TR (BR) 144

SHALLOW WAVE PROPAGATION CHARACTERISTICS IN OPEN CHANNELS

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1995-96

CONTENTS

PREFACE

Our understanding on the flow behaviour in open channels is primarily based on the popular St. Venant's equations since their inception. The application of Chezy's and Manning's friction laws to these equations had been of wide controversy. Most of the theoretical works consider Manning's friction whereas the practical applications consider Manning's friction more appropriate. The former, however, is more suitable to the laminar flow conditions and the latter to the turbulent nature therefore, the Manning's applications are widely preferred and have also resulted in satisfactory applications.

The present work is an attempt in this direction to carry out an analysis for the Manning's friction and to determine the flood wave propagation characteristics. Its application does not only lead to a significant variation in the propagation characteristics but also leads to widening of the applicability horizon of the approximate kinematic and diffusion wave models in lieu of full dynamic wave model.

This report entitled SHALLOW WAVE PROPAGATION CHARACTERISTICS IN OPEN CHANNELS is prepared by Sh. **SURENDRA KUMAR MISHRA**, Scientist C of this institute. This report would help better understand the flow behaviour in open channels which is of concern not only to the hydraulicians but also to the practicing engineers.

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ABSTRACT

The St. Venant's equations still represent the state-of-the-art for simulating the transient flow in open channels and hence forms the basis of our understanding on the channel flow behaviour. Ponce and Simons(1977) first derived the analytically the wave propagation characteristics in wide rectangular open channels using linear theory under the assumption that the Chezy's friction holds good, and the stage and discharge waves are of sinusoidal form. Based on their works Ponce et al.(1978) developed depth specific criteria for identifying wave kind occuring in the channel using the flow and channel characteristics.

Since the Manning's equation is preferred to Chezy's by practising engineers, the works of Ponce and Simons (1977) and Ponce et al.(1978) on shallow wave propagation in open channels using Chezy's friction are re-analysed in Manning's perspectives. Its use in the development of the applicability criteria for kinematic wave model decreased dimensionless time periods from 873, 171, 83 to 707, 139, 67 corresponding to 99, 95 and 90 percent accuracies, respectively. In case of diffusion wave model, the ratio of dimensionless time period to Froude number comes out to be 22 as against 30. The applicability horizon of these models is widened and the efficiency of the Muskingum-Cunge method is judged.

INTRODUCTION

In most of the practical applications, Manning's friction formula is preferred to Chezy's (Chow, 1959; Giles, 1977; Streeter and Wylie, 1983; French, 1985; Ponce, 1989). However Ponce and Simons (1977) having agreed to above, applied Chezy's friction because of its intrinsic non-dimensional property and derived shallow wave propagation characteristics in open channels using linear perturbation theory. Later, Ponce(1989) applied Manning's friction for determination of the celerity and diffusive characteristics of diffusion waves only. With the aim to make the analysis consistent with the practical applications, these characteristics have been analysed for complete long wave spectrum using Manning's friction law. For defining the threshold Froude number for the attenuation characteristics of the primary waves of dynamic wave, use of Vedernikov number (Chow, 1959; Ponce, 1991), independent of friction laws, has been emphasised over Froude number.

THEORETICAL BACKGROUND

The linearised St. Venant's equations (Liggett, 1975) for Manning's friction are written in the form of perturbed variables as below:

$$u_o \frac{\partial h'}{\partial x} + h_o \frac{\partial u'}{\partial x} + \frac{\partial h'}{\partial t} = 0 \tag{1}$$

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$$\frac{\partial u'}{\partial t} + u_o \frac{\partial u'}{\partial x} + g \frac{\partial h'}{\partial x} + g S_o |2 \frac{u'}{u_o} - \frac{4}{3} \frac{h'}{h_o}| = 0 \tag{2}$$

where u' and h' are small perturbation about normal flow velocity u_0 and normal flow depth h_0 respectively; S_0 is the bed slope; g is the gravitational acceleration; and x and t are the time and space coordinates,respectively. The application of Chezy's friction law replaces the coefficient 4/3 by 1.0 in Eq. 2. for wide rectangular channel the Manning's friction formula is given below:

$$u = (1/n)\, h^{2/3}\, S_f^{1/2} \tag{3}$$

where u $(=u_0 + u')$, h $(=h_0 + h')$,n and S_f are the flow velocity, depth, Manning's roughness and friction slope, respectively. Postulating velocity and depth variation with x and t in the following form(Ponce & Simons, 1977).

$$(u'/\, u_o) = \hat{u} \exp\, [\, i(\hat{\sigma}\hat{x} - \beta \hat{t})\,] \tag{4}$$

$$(h'/\, h_o) = \hat{h} \exp\, [\, i(\hat{\sigma}\hat{x} - \beta \hat{t})\,] \tag{5}$$

and substituting in Eqs. 1 and 2 results in a coefficient matrix the determinant of which is kept equal to zero to arrive at a characteristic equation.

$$F_o^2 \beta^2 - 2(F_o^2 \hat{\sigma} - i)\beta + [\, F_o^2 \hat{\sigma}^2 - \hat{\sigma}^2 - (10/3)\, i\hat{\sigma}] = 0 \tag{6}$$

which is similar to that derived for Chezy's friction by Ponce and Simons (1977). Solution for β can be obtained using elementary comlex algebra. The important definitions of the dimensionless terms used in the Eqs. 4 through 6 and elsewhere in the text are described below:

Definitions

$\hat{\sigma}=(2\pi/L)\,L_o=$ *wave number* (7)

$\beta_R=(2\pi\,L_o)/(T\,u_o)$ (8)

$\beta_I=$ *amplitude propagation factor* (9)

$\hat{x}=x/L_o=$ *space coordinate* (10)

$\hat{t}=t\,(u_o/L_o)=$ *time coordinate* (11)

$L_o=h_o/S_o=$ *reference channel length where the head dro ps by* h_o (12)

$\hat{c}=L/(T\,u_o)=\beta/\hat{\sigma}=$ *celerity* (13)

$\hat{c}_r=\hat{c}-1=$ *relative celerity* (14)

$\delta=2\pi(\beta_I/\beta_R)=$ *logarithmic decrement* (15)

$\hat{\tau}=T(u_o/L_o)=$ *time period* (16)

$F_o=u_o/\sqrt{gh_o}=$ *Froude Number* (17)

All the terms with superscript ^ are dimensionless. The L and T are wave length and time period of the wave (wave celerity c= L/T), respectively. The $\hat{c}$ and δ describe respectively the translation and attenuating chara-cteristics of the wave. Positive $\hat{c}$ shows the downstreem movement and negative, the upstream movement. If δ sets in and if negative, the wave attenuates and dies away.

FEATURES OF MANNING'S FRICTION BASED ANALYSIS

The propagation characteristics of the postulated wave models (Ponce and Simons, 1977), whose formation depend on the inclusion of the terms described in Eq. 2, are summarised in Table 1. This also includes the results worked out using Manning's friction. The salient features are:

- The celerity of kinematic and diffusion waves increases.
- The deviation in the celerities, due to the application of two friction laws, gradually vanishes as the wave number approaches the gravity wave region, where it vanishes completely (Fig.1). The $\hat{c}_r$ is independent of the $\hat{\sigma}$ at $F_0 = 1.5$. For Chezy's friction it is at $F_0 = 2.0$.
- For a given wave number $\hat{\sigma}$ and F_0 (< 1.5) the application of Manning's friction results in lesser attenuation (Fig. 2) of the primary waves of dynamic wave than that due to Chezy's friction. However, for $F_0 > 1.5$, the converse is true (Fig. 3).
- A summary of propagation characteristics (Table 2) shows that the threshold F_0 defining the attenuating characteristics of primary waves sets in at 1.5 (for Manning's friction) instead of 2.0 (for Chezy's friction). The primary waves attenuate at $F_0 < 1.5$ and amplify at $F_0 > 1.5$.These waves neither amplify nor attenuate at $F_0 = 1.5$.
- The Vedernikov number V (= $\chi\gamma / \hat{c}_r$; where χ is the exponent of the hydraulic radius in the uniform flow formula, and γ is the shape factor of channel section which is 1.00 for wide rectangular channels). It is apparent from Fig. 1 that the values of $\hat{c}_r$ for Chezy's and Manning's friction in kinematic wave region are 1/2 and 2/3, respectively. Using either of the friction formula and suitably substituting the values, the Vedernikov number sets in at 1.00. In the gravity wave region too it is 1.00 for both $F_0 = 2.0$ (Chezy's friction) and $F_0 = 1.5$ (Manning's friction). Further, it can be shown that the primary waves always attenuate at $V < 1.00$ and amplify at $V > 1.00$. At $V = 1.00$ these neither attenuate nor amplify.

APPLICABILITY OF KINEMATIC AND DIFFUSION WAVE MODELS

The applicability criteria are analysed using the shallow wave propagation properties derived for the Manning's friction. The analysis follow the same procedure adopted by Ponce et al (1978). The results are compiled in Table 3. For the kinematic wave model the criteria are derived for 99, 95 and 90 percent accuracies. A comparison of the results shows that the values of the dimensionless time period $\hat{\tau}$ are significantly reduced (for Manning's friction) from those derived for Chezy's friction for the above accuracies. For the kinematic wave model the modified applicability criteria at 95 percent accuracy is given by

$$\hat{\tau} = \frac{TS_o U_o}{h_o} \geq 139 \tag{18}$$

For the diffusion wave model the analysis for applicability criteria is carried out and presented in Fig. 4. The plot $\hat{\tau}/F_0$ Vs $e^{(\delta_1 - \delta_2)}$ for both Chezy's and Manning's friction laws shows that the whole spectrum due to the latter is shifted to the right of the former (i.e. decreasing $\hat{\tau}/F_0$). The postulated value of $\hat{\tau}/F_0 = 22$ (also compiled in Table 3) for ± 5 percent inaccuracy. This is lower than that arrived at by Ponce et al (1978). Thus, the modified diffusion wave criterion is given by

$$\frac{\hat{\tau}}{F_o} = TS_o \sqrt{\frac{g}{h_o}} \geq 22 \tag{19}$$

TABLE 1. PROPAGATION CHARACTERISTICS OF SHALLOW WAVES

WAVE TYPE	$\hat{c}$ and δ_d	RESULTS OF LINEAR ANALYSIS BASED ON	
		CHEZY'S FORMULA (Ponce & Simons, 1977)	MANNING'S FORMULA (Present)
Kinematic Wave (I term of Eq. 2)	$\hat{c}_k$	3/2	5/3
	δ_d	0	0
Diffusion Wave (I+II terms of Eq. 2)	$\hat{c}_d$	3/2	5/3
	δ_d	$-2\pi(\hat{\sigma}/3)$	$-2\pi\,(3\hat{\sigma}/10)$
Steady Dynamic Wave (I+II+III terms of Eq. 2)	$\hat{c}_{sd}$	$1+[2-\hat{\sigma}^2 F_o^2]/[4+\hat{\sigma}^2 F_o^4]$	$1+[8-3\hat{\sigma}^2 F_o^2]/[12+3\hat{\sigma}^2 F_o^4]$
	δ_{sd}	$-2\pi[\hat{\sigma}(2+F_o^2)]/[6-\hat{\sigma}^2 F_o^2(1-F_o^2)]$	$-2\pi\,[2\hat{\sigma}(3+2F_o^2)]/[20-3\hat{\sigma}^2 F_o^2(1-F_o^2)]$
Dynamic Wave (All terms of Eq. 2)	$\hat{c}_1$	$1+[(C+A)/2]^{1/2}$	$1+[(C+A)/2]^{1/2}$
	$\hat{c}_2$	$1-[(C+A)/2]^{1/2}$	$1-[(C+A)/2]^{1/2}$
	δ_1	$-2\pi\,[\zeta+E]/\lvert 1+D\rvert$	$-2\pi\,[\zeta+E]/\lvert 1+D\rvert$
	δ_2	$-2\pi\,[\zeta+E]/\lvert 1-D\rvert$	$-2\pi\,[\zeta+E]/\lvert 1-D\rvert$
Gravity Wave (II+III+IV terms of Eq. 2)	$\hat{c}_g$	$1\pm(1/F_o)$	$1\pm(1/F_o)$
	δ_g	0	0
NOTATIONS	ζ	$1/(\hat{\sigma} F_o^2)$	$1/(\hat{\sigma} F_o^2)$
	A	$1/F_o^2-\zeta^2$	$1/F_o^2-\zeta^2$
	B	ζ	$4/3\zeta$
	C	$[A^2+B^2]^{1/2}$	$[A^2+B^2]^{1/2}$
	D	$[(C+A)/2]^{1/2}$	$[(C+A)/2]^{1/2}$
	E	$[(C-A)/2]^{1/2}$	$[(C-A)/2]^{1/2}$
Subscripts 1 and 2 refer to Primary and Secondary waves, respectively.			

TABLE 2.CELERITY AND ATTENUATION CHARACTERISTICS OF DYNAMIC WAVE

FROUDE NUMBER	PRIMARY WAVE				SECONDARY WAVE			
	c_1		δ_1		c_1		δ_2	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
$F_o < 1$	+	+	-	-	-	-	-	-
$F_o = 1$	+	+	-	-	0	0	-	-
$F_o = 1.5$	+	+	-	0	+	+	-	-
$F_o = 2$	+	+	0	+	+	+	-	-
$F_o > 2$	+	+	+	+	+	+	-	-
Downstream celerity + ; upstream celerity -; Attenuation -; amplification +								
(a) Chezy formula based (Ponce&Simons(1977); (b) Manning formula based (Present)								

TABLE 3. APPLICABILITY CRITERIA OF KINEMATIC AND DIFFUSION WAVE MODELS

WAVE MODEL	ACCURACY	CHEZY BASED (Ponce&Simons,1977)	MANNING BASED (Present)
Kinematic Wave	$e^{\delta_d} = 0.99$	$\hat{\tau} \geq 873$	$\hat{\tau} \geq 707$
	$e^{\delta_d} = 0.95$	$\hat{\tau} \geq 171$	$\hat{\tau} \geq 139$
	$e^{\delta_d} = 0.90$	$\hat{\tau} \geq 83$	$\hat{\tau} \geq 67$
Diffusion Wave	$e^{(\delta_1-\delta_d)} = 1 \pm .05$	$\hat{\tau}/F_o \geq 30^*$	$\hat{\tau}/F_o \geq 22^*$
* postulated values			

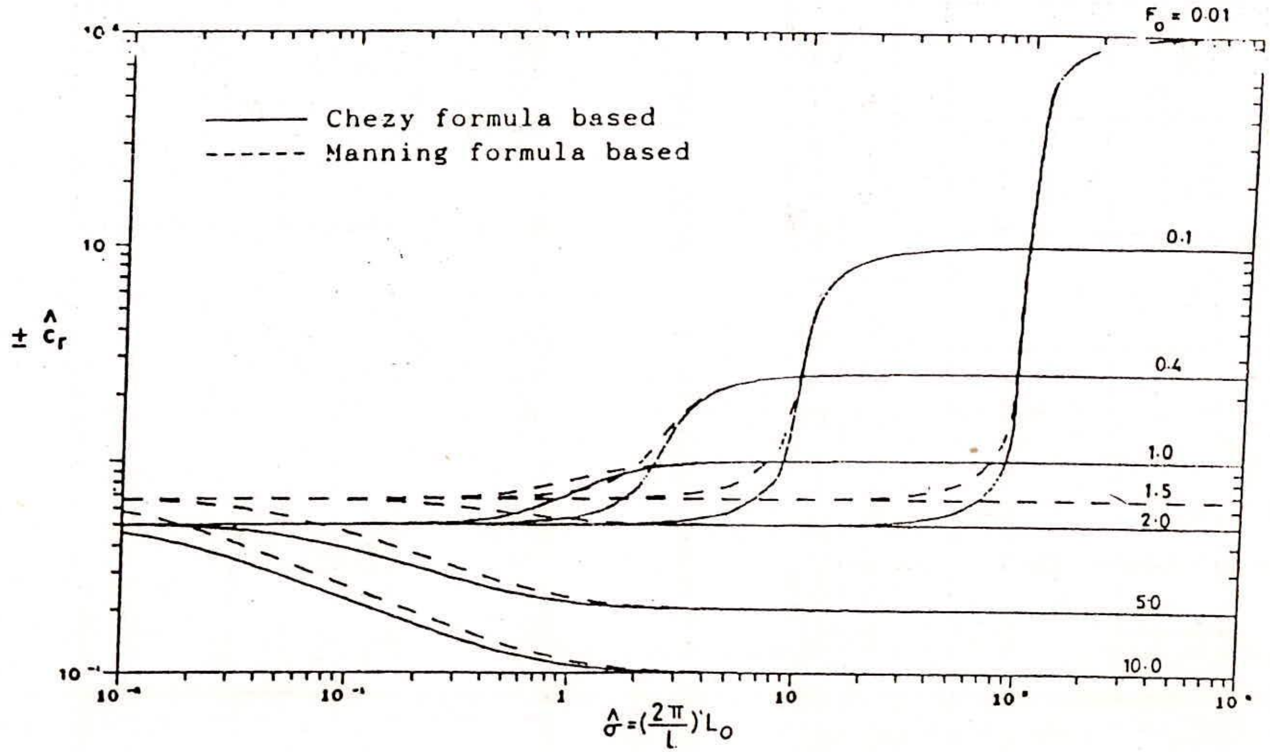

Fig. 1. Dimensionless relative celerity versus dimensionless wave number; Curve Parameter = Froude Number F_o $(0.01 \leq F_o \leq 10.0)$

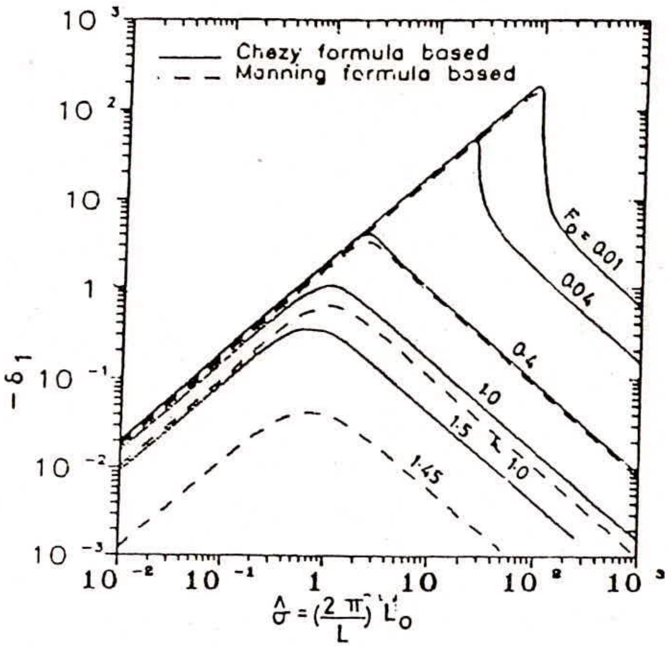

Fig. 2. Primary wave logarithmic decrement $-\delta_1$ versus dimensionless wave number $\hat{\sigma}$; Curve parameter = Froude number F_o ($F_o < 1.5$)

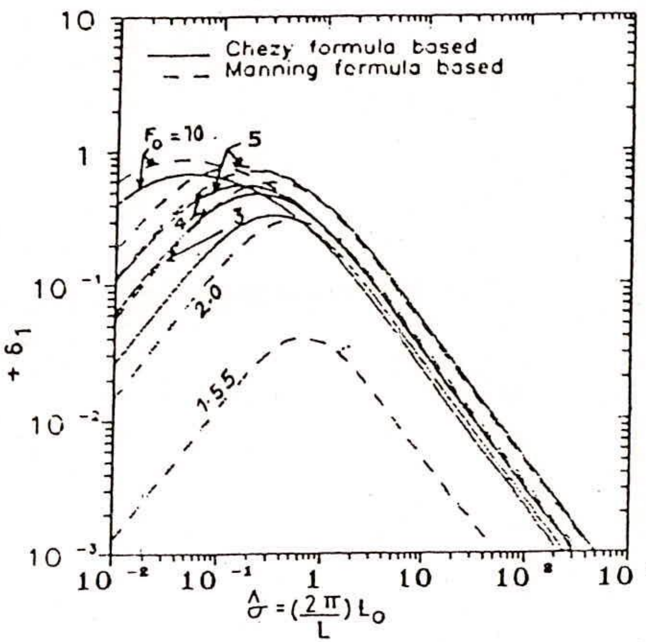

Fig. 3. Primary wave logarithmic decrement $+\delta_1$ versus dimensionless wave number $\hat{\sigma}$. Curve parameter = Froude number F_o ($F_o > 1.5$)

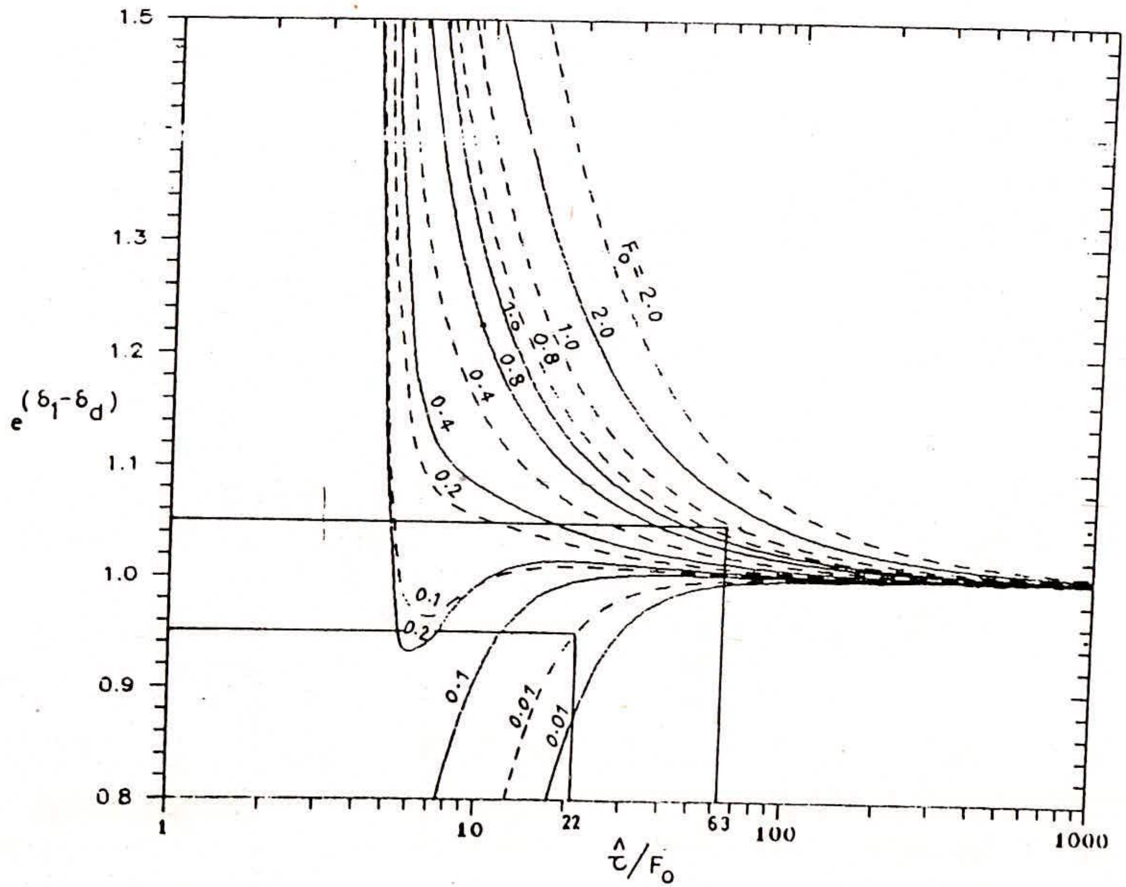

Fig. 4. Ratio of attenuation factors $e^{(\delta_1-\delta_d)}$ versus $\hat{\tau}/F_o$

EXAMPLE APPLICATION

An example problem of a sinusoidal wave travelling in unit width rectangular channel is considered for sensitivity analysis. The considered input values are: peak inflow (q_{pl})=200 cfs/ft; base flow (q_b) = 50 cfs / ft; reference flow (q_0) =125 cfs / ft; time period (time base, T_b, of the sinusoidal wave)=96 hrs; bed slope of the channel (S_0)= 1 ft / mi ; Mannings roughness (n)=0.0297; and length of the reach (L_r) = 500 mi. Ponce et al (1996) showed that the considered wave was diffusion wave. The sensitivity analysis of the variable of Eqs. 18 and 19 using the Manning's formula (Eq. 3; suitably modified for FPS system of units) is carried out and the results are plotted in Figs 5 and 6. In Eq. 19, the ratio of $\hat{\tau}$ and F_0 is most sensitive to S_0 and least to q_o and n whereas in Eq. 18, the $\hat{\tau}$ is as sensitive to variation in n as in S_0 and T_b . Increase in T_b and/or S_0 tends to change sharply the diffusion wave towards kinematic wave. However, the increase in n shows a reverse trend. As a corollary, the smoother channel bed favours the development of kinematic wave. In both the Eqs. 18 and 19, the T_b has less bearing on $\hat{\tau}$ or $\hat{\tau}/F_0$ than the S_0.

For the applicability of kinematic and diffusion wave models the example problem is hypothetically extended to the limiting zones (Figs. 5 and 6). The computation for the left hand sides of Eqs. 18 and 19 comes out to be 140 and 23, respectively and thus confirm the waves to be kinematic and diffusion as shown in Table 4. These selected typical examples, summarised in Table 4, are applied to the Constant Parameter Muskingum-Cunge (CPMC) method of routing and the results are compared with those computed analytically. Analytical compution for peak outflow is based on Eq. 20 (Ponce et al, 1996) given below:

$$q_{po} = q_o + (q_{pi} - q_o)\ e^{-\beta_r \hat{t}} \tag{20}$$

where, q_{p0} = peak outflow (cfs/ft); and q_0 = (q_{pl} +q_b) /2 = reference discharge (cfs/ft). For diffusion wave the $\beta_r \hat{t}$ is given by

$$\beta_r \hat{t} = [\, 2\pi / (c T_b) \,]^2\, \nu t \tag{21}$$

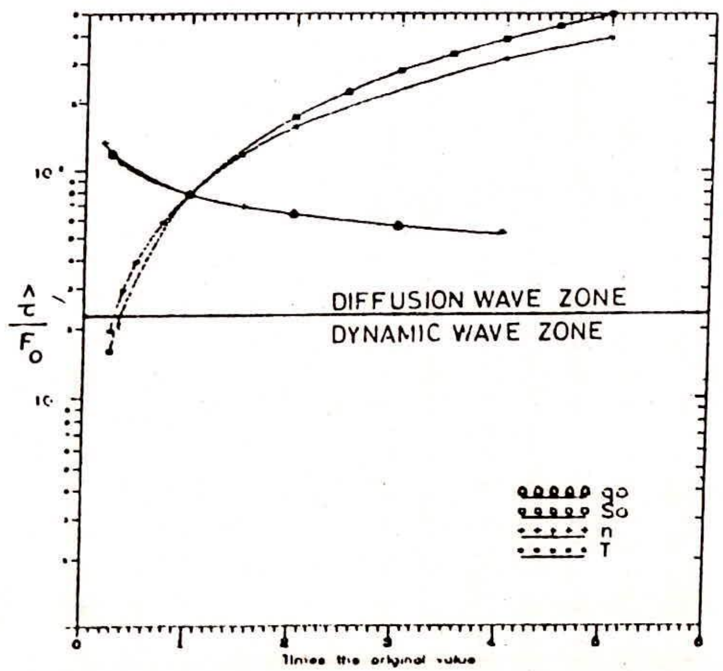

Fig. 5. Sensitivity analysis for diffusion wave criterion (Eq. 19)

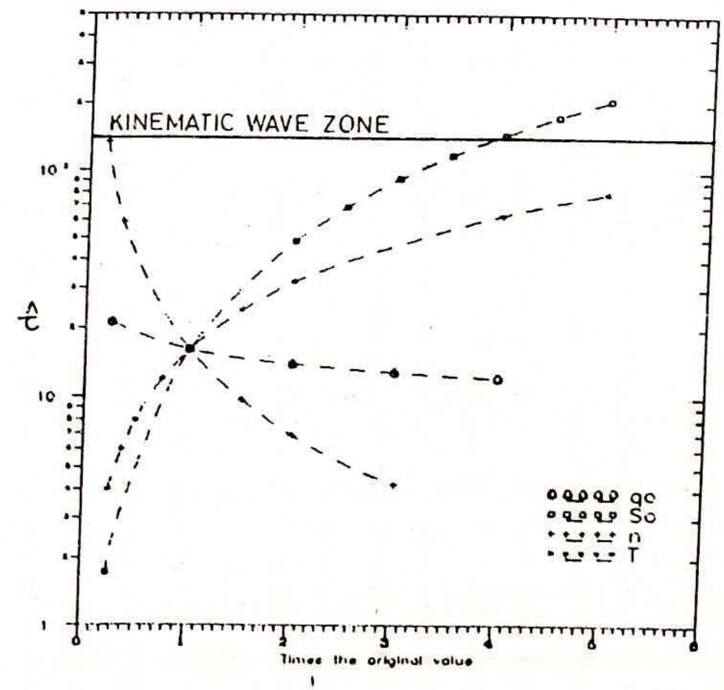

Fig. 6. Sensitivity analysis for kinematic wave criterion (Eq. 18)

in which $v = q_0/(2S_0)$ is the hydraulic diffusivity (Hayami, 1951); and $c = (5/3)\, u_0$ for kinematic and diffusion waves. The time of travel (tt) is given by

$$tt = L_r \;/\; c \tag{22}$$

TABLE 4. TYPICAL CASES OF KINEMATIC AND DIFFUSION WAVE ROUTING

(T_b=96hr; q_{pi}=200 cfs/ft; q_b=50 cfs/ft; q_o=125 cfs/ft; L_r=500 mi; n=0.0297)

WAVE	S_o	Δt	Δx	CPMC		ANALYTICAL	
				q_{po}	tt	q_{po}	tt
	(ft/mi)	(hr)	(mi)	(cfs/ft)	(hr)	(cfs/ft)	(hr)
Kinematic	3.900	2.0	20.0	197.81	53.0	196.25	53.06
Diffusion	0.345	2.0	10.0	122.60	101.0	124.89	109.83

A comparison of the routing results (Table 4) shows that the outflow peak, by the CPMC method, is in error (at ±5 percent inaccuracy) by +0.79 percent in routing of kinematic wave and by -1.83 percent in routing of diffusion wave. The computed time of travel for the kinematic wave by the two approaches are close to each other. However, the time of travel by CPMC method faithfully simulates the attenuation characteristics of both the waves, but fails to simulate the translation characteristics of the diffusion wave in its limiting applicability horizon.

SUMMARY

Propagation characteristics of the shallow waves for Manning's friction were analysed, presented and compared with those derived for Chezy''s friction. The application of Manning's friction results in

- greater celerities of all the shallow waves except the gravity wave. For kinematic and diffusion waves the relative dimensionless celerity is 5/3 instead of 3/2; the latter is due to Chezy's friction whereas the former is consistent with the routine practical applications.

- threshold Froude Number F_o = 1.5 as against 2.0 (for Chezy's friction). The use of Vedernikov Number, V, eliminates this anomaly, and it sets in at 1.0 irrespctive of the application of either of the friction formulae.
- the extension of applicability horizon of kinematic and diffusion wave models.

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