9.39	9.80	10.15	10.82
15.5	4.84	5.53	6.14	6.70	7.65	8.42	9.04	9.57	10.00	10.37	11.08
16.0	4.86	5.56	6.19	6.76	7.74	8.54	9.19	9.74	10.20	10.58	11.33
16.5	4.88	5.59	6.24	6.82	7.83	8.65	9.34	9.90	10.38	10.79	11.58
17.0	4.89	5.62	6.28	6.88	7.91	8.77	9.47	10.06	10.57	10.99	11.82
17.5	4.91	5.65	6.32	6.93	7.99	8.87	9.61	10.22	10.74	11.19	12.06
18.0	4.92	5.67	6.36	6.98	8.07	8.97	9.73	10.37	10.91	11.38	12.29
18.5	4.94	5.69	6.39	7.03	8.14	9.07	9.85	10.52	11.08	11.56	12.51
19.0	4.95	5.72	6.42	7.07	8.21	9.17	9.97	10.65	11.24	11.74	12.73
19.5	4.96	5.73	6.45	7.11	8.27	9.25	10.08	10.79	11.40	11.92	12.95
20.0	4.97	5.75	6.48	7.15	8.33	9.34	10.19	10.92	11.55	12.09	13.16

Table 6: Values of the equivalent depth of rigorous method (r_o = 0.1 m; R/K = 0.25)

D	L, Spacing of the Drainage (m)									
	5	10	15	20	25	30	35	40	45	50
0.50	0.50	0.51	0.51	0.52	0.52	0.52	0.52	0.52	0.53	0.53
0.75	0.64	0.68	0.70	0.71	0.72	0.73	0.73	0.74	0.74	0.74
1.00	0.74	0.83	0.87	0.90	0.91	0.92	0.93	0.94	0.94	0.95
1.25	0.81	0.96	1.02	1.06	1.09	1.11	1.12	1.13	1.14	1.15
1.50	0.85	1.06	1.16	1.21	1.25	1.28	1.30	1.31	1.33	1.34
1.75	0.88	1.15	1.28	1.35	1.41	1.44	1.47	1.49	1.51	1.52
2.00	0.90	1.22	1.39	1.48	1.55	1.60	1.63	1.66	1.68	1.70
2.25	0.92	1.28	1.48	1.60	1.68	1.74	1.79	1.82	1.85	1.87
2.50	0.92	1.33	1.56	1.71	1.81	1.88	1.93	1.98	2.01	2.04
2.75	0.93	1.37	1.64	1.81	1.92	2.01	2.07	2.12	2.17	2.20
3.00	0.93	1.41	1.70	1.90	2.03	2.13	2.21	2.27	2.32	2.36
3.25	-	-	1.76	1.98	2.13	2.25	2.33	2.40	2.46	2.51
3.50	-	-	1.81	2.05	2.23	2.35	2.45	2.53	2.60	2.65
3.75	-	-	1.86	2.12	2.31	2.46	2.57	2.66	2.73	2.79
4.00	-	-	1.89	2.18	2.39	2.55	2.68	2.78	2.86	2.93
4.25	-	-	1.93	2.24	2.47	2.64	2.78	2.89	2.98	3.06
4.50	-	-	1.96	2.29	2.54	2.73	2.88	3.00	3.10	3.18
4.75	-	-	1.99	2.34	2.60	2.81	2.97	3.10	3.21	3.30
5.00	-	-	2.01	2.38	2.66	2.88	3.06	3.20	3.32	3.42
5.25	-	-	2.03	2.42	2.72	2.95	3.14	3.30	3.43	3.53
5.50	-	-	2.05	2.45	2.77	3.02	3.22	3.39	3.53	3.64
5.75	-	-	2.06	2.48	2.82	3.08	3.30	3.48	3.62	3.75
6.00	-	-	2.07	2.51	2.86	3.14	3.37	3.56	3.72	3.85
6.25	-	-	2.09	2.54	2.90	3.20	3.44	3.64	3.81	3.95
6.50	-	-	2.10	2.56	2.94	3.25	3.50	3.71	3.89	4.04
6.75	-	-	2.10	2.58	2.97	3.30	3.56	3.78	3.97	4.13
7.00	-	-	2.11	2.60	3.01	3.34	3.62	3.85	4.05	4.22
7.25	-	-	2.12	2.62	3.04	3.38	3.67	3.92	4.13	4.31
7.50	-	-	2.12	2.63	3.06	3.42	3.73	3.98	4.20	4.39
7.75	-	-	2.13	2.65	3.09	3.46	3.78	4.04	4.27	4.47
8.00	-	-	2.13	2.66	3.11	3.50	3.82	4.10	4.34	4.54
8.25	-	-	2.13	2.67	3.13	3.53	3.87	4.15	4.40	4.62
8.50	-	-	2.14	2.68	3.15	3.56	3.91	4.20	4.46	4.69
8.75	-	-	2.14	2.69	3.17	3.59	3.95	4.25	4.52	4.75
9.00	-	-	2.14	2.70	3.19	3.61	3.98	4.30	4.58	4.82
9.25	-	-	2.14	2.70	3.20	3.64	4.02	4.35	4.63	4.88
9.50	-	-	2.14	2.71	3.22	3.66	4.05	4.39	4.68	4.94
9.75	-	-	2.14	2.72	3.23	3.69	4.08	4.43	4.73	5.00
10.00	-	-	2.14	2.72	3.24	3.71	4.11	4.47	4.78	5.06

Table 6 Cont.

D	Drainage Spacing, L (m)									
	50	60	70	80	100	120	140	160	180	200	250
0.5	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.54	0.54	0.54	0.54
1.0	0.95	0.96	0.96	0.97	0.98	0.98	0.99	0.99	0.99	1.00	1.00
1.5	1.34	1.36	1.37	1.38	1.40	1.41	1.42	1.43	1.44	1.44	1.45
2.0	1.70	1.73	1.76	1.78	1.80	1.83	1.84	1.85	1.87	1.87	1.89
2.5	2.04	2.09	2.12	2.15	2.19	2.22	2.25	2.27	2.28	2.30	2.32
3.0	2.36	2.42	2.47	2.51	2.57	2.61	2.64	2.67	2.69	2.71	2.75
3.5	2.65	2.74	2.80	2.85	2.93	2.99	3.03	3.06	3.09	3.11	3.16
4.0	2.93	3.03	3.12	3.18	3.28	3.35	3.40	3.45	3.48	3.51	3.57
4.5	3.18	3.31	3.42	3.5	3.62	3.70	3.77	3.82	3.86	3.90	3.97
5.0	3.42	3.58	3.70	3.8	3.94	4.05	4.12	4.19	4.24	4.28	4.36
5.5	3.64	3.83	3.97	4.09	4.26	4.38	4.47	4.55	4.61	4.66	4.75
6.0	3.85	4.07	4.23	4.36	4.56	4.70	4.81	4.90	4.97	5.02	5.14
6.5	4.04	4.29	4.48	4.63	4.85	5.02	5.14	5.24	5.32	5.39	5.51
7.0	4.22	4.50	4.71	4.88	5.14	5.32	5.46	5.57	5.66	5.74	5.89
7.5	4.39	4.69	4.93	5.12	5.41	5.62	5.78	5.90	6.00	6.09	6.25
8.0	4.54	4.88	5.14	5.36	5.68	5.91	6.08	6.22	6.34	6.43	6.61
8.5	4.69	5.05	5.34	5.58	5.93	6.19	6.38	6.54	6.66	6.77	6.97
9.0	4.82	5.22	5.53	5.79	6.18	6.46	6.67	6.84	6.98	7.10	7.32
9.5	4.94	5.37	5.71	5.99	6.42	6.72	6.96	7.14	7.30	7.42	7.67
10.0	5.06	5.52	5.89	6.19	6.64	6.98	7.24	7.44	7.60	7.74	8.01
10.5	5.16	5.65	6.05	6.37	6.87	7.23	7.51	7.73	7.91	8.06	8.34
11.0	5.26	5.78	6.20	6.55	7.08	7.47	7.77	8.01	8.20	8.37	8.67
11.5	5.35	5.90	6.35	6.72	7.29	7.71	8.03	8.29	8.49	8.67	9.00
12.0	5.43	6.01	6.49	6.88	7.48	7.93	8.28	8.55	8.78	8.97	9.32
12.5	5.51	6.12	6.62	7.03	7.68	8.16	8.52	8.82	9.06	9.26	9.64
13.0	5.58	6.22	6.74	7.18	7.86	8.37	8.76	9.08	9.33	9.55	9.96
13.5	5.65	6.31	6.86	7.32	8.04	8.58	9.00	9.33	9.60	9.83	10.27
14.0	5.71	6.40	6.97	7.45	8.21	8.78	9.22	9.58	9.87	10.11	10.57
14.5	5.77	6.48	7.08	7.58	8.38	8.98	9.44	9.82	10.13	10.38	10.87
15.0	5.82	6.56	7.18	7.70	8.54	9.17	9.66	10.06	10.38	10.65	11.17
15.5	5.87	6.63	7.27	7.82	8.69	9.35	9.87	10.29	10.63	10.91	11.46
16.0	5.91	6.70	7.36	7.93	8.84	9.53	10.08	10.51	10.87	11.17	11.75
16.5	5.96	6.76	7.44	8.03	8.98	9.71	10.27	10.73	11.11	11.43	12.04
17.0	5.99	6.82	7.52	8.13	9.12	9.87	10.47	10.95	11.35	11.68	12.32
17.5	6.03	6.87	7.60	8.23	9.25	10.04	10.66	11.16	11.58	11.92	12.59
18.0	6.06	6.92	7.67	8.32	9.38	10.20	10.84	11.37	11.80	12.17	12.87
18.5	6.09	6.97	7.74	8.41	9.50	10.35	11.02	11.57	12.02	12.40	13.14
19.0	6.12	7.02	7.80	8.49	9.62	10.50	11.20	11.77	12.24	12.64	13.40
19.5	6.14	7.06	7.86	8.57	9.73	10.64	11.37	11.96	12.45	12.87	13.66
20.0	6.16	7.10	7.92	8.64	9.84	10.78	11.53	12.15	12.66	13.09	13.92

Table 7: Values of the equivalent depth of rigorous method
(r_o = 0.1 m; R/K = 0 61685)

D	Drainage Spacing, L (m)									
	5	10	15	20	25	30	35	40	45	50
0.50	0.52	0.53	0.54	0.54	0.55	0.55	0.55	0.55	0.55	0.55
0.75	0.66	0.71	0.73	0.74	0.75	0.75	0.76	0.76	0.77	0.77
1.00	0.76	0.85	0.89	0.92	0.93	0.95	0.96	0.96	0.97	0.97
1.25	0.83	0.98	1.05	1.08	1.11	1.13	1.14	1.15	1.16	1.17
1.50	0.88	1.09	1.18	1.24	1.27	1.30	1.32	1.34	1.35	1.36
1.75	0.92	1.18	1.31	1.38	1.43	1.46	1.49	1.51	1.53	1.55
2.00	0.94	1.26	1.42	1.51	1.57	1.62	1.65	1.68	1.71	1.72
2.25	0.96	1.33	1.52	1.63	1.71	1.77	1.81	1.84	1.87	1.90
2.50	0.97	1.38	1.61	1.75	1.84	1.91	1.96	2.00	2.04	2.06
2.75	0.98	1.43	1.69	1.85	1.96	2.04	2.10	2.15	2.19	2.23
3.00	0.98	1.47	1.76	1.95	2.07	2.17	2.24	2.30	2.34	2.38
3.25	-	-	1.83	2.04	2.18	2.29	2.37	2.44	2.49	2.53
3.50	-	-	1.89	2.12	2.28	2.40	2.49	2.57	2.63	2.68
3.75	-	-	1.94	2.19	2.37	2.51	2.61	2.70	2.77	2.82
4.00	-	-	1.98	2.26	2.46	2.61	2.73	2.82	2.90	2.96
4.25	-	-	2.03	2.33	2.55	2.71	2.84	2.94	3.03	3.10
4.50	-	-	2.06	2.39	2.62	2.80	2.94	3.06	3.15	3.23
4.75	-	-	2.09	2.44	2.70	2.89	3.04	3.17	3.27	3.35
5.00	-	-	2.12	2.49	2.76	2.97	3.14	3.27	3.38	3.48
5.25	-	-	2.15	2.54	2.83	3.05	3.23	3.37	3.49	3.60
5.50	-	-	2.17	2.58	2.89	3.13	3.32	3.47	3.60	3.71
5.75	-	-	2.19	2.62	2.94	3.20	3.40	3.57	3.71	3.82
6.00	-	-	2.21	2.65	2.99	3.26	3.48	3.66	3.81	3.93
6.25	-	-	2.22	2.68	3.04	3.33	3.56	3.75	3.90	4.04
6.50	-	-	2.24	2.71	3.09	3.39	3.63	3.83	4.00	4.14
6.75	-	-	2.25	2.74	3.13	3.44	3.70	3.91	4.09	4.24
7.00	-	-	2.26	2.76	3.17	3.50	3.77	3.99	4.17	4.33
7.25	-	-	2.26	2.79	3.21	3.55	3.83	4.06	4.26	4.42
7.50	-	-	2.27	2.81	3.24	3.60	3.89	4.13	4.34	4.51
7.75	-	-	2.28	2.82	3.27	3.64	3.95	4.20	4.42	4.60
8.00	-	-	2.28	2.84	3.30	3.68	4.00	4.27	4.49	4.68
8.25	-	-	2.29	2.86	3.33	3.72	4.05	4.33	4.56	4.77
8.50	-	-	2.29	2.87	3.36	3.76	4.10	4.39	4.63	4.84
8.75	-	-	2.29	2.88	3.38	3.80	4.15	4.45	4.70	4.92
9.00	-	-	2.30	2.89	3.40	3.83	4.19	4.50	4.77	4.99
9.25	-	-	2.30	2.90	3.42	3.86	4.24	4.56	4.83	5.07
9.50	-	-	2.30	2.91	3.44	3.89	4.28	4.61	4.89	5.14
9.75	-	-	2.30	2.92	3.46	3.92	4.32	4.66	4.95	5.20
10.00	-	-	2.30	2.92	3.47	3.95	4.35	4.70	5.00	5.27

Table 7 cont.

D	Drainage Spacing, L (m)										
	50	60	70	80	100	120	140	160	180	200	250
0.5	0.55	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.57
1.0	0.97	0.98	0.99	0.99	1.00	1.01	1.01	1.02	1.02	1.02	1.03
1.5	1.36	1.38	1.40	1.41	1.42	1.44	1.45	1.46	1.46	1.47	1.48
2.0	1.72	1.76	1.78	1.80	1.83	1.85	1.87	1.88	1.89	1.90	1.92
2.5	2.06	2.11	2.15	2.17	2.22	2.25	2.27	2.29	2.31	2.32	2.35
3.0	2.38	2.45	2.49	2.53	2.59	2.63	2.67	2.69	2.72	2.73	2.77
3.5	2.68	2.76	2.83	2.88	2.95	3.01	3.05	3.09	3.11	3.14	3.18
4.0	2.96	3.07	3.15	3.21	3.30	3.37	3.43	3.47	3.50	3.53	3.59
4.5	3.23	3.35	3.45	3.53	3.64	3.73	3.79	3.84	3.89	3.92	3.99
5.0	3.48	3.63	3.74	3.83	3.97	4.07	4.15	4.21	4.26	4.30	4.39
5.5	3.71	3.89	4.02	4.13	4.29	4.41	4.50	4.57	4.63	4.68	4.77
6.0	3.93	4.13	4.29	4.41	4.60	4.73	4.84	4.92	4.99	5.05	5.16
6.5	4.14	4.37	4.54	4.68	4.90	5.05	5.17	5.26	5.34	5.41	5.53
7.0	4.33	4.59	4.79	4.95	5.19	5.36	5.50	5.60	5.69	5.76	5.91
7.5	4.51	4.80	5.02	5.20	5.47	5.66	5.81	5.93	6.03	6.11	6.27
8.0	4.68	5.00	5.24	5.44	5.74	5.96	6.13	6.26	6.37	6.46	6.64
8.5	4.84	5.19	5.46	5.68	6.01	6.25	6.43	6.58	6.70	6.80	6.99
9.0	4.99	5.37	5.66	5.90	6.26	6.53	6.73	6.89	7.02	7.13	7.35
9.5	5.14	5.54	5.86	6.12	6.51	6.80	7.02	7.19	7.34	7.46	7.69
10.0	5.27	5.70	6.05	6.33	6.75	7.06	7.30	7.50	7.65	7.78	8.04
10.5	5.39	5.86	6.23	6.53	6.99	7.32	7.58	7.79	7.96	8.10	8.38
11.0	5.51	6.00	6.40	6.72	7.21	7.58	7.86	8.08	8.26	8.41	8.71
11.5	5.61	6.14	6.56	6.90	7.43	7.82	8.12	8.36	8.56	8.72	9.04
12.0	5.71	6.27	6.71	7.08	7.65	8.06	8.38	8.64	8.85	9.03	9.37
12.5	5.81	6.39	6.86	7.25	7.85	8.30	8.64	8.91	9.14	9.33	9.69
13.0	5.90	6.51	7.01	7.42	8.05	8.52	8.89	9.18	9.42	9.62	10.01
13.5	5.98	6.62	7.14	7.57	8.25	8.75	9.13	9.44	9.70	9.91	10.32
14.0	6.05	6.72	7.27	7.72	8.43	8.96	9.37	9.70	9.97	10.20	10.63
14.5	6.12	6.82	7.39	7.87	8.62	9.17	9.61	9.95	10.24	10.48	10.94
15.0	6.19	6.91	7.51	8.01	8.79	9.38	9.83	10.20	10.50	10.75	11.24
15.5	6.25	7.00	7.62	8.14	8.96	9.58	10.06	10.44	10.76	11.03	11.54
16.0	6.31	7.08	7.73	8.27	9.13	9.77	10.28	10.68	11.01	11.30	11.84
16.5	6.36	7.16	7.83	8.39	9.29	9.96	10.49	10.92	11.27	11.56	12.13
17.0	6.41	7.24	7.93	8.51	9.44	10.15	10.70	11.14	11.51	11.82	12.42
17.5	6.46	7.31	8.02	8.63	9.59	10.33	10.90	11.37	11.75	12.08	12.70
18.0	6.50	7.37	8.11	8.73	9.74	10.50	11.10	11.59	11.99	12.33	12.98
18.5	6.54	7.43	8.19	8.84	9.88	10.67	11.30	11.81	12.22	12.58	13.26
19.0	6.58	7.49	8.27	8.94	10.01	10.84	11.49	12.02	12.45	12.82	13.53
19.5	6.61	7.54	8.35	9.03	10.15	11.00	11.68	12.23	12.68	13.06	13.80
20.0	6.64	7.60	8.42	9.13	10.27	11.16	11.86	12.43	12.90	13.30	14.07

Table 8: Values of the equivalent depth of rigorous method
(r_o = 0.1 m; R/K = 1.0)

D	Drainage Spacing, L (m)									
	5	10	15	20	25	30	35	40	45	50
0.50	0.54	0.55	0.56	0.56	0.57	0.57	0.57	0.57	0.57	0.57
0.75	0.68	0.72	0.74	0.76	0.77	0.77	0.78	0.78	0.79	0.79
1.00	0.78	0.87	0.91	0.94	0.95	0.97	0.97	0.98	0.99	0.99
1.25	0.85	1.00	1.06	1.10	1.13	1.15	1.16	1.17	1.18	1.19
1.50	0.91	1.11	1.20	1.26	1.29	1.32	1.34	1.36	1.37	1.38
1.75	0.95	1.21	1.33	1.40	1.45	1.48	1.51	1.53	1.55	1.57
2.00	0.97	1.29	1.44	1.53	1.59	1.64	1.67	1.70	1.72	1.74
2.25	0.99	1.36	1.54	1.66	1.73	1.79	1.83	1.86	1.89	1.92
2.50	1.00	1.42	1.64	1.77	1.86	1.93	1.98	2.02	2.05	2.08
2.75	1.01	1.47	1.72	1.88	1.98	2.06	2.12	2.17	2.21	2.25
3.00	1.01	1.51	1.80	1.98	2.10	2.19	2.26	2.32	2.36	2.40
3.25	-	-	1.87	2.07	2.21	2.31	2.39	2.46	2.51	2.56
3.50	-	-	1.93	2.15	2.31	2.43	2.52	2.59	2.65	2.70
3.75	-	-	1.98	2.23	2.41	2.54	2.64	2.72	2.79	2.85
4.00	-	-	2.03	2.31	2.50	2.65	2.76	2.85	2.93	2.99
4.25	-	-	2.08	2.37	2.59	2.75	2.87	2.97	3.05	3.12
4.50	-	-	2.12	2.44	2.67	2.84	2.98	3.09	3.18	3.26
4.75	-	-	2.15	2.49	2.74	2.93	3.08	3.20	3.30	3.38
5.00	-	-	2.18	2.55	2.82	3.02	3.18	3.31	3.42	3.51
5.25	-	-	2.21	2.60	2.88	3.10	3.27	3.42	3.53	3.63
5.50	-	-	2.24	2.64	2.95	3.18	3.37	3.52	3.64	3.75
5.75	-	-	2.26	2.68	3.01	3.25	3.45	3.61	3.75	3.86
6.00	-	-	2.28	2.72	3.06	3.33	3.54	3.71	3.85	3.97
6.25	-	-	2.30	2.76	3.11	3.39	3.62	3.80	3.95	4.08
6.50	-	-	2.31	2.79	3.16	3.46	3.69	3.89	4.05	4.18
6.75	-	-	2.32	2.82	3.21	3.52	3.76	3.97	4.14	4.29
7.00	-	-	2.34	2.85	3.25	3.57	3.83	4.05	4.23	4.38
7.25	-	-	2.35	2.87	3.29	3.63	3.90	4.13	4.32	4.48
7.50	-	-	2.35	2.90	3.33	3.68	3.97	4.20	4.40	4.57
7.75	-	-	2.36	2.92	3.36	3.73	4.03	4.27	4.48	4.66
8.00	-	-	2.37	2.93	3.40	3.77	4.09	4.34	4.56	4.75
8.25	-	-	2.37	2.95	3.43	3.82	4.14	4.41	4.64	4.84
8.50	-	-	2.38	2.97	3.46	3.86	4.19	4.48	4.71	4.92
8.75	-	-	2.38	2.98	3.48	3.90	4.25	4.54	4.78	5.00
9.00	-	-	2.38	2.99	3.51	3.94	4.29	4.60	4.85	5.08
9.25	-	-	2.38	3.00	3.53	3.97	4.34	4.65	4.92	5.15
9.50	-	-	2.39	3.01	3.55	4.00	4.39	4.71	4.98	5.22
9.75	-	-	2.39	3.02	3.57	4.04	4.43	4.76	5.05	5.29
10.00	-	-	2.39	3.03	3.59	4.06	4.47	4.81	5.11	5.36

Table 8 cont.

D	Drainage Spacing, L (m)									
	50	60	70	80	100	120	140	160	180	200	250
0.5	0.57	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.59	0.59	0.59
1.0	0.99	1.00	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.05	1.05
1.5	1.38	1.40	1.41	1.43	1.44	1.46	1.47	1.48	1.48	1.49	1.50
2.0	1.74	1.77	1.80	1.82	1.85	1.87	1.89	1.90	1.91	1.92	1.94
2.5	2.08	2.13	2.16	2.19	2.24	2.27	2.29	2.31	2.33	2.34	2.37
3.0	2.40	2.47	2.51	2.55	2.61	2.65	2.69	2.71	2.73	2.75	2.79
3.5	2.70	2.78	2.85	2.90	2.97	3.03	3.07	3.10	3.13	3.16	3.20
4.0	2.99	3.09	3.17	3.23	3.32	3.39	3.44	3.49	3.52	3.55	3.61
4.5	3.26	3.38	3.47	3.55	3.66	3.75	3.81	3.86	3.90	3.94	4.01
5.0	3.51	3.65	3.77	3.86	3.99	4.09	4.17	4.23	4.28	4.32	4.40
5.5	3.75	3.92	4.05	4.15	4.31	4.43	4.52	4.59	4.65	4.70	4.79
6.0	3.97	4.17	4.32	4.44	4.62	4.76	4.86	4.94	5.01	5.06	5.18
6.5	4.18	4.41	4.58	4.71	4.92	5.07	5.19	5.29	5.36	5.43	5.55
7.0	4.38	4.63	4.83	4.98	5.22	5.39	5.52	5.62	5.71	5.78	5.92
7.5	4.57	4.85	5.06	5.24	5.50	5.69	5.84	5.96	6.05	6.13	6.29
8.0	4.75	5.06	5.29	5.48	5.78	5.99	6.15	6.28	6.39	6.48	6.65
8.5	4.92	5.25	5.51	5.72	6.04	6.28	6.46	6.60	6.72	6.82	7.01
9.0	5.08	5.44	5.72	5.95	6.30	6.56	6.76	6.92	7.04	7.15	7.36
9.5	5.22	5.62	5.93	6.18	6.56	6.84	7.05	7.22	7.36	7.48	7.71
10.0	5.36	5.79	6.12	6.39	6.80	7.11	7.34	7.53	7.68	7.81	8.06
10.5	5.49	5.95	6.30	6.60	7.04	7.37	7.62	7.82	7.99	8.13	8.40
11.0	5.62	6.10	6.48	6.79	7.27	7.62	7.90	8.11	8.29	8.44	8.73
11.5	5.73	6.24	6.65	6.99	7.50	7.87	8.17	8.40	8.59	8.75	9.07
12.0	5.84	6.38	6.82	7.17	7.72	8.12	8.43	8.68	8.89	9.06	9.39
12.5	5.94	6.51	6.97	7.35	7.93	8.36	8.69	8.96	9.18	9.36	9.72
13.0	6.04	6.64	7.12	7.52	8.13	8.59	8.94	9.23	9.46	9.66	10.04
13.5	6.13	6.76	7.26	7.68	8.33	8.82	9.19	9.49	9.74	9.95	10.35
14.0	6.21	6.87	7.40	7.84	8.53	9.04	9.44	9.75	10.02	10.24	10.66
14.5	6.29	6.97	7.53	8.00	8.72	9.26	9.68	10.01	10.29	10.52	10.97
15.0	6.36	7.07	7.66	8.14	8.90	9.47	9.91	10.26	10.56	10.80	11.28
15.5	6.43	7.17	7.78	8.28	9.08	9.67	10.14	10.51	10.82	11.08	11.58
16.0	6.49	7.26	7.89	8.42	9.25	9.88	10.36	10.75	11.08	11.35	11.88
16.5	6.55	7.35	8.00	8.55	9.42	10.07	10.58	10.99	11.33	11.62	12.17
17.0	6.61	7.43	8.11	8.68	9.58	10.26	10.80	11.23	11.58	11.88	12.46
17.5	6.66	7.50	8.21	8.80	9.74	10.45	11.01	11.46	11.83	12.14	12.75
18.0	6.71	7.58	8.30	8.92	9.89	10.63	11.21	11.68	12.07	12.40	13.03
18.5	6.75	7.64	8.39	9.03	10.04	10.81	11.42	11.90	12.31	12.65	13.31
19.0	6.80	7.71	8.48	9.14	10.18	10.98	11.61	12.12	12.54	12.90	13.59
19.5	6.83	7.77	8.56	9.24	10.32	11.15	11.81	12.34	12.78	13.15	13.87
20.0	6.87	7.83	8.64	9.34	10.46	11.32	12.00	12.55	13.00	13.39	14.14

5.0 CONCLUSIONS

In this report, a drain spacing formula has been derived rigorously considering the variation in radial flow and the area in the radial flow zone above the drain level. The maximum water table height computed rigorously is compared with that obtained by Hooghoudt's formula. The Hooghoudt's formula computes maximum water table height, which is very close to the value computed by the rigorous method. For normal recharge rate and shallow draining layer, the error involved in computation of maximum water table height by Hooghoudt's formula is less than 1 %. The water table height near the drains computed using Hooghoudt's equivalent depth differs considerably from the one computed by the rigorous method. The simple Hooghoudt's drain spacing formula based on equivalent depth is accurate enough to be used for normal recharge rate.

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APPENDIX I. NOTATION

The following symbols are used in this report:

A	: area through which radial flow takes place (m^2)
D	: height above the impervious floor of the water level in the drains i.e. thickness of aquifer below drain level (m)
h	: hydraulic head (m)
h_1	: hydraulic head dissipated in horizontal flow zone (m)
h_2	: hydraulic head dissipated in radial flow zone (m)
h_{max}	: maximum water level height at the middle of the two-drainpipe (m)
K	: hydraulic conductivity of the soil (m/day)
L	: drainage spacing (m)
q_r	: radial flux (m/day)
R	: recharge rate per unit surface area (m/day)
r	: radial distance from the center of the drain pipe (m)
r_o	: radius of the drainage pipe (m)

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