ASSESSMENT OF RECHARGE FROM PARTIALLY PENETRATING IN FLUENT STREAM

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PREFACE

Stream aquifer interaction has been studied in greater details in recent years. There are two aspects of the process, the exchange of flow during the passage of a flood wave and the effluent or influent flow during lean flow period. The groundwater flow during the passage of a flood remains in an unsteady state where as the flow during the lean flow can be regarded as steady. The steady recharge from or discharge to a stream can be evaluated using analytical approach for idealised stream aquifer system. Rigorous groundwater modelling considering the anisotropy and non homogeneity of the system and the time varying boundary condition can predict the flow in complex stream aquifer system.

The recharge from a partially penetrating stream can also be predicted from the observation of piezometric level in the vicinity of the stream. In the present study the parameter that is required to find the seepage from a stream has been derived. Using this parameter , the stream stage and piezometric level at a piezometer in the vicinity of the stream the recharge from a stream can be computed for steady state groundwater flow condition.

The present study on assessment of recharge from a partially penetrating stream has been carried out by Dr.G.C.Mishra, Scientist F as a part of the work programme of Ground Water Assessment Division.

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ABSTRACT

The exchange of flow between a stream and an aquifer can be estimated solving the differential equation governing the flow using either an analytical or a numerical method. Solution of Laplace equation, which satisfies the boundary conditions prevailing at the flow domain boundaries, enables quantification of steady state seepage from the stream. For unsteady state condition Boussinesq's equation is solved satisfying the initial and the boundary condition. Using analytical methods the exchange of flow is obtained as a nondimensional term. To obtain the numerical value, the different terms such as hydraulic conductivity, storativity, and potential difference and stream geometry factor which appear in the dimensionless term are required to be quantified.

Using conformal mapping technique an expression of steady state seepage from a partially penetrating stream has been derived in this study. The expression relates the seepage to the potential difference at an observation well and the geometry of the flow domain. The conductivity can be estimated from dilution technique if the gradient of the flow line at the observation well is monitored.

1.0 INTRODUCTION

The exchange of flow between a stream and an aquifer can be estimated solving the differential equation governing the flow using either an analytical or a numerical method. Solution of Laplace equation, which satisfies the boundary conditions prevailing at the flow domain boundaries, enables quantification of steady state seepage from the stream. For unsteady state condition Boussinesq's equation is solved satisfying the initial and the boundary condition. Using analytical methods the exchange of flow is obtained as a nondimensional term. To obtain the numerical value, the different terms such as hydraulic conductivity, storativity, and potential difference and stream geometry factor which appear in the dimensionless term are required to be quantified.

The insitu hydraulic conductivity and storativity of an aquifer can be determined in various ways. The one which is mostly used is aquifer test. However aquifer test is expensive as well as time consuming. The hydraulic diffusivity is also determined using unsteady stream stages and corresponding water level fluctuations in an observation well. The transmissivity and storativity can not be identified separately from the study of unsteady flow in a stream aquifer system. Experimental method such as the dilution technique can be used to determine the insitu hydraulic conductivity. The dilution test is convenient and less time consuming.

Using conformal mapping technique an expression of steady state seepage from a partially penetrating stream has been derived in this study. The expression relates the seepage to the potential difference at an observation well and the geometry of the flow domain. The conductivity can be estimated from dilution technique

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2.0   REVIEW

2.1   Estimation of Insitu Hydraulic Conductivity:

The transmissivity and storage coefficient of an aquifer are determined by analyzing its response to some known perturbation. The perturbation may be imparted either by artificial means, such as pumping, or by natural ways, such as passage of a flood in an adjoining stream. The first type of test, in which water is pumped from a well and the consequent changes in piezometric surface are observed, enables determination of both transmissivity and storage coefficient separately. The second type of test, in which the changes in water level in an observation well occur consequent to the change in stream stage, requires data collection over a longer time duration. Such a test in a stream-aquifer system enables determination of the hydraulic diffusivity only; the transmissivity and the storage coefficient can not be estimated individually (Hall and Moench, 1972).

It is possible to determine hydraulic conductivity in a piezometer or single well by the introduction of a tracer into the well bore. The tracer concentration decreases with time under the influence of the natural hydraulic gradient that exists in the vicinity of the well. This approach is known as the bore hole dilution. Bore hole dilution tests can be performed in relatively short periods of time in a single well or piezometer. The test provides an estimate of the horizontal average linear velocity of the groundwater in the formation near the well screen. The test is performed in a segment of a well screen that is isolated by packers from overlying and underlying portions of an observational

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well. Into this isolated well segment a tracer is quickly introduced and is then subjected to continual mixing as lateral groundwater flow gradually removes the tracer from the well bore. The combined effect of groundwater through-flow and mixing within the isolated well segment produces a dilution versus time relation. From this relation the average horizontal velocity of groundwater in the formation beyond the sand or gravel pack but close to the well screen is computed. The theory on which the computational methods are based is described in text book (Freeze and Cherry, 1979) and it is described below.

The effect of the well bore and sand pack in a lateral flow regime is shown in Fig.1. The average linear velocity of the groundwater in the formation beyond the zone of disturbance is $\bar{v}$. The average bulk velocity across the center of the well bore is denoted by $\bar{v}^*$. It is assumed that the tracer is nonreactive and that it is introduced instantaneously at concentration C_o into the isolated segment of the well screen. The vertical cross sectional area through the center of the isolated segment is denoted as A which is equal to the product of diameter of the bore hole and spacing of the packers. The volume of this well segment is W which is equal to cross sectional area of the bore hole multiplied by the distance between the two packers. At time $t>0$, the concentration C in the well decreases at a rate

$$\frac{dC}{t} = - \frac{A \cdot v^* \cdot C}{W}$$

which, upon rearrangement, yields

$$\frac{dC}{C} = - \frac{A \cdot v^* \cdot dt}{W}$$

Integration and use of the initial condition, $C = C_0$ at$=0$, leads to

$$\bar{v}^* = - \frac{W}{A \cdot t} \ln (C/C_0)$$
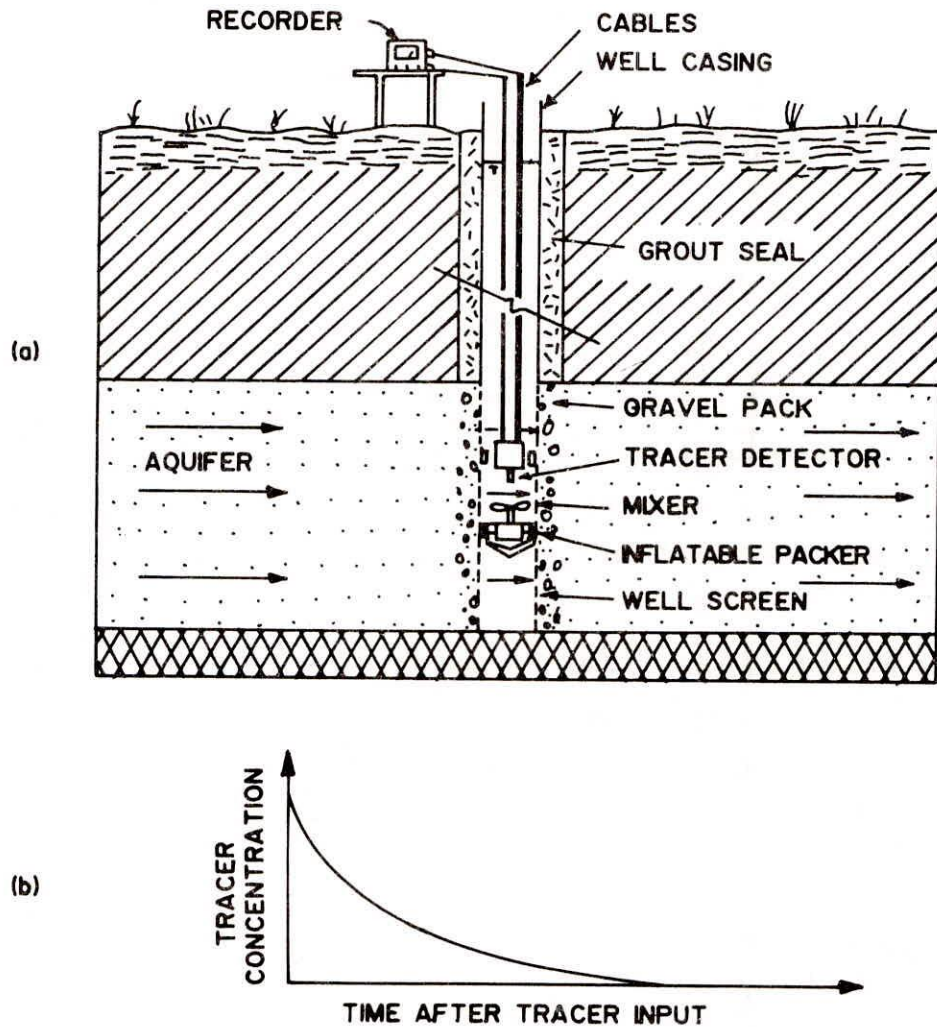
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(a)

(b)

FIGURE I. BOREHIOLE DILUTION TEST:
    (a) SCHEMATIC DIAGRAM OF APPARATUS;
    (b) DILUTION OF TRACER WITH TIME

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From the concentration versus time data obtained during bore hole dilution tests values of v^* is computed.

The objective of the test, however, is to obtain estimates of $\bar{v}$. This is accomplished using the relation $\bar{v} = \bar{v}^* / \bar{\alpha}$, where $\bar{\alpha}$ is an adjustment factor that depends on the geometry of the well screen, and on the radius and hydraulic conductivity of the sand or gravel pack around the screen. The usual range of $\bar{\alpha}$ for tests in sand or gravel aquifer is from 0.5 to 4 (Drost et al. ,1968).

2.2 Reach Transmissivity Constant:

Using a simple potential theory Morel-Seytoux et al. (1979) have derived the following expression for influent seepage from a partially penetrating stream.

$$Q = k \left[(0.5 W_p + e)/ (5 W_p + 0.5 e) \right] \Delta h$$

in which Q is exchange of flow between the stream and the aquifer per unit length of stream, k = hydraulic conductivity, W_p is the wetted perimeter of the stream, e is the saturated thickness of the aquifer below the stream bed, Δh is the difference in water level in the stream and in an observation well which is located at a distance of $5 W_p$ from the centre of the stream. If the stream stage is higher than the water level in the observation well the flow will take place from the stream to the aquifer, otherwise the aquifer contributes to the flow in the stream. This expression has been used in many stream aquifer interaction studies. The term $L_r k \left[(0.5 W_p + e)/(5 W_p + 0.5 e) \right]$ is known as reach the transmissivity which is the constant of proportionality between the exchange of flow and the potential difference between the stream and the aquifer in the vicinity of the stream. L_r is the length of the reach.

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The reach transmissivity term can be derived rigorously using conformal mapping. In the present study the reach transmissivity has been derived for a partially penetrating stream of large width for a confined flow problem.

3.0 PROBLEM DEFINITION

A partially penetrating stream of large width forms the boundary of a semi infinite isotropic confined aquifer. The two dimensional flow is in a steady state condition. An observation well is located at a distance L from the stream bank in which the depth to water level is measured from a high datum. The level of water in the stream is also measured from the same high datum. It is required to estimate the seepage making use of the potential difference between the stream and the aquifer at the piezometer.

4.0 ANALYSIS

A Schematic section of the partially penetrating stream with large width is shown in Fig.2(a) in z-plane (z=x+iy). The complex potential plane w (w=ϕ+iψ) for the flow domain is shown in Fig.2(b). ψ is the stream function and velocity potential function, ϕ, is defined as ϕ =-k(p/γ_v+y) + c_1 (Harr, 1962) in which k=hydraulic conductivity, p=pressure, γ_v=unit weight of water, c_1 is an arbitrary constant. The complex potential plane has been drawn assuming the constant c_1=0. According to Schwarz-Christoffel transformation, the conformal mapping of the flow domain onto the lower half of the ζ plane is given by (vide, Harr, 1962),
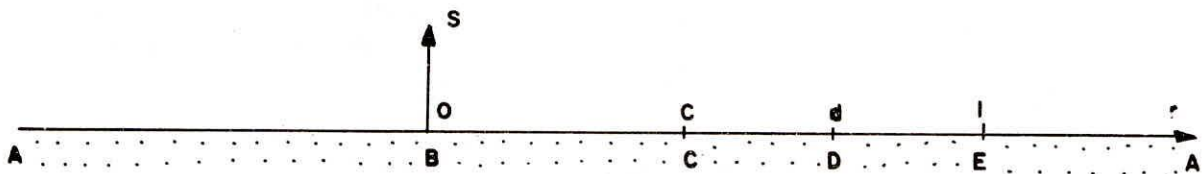
$$dz = M \, d\zeta \; / \; [(\zeta - c)^{1/2} (1 - \zeta) \zeta^{1-9/2}] \qquad \ldots (1)$$

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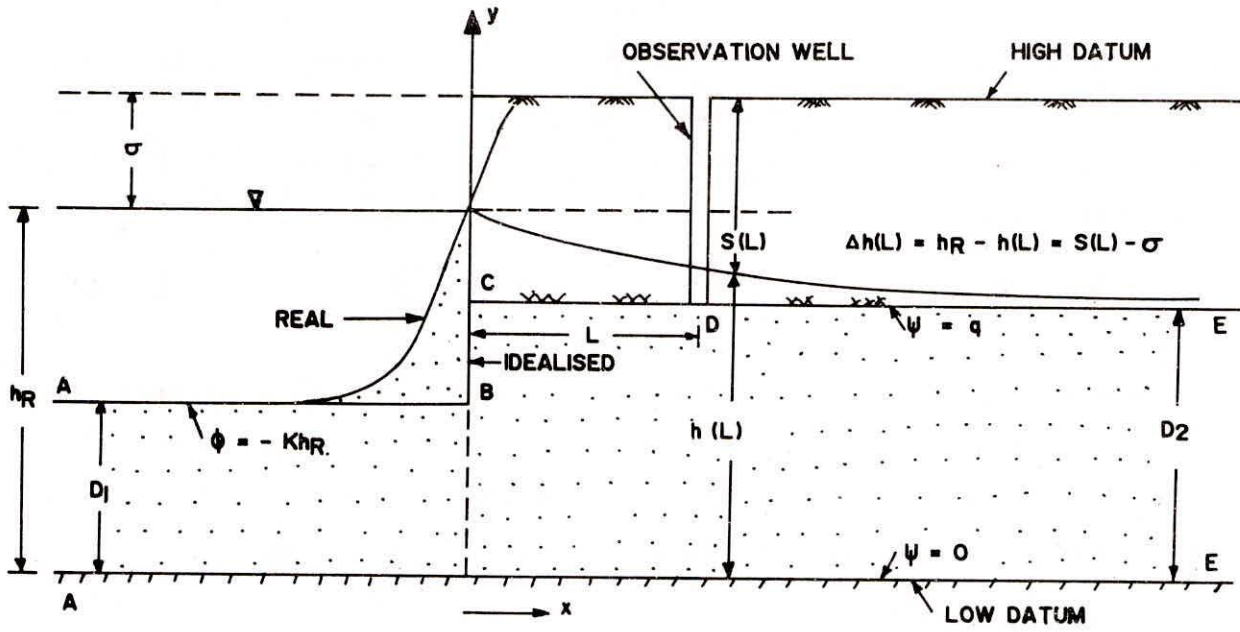
(a). PHYSICAL FLOW DOMAIN IN z-PLANE, z = x + iy
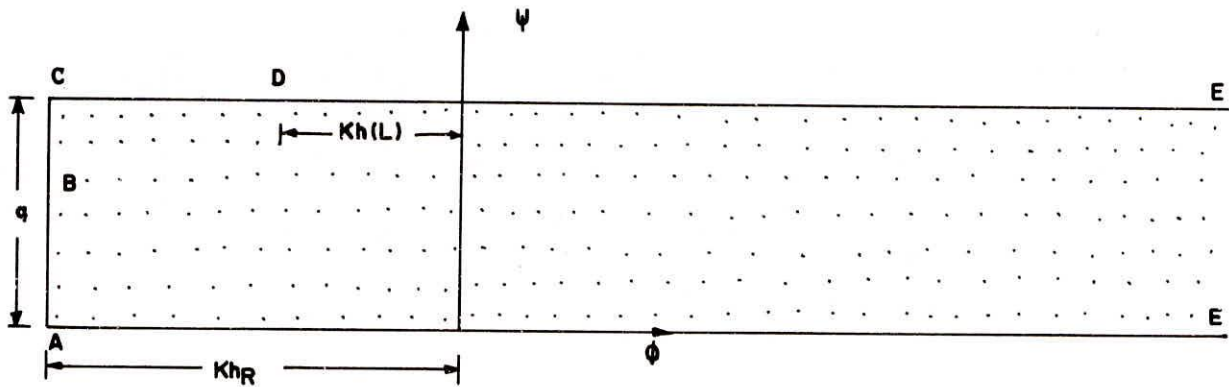
(b)   w - PLANE (w = ϕ + iψ)

(c) ξ - PLANE (ξ = r + is)

FIG.2- PHYSICAL FLOW DOMAIN

COMPLEX POENTIAL (w) AND AUXILIARY PLANE (ξ)

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the vertices A, B, C, and E in z-plane having been mapped onto points $-\infty$, 0, c and 1 respectively of the ξ-plane. Substituting $\xi = Re^{i\theta}$, $d\xi = R e^{i\theta} d\theta$ and applying the condition that as one traverses in ξ-plane from $\theta = \pi$ to $\theta = 2\pi$ along a radius R, $R \to \infty$, the jump in z-plane is $-iD_1$, the constant M is found to be

$$M = D_1/\pi \qquad \qquad \ldots(2)$$

Substituting $(1-\xi) = Re^{i\theta}$, $d\xi = - R e^{i\theta} d\theta$, and applying the condition that as one traverses in ξ-plane around point $\xi = 1$ from $\theta = \pi$ to $\theta = 2\pi$ along a small circle of radius R, $R \to 0$, the jump in z-plane is $-iD_2$, the parameter c is found to be

$$c = 1 - (D_1/D_2)^2 \qquad \qquad \ldots(3)$$

For $c < \xi < 1$, the relation between z and ξ is given by

$$z = M \int_c^{\xi} [\, \xi^{1/2}/\{(1-\xi)(\xi-c)^{1/2}\}\,] \, d\xi + z_c \qquad \ldots(4)$$

At $\xi = d$, $z = z_d$; hence,

$$z_d - z_c = L = M \int_c^d [\, \xi^{1/2}/\{(1-\xi)(\xi-c)^{1/2}\}\,] \, d\xi$$

$$\qquad \qquad \ldots(5)$$

Substituting $\xi - c = v^2$, the improper integral appearing in (5) is converted to the following proper integral.

$$\frac{L}{D_1} = \frac{1}{\pi} \int_0^{\sqrt{d-c}} [\, 2\sqrt{(c+v^2)}/(1-c-v^2)\,] \, dv \qquad \ldots(6)$$

The above integral can be evaluated using Gauss-Quadrature formula. For a given value of L, the parameter d can be obtained by an iterative procedure.

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The conformal mapping of the w-plane onto the lower half of the ζ-plane is given by the Schwarz-Christoffel transformation

$$dw = M_1 \, d\zeta / [(\zeta - c)^{1/2}(1 - \zeta)] \qquad \ldots (7)$$

Substituting, $1 - \zeta = R e^{i\theta}$, and $d\zeta = -R e^{i\theta} i \, d\theta$ and applying the condition that as one traverses in ζ-plane from $\theta = \pi$ to $\theta = 2\pi$ around point $\zeta = 1$ along a small circle of radius R, $R \to 0$, the jump in w-plane is $-iq$, the constant M_1 is found to be

$$M_1 = (q/\pi)(1 - c)^{1/2} \qquad \ldots (8)$$

For $c \leq \zeta \leq d$

$$w = M_1 \int_c^\zeta d\zeta / [(1 - \zeta)(\zeta - c)^{1/2}] - kh_R + iq \qquad \ldots (9)$$

Substituting $\zeta - c = v^2$ in (9) and integrating

$$w = \{M_1/(1.-c)^{1/2}\} \log_e [\{(1-c)^{1/2} + \zeta\}/(1-c)^{1/2} - \zeta\}] \Big|_0^{(\zeta - c)^{1/2}}$$

$$-kh_R + iq \qquad \ldots (10)$$

Applying the condition that at $\zeta = d$, $w = -kh(L) + iq$, we get

$$k[h_R - h(L)] = q / \pi \log [\{(1-c)^{1/2} + d\}/(1-c)^{1/2} - d\}] \qquad \ldots (11)$$

Solving for q

$$q = \pi k [h_R - h(L)]/\ln[(\sqrt{1-c} + \sqrt{d-c})/(\sqrt{1-c} - \sqrt{d-c})] \qquad \ldots (12)$$

where, h_R and $h(L)$ are the hydraulic heads in the stream and at the observation well respectively. The flow being steady, q at any section is constant. The flow can be known for a known value of $h(L)$, which has to be recorded at an observation well. Equation

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(11) shows that the variation of head in the aquifer with distance from the stream bank is nonlinear. Equation(12) can be written as

$$q = \Gamma_r \, \Delta h \qquad \qquad \ldots(13)$$

in which the potential difference $\Delta h = h_R - h(L)$ and the reach transmissivity per unit length of stream from equation(12) is given by

$$\Gamma_r = \pi k / \ln[\, (\sqrt{1-c} + \sqrt{d-c}) / (\sqrt{1-c} - \sqrt{d-c}) \,] \qquad \ldots(14)$$

The reach transmissivity is dependent on the location of the observation well besides the depth of penetration of the stream.

The gradient along the top flow line can be derived as follows:

The derivative of the complex potential can be expressed as (vide Harr,1962)

$$\frac{dw}{dz} = \frac{dw}{d\zeta} \frac{d\zeta}{dz} \qquad \qquad (15)$$

Replacing expression of $\frac{dw}{d\zeta}$ and $\frac{dw}{dz}$ from (7) and (1) respectively in (13) and simplifying

$$\frac{dw}{dz} = q/D_2 \; \zeta^{-1/2} = u - iv \qquad \qquad (16)$$

Along C D E A the component of velocity in y direction is zero. Hence,

$$\frac{dw}{dz} = u = -k \frac{dh}{dx} = q/D_2 \; \zeta^{-1/2} \qquad \qquad (17)$$

$$(kD_2)/q \; \frac{dh}{dx} = -\zeta^{-1/2} \qquad \qquad (18)$$

5.0 RESULT AND DISCUSSION

The variation of $q/(k\Delta h)$ with distance of the observation well from the stream bank is shown in Fig.3 for different
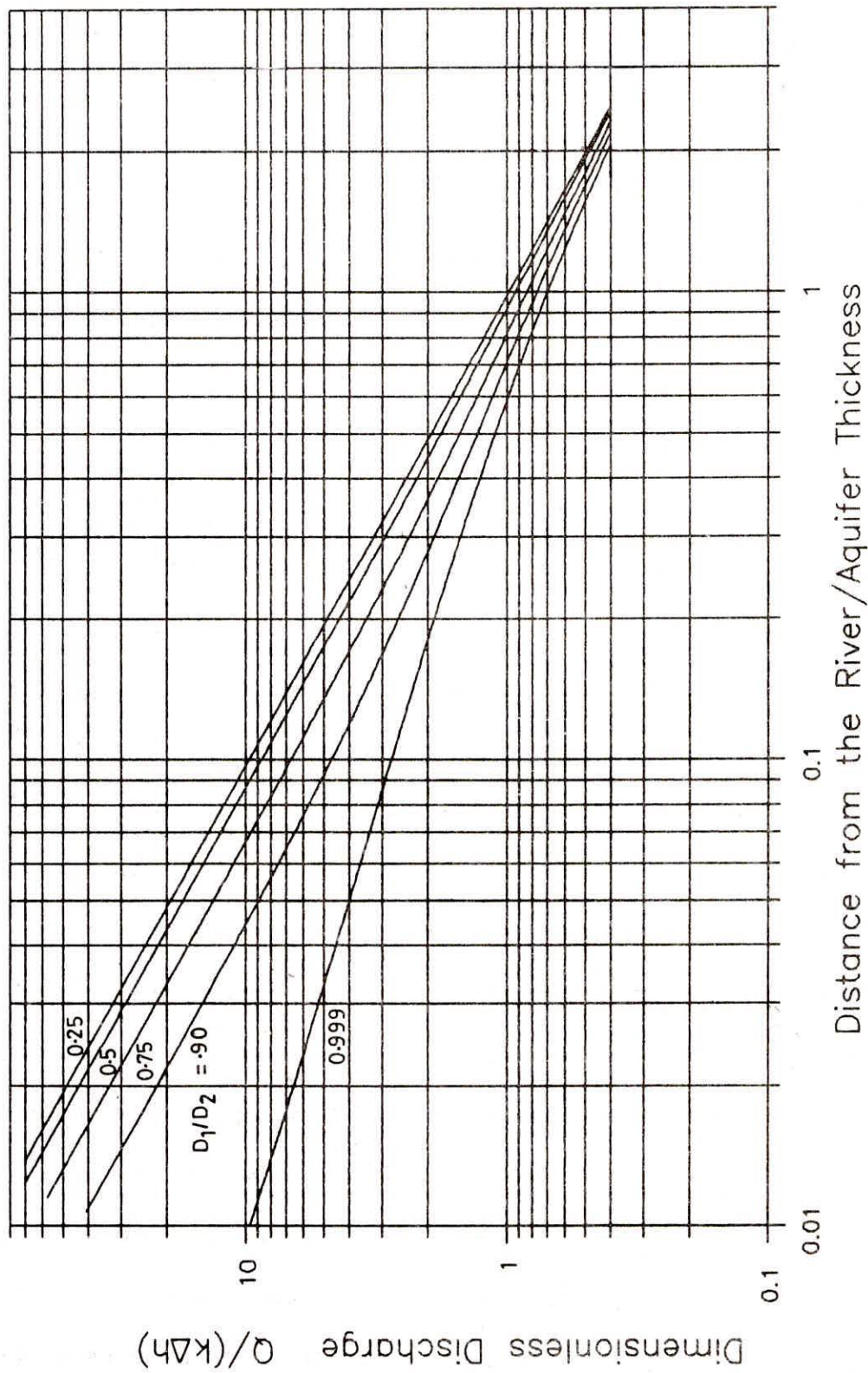
FIG. 3. VARIATION OF Q/(k h) WITH DISTANCE FROM THE RIVER
FOR DIFFERENT PENETRATION OF THE RIVER

Distance from the River/Aquifer Thickness
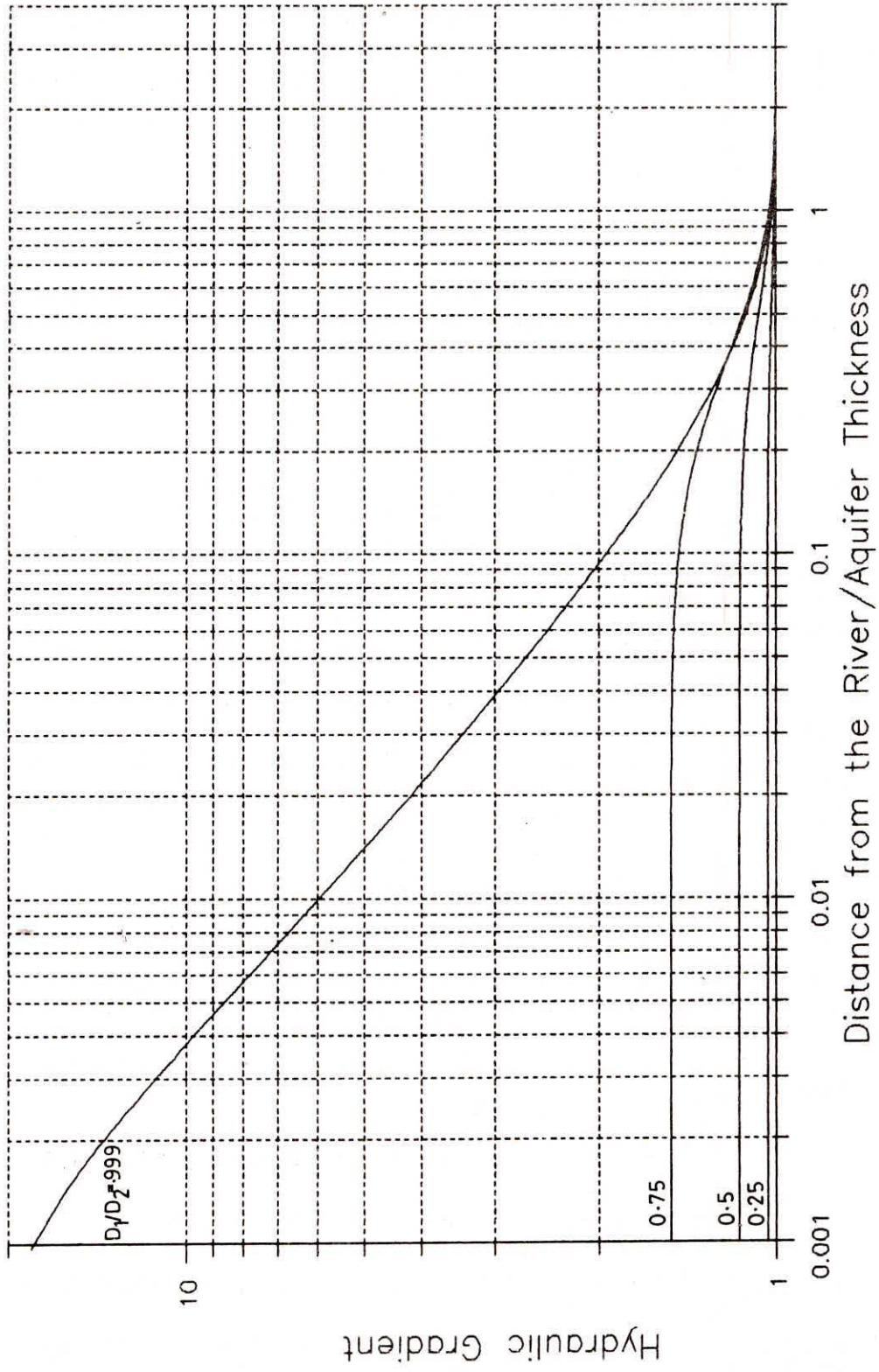
Dimensionless Discharge Q/(kΔh)

$D_1/D_2 = \cdot 90$

0·25
0·5
0·75
0.999

FIG. 4. VARIATION OF HYDRAULIC GRADIENT WITH DISTANCE FROM RIVER FOR DIFFERENT PENETRATION OF THE RIVER

Distance from the River/Aquifer Thickness

Hydraulic Gradient

$D_1/D_2 = .999$

0.75

0.5

0.25

penetration depth of the stream. The reach transmissivity is found to vary with distance from the stream for any depth of penetration. The variation is linear for a fully penetrating stream only, otherwise the relationship is nonlinear. From the graph it can be seen that beyond 1.5 times the aquifer depth, the partial penetration has no influence on the reach transmissivity. Using this graph the seepage from a stream can ascertained provided the hydraulic conductivity, the stream stage and the piezometric level in the observation well are available. The programme for computation of $q/(k\Delta h)$ is listed in Appendix I.

The variation of gradient along the top flow line is presented in Fig.4 for different depth of penetration. As seen from the figure the gradient tends to 1 at a distance equal to thickness of the aquifer. For a fully penetrating stream the gradient is 1.

6.0 CONCLUSION

For steady state flow the influent or effluent flow between a stream and a confined aquifer can be estimated from observation of stream stage, piezometric level in a piezometer in the vicinity of the stream and hydraulic conductivity. The insitu hydraulic conductivity can be found from dilution test. The geometry factor has been derived using a conformal mapping. The method suggested is not applicable for unsteady flow.

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```fortran
      DIMENSION GW(100),GX(100),CQBKHI(500)
      open(1,status='old',file='GANESH.dat')
      open(2,status='new',file='GANESH.out')
      nx=96
      read(1,*) D1,D2
C     PART=THICKNESS OF AQUIFER BELOW STREAM/ THICKNESS OF AQUIFER
C     D2=THICKNESS OF AQUIFER
C     D1=THICKNESS OF AQUIFER BELOW STREAM BED
C     gw(i) and gx(i) ARE GAUSSIAN WEIGHT AND ABSCISSAS
      read(1,*) (gw(i),i=1,nx)
      read(1,*) (gx(i),i=1,nx)
      PAI=3.14159265
200   CONTINUE
      D1BYD2=D1/D2
      WRITE(2,3)
3     FORMAT(5X,'D1',8X,'D2',9X,'D1BYD2')
      WRITE(2,4)D1,D2,D1BYD2
4     FORMAT(3E16.7)
      WRITE(2,33)
33    FORMAT(3X,'AL',9X,'QBYKH')
      C=1.-D1BYD2**2
      DELC=(1.0-C)*0.001
      D=C+DELC
300   CONTINUE
      SUM1=0.
      sum5=0.
      DO 100 I=1,NX
      x=gx(i)
      v=0.5*sqrt(c)+0.5*sqrt(c)*x
```

```fortran
      f5=sqrt(c)*sqrt(c-v*v)/(1.-c+v*v)
      F1=SQRT(C*(D-C)+((1.+x)*(D-C))**2/4.0)/
     1(1.-C-(1.+x)**2*(D-C)/4.)
      SUM1=SUM1+F1*GW(I)
      sum5=sum5+f5*gw(i)
100   CONTINUE
      AL=D1/PAI*SUM1
      d2md1=d1/pai*sum5
      term1=sqrt(1.-c)+sqrt(d-c)
      term2=sqrt(1.-c)-sqrt(d-c)
      term3=alog(term1/term2)
      qbykh=pai/term3
      WRITE(2,2)AL,QBYKH
2     FORMAT(2E12.4)
      D=D+DELC
      IF(D.LT.1.0) GO TO 300
      STOP
      END
```

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