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Stream-Aquifer Interaction in Multi-Laycred System

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Abstract

A system of extensive aquifers sandwiched between silty or clayey beds are common in alluvial basins. Such a system is known as multi-aquifer or multi-layered system. Often a river penetrates fully or partially the upper aquifer in such multi-layered system. When the river stage rises during the passage of a flood, the upper aquifer is recharged through the bed and banks of the river. The lower aquifers are recharged through the intervening aquitards.

In the present report, a mathematical model has been developed for study of stream-aquifer interaction in (multi-layered system) three aquifer-system in which aquifers are separated by aquitards, considering varying stream-stage and vertical flow through aquitards. The model enables the computation of rate of recharges to the upper aquifer and exchanges of flow among aquifers along with the spatial and temporal distributions of piezomeric heads in the aquifers. The methodology presented by Mishra(1987b) for two-aquifer system, has been shown to be the special case of present formulation.

The model has been validated against the numerical simulation using MODFLOW for different cases. MODFLOW source code was modified to get the exchange of flow from each strip and its spatial and temporal summation. Close reproduction of results using MODFLOW show that the methodology based on discrete kernel approach, is valid for the analysis of the interaction of stream and multi-aquifer system.

Contents

1.0 INTRODUCTION

Stratified soil deposits having different soil and hydraulic properties are common in both alluvial and coastal basins. These basins generally have permeable strata consisting of sand and gravel deposits (aquifers) which are sandwiched between silty and clayey beds (less permeable strata). Such leaky multi-aquifer systems idealized as a sequence of alternating aquifers and aquitards, are common in alluvial basins. A stream in a multi-aquifer system may fully or partially penetrate the upper aquifer and thus, recharges the upper aquifer due to seepage occurring through its bed and bank. This excitation in the top aquifer induces exchange of flow between aquifers through intervening aquitards. When the stream-stage varies during the passage of a flood, rate of recharge to the upper aquifer varies and consequently, the recharge to the lower aquifers varies with time.

Though, a single aquifer river interaction problem has been studied analytically by several investigators (Morel-Seytoux and Daly, 1975; Todd, 1955; Cooper and Rorabaugh, 1963), very studies have been done for analyzing stream-aquifer interaction in a multi-layered system. A digital model of multi-aquifer system has been developed by Bredehoeft and Pinder (1970) assuming horizontal flow in the aquifers and vertical flow through the confining layers which separate the aquifers. Mass (1986) have presented the use of matrix differential calculus in problems of multi-aquifer flow. Thus, there is a need to study the interaction of stream and multi-aquifer system.

In the present report a mathematical model has been developed for the study of stream-aquifer interaction in (multi-layered system) three aquifer-system in which aquifers are separated by aquitards, considering varying stream-stage and vertical flow through aquitards. The model enables the computation of rate of recharges to the upper aquifer and exchanges of flow among aquifers along with the spatial and temporal distributions of piezomeric heads in the aquifers. The model has been validated against the numerical simulation using MODFLOW for different cases.

2.0 REVIEW

A brief review of aquifer recharge studies on account of time-varying stream stage are presented below.

2.1 Interaction of Single aquifer and stream:

The equation for piezometric head variation in an homogeneous and isotropic infinite confined aquifer due to a step rise in the stage of the stream which penetrates the full depth of the aquifer, can be obtained from the solution of heat conduction problem in a semi-infinite bar (Carslaw and Jaeger,1959) and is expressed as:

$$h(x,t) = H \, \mathrm{erf}\left(\frac{x}{4\alpha t} \right) \tag{2.1}$$

where,

$h(x,t)$ = head in the aquifer at x after time t, (L);
x = distance from the stream-bank, (L);
α = aquifer diffusivity = T/ϕ , (L^2T^{-1});
H = step rise in the river stage, (L);
t = time, (T).

Cooper and Rorabaugh(1963) have given the solution of stream-aquifer interaction problem for a generalized flood wave in a stream which penetrates the full depth of an homogeneous and isotropic aquifer. They proposed the analytical form of generalized flood wave in the stream. They derived expressions for piezometric head in the aquifer at any section, discharge from the stream, and bank storage volumes when a generalized flood wave passes in the stream for both finite and infinite confined aquifers. Verma and Brutsaert(1970) developed a numerical scheme to analyze the flow in a two-dimensional unconfined aquifer. They determined the fall of water table and rate of outflow into an adjoining fully penetrating stream. Marino(1973) has developed analytical expressions which describe the water table fluctuations in a semi-pervious stream and unconfined aquifer system. He has considered semi-infinite and finite aquifer systems, with and without semi-pervious stream banks. Singh and Sagar (1979) have studied the flow in a semi-infinite aquifer with a fully penetrating stream as its boundary using linearized Boussinesq equation.

Morel-Seytoux and Daly(1975) proposed a discrete impulse kernel approach for stream-aquifer interaction studies for time-varying stage in a partially penetrating stream. They have assumed that the recharge from a reach at any time is directly proportional to the difference between the stream-stage and head in the aquifer in the vicinity of the stream. Using discrete kernel approach Morel-Seytoux(1988) has presented the expression for the mean return flow during n^{th} time interval from a fully penetrating stream due to a pattern of river drawdown characterized by mean values over the time interval. Mishra(1987a) has studied the exchange of flow between a partially penetrating river and a homogeneous aquifer for time varying river-stage using discrete kernel approach employing the concept of reach transmissivity as proposed by Morel-Seytoux and Daly (1977). He used the analytical form of Cooper and Rorabaugh's flood wave. Detailed review on the subject can be had from Singh(1994).

2.2 Interaction of Multi aquifer and stream:

A digital model of multi-aquifer system has been developed by Bredehoeft and Pinder (1970) assuming horizontal flow in the aquifers and vertical flow through the confining layers which separate the aquifers. These assumptions have reduced the mathematical problem to one of solving coupled two dimensional equation for each aquifer in the system. An interactive, alternating-direction-implicit scheme has been used to solve the system of simultaneous finite difference equations which describe the response of the aquifer system to applied stresses. The quasi three-dimensional model has been developed to simulate a groundwater system having any number of aquifers.

Mass (1986) have presented the use of matrix differential calculus in problems of multi-aquifer flow. Steady and unsteady multi-aquifer flow near a river fully penetrating all the aquifers, fully penetrating upper few aquifers have been discussed. Mishra(1987b) has given a discrete kernel approach for solving the problem of partially penetrating stream in two-aquifer system.

2.3 Concluding Remark:

From review of literature, it is observed that considerable research has been done on the interaction of stream and single aquifer. In spite of the early recognition of stream-aquifer interaction studies, very few attempts have been made towards the understanding of interaction of stream and multi-aquifer system. Therefore, there is a need to study in detail the problem of interaction of stream and multi-aquifer system.

3.0 STATEMENT OF THE PROBLEM

A schematic section of a partially penetrating river in a five-layer multi-aquifer system is shown in Fig.3.1. The three aquifers are separated by two aquitards. Each aquifer is considered homogeneous, isotropic and infinite in areal extent. The aquitards can have variable thickness and vertical permeability in the direction perpendicular to the stream. However, in the direction aligned to the stream, aquitard's properties are assumed constant. The stream and the three aquifers are initially in equilibrium. Due to passage of a flood, the changes in the stream-stage are assumed identical over a long reach of the stream.

It is required,

1. to develop a mathematical model to find the recharge from the river to the top aquifer; the exchange of flow between the aquifers through the intervening aquitards; and the piezomeric head distributions in the aquifers.

2. to validate the model against the results of simulation of the problem using a three dimensional groundwater flow model, MODFLOW.

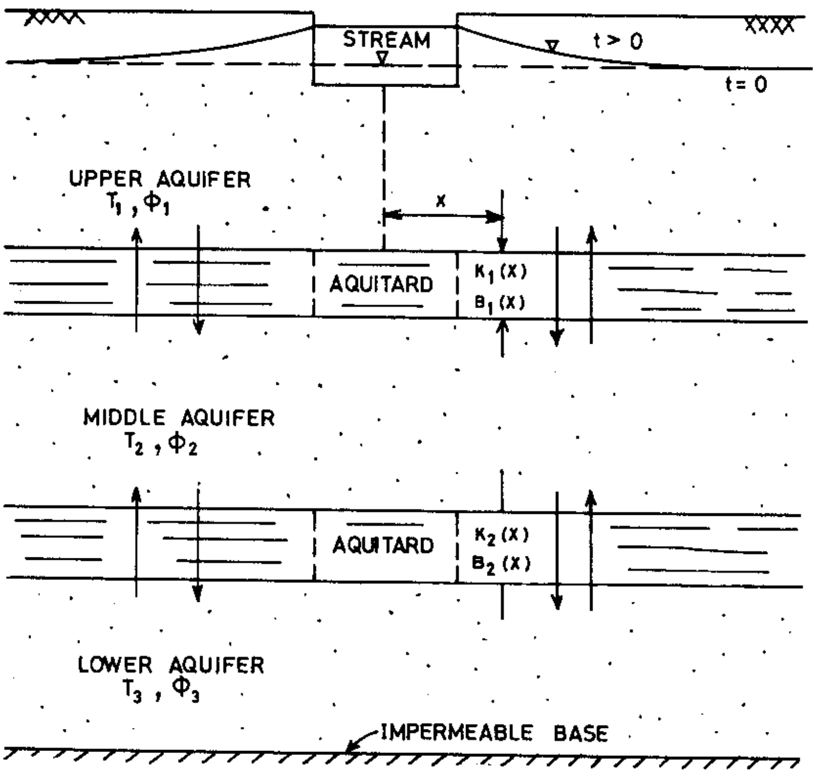

FIG. 3·1: SCHEMATIC SECTION OF STREAM IN MULTI-AQUIFER SYSTEM

4.0 METHODOLOGY

4.1 Assumptions:

The following assumptions are made for the analysis.

1. The flow in each aquifer is in horizontal direction and one dimensional Boussinesq's equation governs the flow in each aquifer.

2. The flow in the aquitards are essentially vertical and there are no release of water from storage of aquitards. The variation in the thicknesses and vertical conductivities of aquitards over a large length parallel to the stream is negligible. However, the variation in the thicknesses and vertical conductivities of aquitards along a line normal to the stream has been taken care of.

3. The exchange of flow between the stream and the upper aquifer is proportional to the difference in the potentials at the river boundary and in the upper aquifer below the river bed.

4.2 Analysis:

The differential equations which govern the flow in the upper, the middle and the lower aquifer respectively are,

$$T_1 \frac{\partial^2 s_1}{\partial x^2} = \phi_1 \frac{\partial s_1}{\partial t} - q_1 \quad ; \qquad (4.1)$$

$$T_2 \frac{\partial^2 s_2}{\partial x^2} = \phi_2 \frac{\partial s_2}{\partial t} + q_1 - q_2 \qquad (4.2)$$

and,

$$T_3 \frac{\partial^2 s_3}{\partial x^2} = \phi_3 \frac{\partial s_3}{\partial t} + q_2 \qquad (4.3)$$

where,

$$s_1 = s_1(x,t) \quad ; \quad s_2 = s_2(x,t) \quad ; \quad s_3 = s_3(x,t) \tag{4.4}$$

$$q_1 = q_1(x,t) \quad ; \quad q_2 = q_2(x,t) \tag{4.5}$$

If the water level in the stream and the piezometric head in every aquifer are assumed at the same level initially, the initial conditions and the boundary conditions to be satisfied can be expressed as,

$$s_1(x,0) = 0 \quad ; \quad s_2(x,0) = 0 \quad ; \quad s_3(x,0) = 0 \tag{4.6}$$

$$s_1(\pm\infty,t) = 0 \quad ; \quad s_2(\pm\infty,t) = 0 \quad ; \quad s_3(\pm\infty,t) = 0 \tag{4.7}$$

where,

$s_1(x,t)$ = water table rise in the upper aquifer;
$s_2(x,t)$ = water table rise in the middle aquifer;
$s_3(x,t)$ = water table rise in the lower aquifer;
T_1 = transmissivity of the upper aquifer;
T_2 = transmissivity of the middle aquifer;
T_3 = transmissivity of the lower aquifer;
ϕ_1 = storage coefficient of the upper aquifer;
ϕ_2 = storage coefficient of the middle aquifer;
ϕ_3 = storage coefficient of the lower aquifer;
$q_1(x,t)$ = recharge rate from the upper aquifer to the middle aquifer per unit area;
$q_2(x,t)$ = recharge rate from the middle aquifer to the lower aquifer per unit area.

The exchange of flow between the upper aquifer and the middle aquifer is assumed essentially vertical through the intervening aquitard. Therefore, the recharge rate per unit area from the upper aquifer to the middle aquifer at section x after time t, i.e., $q_1(x,t)$, is given by,

$$q_1(x,t) = \frac{K_1(x)}{B_1(x)} \left[s_1(x,t) - s_2(x,t)\right] \tag{4.8}$$

Similarly, since, the exchange of flow between the middle aquifer and the lower aquifer is in vertical direction through the intervening aquitard, the recharge rate per unit area from the middle aquifer to the lower aquifer at section x after time t, i.e., $q_2(x,t)$, is given by,

$$q_2(x,t) = \frac{K_2(x)}{B_2(x)} \left[s_2(x,t) - s_3(x,t)\right] \tag{4.9}$$

where,

$K_1(x)$ = vertical hydraulic conductivity of the upper aquitard at section x;
$K_2(x)$ = vertical hydraulic conductivity of the lower aquitard at section x;
$B_1(x)$ = thickness of upper aquitard at section x, and;
$B_2(x)$ = thickness of lower aquitard at section x.

4.3 Discretization:

The upper, the middle and the lower aquifer along with the intervening aquitards have been divided identically into a number of strips of varying widths as shown in Fig.4.1. Width of i^{th} strip is w(i). Recharge to the upper aquifer takes place from the stream. The rate of recharge to the upper aquifer at any time has been considered directly proportional to the difference between the river stage and the rise of water table in the upper aquifer at the strip below the stream. If the stream fully penetrates the upper aquifer, the rise in water table at the river node in the upper aquifer is equal to rise in river stage at any time. For a partially penetrating stream, rise in the piezometric level of upper aquifer at the strip below the stream is not equal to the rise in the stream stage and is to be determined as a part of the solution.

Let the time span be discretized into a number of uniform time-steps of Δt duration. It is assumed that the recharges are constant within a time step and within the span of a strip.

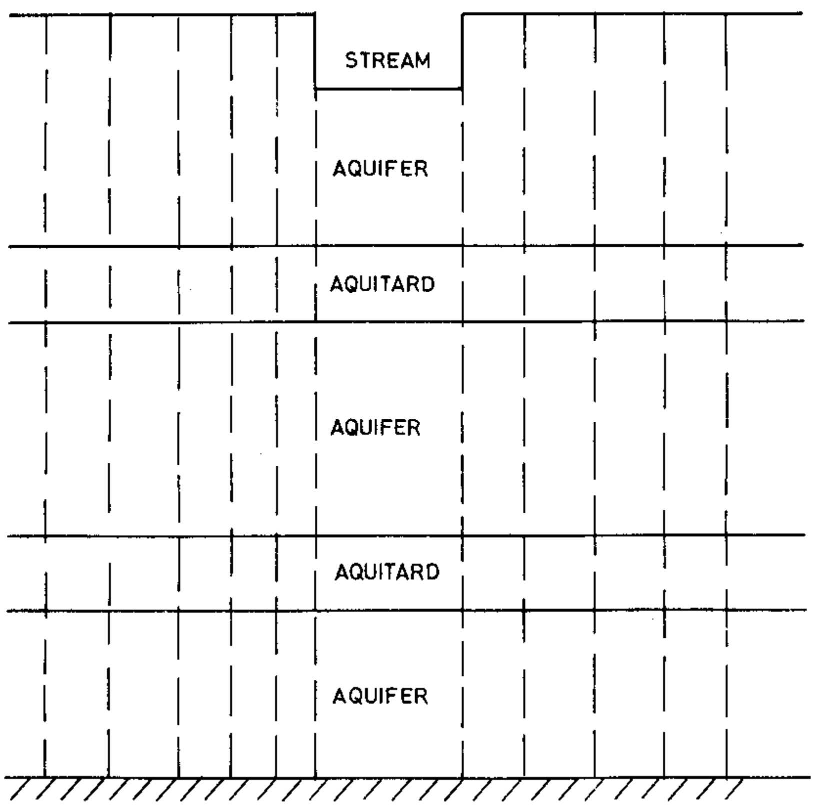

FIG. 4·1 : DISCRETIZED MULTI AQUIFER SYSTEM

4.4 Expressions for s_2, s_3, and s_1:

The rise in piezometric surface at j^{th} strip in the middle aquifer after time t due to the unit step rise in the rate of recharge per unit area from the i^{th} strip, $K_2(i,j,t)$, is given by:

For $r_{ij} < w(i)/2$;

$$K_2(i,j,t) = F\left[r_{ij}, T_2, \phi_2, w(i), t\right] - \frac{r_{ij}^2 + 0.25\ \{w(i)\}^2}{2T_2} \tag{4.10}$$

and for $r_{ij} > w(i)/2$;

$$K_2(i,j,t) = F\left[r_{ij}, T_2, \phi_2, w(i), t\right] - \frac{r_{ij}\ w(i)}{2T_2} \tag{4.11}$$

in which function F[.] is given by,

$$\begin{aligned} F\left[r, T, \phi, w(i), t\right] = & \left(\frac{(r+0.5w(i))^2}{4T} + \frac{t}{2\phi}\right) \mathrm{erf}\left(\frac{r+0.5w(i)}{\sqrt{4\alpha t}}\right) - \\ & \left(\frac{(r-0.5w(i))^2}{4T} + \frac{t}{2\phi}\right) \mathrm{erf}\left(\frac{r-0.5w(i)}{\sqrt{4\alpha t}}\right) + \\ & \left(\frac{(r+0.5w(i))\sqrt{\alpha t}}{2T\sqrt{\pi}}\right) \exp\left(-\frac{(r+0.5w(i))^2}{4\alpha t}\right) - \\ & \left(\frac{(r-0.5w(i))\sqrt{\alpha t}}{2T\sqrt{\pi}}\right) \exp\left(-\frac{(r-0.5w(i))^2}{4\alpha t}\right) \ ; \end{aligned} \tag{4.12}$$

Where,

r_{ij} = centre to centre distance from i^{th} strip to j^{th} strip ;

$\alpha = \frac{T}{\phi}$, and;

$$\text{erf}(u) = \text{error function} = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-x^2)dx$$

Rise in the piezometric surface at j^{th} strip in the middle aquifer due to time variant recharge taking place through all the strips can be expressed as,

$$s_2(j,n) = \sum_{\gamma=1}^{n} \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,n-\gamma+1) \qquad (4.13)$$

where,

$$\delta_2(i,j,m) = K_2(i,j,m) - K_2(i,j,m-1) \qquad (4.14)$$

in which,

N = number of strips;
n = index denoting time step.

Similarly, rise in the piezometric surface in j^{th} strip of the lower aquifer due to time variant recharge taking place through all the strips can be expressed as,

$$s_3(j,n) = \sum_{\gamma=1}^{n} \sum_{i=1}^{N} q_2(i,\gamma)\delta_3(i,j,n-\gamma+1) \qquad (4.15)$$

where,

$$\delta_3(i,j,m) = K_3(i,j,m) - K_3(i,j,m-1) \qquad (4.16)$$

$K_3(i,j,m)$ is the rise in piezometric surface at j^{th} strip in lower aquifer at the end of m^{th} time step on account of recharge occurring at unit rate per unit area through i^{th} strip. $K_3(i,j,m)$ can be expressed as, For $r_{ij} < w(i)/2$;

$$K_3(i,j,m) = F\left[r_{ij}, T_3, \phi_3, w(i), m\right] - \frac{r_{ij} + 0.25\,\{w(i)\}^2}{2T_3} \tag{4.17}$$

and for $r_{ij} > w(i)/2$;

$$K_3(i,j,m) = F\left[r_{ij}, T_3, \phi_3, w(i), t\right] - \frac{r_{ij}\,w(i)}{2T_3} \tag{4.18}$$

Let the bottom of the river be designated by i_0^{th} strip. The rise in the piezometric surface in i^{th} strip of the upper aquifer at the end of n^{th} time step can be expressed as,

$$s_1(j,n) = \sum_{\gamma=1}^{n} \frac{Q(\gamma)}{w(i_0)}\,\delta_1(i_0,j,n-\gamma+1) - \sum_{\gamma=1}^{n}\sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,j,n-\gamma+1) \tag{4.19}$$

Where,

$$\delta_1(i,j,m) = K_1(i,j,m) - K_1(i,j,m-1) \tag{4.20}$$

$K_1(i,j,m)$ is the rise in piezometric surface at j^{th} strip in the lower aquifer at the end of m^{th} time step on account of recharge occurring at unit rate per unit area through i^{th} strip. $Q(\gamma)$ is the rate of recharge from the stream to the upper aquifer during γ^{th} time step, and; $w(i_0)$ is the width of the stream. $K_3(i,j,m)$ can be expressed as,

For $r_{ij} < w(i)/2$;

$$K_1(i,j,m) = F\left[r_{ij}, T_1, \phi_1, w(i), m\right] - \frac{r_{ij}^2 + 0.25\,\{w(i)\}^2}{2T_1} \tag{4.21}$$

and for $r_{ij} > w(i)/2$;

$$K_1(i,j,m) = F\left[r_{ij}, T_1, \phi_1, w(i), t\right] - \frac{r_{ij}\, w(i)}{2T_1} \tag{4.22}$$

Since, the recharge from the stream is proportional to the difference in the stream water level and piezometric surface at the i_0^{th} strip in upper aquifer, expression for recharge from the stream can be written as,

$$Q(n) = \Gamma\left[\sigma(n) - s_1(i_0,n)\right] \tag{4.23}$$

Where,

Γ = reach transmissivity for unit length of the stream;
$\sigma(n)$ = stream-stage during time step n;
$s_1(i_0,n)$ = rise in piezometric surface below stream bed;
$Q(n)$ = rate of recharge from stream during n^{th} time step.

4.5 Solution of the problem:

Substituting s_1 and s_2 from eqs. (4.19) and (4.13) respectively into eq. (4.8), we get,

$$\frac{B_1(j)}{K_1(j)} q_1(j,n) = \sum_{\gamma=1}^{n} \frac{Q(\gamma)}{w(i_0)} \delta_1(i_0,j,n-\gamma+1) - \sum_{\gamma=1}^{n} \sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,j,n-\gamma+1) -$$

$$\sum_{\gamma=1}^{n} \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,n-\gamma+1) \tag{4.24}$$

Substituting s_2 and s_3 from eqs. (4.13) and (4.15) respectively into eq. (4.9), we get,

$$\frac{B_2(j)}{K_2(j)} q_2(j,n) = \sum_{\gamma=1}^{n} \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,n-\gamma+1) - \sum_{\gamma=1}^{n} \sum_{i=1}^{N} q_2(i,\gamma)\delta_3(i,j,n-\gamma+1) \tag{4.25}$$

Substituting the value of s_1 from eq. (4.19) into eq. (4.23), we get,

$$Q(n) = \Gamma \left\{\sigma(n) - \sum_{\gamma=1}^{n} \frac{Q(\gamma)}{w(i_0)} \delta_1(i_0,i_0,n-\gamma+1) + \sum_{\gamma=1}^{n} \sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,i_0,n-\gamma+1)\right\} \tag{4.26}$$

Rearranging the temporal terms of eqs. (4.24), (4.25) & (4.26) respectively, we get,

$$\frac{B_1(j)}{K_1(j)} q_1(j,n) - \frac{\delta_1(i_0,j,1)}{w(i_0)} Q(n) + \sum_{i=1}^{N} q_1(i,n)\delta_1(i,j,1) + \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,1) =$$

$$\sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{w(i_0)} \delta_1(i_0,j,n-\gamma+1) - \sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,j,n-\gamma+1) -$$

$$\sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,n-\gamma+1) \tag{4.27}$$

$$\frac{B_2(j)}{K_2(j)} q_2(j,n) - \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,1) + \sum_{i=1}^{N} q_2(i,n)\delta_3(i,j,1) =$$

$$\sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} \left[q_1(i,\gamma)-q_2(i,\gamma)\right]\delta_2(i,j,n-\gamma+1) - \sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} q_2(i,\gamma)\delta_3(i,j,n-\gamma+1) \qquad (4.28)$$

$$\left[\frac{1}{r} + \frac{\delta_1(i_0,i_0,1)}{w(i_0)}\right] Q(n) - \sum_{i=1}^{N} q_1(i,n)\delta_1(i,i_0,1) = - \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{w(i_0)} \delta_1(i_0,i_0,n-\gamma+1) +$$

$$\sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,i_0,n-\gamma+1) \qquad (4.29)$$

N number of equations one each for a recharge strip can be obtained making use of (4.27) for each time step. Another N number of equations one each for a recharge strip can be obtained making use of eq. (4.28) for each time-step. Besides, these 2N equations, one equation can be written making use of eq.(4.29) for each time-step. Thus, for each time step, 2N+1 equations can be written in 2N+1 unknowns [i.e., $q_1(i,n)$, i= 1, 2 ...N, $q_2(i,n)$, i= 1, 2 ...N, and Q(n)]. Therefore, the recharges from all the strips and the recharge from stream-bed can be solved for each time step in succession, starting from first time step making use of eqs. (4.27), (4.28), and (4.29) as stated above.

The above methodology can easily be extended to account for more than three aquifers on similar lines as explained in sections 4.4 & 4.5.

4.6 Reduction to two Aquifer System:

The problem solved above is stream-aquifer interaction in a three aquifer system in which aquifers are separated by aquitards and where cross flow through aquitards have been considered. For recharge from stream in a two-aquifer system in which aquifers are separated by an aquitard, $q_2(i,n)=0$ for each time-step and $K_2(j)/B_2(j)=0$ for each strip. Therefore, putting $q_2(i,n)$, i= 1, 2, ...N equals to zero for each time step, and letting $K_2(j)/B_2(j) = 0$ for each strip; one can get the solution for two-aquifer problem. Thus, eqs. (4.28) which is obtained from eq. (4.25) reduces to null equation and; eq. (4.27), & (4.29) reduce to following forms which are applicable to two-aquifer system.

$$\frac{B_1(j)}{K_1(j)} q_1(j,n) - \frac{\delta_1(i_0,j,1)}{w(i_0)} Q(n) + \sum_{i=1}^{N} q_1(i,n)\left[\delta_1(i,j,1) + \delta_2(i,j,1)\right] =$$

$$\sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{w(i_0)} \delta_1(i_0,i_0,n-\gamma+1) - \sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,i_0,n-\gamma+1) -$$

$$\sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} q_1(i,\gamma)\delta_2(i,i_0,n-\gamma+1) \tag{4.30}$$

$$\left(\frac{1}{\Gamma} + \frac{\delta_1(i_0,i_0,1)}{w(i_0)}\right) Q(n) - \sum_{i=1}^{N} q_1(i,n)\delta_1(i,i_0,1) = \sigma(n) - \sum_{\gamma=1}^{n-1} \frac{Q(\gamma)}{w(i_0)} \delta_1(i_0,i_0,n-\gamma+1) +$$

$$\sum_{\gamma=1}^{n-1} \sum_{i=1}^{N} q_1(i,\gamma)\delta_1(i,i_0,n-\gamma+1) \tag{4.31}$$

These equations (eqs. 4.30 & 4.31) are the same as obtained by Mishra(1987b) while solving for recharge from a river in a two-aquifer system in which aquifers are separated by an aquitard through which he considered vertical exchange of flow between aquifers.

4.7 Application of MODFLOW:

USGS modular three dimensional flow model (MacDonald and Harbaugh, 1988) commonly known as MODFLOW has also been used to model the aquifer recharge for varying stream stage for a two-aquifer system. The problems studied analytically by Mishra (1987b) were simulated using MODFLOW. A brief description of MODFLOW structure is given at Appendix.

5.0 RESULTS AND DISCUSSION

The following four cases were simulated.

Case I:

$T_1 = 500\ m^2/d$ $\quad$ $T_2 = 700\ m^2/d$

$\phi_1 = 0.10$ $\quad$ $\phi_2 = 0.01$

$K_1/B_1 = 0.001\ /d$

Case II:

$T_1 = 500\ m^2/d$ $\quad$ $T_2 = 700\ m^2/d$

$\phi_1 = 0.10$ $\quad$ $\phi_2 = 0.01$

$K_1/B_1 = 0.01\ /d$

Case III:

$T_1 = 500\ m^2/d$ $\quad$ $T_2 = 700\ m^2/d$

$\phi_1 = 0.10$ $\quad$ $\phi_2 = 0.01$

$K_1/B_1 = 0.001\ /d$ for $x > 150m$ and $x < -150\ m$

$K_1/B_1 = 0.01\ /d$ for $-150 \leq x \leq 150m$ and $x < -150\ m$

Case IV:

$T_1 = 500\ m^2/d$ $\quad$ $T_2 = 700\ m^2/d$

$\phi_1 = 0.10$ $\quad$ $\phi_2 = 0.01$

$K_1/B_1 = 0.001\ /d$ for $x > 150m$ and $x < -150\ m$

$K_1/B_1 = 0.1\ /d$ for $-150 \leq x \leq 150m$ and $x < -150\ m$

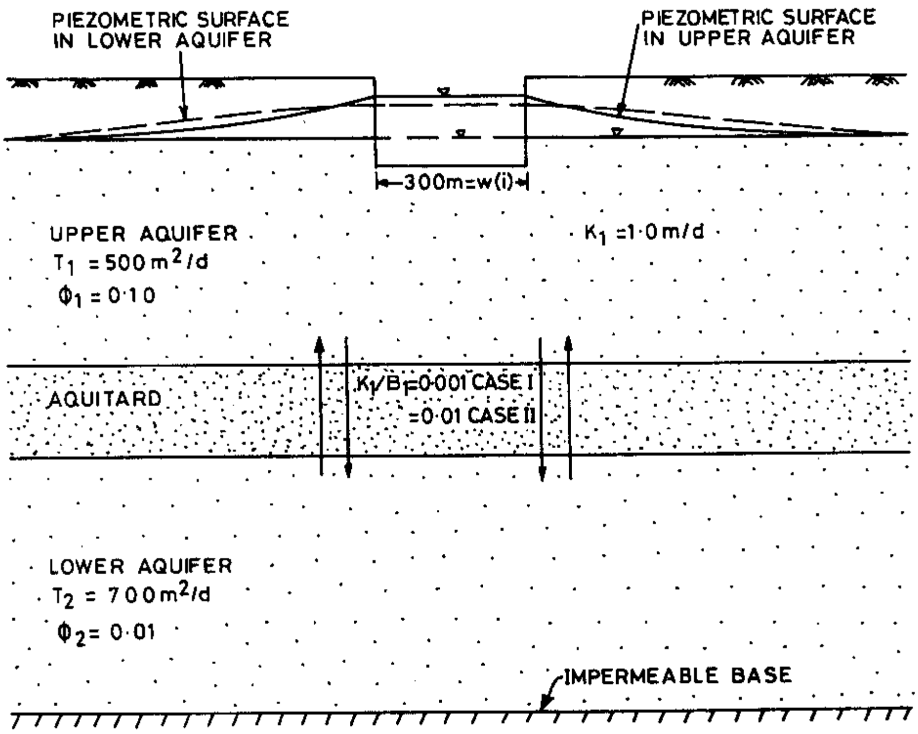

FIG. 4·2 DEFINITION SKETCH FOR CASES I AND II

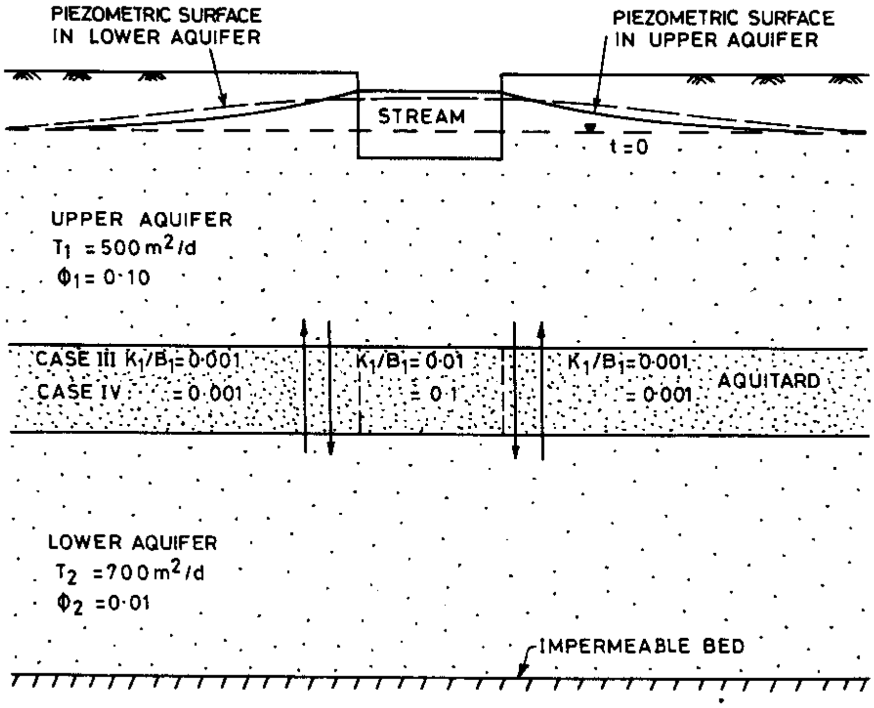

FIG. 4·3 DEFINITION SKETCH FOR CASES III AND IV

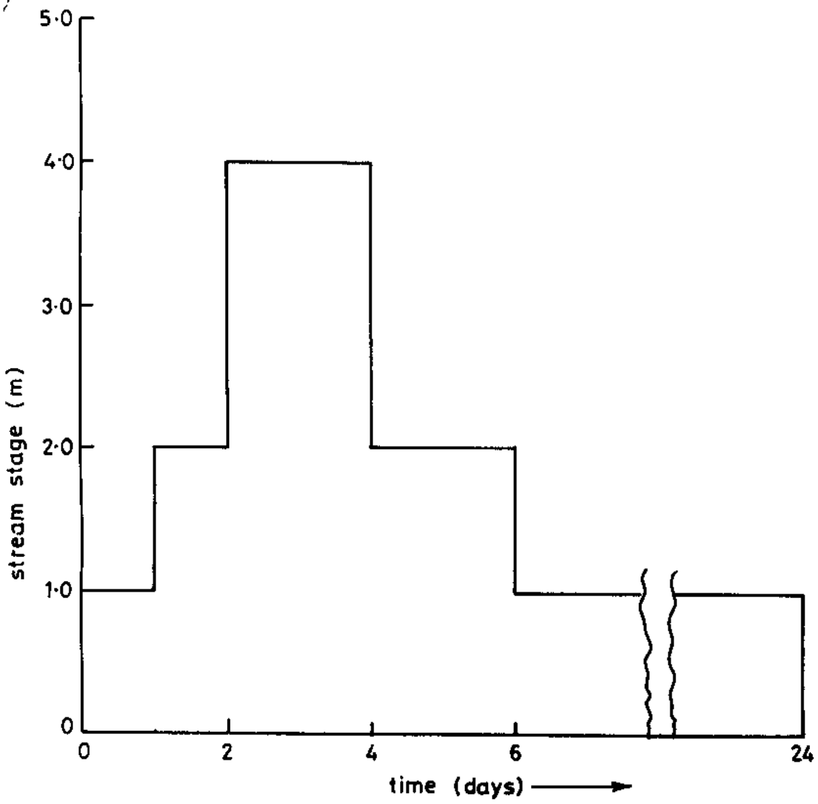

FIG. 4·4: VARIATION OF STREAM STAGE WITH TIME (FOR ALL CASES)

Fig. 4.2 & 4.3 show the definition sketches of the problems for different cases detailed above. In all the cases, time-varying stream-stage given in Fig. 4.4 have been considered. River Package of MODFLOW has been employed for simulating recharges from the stream. Simulation was carried out for five stress periods of 1, 1, 2, 2, and 18 days respectively. Uniform time-step size of 1 day has been adopted. The upper aquifer was simulated as unconfined and the lower aquifer was simulated as confined. Variable spacing of columns along row direction has been used. For all the cases, reach transmissivity of the stream is assumed as 1.54 m^2/d per m length of the stream. Width of the stream was taken as 300 m.

Table 5.1 : RECHARGE FROM UNIT LENGTH OF STREAM AND RECHARGE FROM UPPER AQUIFER TO LOWER AQUIFER PER UNIT WIDTH OF AQUIFERS.

(Legend: All the figures are in m^2/d ; Figures in italics correspond to the recharge from upper aquifer to lower aquifer; Recharge; Figures in brackets correspond to the results of Mishra, 1987b)

Time (d) Case	1	2	3	4	5	10	15	20	24
I	1.48 (1.47)	4.39 (4.35)	10.17 (10.08)	15.78 (15.61)	18.28 (18.05)	24.64 (24.24)	29.57 (29.09)	34.45 (33.87)	38.22 (37.61)
	0.014 (0.015)	*0.053 (0.058)*	*0.140 (0.154)*	*0.269 (0.296)*	*0.408 (0.447)*	*1.107 (1.216)*	*1.759 (1.963)*	*2.419 (2.713)*	*2.969 (3.319)*
II	1.48 (1.47)	4.40 (4.37)	10.21 (10.11)	15.86 (15.68)	18.41 (18.17)	25.09 (24.65)	30.39 (29.78)	35.61 (34.83)	39.72 (38.78)
	0.071 (0.080)	*0.244 (0.279)*	*0.603 (0.695)*	*1.044 (1.215)*	*1.378 (1.629)*	*2.242 (2.931)*	*2.852 (3.959)*	*3.519 (4.981)*	*4.114 (5.789)*
III	1.48 (1.47)	4.40 (4.37)	10.21 (10.12)	15.85 (15.70)	18.40 (18.19)	25.06 (24.72)	30.35 (29.90)	35.52 (34.99)	39.58 (38.98)
	0.078 (0.087)	*0.277 (0.312)*	*0.700 (0.788)*	*1.252 (1.409)*	*1.725 (1.944)*	*3.212 (3.782)*	*4.172 (4.979)*	*5.119 (6.147)*	*5.914 (7.057)*
IV	1.49 (1.48)	4.42 (4.39)	10.26 (10.18)	15.95 (15.81)	18.55 (18.36)	25.41 (25.12)	30.82 (30.47)	36.16 (35.73)	40.36 (39.84)
	0.192 (0.201)	*0.619 (0.666)*	*1.484 (1.620)*	*2.446 (2.732)*	*3.046 (3.496)*	*4.481 (5.470)*	*5.491 (6.811)*	*6.567 (8.105)*	*7.481 (9.120)*

In order to compare the results of simulations to those obtained by Mishra(1987b), MODFLOW source code was suitably modified to get the exchange of flow and cumulative exchange of flow through the aquitard from upper aquifer to lower aquifer for all the strips. Results of simulations are presented and compared with those of Mishra(1987b) in Table 5.1.

It is observed from Table 5.1 that MODFLOW simulation of recharge from stream differ by a maximum of 3.7% from those obtained by Mishra(1987b) for all the cases. However, MODFLOW simulation of recharge from upper aquifer to lower aquifer differ by more than 10% for few cases at large time. The reasons for this may be the coarser strip, and larger time step size; thus, this difference can further be reduced by taking smaller time-step size and finer strips. In fine, it can be stated that the present methodology is valid for the analysis of the interaction of stream and multi-aquifer system and stands validated against numerical solution obtained using MODFLOW.

6.0 CONCLUSIONS

The following conclusions are drawn from the study.

1. A methodology based on discrete kernel approach has been evolved for solving stream-aquifer interaction in multi-layered aquifer system in which three aquifers are separated by intervening two aquitards. In this, time varying stream-stage and vertical transfer of flow between aquifers through aquitards, can be accounted. Using the present approach, recharge from stream and exchange of flow between aquifers along with the rise in piezometric surface in each aquifer, can be obtained. The methodology can easily be extended to more than three aquifers.

2. The methodology presented by Mishra(1987b) for two-aquifer system, has been shown to be the special case of present formulation.

3. Three dimensional groundwater flow model (MODFLOW) has been used to reproduce the results of stream-aquifer interaction in two-aquifer system using discrete kernel approach for four different cases as considered by Mishra(1987b). MODFLOW source code was modified to get the exchange of flow from each strip and its spatial and temporal summation. Close reproduction of results using MODFLOW show that the methodology based on discrete kernel approach, is valid for the analysis of the interaction of stream and multi-aquifer system.

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APPENDIX

USGS GROUND WATER FLOW MODEL (MODFLOW)

MODFLOW (MODular three dimensional finite difference FLOW model) was originally Developed by McDonald and Harbaugh, USGS, USA in 1984. It simulates three dimensional groundwater flow using a block-centred finite difference approach. It can simulate aquifer-layers as confined, unconfined or changing from unconfined to confined during the course of simulation. Stresses from external sources, such as, wells, areal recharge, evapotranspiration, leakage from drains and riverbeds, and flow from flow-controlled or head controlled boundaries. can be simulated individually or in combination through the modular structural of the model. It has two modules for the solution of finite difference flow equations, viz., Strongly Implicit Procedure(SIP) and Slice Successive Over-relaxation method(SSOR), either of which can be selected.

Governing Equation:

Boussinesq equation governing the three dimensional unsteady groundwater flow in a heterogeneous and anisotropic medium is,

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right) - W = S_s\frac{\partial h}{\partial t} \qquad \text{(A-11}$$

Where, K_{xx}, K_{yy}, and K_{zz} are the hydraulic conductivities along the x, y, and z axes respectively; h is the piezometric head; W is a volumetric flux per unit volume; S_s is the specific storage of the porous material; and t is the time.

Discretization:

For the formulation of finite difference equation, aquifer system is generally discretized into a number of elements called cells, the locations of which are described by indices denoting rows, column and layers. To conform with computer array convention, an i,j,k, indexing system is used. A discretized hypothetical system with

5 rows, 9 columns and 5 layers, is shown in Fig. A-1. The width of the cells in row direction at a given column, j, is designated as Δr_j; width of cells in the column direction at a given row, i, is designated as Δc_i; and thickness of cells in a given layer, k, is designated Δv_k.

A simulation period is discretized into a number of stress periods and each stress period into a number of time steps. In each stress period, all the external stresses are assumed constant. To simulate the boundary conditions 'constant head cells' and 'inactive cells' are specified in advance. Constant head cells are those for which head is specified in advance, and is held at this particular value through all time step of simulation. Inactive cells are those no flow into or out of cell is permitted, in any time step of simulation. the remaining cells of the mesh, termed variable head cells are characterized by heads that are unspecified and free to vary with time.

Finite Difference Equations:

In the development of finite difference equation, water balance within a cell i,j,k using Darcy's law is obtained. Total internal groundwater flow to the cell i,j,k is the flow from all the six adjacent cells. The difference equation for the cell i,j,k in backward difference form as used in the model is,

$$CR_{i,j-1/2,k}[h^m_{i,j-1,k} - h^m_{i,j,k}] + CR_{i,j+1/2,k}[h^m_{i,j+1,k} - h^m_{i,j,k}] +$$

$$CC_{i-1/2,j,k}[h^m_{i-1,j,k} - h^m_{i,j,k}] + CC_{i+1/2,j,k}[h^m_{i+1,j,k} - h^m_{i,j,k}] +$$

$$CV_{i,j,k-1/2}[h^m_{i,j,k-1} - h^m_{i,j,k}] + CV_{i,j,k+1/2}[h^m_{i,j,k+1} - h^m_{i,j,k}]$$

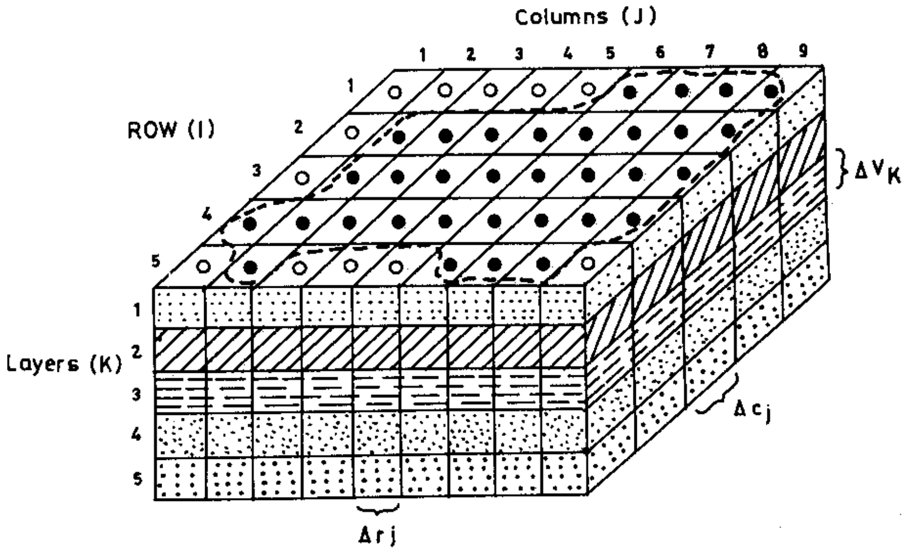
$$P_{i,j,k}h^m_{i,j,k} + Q_{i,j,k} = S_{s_{i,j,k}}(\Delta r_j \Delta c_i \Delta v_k)\frac{h^m_{i,j,k} - h^{m-1}_{i,j,k}}{t_m - t_{m-1}} \qquad \text{(A-2)}$$

Where,

$S_{s_{i,j,k}}$ = specific storage of cell i,j,k;

m = index denoting time step;

t_m = time at the end of m^{th} time-step;



LEGEND

- - - Aquifer Boundary
● Cell
○ Inactive cell

FIG. A·1 : A DISCRETIZED HYPOTHETICAL SYSTEM

t_{m-1} = time at the end of $(m-1)^{th}$ time-step;
$P_{i,j,k}$ = a constant.

$P_{i,j,k}h^m_{i,j,k} + Q_{i,j,k}$ represents total flow to the cell, i,j,k due to all external excitations. CR's, CC's, and CV's are the conductances in row, column., and vertical directions respectively. Conductance is defined as the product of hydraulic conductivity and the area through which the flow occurs divided by the length of flow path. An equation similar to eq. (A-2) can be written for each active cell of the system and thus, a set of equations can be formed for the system. This set of simultaneous equations is solved to get values of heads at all nodes.

Simulation Packages:

The modular structure of the model consist of a main program and a series of highly independent subroutines called 'modules'. The modules are grouped into 'packages'. Each package deals with a specific feature of the groundwater system. Four packages have been used for the present analysis brief description of which are given below.

The **'Basic Package'** handles a number of administrative tasks for the model. It reads data on the number of rows, columns, layers and stress periods, on the major options to be used, and on the location of input data for those options. it allocates space in the computer memory for model arrays; it reads data specifying initial and boundary conditions; it reads and implements data establishing the discretization of time, it sets up starting head arrays for each time step.

The **'Block-Centered Package'** computes the conductance components of the finite difference equation which determine flow between adjacent cells. It also computes the term which determines the rate and movement of water to and from the storage. To make the required calculations, it is assumed that a node is located at the centre of each model cell; thus the name Block-Centered Flow is given to the package. It calculates terms of finite difference equations which represents flow within porous medium, specifically, flow from cell to cell and flow into storage. The **'Strongly Implicit Procedure Package'** is a package for solving a large system of simultaneous linear equations by iteration.

The **'Well Package'** is designated to simulate features such as wells which withdraw water from the aquifer(or add water to it) at a specified rate during a given stress period, where the rates are independent of both the cell area and head in the cell. Well discharge is handled in the package by specifying the rate, Q, at which each individual well adds water to the aquifer or removes water from it, during each stress period of simulation. Negative values of Q are used to indicate well discharge, while positive values of Q indicate a recharging well. The well Package, as it is presently formulated in the model, does not accommodate wells, which are open to more than one layers of the aquifer. However, a well of this type can be represented as a group of single layer wells, each open to only one of the layers tapped by the multi-layered well, and each having an individual Q term specified for each stress period. This approach to represent a multi-layer well fails to take into account interconnection between various layers provided by the well itself and is thus an incomplete representation of the problem.

The **'Evapotranspiration Package'** simulates the effects of plant transpiration and direct evaporation in removing water from the saturated groundwater regime. The purpose of **'River Package'** is to simulate the effects of flow between surface-water features and ground-water system. The **'Recharge Package'** is designed to simulate areally distributed recharge to groundwater system. Most commonly, areal recharge occurs as a result of precipitation that percolates the groundwater system. The **'Drain Package'** is designed to simulate the effects of features such as agricultural drains, which remove water from the aquifer at a rate proportional to the difference between the head in the aquifer and some fixed head or elevation, so long as the head in the aquifer is above that elevation, but which have no effect if head falls below that level.

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