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SURFACE WATER - GROUNDWATER INTERACTION DUE TO PUMPING NEAR A RECHARGE BOUNDARY

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Preface

With rapid increase in population, the ground water exploitation is increasing at a much faster rate. This warrants groundwater hydrologists for maintaing huge and perennial groundwater supplies so as groundwater mining can be avoided. One of the alternatives for creatting additional groundwater potential may be the induced recharge from a perennial stream by pumping near the stream in its effluent reaches where it is loosing water to the aquifer.

In the present report analytical solutions for unsteady drawdown, and rate and volume of stream depletion for variable rate pumping has been derived. A methodology has been evolved for estimation of the parameters (transmissivity, storage coefficient, and effective distance to the recharge boundary utilizing the drawdown data obtained during a variable rate pump test. Expressions for analytical derivatives w.r.t. the parameters have been derived and used for the non-linear optimization. Computer codes in Fortran for both direct problem and inverse problem for variable rate pumping near a recharge boundary, have been developed. This report has been prepared by **Mr. S.K. Singh, Scientist 'C'** as per the Work Plan of Groundwater Modelling and Conjunctive Use Division.

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Abstract

Extensive exploitation of groundwater during past two decades, has drawn the attention of groundwater hydrologists towards the problem of maintaining perennial supply of groundwater. Recharge to groundwater on account of percolation from precipitation falling on the outcrop of an aquifer is limited. Since, a stream derives its flow from precipitation over a relatively larger drainage area, large and perennial groundwater supply can better be maintained by inducing percolation from a perennial stream hydraulically connected with the aquifer. With favourable hydrogeological condition and permeable stream-bed, induced infiltration into an aquifer takes place when wells are pumped near the stream. Thus, pumping near effluent reaches of the stream may form an important factor in planning conjunctive use practices. The pumping rate may vary with time from conjunctive use management point of view, hence, analytical solutions for drawdown, and rate and volume of stream depletion for variable/intermittent pumping are required.

Application of the analytical solutions to field problems or for conjunctive use planning, requires that the parameters, (i.e., transmissivity, storage coefficient, and effective distance to the recharge boundary), are known before hand. These parameters can be determined utilizing the drawdown data obtained during a pump test conducted near the recharge boundary. Methods developed so far, for determination of the parameters either use curve matching, or involve the procedure of finding inflexion point on drawdown vs. time graph. Therefore, use of these methods suffers from errors due to personal judgement.

The present report deals with the development of analytical solutions for unsteady drawdown, and rate and volume of stream depletion for variable rate pumping. A method has been evolved for estimation of the parameters making use of the drawdown data obtained during a pump test. The method takes into account the variable pumping rate. A derivative-based non-linear optimization technique has been used for the estimation of the parameters. Analytical derivatives w.r.t. the parameters have been used in the optimization. The proposed method do not require use of tables or curve matching as it is based on objective criteria of minimization of sum of the squares of errors between observed and computed drawdowns.

Computer codes in Fortran for both direct problem and inverse problem for variable rate pumping near a recharge boundary, have been developed. The estimated parameters using present methodology are found to be more reliable as compared to that obtained using traditional method.

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1.0 INTRODUCTION

Increasing groundwater withdrawals during recent past for meeting irrigation, industrial and municipal water demands, have given rise to the problem of maintaining perennial groundwater supply. Sources of groundwater recharge are percolation from surface water in one or other form. Percolation from precipitation falling on the outcrop of an aquifer has three main limiting factors for perennial groundwater supply. These factors are i) percolation takes place from a relatively small area, ii) percolated water takes a long time to reach the place of utilization, iii) precipitation is not continuous but is concentrated only during monsoon. In view of the factors listed above, perennial supply of groundwater can better be maintained by inducing percolation from other surface sources such as perennial streams hydraulically connected with the aquifer. The advantages of this induced percolation are i) the stream derives its flow from precipitation over a larger drainage area, ii) a perennial stream offers a favourable condition for maintaining perennial ground water supply.

With favourable hydrogeological condition and permeable stream-bed, infiltration of surface water will be induced into an aquifer when wells are pumped near the stream. Fine deposits of silts in the stream bed and clay lenses immediately below the stream bed prevent induced infiltration. Since, a part of the pumped discharge comes from the stream, groundwater withdrawal will reduce the stream flow. Such reduction in stream flow is advantageous during floods as excess discharge in the stream which otherwise goes waste, can be made use of for irrigation. A perennial stream may be gaining (influent) in some reaches and loosing (effluent) in other reaches. Pumping may augment ground water through induced recharge only in the effluent reaches of the stream. Thus, pumping near effluent reaches of the stream may form an important factor in planning conjunctive use practices.

Analytical solutions for drawdown and rate and volume of stream depletion due to constant rate of pumping are available (Theis, 1941, Jenkin, 1968). The actual distance to the recharge boundary from pumping well should be replaced by an effective distance while using the analytical solutions. Because of the low conductivity of stream-bed sediments, and partial penetrating nature of the stream, the effective distance is more than the actual distance. Conjunctive use management requires the pumped water to be utilized for irrigation and other purposes. Therefore, pumping rate may vary with time and this requires analytical solutions for drawdown, and rate and volume of stream depletion for variable/intermittent pumping.

Application of the analytical solutions to field problems or for conjunctive use planning, requires that the values of the parameters. (transmissivity, storage coefficient, and effective distance), are known before hand. It is possible to determine these parameters utilizing the drawdown data obtained during a pump test conducted near the recharge boundary. Methods developed so far, for determination of the parameters either use curve matching, or involve the procedure of finding inflexion point on drawdown vs. time graph. Therefore use of these methods suffers from errors due to personal judgement.

The present report deals with the development of analytical solutions for unsteady drawdown, and rate and volume of stream depletion for variable rate pumping using convolution with time. A method has been evolved for the estimation of the parameters, based on objective criteria of minimization of sum of the squares of errors between observed and computed drawdowns.

2.0 REVIEW OF LITERATURE

Effect of recharge boundary has mostly been analyzed using method of image assuming that the aquifer is homogeneous and isotropic and the stream water level remains constant during pumping. A detailed review may be found in Walton(1970).

Theis (1941) gave the following expression for the ratio of stream depletion rate to pumping rate (Walton.1970,pp. 161-165).

$$P = \frac{q(t)}{Q} = \frac{2}{\pi} \int_0^{\infty} \frac{\exp\{k(1+z^2)\}}{(1+z^2)} dz \qquad ...(2.1)$$

$$k = \frac{a^2 S}{4Tt} \qquad ...(2.2)$$

Where.

q(t) = stream depletion rate at time t;

Q = constant rate of pumping;

a = effective distance to the recharge boundary from pumping well:

z = a dummy variable:

deHan.1939 (in Walton.1970) has given approximate expression for the integral appearing in eq. (2.1). which may be expressed as.

$$P = 1 - \left(\frac{2\exp(-k/2)}{\sqrt{\pi}}\right)\left[\sum_{1}^{\infty} \frac{k^n}{n!} \left\{\frac{(-1)^{m-1}}{2m-1}\right\}\right] = 1 - \exp(-k/2)$$

$$\left\{ a_1 + a_2k^2 + a_3k^3 + a_4k^4 + a_5k^5 + a_6k^6 + a_7k^7 + a_7k^8 + \ldots \right\}^{1/2} \qquad ...(2.3)$$

where,

$a_1 = 1.273239$ $a_2 = 0.424413$ $a_3 = 0.183912$ $a_4 = 0.038399$

$a_5 = 0.008859$ $a_6 = 0.001316$ $a_7 = 0.000207$ $a_8 = 0.000024$

The above series converses for fractional values of k. For large values of k the integral can be more easily evaluated graphically or by Simpson's role in the following form.

$$P = \frac{2}{\pi} \int_0^{\pi/2} e^{-k\sec^2 u}\, du \qquad \ldots(2.4)$$

Where, u is a dummy variable.

Glover (1954), (in Glover, 1978) has given the following equation for computation of P.

$$P = \operatorname{erfc}\left(\frac{a}{\sqrt{4\alpha t}}\right) \qquad \ldots(2.5)$$

where,

α = diffusivity of the aquifer $= T/S$;

$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-u^2} du$ = error fu..ction, and;

$\operatorname{erfc}(x) = 1-\operatorname{erf}(x)$ = complementary error function.

Kazmann(1946) proposed a method to determine the effective distance by matching the family of type curves with data obtained from longer period of pumping. He recommended Theis graphical method to compute transmissivity from data obtained in the first few minutes of the pumping test before the recharge from river affects water level in the observation well.

Hantush (1959) have outlined the procedures to determine effective distance to the recharge boundary and T & S from drawdown data of constant rate pumping tests. The procedure involves determination of inflexion point on drawdown vs. time curve, and use of tables.

Review of literature shows that the equations for drawdown and rate and volume of stream depletion resulting from variable rate pumping have not been proposed. Methods for the estimation of parameters (T, S, and a) proposed so far, require curve matching, use of tables or determination of inflexion point. The values of the parameters estimated using these methods are likely to be erroneous due to personal judgement. Estimation of the parameters based on minimization of integral sum of the squares of errors between observed and calculated drawdown has not been attempted.

3.0 STATEMENT OF THE PROBLEM

A tubewell near a fully penetrating stream in a confined aquifer is being pumped. The stream is hydraulically connected to the aquifer and is straight in a fairly long reach. The aquifer is homogeneous, isotropic and infinite. Definition sketch of the problem is shown in Fig.3.1. Assuming that the i) Dupuits assumptions are valid and ii) stream water level does not change with time, it is required to develop,

1. analytical equations for drawdown and rate and volume of stream depletion due to variable rate pumping, when parameters (aquifer parameters and effective distance) are known.

2. a methodology using a non-linear optimization technique for determination of the parameters, when drawdowns in an observation well due to a variable rate pumping, are known.

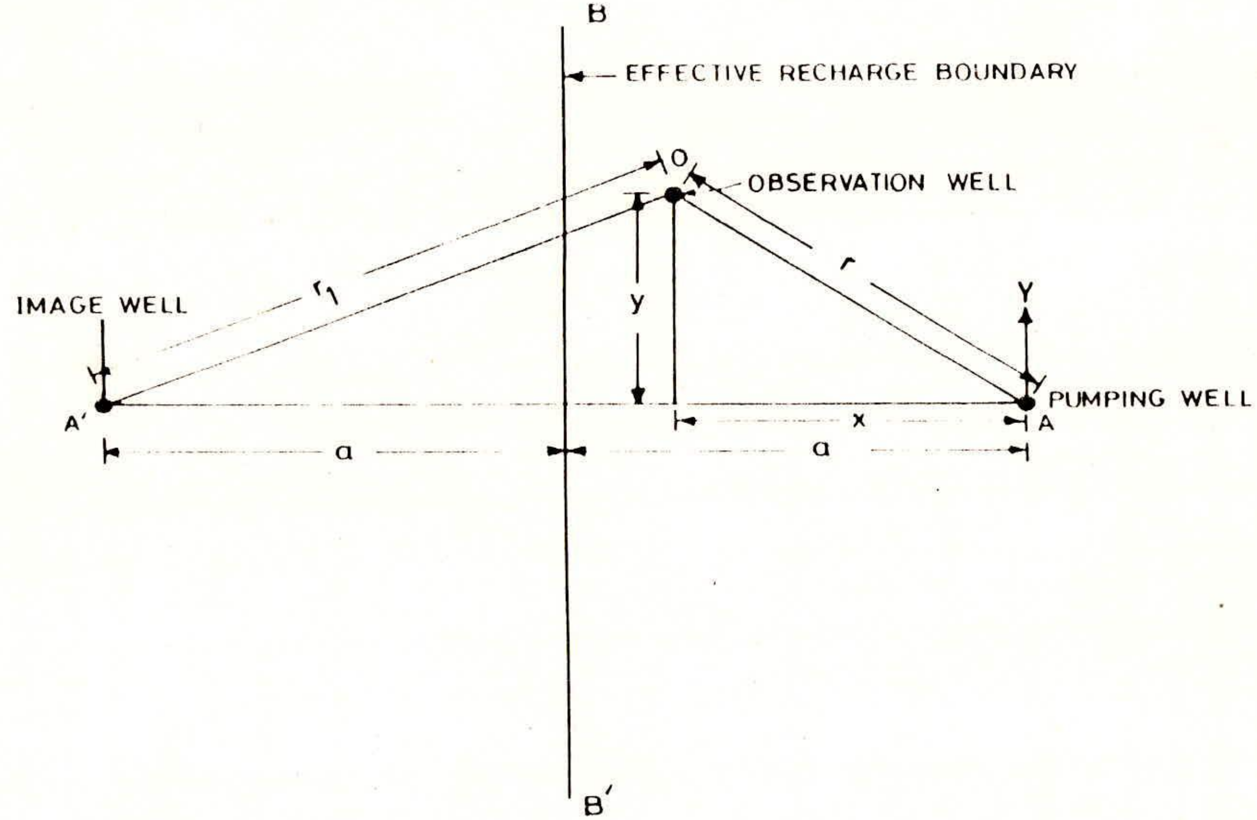

FIG. 3·1 DEFINITION SKETCH OF THE PROBLEM

4.0 METHODOLOGY

4.1 Direct Problem

The direct problem in present study is defined as 'to find the variation of drawdown, and rate and volume of stream depletion with time when the parameters (T, S, and a, are known. The basic approach for convolution as proposed by Morel-Seytoux and Daly(1975) has been used for the solution of the present problem. Convolution equations proposed here are dimensionally homogeneous.

4.1.1. DRAWDOWN

Drawdown at O(x,y) due to constant rate pumping at A(0,0) (Fig.3.1) is given by,

$$s(t) = \frac{Q}{4\pi T}\left[W\left\{r^2 S/(4Tt)\right\} - W\left\{r_1^2 S/(4Tt)\right\}\right] \qquad \ldots(4.1)$$

where,

$$r_1^2 = (2a-x)^2 + y^2 \;;$$

$$r = \sqrt{x^2+y^2} \;;$$

$$W(u) = \text{Well function} = \int_u^\infty (e^{-x}/x)\,dx$$

and,

$s(t)$ = drawdown in the observation well at time t;

r = distance to the observation well from pumping well;

r_1 = distance to the observation well from image well;

S = storage coefficient of the aquifer;

T = transmissivity of the aquifer;

t = time since start of the pumping;

Q = constant rate of pumping;

x,y = Cartesian coordinates of observation well location, origin being at pumping well location.

Unit impulse response function u(t), for drawdown at O can be obtained by differentiating eq. (4.1) w.r.t. t and then dividing by Q.

$$u(t) = \frac{1}{4\pi Tt}\left[\exp\left\{r^2 S/(4Tt)\right\} - \exp\left\{r_1^2 S/(4Tt)\right\}\right] \qquad \ldots(4.3)$$

Employing the principle of superposition which is valid for a linear system, drawdown at time t due to time-varying rate of pumping can be expressed as,

$$s(t) = \int_0^t Q(\tau)u(t-\tau)d\tau \qquad \ldots(4.4)$$

In discretized form, above eq. can be written as,

$$s_n = \sum_{i=1}^{n} \bar{Q}_1 \delta_1(n-i+1) \qquad \ldots(4.5)$$

where,

$$\delta_1(m) = \int_0^{\Delta t} u(m\Delta t-\tau)d\tau \qquad \ldots(4.6)$$

and,

Δt = uniform time step size;

i, n, & m = indices denoting number of time steps;

$\bar{Q}_i$ = average pumping rate during i^{th} time step;

s_n = drawdown at the end of n^{th} time step.

Combining eqs. (4.3) and (4.6) and solving the integral, we get,

$$\delta_1(m) = F_1(m) - F_1(m-1) \qquad \ldots(4.7)$$

where,

$$F_1(m) = \frac{1}{4\pi T}\left[W\left\{r^2 S/(4Tm\Delta t)\right\} - W\left\{r_1^2 S/(4Tm\Delta t)\right\}\right] \qquad \ldots(4.8)$$

Time distribution of drawdown at the observation well due to variable rate pumping can be obtained making use of eqs. (4.5) through (4.8). Since, time span is discretized, drawdown due to intermittent pumping or cyclic pumping can also be obtained using above equations by specifying actual discharge during each time step

4.1.2 RATE OF STREAM DEPLETION:

Stream depletion rate, q(t), due to constant rate pumping is given by (Glover,1978).

$$q(t) = Q\ \mathrm{erfc}\left\{\frac{a}{\sqrt{4\alpha t}}\right\} \qquad \ldots(4.9)$$

Therefore, unit step response function, U(t) for stream depletion rate can be expressed as,

$$U(t) = \mathrm{erfc}\left\{\frac{a}{\sqrt{4\alpha t}}\right\} \qquad \ldots(4.10)$$

From principle of superposition, stream depletion rate at time t due to variable rate pumping is given by,

$$q(t) = \int_0^t \frac{\partial Q(\tau)}{\partial \tau}\, U(t-\tau)d\tau \qquad \ldots(4.11)$$

In discretized form, above can be expressed as,

$$q_n = \sum_{i=1}^{n} \left(\Delta Q_i / \Delta t\right) \delta_2(n-i+1) \qquad \ldots(4.12)$$

$$\delta_2(m) = \int_0^{\Delta t} U(m\Delta t-\tau)d\tau \; ; \qquad \ldots(4.13)$$

and,

$\Delta Q_i = Q_i - Q_{i-1}$

q_n = stream depletion rate at the end of n^{th} time step.

Combining eqs. (4.10) and (4.13), and solving the integral, we get,

$$\delta_2(m) = \Delta t\left\{1-F_2(m-1)\right\} - \left\{a^2/(2\alpha)+m\Delta t\right\}\Delta F_2(m) - a\sqrt{\Delta t/(\pi\alpha)}\,\Delta F_3(m) \qquad \ldots(4.14)$$

where,

$$F_2(m) = \operatorname{erf}\left[a/\sqrt{4\alpha m\Delta t}\right] \; ; \qquad \ldots(4.15)$$

$$F_3(m) = \sqrt{m}\,\exp\left(-a^2/(4\alpha m\Delta t)\right) \; ; \qquad \ldots(4.16)$$

and,

$\Delta F_2(m) = F_2(m) - F_2(m-1)$;

$\Delta F_3(m) = F_3(m) - F_3(m-1)$.

Using eq. (4.12) and eqs. (4.14) through (4.16), variation of rate of stream depletion with time can be obtained for variable rate pumping.

4.1.3. VOLUME OF STREAM DEPLETION:

With calculated values of q_i (vide section 4.1.2), the cumulative volume of stream depletion up to the end of n^{th} time step, is given by,

$$V_n = \sum_{i=1}^{n} \bar{q}_i \Delta t \qquad \ldots(4.17)$$

where,

V_n = Cumulative volume of stream depletion up to the end of n^{th} time step and;
$\bar{q}_i$ = average stream depletion rate during i^{th} time step.

4.2 Inverse Problem

The inverse problem in present study is defined as, 'to find the parameters (T, S, and a) utilizing the drawdown in an observation well obtained during a variable rate pump test conducted near a recharge boundary'.

In order to solve the problem, a derivative based nonlinear optimization method proposed by Marquardt(1963) has been used. A description of the algorithm for a general problem may be found in Singh(1995). Present problem involves the determination of three parameters, viz., T, S, and a. The associated direct problem, has been discussed in section 4.1.1. In application of Marquardt algorithm to the present problem, analytical derivatives of calculated drawdown w.r.t. the parameters have been used.

4.2.1. ANALYTICAL DERIVATIVES:

Substituting eqs. (4.7) and (4.8) in eq. (4.5) and differentiating w.r.t. T, S, and a, respectively, we get,

$$\left.\frac{\partial s}{\partial S}\right\}_n = \sum_{i=1}^{n} \bar{Q}_i \delta_3(n-i+1) \; ; \qquad \ldots(4.18)$$

$$\left.\frac{\partial s}{\partial T}\right\}_n = \sum_{i=1}^{n} \bar{Q}_i \delta_4(n-i+1) \; ; \qquad \ldots(4.19)$$

and,

$$\left.\frac{\partial s}{\partial a}\right\}_n = \sum_{i=1}^{n} \bar{Q}_i \delta_5(n-i+1) \; ; \qquad \ldots(4.20)$$

where,

$$\delta_3(m) = \frac{1}{4\pi TS} \left\{\Delta F_4(m) - \Delta F_5(m)\right\} \; ; \qquad \ldots(4.21)$$

$$\delta_4(m) = -\frac{1}{T} \left\{S\delta_3(m) + \delta_1(m)\right\} \; ; \qquad \ldots(4.22)$$

$$\delta_5(m) = \frac{(2a-x)}{\pi T r_1^2} \Delta F_4(m) \; ; \qquad \ldots(4.23)$$

in which,

$$F_4(m) = \exp\left\{-r_1^2 S/(4Tm\Delta t)\right\} \; ; \qquad \ldots(4.24)$$

$$F_5(m) = \exp\left\{-r^2 S/(4Tm\Delta t)\right\} \; ; \qquad \ldots(4.25)$$

$$\Delta F_4(m) = F_4(m) - F_4(m-1) \; ; \qquad \ldots(4.26)$$

$$\Delta F_5(m) = F_5(m) - F_5(m-1) \qquad \ldots(4.27)$$

Using eqs. (4.18) through (4.27), analytical derivatives of calculated drawdown w.r.t. the parameters can be obtained.

5.0 RESULTS

The results of the application of methodologies developed in chapter 4, have been presented here. Results concerning with direct problem and inverse problem have been discussed in section 5.1 and 5.2 respectively.

5.1 Direct Problem

Results for the direct problem have been obtained for a hypothetical problem with variable rate pumping near a stream. Variation of pumping rate with time is shown in fig.5.1. Values of the parameters have been taken as $T = 0.15m^2/min.$, $S=5x10^{-4}$ and a=200m. The coordinate of the observation well location have been taken as x=50m, y=50m, origin being at pumping well location.

Drawdowns at the observation well due to above hypothetical variable rate pumping, have been obtained using eqs. (4.5) through (4.8), at 15 min. intervals. Δt has been taken equals to 5 min. The variation of drawdown with time is shown in fig.5.2. Rate of stream depletion at an interval of 15 min., have been calculated using eqs.(4.12) through (4.16). Fig.5.3 shows the variation of stream depletion rate with time. Using the above calculated values of rate of stream depletion, cumulative volume of stream depletion have been obtained using eq. (4.17). The variation of cumulative volume of stream depletion with time is shown in fig.5.4. For the present hypothetical problem, the total volume of pumped water is 2460 m^3 while cumulative volume of stream depletion after 3000 min. is 2066m^3.

5.2 Inverse Problem

5.2.1 APPLICATION TO SYNTHETIC DATA:

The drawdown data obtained at the observation well (vide section 5.1) for the hypothetical problem, were used to determine the aquifer parameters (T & S) and effective distance (a) using the methodology discussed in section 4.2. Starting from the initial guesses of the parameters, the step changes in the parameter values were calculated successively using Marquardt(1963) criteria, to arrive at the optimal values of the parameters for which the sum of the squares of the errors between observed and calculated drawdowns is minimum. At each iteration, the analytical derivatives of calculated drawdown w.r.t. each parameter were calculated using the expressions derived in section 4.2.1.

The optimized values of the parameters for different initial guesses along with the sum of squared errors are given in table 5.1. The table shows that irrespective of the initial guesses, the optimized values of the parameters come to be exactly equal to those used for generating synthetic data (i.e., $T=0.15m^2/min.$, $S=5x10^{-4}$ and $a=200m$). This does not fully justifies the applicability of the methodology for inverse problem because the synthetic data taken were free from observational errors.

Table 5.1 Hypothetical Problem: Results of Optimization

Sl. No.	Initial guesses of the Parameters T (m^2/min.)	S	a (m)	Optimized values of the Parameters T (m^2/min.)	S	a (m)	Sum of squared errors (m^2)
1.	1.0	0.01	140	0.15	$5x10^{-4}$	200	$7.50x10^{-10}$
2.	10.0	0.10	50	0.15	$5x10^{-4}$	200	$7.63x10^{-10}$
3.	100.0	0.10	500	0.15	$5x10^{-4}$	200	$7.51x10^{-10}$
4.	0.01	0.10	4000	0.15	$5x10^{-4}$	200	$7.56x10^{-10}$

Application of the methodology to a field example in which the drawdown data may contain errors, is discussed in next section.

5.2.2 APPLICATION TO FIELD DATA:

In this section, the optimized values of the parameters (T, S, & a) were obtained using the drawdown data recorded during a pump test in field. Data from an example in Chawla and Sharma(1977) has been used to demonstrate the applicability of the methodology developed in section 4.2.

Example: A well 40cm in diameter and 8.8m depth is installed at a distance of 41.15m from a river nearby. Whereas the well completely penetrates the 6.7m thick layer of saturated alluvium. The well was pumped to give a constant discharge of

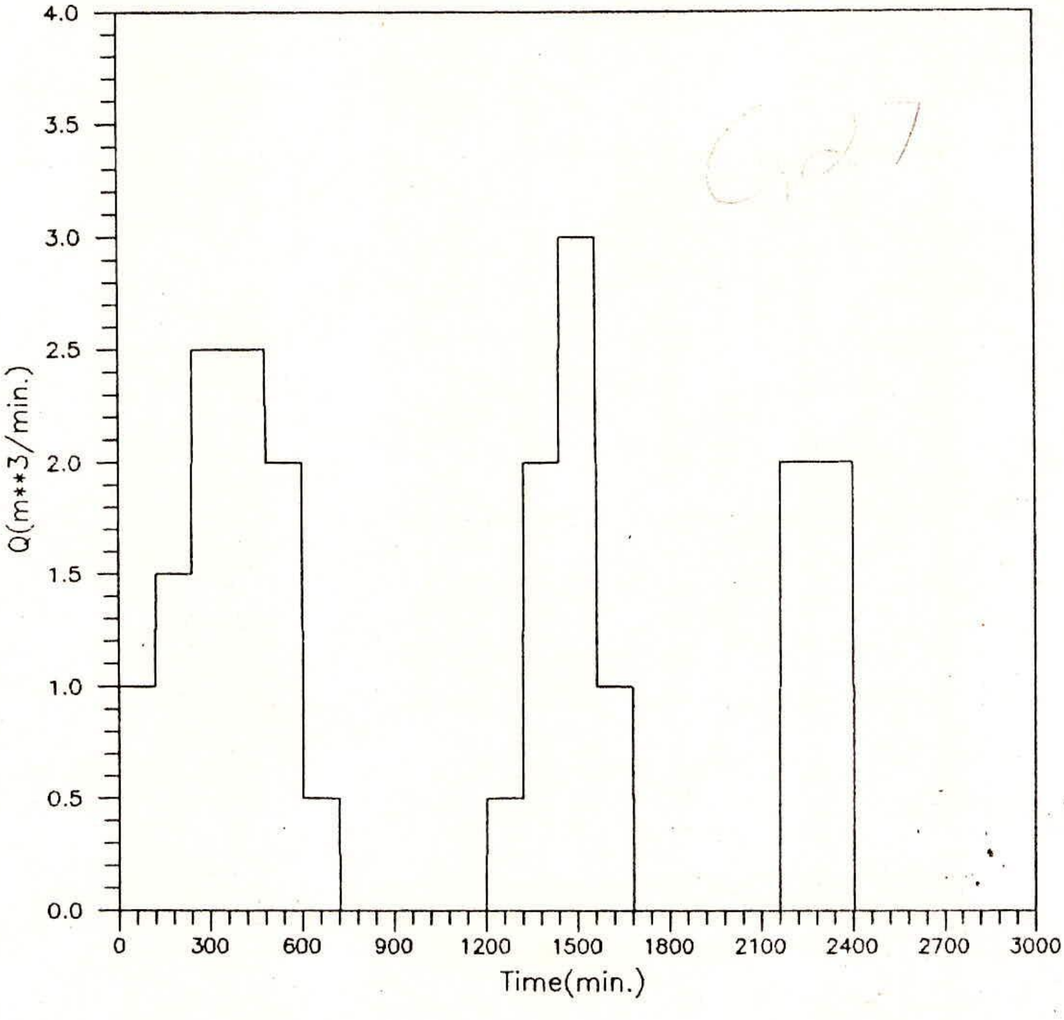

FIG.5.1 Hypothetical Problem: Variation of Q with time

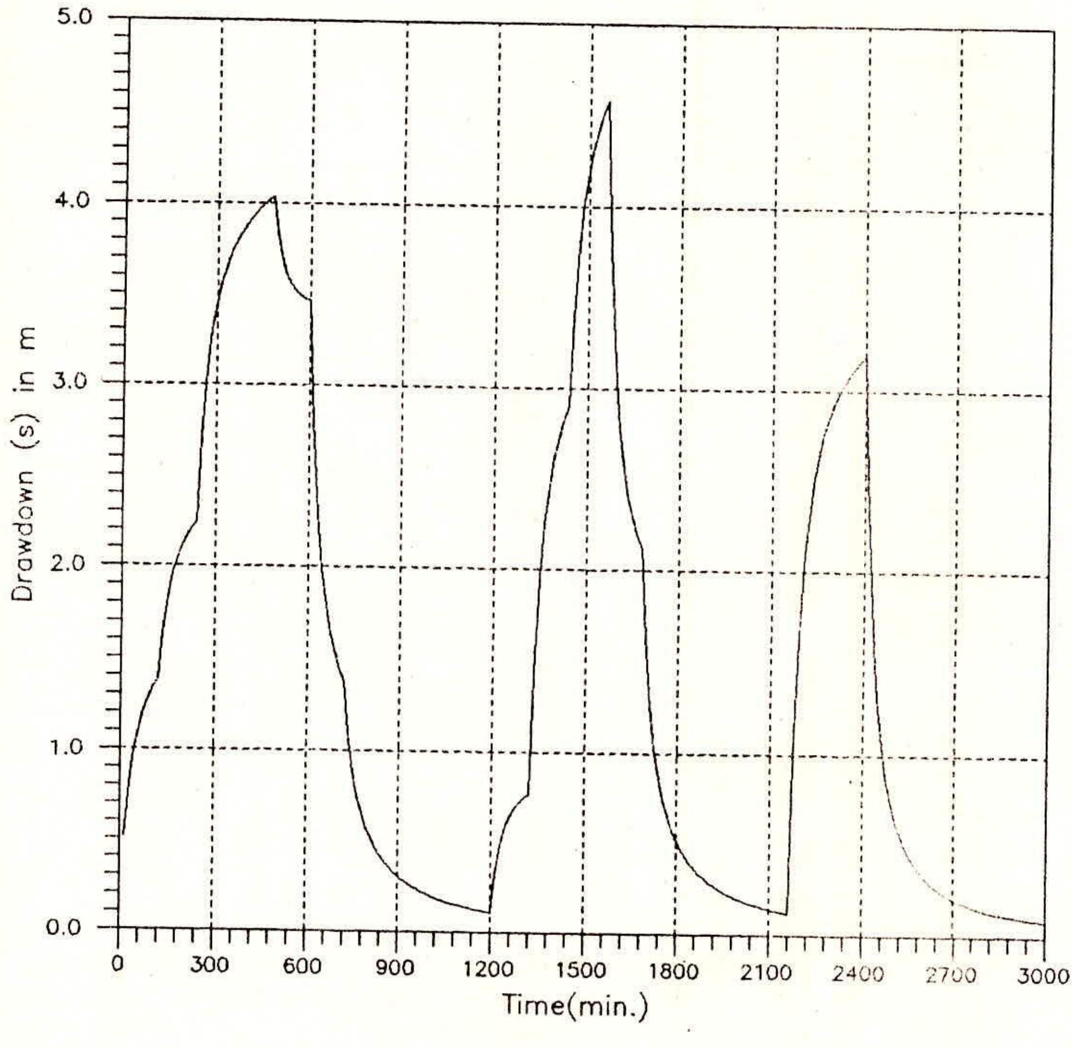

FIG.5.2 Variation of Drawdown with Time

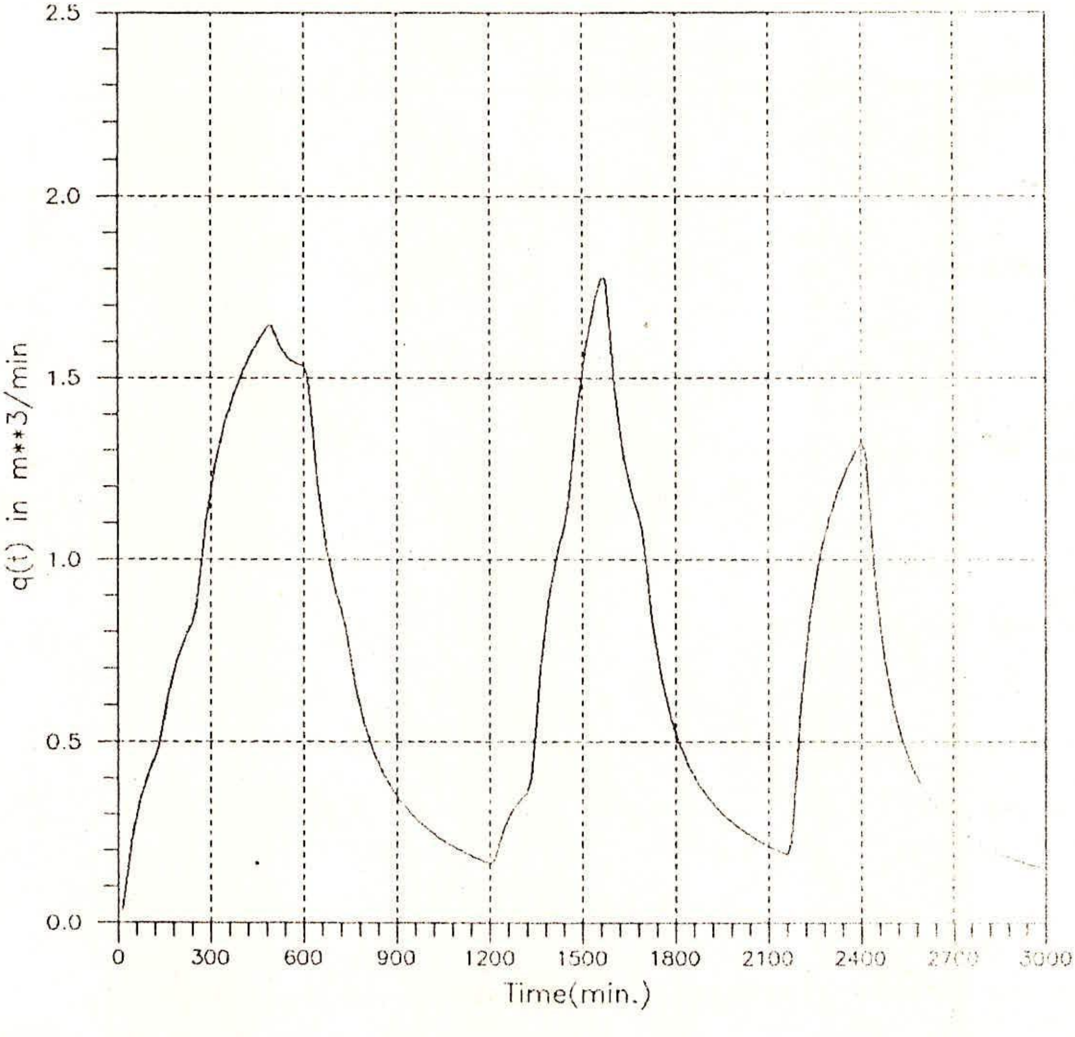

FIG.5.3 Variation of Stream Depletion Rate with Time

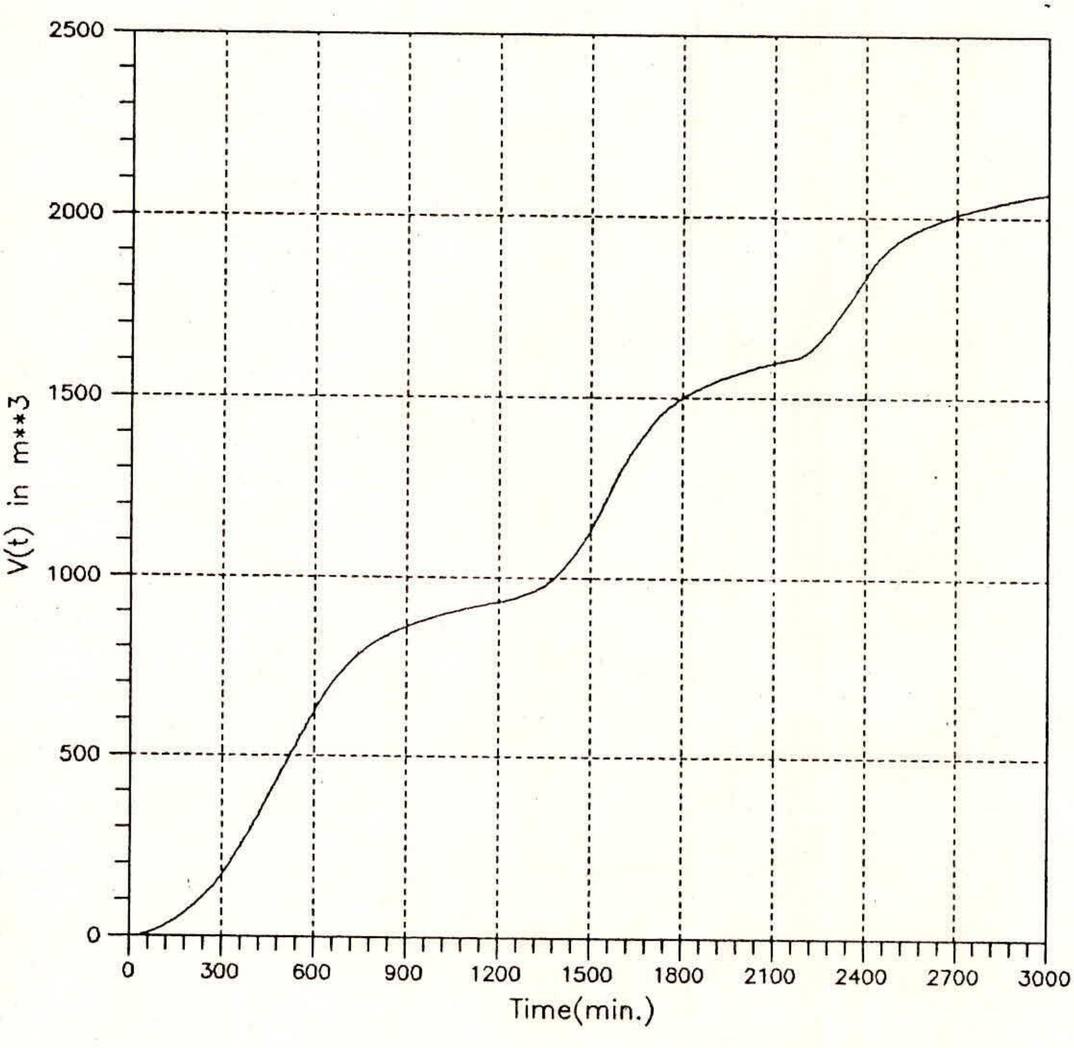

FIG.5.4 Variation of Cumulative Volume of Stream Depletion with time

$0.043 m^3/sec$. Drawdowns at the two observation well (vide fig. 5.5), are given in Table 5.2. The drawdowns given in the table has already been reduced by $s^2/(2x6.7)$ to account for the water table conditions. The optimized values of the parameters along with the values obtained using Hantushmethod, are given in table 5.3. The sum of the squares of errors between observed and computed drawdowns, is much smaller in present study than that obtained using Hantush method. This shows that the present methodology is applicable to the above type of problems and gives more reliable estimates of parameters than those obtained using traditional method.

Table 5.2 Field Problem: Observed Drawdown

Sl. No.	Time since pumping started (min)	Observation Well O_1 (x=-29.5m, y=-7.7m) Drawdown in m	Observation Well O_2 (x=-38.7m, y=-8.4m) Drawdown in m
1.	75	0.137	--
2.	90	0.146	--
3.	110	0.152	0.125
4.	130	0.168	0.137
5.	150	0.171	0.143
6.	180	0.186	0.149
7.	210	0.198	0.158
8.	240	0.207	0.165
9.	300	0.206	0.180
10.	390	0.235	0.192
11.	480	0.244	0.201
12.	600	0.256	0.210
13.	720	0.262	0.216
14.	900	0.271	0.219

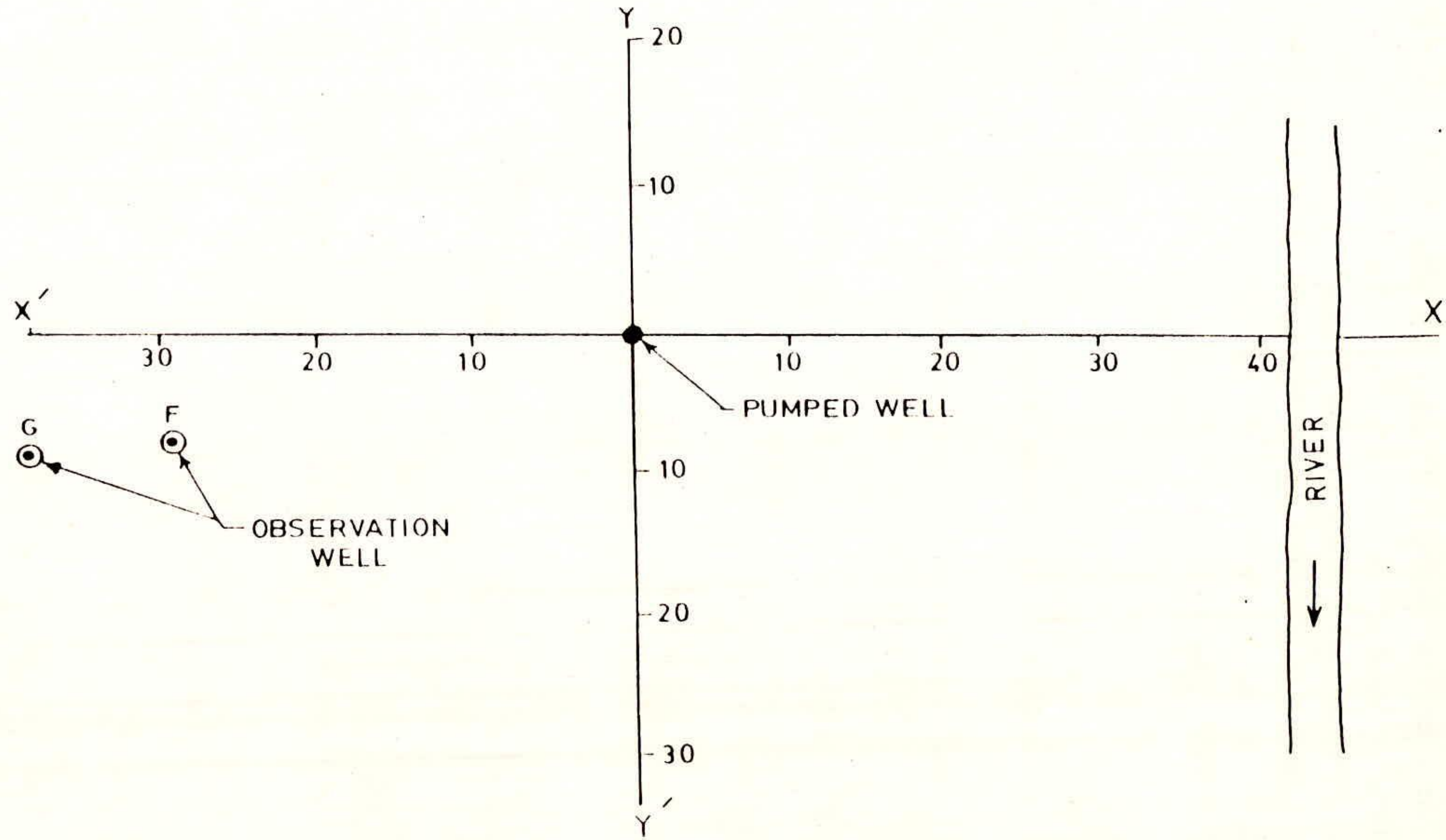

Fig. 5·5 - Field Example: Definition sketch

Table 5.3 Field Problem: Optimized Values of the Parameters

Well No.	Location x (m)	Location y (m)	Method	Optimized Values of the Parameters T (m^2/min)	S	a (m)	Sum of Squared Errors (m^2)
O_1	-29.5	-7.7	Present	3.1927	0.0781	172.6	8.03×10^{-5}
			Hantush's	1.38	0.137	25.0	7.73×10^{-3}
O_2	-38.7	-8.4	Present	3.5453	0.0687	169.4	4.19×10^{-5}
			Hantush's	1.38	0.138	26.0	5.57×10^{-3}

6.0 CONCLUSIONS

The following conclusions are drawn from the study.

1. Equations for calculating unsteady drawdown. and rate and volume of stream depletion have been developed which are applicable for variable rate pumping near the stream.

2. Expressions for analytical derivatives of unsteady drawdown w.r.t. the parameters (T. S. & a) have been derived.

3. A methodology has been evolved for the estimation of the aquifer parameters (T & S) and effective distance (a) utilizing the drawdown data obtained at an observation well due to a variable rate pump test conducted near a recharge boundary. A nonlinear optimization algorithm developed by Marquardt has been used with analytical derivatives.

4. The parameters estimated using above methodology have been found to be more reliable as compared to those obtained using traditional method.

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