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DETERMINATION OF SPECIFIC YIELD IN THE ZONE OF WATER TABLE FLUCTUATION

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PREFACE

The Ground Water Estimation Committee has recommended that where the stage of ground water development in a block is over 60 percent of the recoverable recharge, the ground water should be evaluated by water table fluctuation and specific yield approach. The specific yield values for different types of geological formations in the zones of fluctuation of water table has been proposed by the committee and these values may be adopted.

There are different methods by which the specific yield in the zone of water table fluctuation can be determined. The simplest method is soil moisture measurement in the zone of fluctuation. Long duration pumping tests in unconfined aquifer are conducted to find the delayed yield and this value can be taken as the specific yield. In the proposed study a mathematical groundwater flow model has been developed for determining the specific yield in the zone of water table fluctuation. A family of type curves for different ratios of specific yield to elastic storage coefficient have been obtained using which the specific yield and the elastic storage can be obtained from aquifer test data.

The study has been carried out under the work programme of Groundwater Assessment Division for the year 1993-94 by Dr.G.C.Mishra, Scientist of NIH.

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Director

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ABSTRACT

Regional dynamic ground water potential is evaluated from observed water table fluctuation and specific yield. There are different methods by which the specific yield in the zone of water table fluctuation can be determined. The simplest method is soil moisture measurement in the zone of fluctuation. Long duration pumping tests in unconfined aquifer are conducted to find the delayed yield and this value can be taken as the specific yield.

In the present study a mathematical groundwater flow model has been developed for determining the specific yield in the zone of water table fluctuation. A discrete kernel approach has been used to solve the water released owing to specific yield from the dewatered zone. A family of type curves for different ratios of specific yield to elastic storage coefficient have been obtained using which the specific yield and the elastic storage can be obtained from aquifer test data. The variation of well function with nondimensional time parameter exhibits three distinct parts. The early part like Theis curve has steep slope. The middle portion is flat due to the recharge from the dewatered zone. The latter part is again similar to Theis curve. As time increases the Theis well function and well function of the present analysis converge.

1.0 INTRODUCTION

In areas with well-defined seasonal rainfall the water table rises and falls in annual cycles, the rise corresponding to the rainfall period, and the low stage corresponding to the dry period. In coastal tracts cyclic semi-diurnal fluctuations in the water table are caused by tidal changes in sea level. Such fluctuations of the water table may extend inland across tidal streams also. The amplitude of the water-table fluctuations is always smaller than the tidal amplitude. Moreover, there is a considerable time lag in the propagation of tidal effect on the water table. The decrease in the amplitude and the increase in time lag are largely governed by the permeability and storativity of the aquifer and the distance from the shore line (Karanth, 1987). Water table also fluctuates near a stream in response to changes in the stream stage. The magnitude and extent of fluctuation depends on the relative head differences between the gorundwater and surface water levels, slope of the water table, permeability and specific yield of the materials and duration of changes in the stream stage. The magnitude of the water-table fluctuation also depends on climatic factors, drainage, topography, and geological conditions. Primarily, water-table fluctuation is governed by the specific yield of the material in the zone of water-table fluctuation.

The water table fluctuation and the corresponding boundary perturbation can be used for estimation of the parameters of the aquifer. Observed water table fluctuation through an adequate observation network and specific yield in the vicinity of the observation well enable evaluation of the dynamic ground water potential in a region.

There are different methods by which the specific yield in the zone of water table fluctuation can be determined. The simplest method is soil moisture measurement in the zone of fluctuation. Long duration pumping test in unconfined aquifer are conducted to find the delayed yield and this value can be taken as the specific yield. A review on determination of specific yield has been carried in the following paragraphs.

2.0 REVIEW

In the zone of saturation, groundwater fills all of the interstices; hence, the porosity provides a direct measure of the water contained per unit volume of soil. A portion of the water can be removed from subsurface strata by drainage or by pumping of a well; however, molecular and surface tension forces hold the remainder of the water in place. The specific yield is defined as the volume of water derived from storage in response to a change in water level over a given area of aquifer and it has been expressed as: $S_y = \Delta V/(\Delta h\ A)$, where S_y is specific yield, ΔV is the volume of water drained in response to the change in water level, Δh, which is effective over the area A (Domenico, 1972). It is the volume of water per unit area of soil, drained from a soil column extending from the water table to the ground surface, per unit lowering of the water table (Bear,1979).

Some water is retained in the pores by molecular attraction and capillarity. The amount of water that a unit volume of aquifer retains after gravity drainage is called its specific retention. The smaller the average grain size, the greater is the percent of retention; the coarser the sediment, the greater will be the specific yield. As the surface area increases, a larger percentage of the water in the pores is held by surface tension or other adhesive forces. Therefore, finer sediments have lower specific yields compared to coarser sediments, even if they both have the same porosity. Representative specific yields for various geologic materials are listed in Table 1 (Johnson, 1967). Individual values for a soil or rock can vary considerably from these values. It should be noted that fine-grained materials yield little water, where as coarse-grained materials permit a substantial release of water and hence serve as aquifers. In general, specific yields for thick unconsolidated formations tend to fall in the range off 7-15 percent, because of the mixture of grain sizes present in the various strata; furthermore, they normally decrease with depth due to compaction. Storage coefficients are much lower in confined aquifers because they are not drained during pumping, and any water released from storage is obtained primarily by compression of the aquifer and expansion of the water when pumped.

Table 1 Specific Yield in %for Selected Earth Materials

Material	Specific Yield Maximum	Minimum	Average
Clay	5	0	2
Sandy clay	12	3	7
Silt	19	3	18
Fine sand	28	10	21
Medium sand	32	15	26
Coarse sand	35	20	27
Gravelly sand	35	20	25
Fine gravel	35	21	25
Medium gravel	26	13	23
Coarse gravel	26	12	22

Values of specific yield depend on grain size, shape and distribution of pores, compaction of the stratum, and time of drainage. For comparatively uniform soils the specific yield is $S_y=\alpha-S_r$, where α is equal to maximum moisture capacity equal to porosity less entrapped air. Usually α equals the capillary moisture capacity a the base of the capillary fringe near the water table. S_r is the volume moisture content when gravity water has drained after lowering of the ground-water table. Usually it equals the volume of moisture content above the capillary fringe in the lower part of the zone of aeration not subjected to the influence of evaporation from land surface.

The specific yield in the zone of water table fluctuation can be evaluated in several ways. These are:

(i) Aquifer test,

(ii)Moisture measurements in zone of aeration.

2.1 Aquifer Test

Under water-table condition water is derived from storage by gravity drainage of the interstices above the cone of depression by compaction of the aquifer, and by expansion of the water itself as. pressure on the ground water is reduced. The gravity drainage of water through stratified sediments is not immediate and the unsteady flow of water towards a well in an unconfined aquifer is characterised by slow drainage of interstices. The effects of gravity drainage are not considered in Theis' non-equilibrium formula; therefore, application of Theis' formula for the analysis of pumping tests in unconfined aquifers is not justified except under some limiting conditions.

According to Walton (1960) "Three distinct segments of the time-drawdown curve may be recognized under water-table condition. Unconfined stratified sediments often react to pumping for a short time after pumping begins as would an artesian aquifer. Gravity drainage is not immediate but water is released instantaneously from storage by the compaction of the aquifer and its associated beds and by the expansion of the water itself. Under favourable conditions, it is sometimes possible to determine the coefficient of transmissibility of an aquifer by applying the Theis' non-equilibrium formula to the first segment of the time drawdown curve, which may cover little more than the earliest minute or so of data. The coefficient of storage computed from the first segment of the time-draw down curve is in the artesian range and cannot be used to predict long-term drawdowns of the water table. This storage coefficient also can not be used in groundwater balance. The drawdown in distant observation wells during the earliest minute or so of pumping is often too small to be measured and the first segments of the time-drawdown curves for distant wells may not be apparent. Caution must be exercised in using early data collected during tests made in heterogeneous aquifers because the time-drawdown data may deviate from theoretical data as adjustment of flow through regions of different permeability occurs.

"The second segment of the time-drawdown curve represents the intermediate stage in the decline of water levels when the cone of depression slows in its rate of expansion as it is replenished by gravity drainage of the sediments. The slope of the time-drawdown curve decreases as it reflects the presence of 'recharge' in the form of interstitial storage in the vicinity of the pumped well. Test data deviate markedly from the non equilibrium theory during the second segment". Nevertheless, attempts have been made to apply the non equilibrium formula to the second segment of the time-drawdown curve. The result of such an analysis leads one to conclude that there is an apparent change in storage coefficient with respect to time. According to Ferris et al. (1962), "there is little justification for the premise that the storage coefficient of a water-table aquifer varies with the time of pumping in as much as such anomalous data are merely the results of trying to apply a two-dimensional flow formula to a three-dimensional problem".

Walton further states "The third segment, which may start from several minutes to several days after pumping starts depending largely upon aquifer conditions, represents the period during which the time-drawdown curves conform closely to the non-equilibrium type curve". It is possible to determine the coefficient of transmissibility of an aquifer by applying the non equilibrium formula to the third segment of the time-drawdown curve. The coefficient of storage computed from the third segment of the time drawdown curve will be in the water-table range, and can be used to predict the long-term effects of aquifer development. Theis (1935) described the third segment of the time-drawdown curve under water-table conditions: "In as much as the rate of fall of the water table decreases progressively after a short initial period, it seems probable that as pumping continues the rate of drainage of the sediments tends to catch up with the rate of fall of the water table, and hence that the error in the non equilibrium becomes progressively smaller.

Considering the delay in gravity drainage Boulton (1963) has derived analytical solution to the problem of unsteady flow to a well of infinite small diameter which penetrates the full thickness of a homogeneous isotropic and unconfined aquifer. The Boulton's type curves can be used to evaluate the specific yield of unconfined aquifer.

Prickett (1965) has given values of delay index determined from aquifer tests which are presented below.

Table2- Aquifer Materials and their Delay Index

Materials through which gravity drainage takes place (Driller's Log)	Delay index ($1/\alpha$) (in minutes)
Sand: medium to coarse	40
medium	48-83
fine to medium	91
medium, some silty	123
fine	173
medium to coarse, silty	179
silty to medium	233
fine to medium, silty	357
fine to coarse, silty	625
fine with clay streaks	715
fine and silt 2,080	
Clay silt, and fine sand	3,230

The type-curve solution developed by Neuman (1975) can be used conveniently for determination of specific yield. The type curves are based on the assumptions that the production well and the observation well fully penetrate the aquifer and the well is pumped at a constant rate. The flow equation for unconfined aquifers is given by

$$h_0 - h = \frac{Q}{4\pi T} W(u_A, u_B, \Gamma)$$

$W(u_A, u_B, \Gamma)$ = the well function for the water table aquifer;

$u_A = r^2 S_s/(4Tt)$; $u_B = r^2 S_y/(4Tt)$; $\Gamma = (r/b)^2 (k_v/k_h)$;

$h_0 - h$ = drawdown at a radial distance r from the pumping well which is pumped at a constant rate of Q;

b = the initial saturated thickness of the aquifer;

k_v, k_h = the vertical and horizontal hydraulic conductivities of the aquifer;

T = the transmissivity.

Two sets of type curves are used (Figure 1). Type-A curves are good for early drawdown data, when instantaneous release of water from storage is occurring. As time elapses, the effects of gravity drainage and vertical flow cause deviations from the non equilibrium type curve. The Type-B curves are used for late drawdown data, when effects of gravity drainage are becoming smaller. The type-B curves end on a Thies curve. Neuman's type curves do not include such parameter as Boulton's delay index.

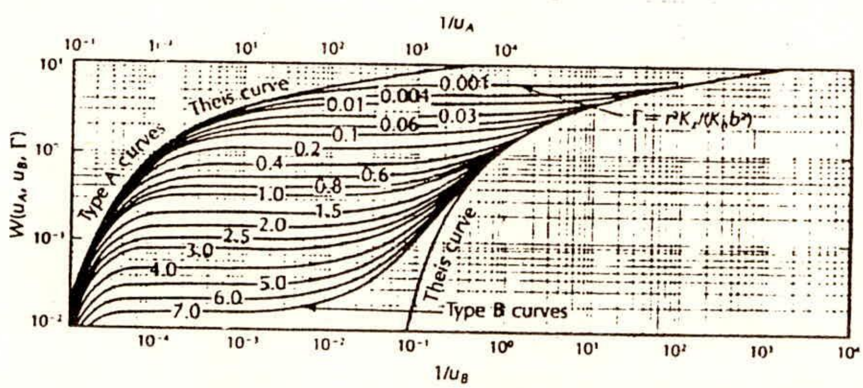


Fig.1- Neuman's Type Curve for Unconfined Aquifer without Delayed Yield

Due to complex effects when the water table moves, such as the capillary fringe and entrapped air, it is has been recommended to estimate the specific yield from observation of the aquifer response (Rushton and Redshaw, 1979).

2.2 Determination of Specific Yield from Moisture Measurement in the Zone of Aeration

The specific yield may be directly determined from the observation of moisture storage in the lower part of the zone of aeration during the period of time Δt during which the unconfined ground-water level changes by the value ΔH (Brown et.al, 1972.)

The initial level of water in a shallow well (depth to water not exceeding 5-6 m) is observed, together with the neighbouring moisture profile by sampling at vertical intervals of 0.1 m. When the ground-water level has risen by ΔH the soil sampling for determination of the moisture profile is repeated on the same site.

The volume moisture profile is drawn as shown in Figure 2.

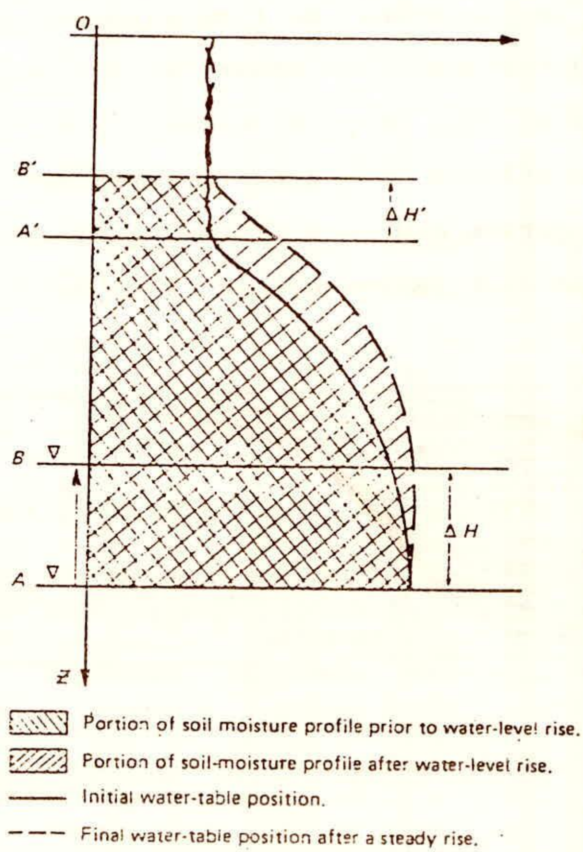


Fig.2 - Soil-moisture profile for the zone of aeration

The difference between the areas enclosed by them represents the increment of gravity water reserves $\mu\Delta H$ for a water-level rise ΔH. The increment in the moisture storage is given by the expression

$$S_y \Delta H = \Sigma_2 \Delta z V_0 - \Sigma_1 \Delta z V_0$$

from which the expression for specific yield has been obtained as:

$$S_y = (\Sigma_2 \Delta z V_0 - \Sigma_1 \Delta z V_0) / \Delta H$$

where:

Δz = vertical interval of soil sampling;

V_0 = the volume moisture content of soil in the middle of that interval (expressed as a decimal fraction).

The subscript 1 indicates the initial soil sampling for moisture content and the subscript 2 the later soil sampling.

Volume moisture content (expressed as a fraction of the soil volume) equals the gravity moisture content multiplied by $G/(1+e)$, where G is the specific gravity of soil and e is the void ratio.

It has been recommended that replication of experimental sites and sampling is required for experimental determination of specific yield because of the variability of soil profile. For sandy soil triple replication has been recommended while for loamy soil four to five replications are necessary.

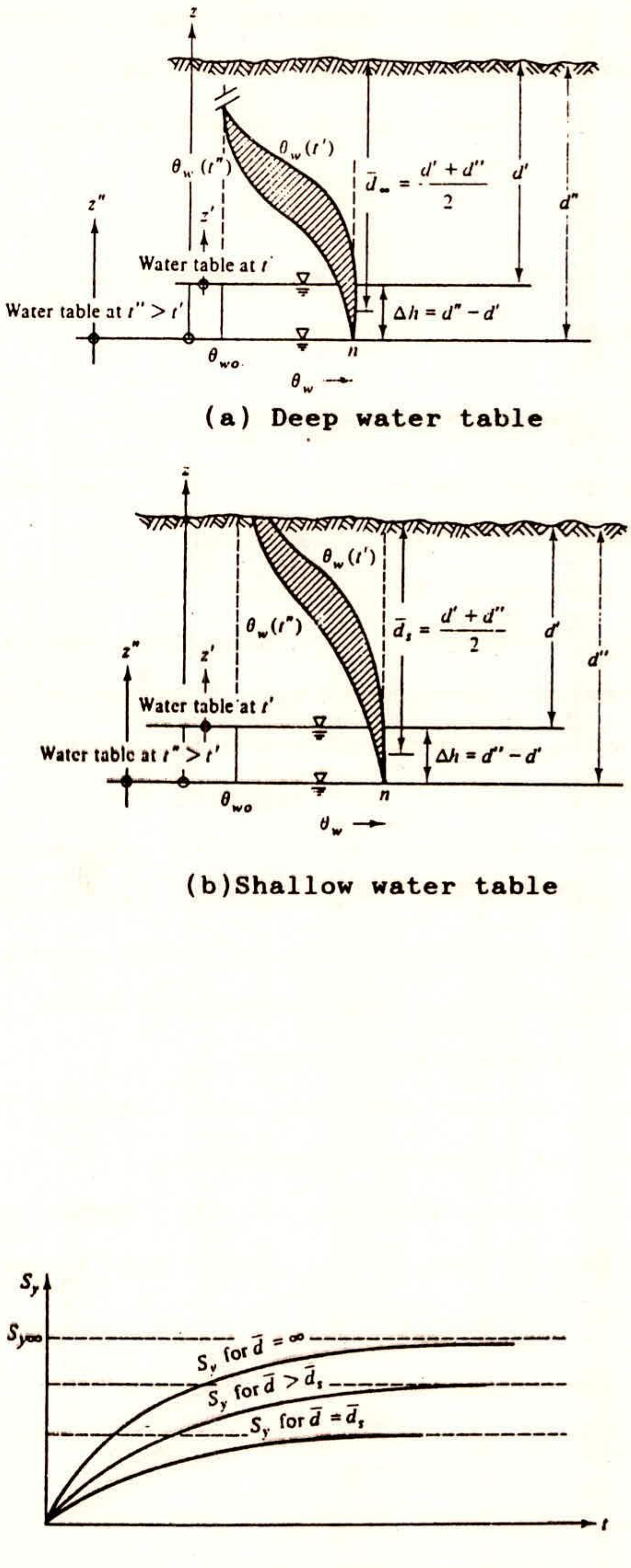
Neutron moisture meters may be used instead of soil sampling for measurement of insitu soil moisture.

The specific yield is a function of the depth of water table and time of drainage (Bear, 1979). Bear has proposed the following expression for specific yield:

$$S_y(d', d", t) = \text{volume of water drained} / (d" - d')$$

$$= [\alpha(d" - d') + \int_{z'=-d'}^{z'=0} \Theta'(z', t)\,dz' - \int_{z"=-d"}^{z"=0} \Theta"(z", t)\,dz"] / (d" - d')$$

The depths d' and d" are shown in Figure 3.



(a) Deep water table

(b)Shallow water table

Fig.3- Effect of depth and time on specific yield

For homogeneous isotropic soil the two curves $\Theta'(z',t)$ and $\Theta''(z'',t)$ are identical in shape. For deep water table positions below ground surface, the two curves will merge at $\Theta = \Theta_{wo}$, the field capacity. For a homogeneous isotropic soil and very deep water table, the specific retention is identical to the field capacity. However, when the soil is inhomogeneous (e.g., composed of layers), or when the water table is at a shallow depth, the moisture distribution curves, corresponding to the two water table positions, are not longer parallel, and the identities $S_y = \alpha - \Theta_{wo}$ is no longer valid. In such case field capacity and specific retention are not same

When the time lag is also taken into account, as it takes time for drainage to be completed, one obtains a specific yield that is time dependent and that approaches asymptotically the values corresponding to the depths considered. The term drainable porosity is sometimes used to denote the instantaneous specific yield. Figure 3(c) shows the effect of time and depth on specific yield.

When the water table is lowered instantaneously (or relatively fast), the corresponding changes in the moisture distribution lag behind and reach a new equilibrium (or practically so) only after a certain time interval that depends on the type of soil. A time lag will also take place when infiltration causes the water table to rise. When the water table is rising or falling slowly, the changes in moisture distribution have sufficient time to adjust continuously and the time lag practically vanishes.

3.0 A DISCRETE KERNEL APPROACH FOR ANALYSIS UNSTEADY FLOW TO A WELL IN AN UNCONFINED AQUIFER AND PREPARATION OF TYPE CURVE FOR ESTIMATION OF SPECIFIC YIELD

The release of water due to the specific yield occurs at the free surface unlike the effect of specific storage which is distributed uniformly throughout the saturated volume of the aquifer. The effective recharge due to specific yield can be expressed as an inflow (Rushton and Redshaw, 1979)

$$q_s = S_y \frac{\partial h_f}{\partial t} \qquad \ldots(1)$$

in which h_f is the free surface elevation. Rushton and Redshaw (1979) have shown that the boundary condition at the free surface can be represented as an inflow at the free surface. In case of an unconfined aquifer the saturated thickness and hence the transmissivity change consequence to the perturbation at the boundary. However, in many practical situations the saturated thickness and therefore the transmissivity are assumed to remain constant, either because the change in saturated thickness is small or because the manner in which the permeability changes with depth is unknown. In the absence of sufficiently precise information , the only practical assumption for many aquifers is that the transmissivity is a constant. With these assumptions the covering differential equation for an unconfined aquifer in the absence of any external recharge becomes

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) = S_c \frac{\partial h}{\partial t} + S_y \frac{\partial h_f}{\partial t} \qquad \ldots(2)$$

Let the time span be discretised into uniform time steps of size Δt. Let the flow domain be discretised into square grids of uniform size Δx.

The recharge which occurs at a node i,j during n^{th} time step can be expressed as

$$Q_r(i,j,n) = S_y\,[s(i,j,n) - s(i,j,n-1)]\Delta x^2 \qquad \ldots(3)$$

where

$Q_r(i,j,n)$ = recharge during the n^{th} time period,

S_y = the specific yield,

s(i,j,n)= drawdown at the end of n^{th} unit time step at the i,j node

s(i,j,n-1)= drawdown at the end of $(n-1)^{th}$ time step at the i,j node,

Δx^2 = area of the square grid.

Refering to Figure 4 the drawdown in terms of discrete kernel coefficients can be written as(Morel-Seytoux,1975):

$$s(i,j,n) = \Sigma_{\gamma=1}^{n} Q_p(i_0,j_0,\gamma)\ \delta_1(i,j;\ i_0,j_0;n-\gamma+1)$$

$$- \Sigma_{\gamma=1}^{n} \Sigma_{\xi=1}^{I} \Sigma_{\vartheta=1}^{J} Q_r(\xi,\ \vartheta,\ \gamma\ )\ \delta_2(i,j;\ \xi,\vartheta;n-\gamma+1) \quad ...(4)$$

in which the discrete kernel coefficient, $\delta_1(i,j;\ i_0,j_0;\ m)$, is the drawdown at the end of m^{th} time step at the i,j node due to a unit pulse withdrawal during the first time step at the i_0j_0 node. The coefficients are given by (Morel-Seytoux,1975)

$$\delta_1(i,j;\ i_0,j_0;\ m)= \frac{1}{4\pi T\Delta t}\ [E_I\{((\ i-i_0)^2\Delta x^2+\ (j-j_0)^2\Delta y^2)/(4m\beta_1\Delta t)\}-$$

$$E_I\{((\ i-i_0)^2\Delta x^2+(j-j_0)^2\Delta y^2)/(4(m-1)\beta_1\Delta t)\}] \quad ...(5)$$

$$\delta_2(i,j;\ \xi,\vartheta;\ m)= \frac{1}{4\pi T\Delta t}\ [E_I\{((\ i-\xi)^2\Delta x^2+\ (j-\vartheta)^2\Delta y^2)/(4m\beta_2\Delta t)\}-$$

$$E_I\{((\ i-\xi)^2\Delta x^2+(j-\vartheta)^2\Delta y^2)/(4(m-1)\beta_2\Delta t)\}]$$

for i,j; not equal to ξ,ϑ ...(6)

$$\delta_2(i,j;\ \xi,\vartheta;\ m)\ =\ {}_0\int^1 \frac{1}{\phi\ \Delta X^2}\ \{\ \text{erf}\ [\frac{\Delta X}{4\{\beta_2(m-\tau)\}^{1/2}}\ ]\}^2\ d\tau$$

for i,j; equal to ξ,ϑ ...(7)

The exponential integral E_I is defined as

$E_I(X)={}_X\int^{\infty}\frac{e^{-u}}{u}du$;

β_1= T/ S_c; β_2 = T/ S_y;

$Q_p(i_0,j_0,\gamma)$ = pumping rate during time step γ;

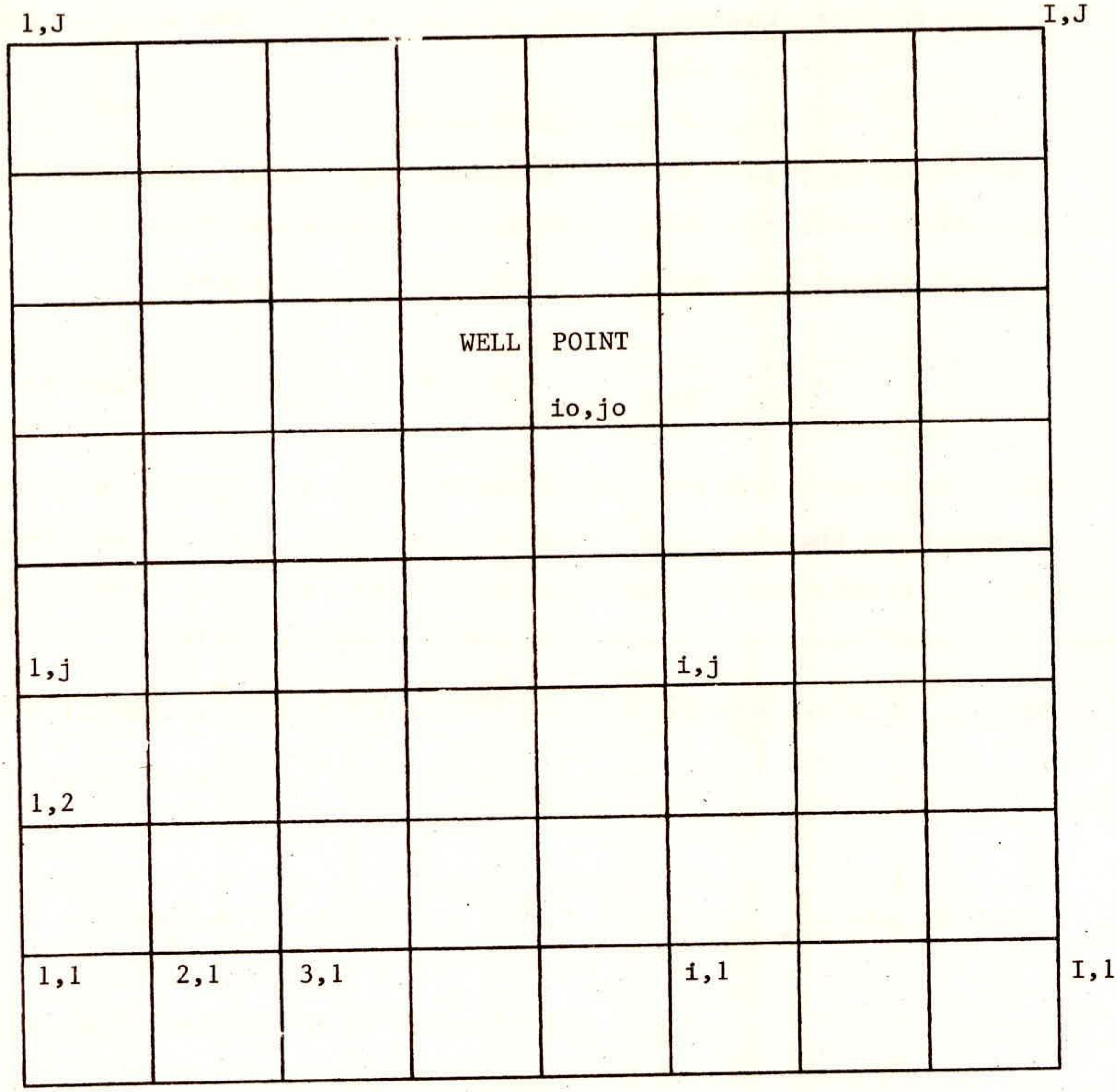


Fig. 4 - Designation of grid nodes used in the derivations

$Q_r(\xi,\vartheta,\gamma)$ = recharge at the node ξ,ϑ during time step γ.

Incorporating equation (4) in (3)

$$Q_r(i,j,n)=S_y\ [\ \Sigma_{\gamma=1}^{n}\ Q_p(i_0,j_0,\gamma)\ \delta_1(i,j;\ i_0,j_0;n-\gamma+1)$$

$$-\ \Sigma_{\gamma=1}^{n}\ \Sigma_{\xi=1}^{I}\ \Sigma_{\vartheta=1}^{J}\ Q_r(\xi,\ \vartheta,\ \gamma\ )\ \delta_2(i,j;\ \xi,\vartheta;n-\gamma+1)$$

$$-s(i,j,n-1)]\Delta X^2 \qquad \ldots(8)$$

Splitting the second temporal summation into two parts, one part containing the all terms up to $(n-1)^{th}$ time step and the other part containing the n^{th} term and simplifying the following equation is obtained from equation(8):

$$Q_r(i,j,n)[1/(S_y\Delta X^2)\ +\ \delta_2(i,j;\ i,j;1)]$$

$$+\ \Sigma_{\xi=1}^{I}\ \Sigma_{\vartheta=1}^{J}\ Q_r(\xi,\ \vartheta,\ n\ )\ \delta_2(i,j;\ \xi,\vartheta;1)$$

$$\xi,\vartheta \neq i,j$$

$$=\Sigma_{\gamma=1}^{n}\ Q_p(i_0,j_0,\gamma)\ \delta_1(i,j;\ i_0,j_0;n-\gamma+1)$$

$$-\ \Sigma_{\gamma=1}^{n-1}\ \Sigma_{\xi=1}^{I}\ \Sigma_{\vartheta=1}^{J}\ Q_r(\xi,\vartheta,\gamma)\ \delta_2(i,j;\ \xi,\vartheta;n-\gamma+1)\ -\ s(i,j,n-1)$$

$$\ldots(9)$$

Linear algebraic equations similar to equation (9) can be written for all (IxJ) number of nodes and the (IxJ) number of unknowns can be solved in succession starting from time step 1 for any time n.

For a homogeneous and isotropic aquifer and assumed grid system (Figure 5) the number of unknowns can be reduced because of the following symmetry in flow:

$$Q_r(1,n)\ =\ Q_r(7,n)\ =\ Q_r(43,n)\ =\ Q_r(49,n); \qquad \ldots(10)$$

$$Q_r(2,n)\ =\ Q_r(6,n)\ =\ Q_r(8,n)\ =\ Q_r(14,n)\ =$$
$$Q_r(36,n)\ =\ Q_r(42,n)\ =\ Q_r(44,n)\ =\ Q_r(48,n); \qquad \ldots(11)$$

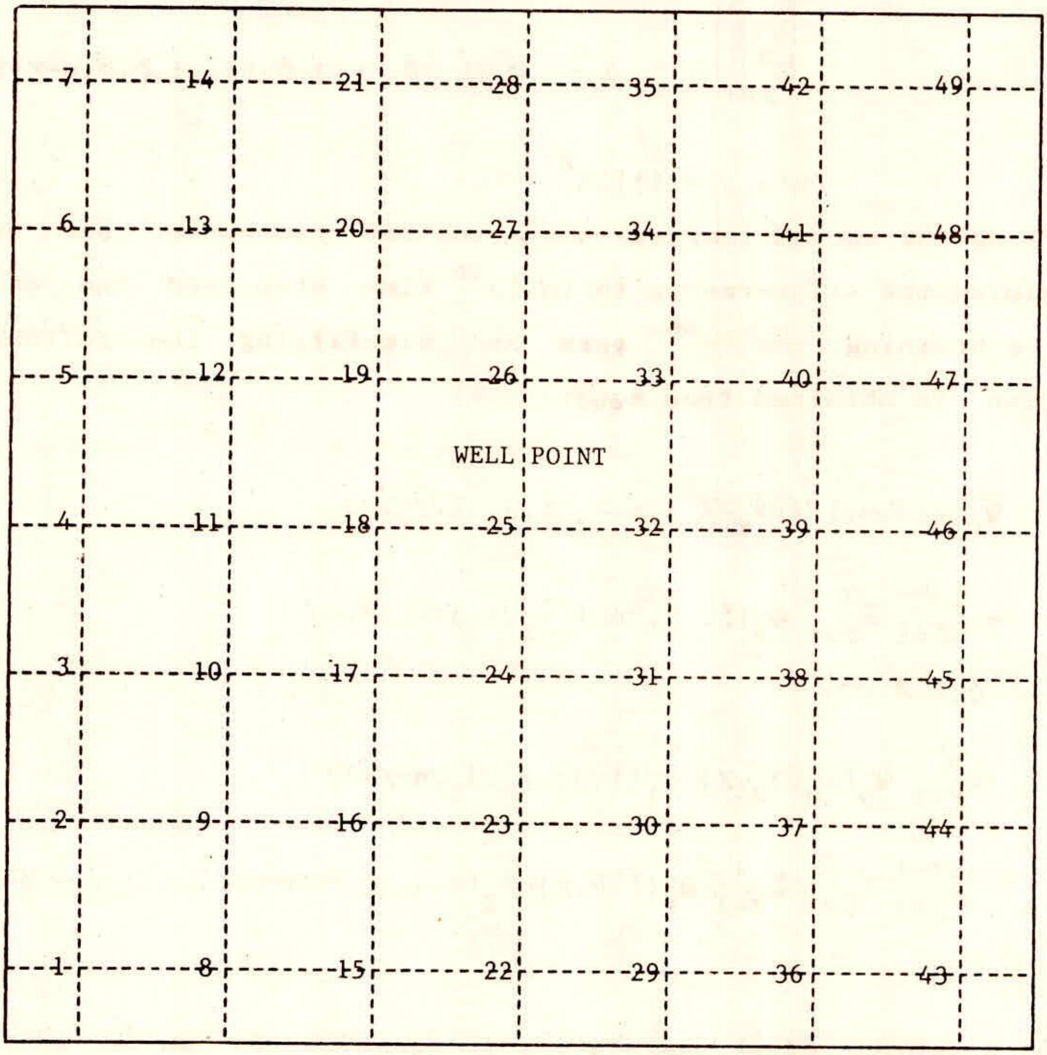


Fig. 5 - Grid network used for computation of well function

$$Q_r(3,n) = Q_r(5,n) = Q_r(15,n) = Q_r(21,n) =$$

$$Q_r(29,n) = Q_r(35,n) = Q_r(45,n) = Q_r(47,n); \qquad \ldots(12)$$

$$Q_r(4,n) = Q_r(22,n) = Q_r(28,n) = Q_r(46,n); \qquad \ldots(13)$$

$$Q_r(9,n) = Q_r(13,n) = Q_r(37,n) = Q_r(41,n); \qquad \ldots(14)$$

$$Q_r(10,n) = Q_r(12,n) = Q_r(16,n) = Q_r(20,n) =$$

$$Q_r(30,n) = Q_r(34,n) = Q_r(38,n) = Q_r(40,n); \qquad \ldots(15)$$

$$Q_r(11,n) = Q_r(23,n) = Q_r(27,n) = Q_r(39,n); \qquad \ldots(16)$$

$$Q_r(17,n) = Q_r(19,n) = Q_r(31,n) = Q_r(33,n); \qquad \ldots(17)$$

$$Q_r(18,n) = Q_r(24,n) = Q_r(26,n) = Q_r(32,n). \qquad \ldots(18)$$

In matrix notation the set of equations can be written as

$$[a].[b] = [c] \qquad \ldots(19)$$

The elements of the matrices are:

$$a(1,1)=\delta_2(1,1,1)+\delta_2(1,7,1)+ \delta_2(1,43,1)+\delta_2(1,49,1)+1/(S_y\Delta X^2)$$

$$a(1,2)= \delta_2(1,2,1) + \delta_2(1,6,1) + \delta_2(1,8,1) + \delta_2(1,14,1) +$$
$$\delta_2(1,36,1) + \delta_2(1,42,1)+ \delta_2(1,44,1 + \delta_2(1,48,1);$$

$$a(1,3)= \delta_2(1,3,1) + \delta_2(1,5,1) + \delta_2(1,15,1) + \delta_2(1,21,1) +$$
$$\delta_2(1,29,1) + \delta_2(1,35,1)+ \delta_2(1,45,1) + \delta_2(1,47,1);$$

$$a(1,4)= \delta_2(1,4,1) + \delta_2(1,22,1)+ \delta_2(1,28,1) + \delta_2(1,46,1);$$

$$a(1,5)= \delta_2(1,9,1) + \delta_2(1,13,1)+ \delta_2(1,37,1) + \delta_2(1,41,1);$$

$$a(1,6)= \delta_2(1,10,1) + \delta_2(1,12,1)+ \delta_2(1,16,1) + \delta_2(1,20,1) +$$
$$\delta_2(1,30,1) + \delta_2(1,34,1)+ \delta_2(1,38,1) + \delta_2(1,40,1);$$

$$a(1,7)= \delta_2(1,11,1) + \delta_2(1,23,1)+ \delta_2(1,27,1) + \delta_2(1,39,1);$$

$$a(1,8)= \delta_2(1,17,1) + \delta_2(1,19,1)+ \delta_2(1,31,1) + \delta_2(1,33,1);$$

$$a(1,9)= \delta_2(1,18,1) + \delta_2(1,24,1)+ \delta_2(1,26,1) + \delta_2(1,32,1);$$

$$a(1,10)= \delta_2(1,25,1)$$

$$a(2,1)= \delta_2(2,1,1) + \delta_2(2,7,1) + \delta_2(2,43,1) + \delta_2(2,49,1);$$

$$a(2,2)= \delta_2(2,2,1) + \delta_2(2,6,1) + \delta_2(2,8,1) + \delta_2(2,14,1) +$$
$$\delta_2(2,36,1) + \delta_2(2,42,1)+ \delta_2(2,44,1 + \delta_2(2,48,1)+$$
$$1/(S_y\Delta X^2)$$

$$a(2,3)= \delta_2(2,3,1) + \delta_2(2,5,1) + \delta_2(2,15,1) + \delta_2(2,21,1) +$$
$$\delta_2(2,29,1) + \delta_2(2,35,1)+ \delta_2(2,45,1) + \delta_2(2,47,1);$$

$$a(2,4)= \delta_2(2,4,1) + \delta_2(2,22,1)+ \delta_2(2,28,1) + \delta_2(2,46,1);$$

$a(2,5)= \delta_2(2,9,1) + \delta_2(2,13,1)+ \delta_2(2,37,1) + \delta_2(2,41,1);$

$a(2,6)= \delta_2(2,10,1) + \delta_2(2,12,1)+ \delta_2(2,16,1) + \delta_2(2,20,1) +$
$\delta_2(2,30,1) + \delta_2(2,34,1)+ \delta_2(2,38,1) + \delta_2(2,40,1);$

$a(2,7)= \delta_2(2,11,1) + \delta_2(2,23,1)+ \delta_2(2,27,1) + \delta_2(2,39,1);$

$a(2,8)= \delta_2(2,17,1) + \delta_2(2,19,1)+ \delta_2(2,31,1) + \delta_2(2,33,1);$

$a(2,9)= \delta_2(2,18,1) + \delta_2(2,24,1)+ \delta_2(2,26,1) + \delta_2(2,32,1);$

$a(2,10)= \delta_2(2,25,1)$

$a(\xi,1)= \delta_2(i,1,1) + \delta_2(i,7,1) + \delta_2(i,43,1) + \delta_2(i,49,1);$

$a(\xi,2)= \delta_2(i,2,1) + \delta_2(i,6,1) + \delta_2(i,8,1) + \delta_2(i,14,1) +$
$\delta_2(i,36,1) + \delta_2(i,42,1)+ \delta_2(i,44,1 + \delta_2(i,48,1);$

$a(\xi,3)= \delta_2(i,3,1) + \delta_2(i,5,1) + \delta_2(i,15,1) + \delta_2(i,21,1) +$
$\delta_2(i,29,1) + \delta_2(i,35,1)+ \delta_2(i,45,1) + \delta_2(i,47,1);$

$a(\xi,4)= \delta_2(i,4,1) + \delta_2(i,22,1)+ \delta_2(i,28,1) + \delta_2(i,46,1);$

$a(\xi,5)= \delta_2(i,9,1) + \delta_2(i,13,1)+ \delta_2(i,37,1) + \delta_2(i,41,1);$

$a(\xi,6)= \delta_2(i,10,1) + \delta_2(i,12,1)+ \delta_2(i,16,1) + \delta_2(i,20,1) +$
$\delta_2(i,30,1) + \delta_2(i,34,1)+ \delta_2(i,38,1) + \delta_2(i,40,1);$

$a(\xi,7)= \delta_2(i,11,1) + \delta_2(i,23,1)+ \delta_2(i,27,1) + \delta_2(i,39,1);$

$a(\xi,8)= \delta_2(i,17,1) + \delta_2(i,19,1)+ \delta_2(i,31,1) + \delta_2(i,33,1);$

$a(\xi,9)= \delta_2(i,18,1) + \delta_2(i,24,1)+ \delta_2(i,26,1) + \delta_2(i,32,1);$

$a(\xi,10)= \delta_2(i,25,1)$

ξ, i= 3,3; 4,4; 5,9; 6,10; 7,11; 8,17; 9,18; 10,25.

With all $a(\xi,\xi)$ elements an extra term $1/(S_y\Delta X^2)$ is to be added.

$[b] = [Q_r(1,n),\ Q_r(2,n),\ Q_r(3,n)\ Q_r(4,n),\ Q_r(9,n),$
$Q_r(10,n),\ Q_r(11,n), Q_r(17,n), Q_r(18,n),\ Q_r(25,n)\ ]'$ ...(20)

$$c(\xi)=\sum_{\gamma=1}^{n} Q_p(\gamma)\delta_1(\xi,25,n-\gamma+1) - \sum_{\gamma=1}^{n-1}\ \sum_{i=1}^{49} Q_r(i,\gamma)\delta_2(\xi,i,n-\gamma+1)$$

...(21)

ξ= 1,2,3,4,9,10,11 ,17,18,25

4.0 RESULTS AND DISCUSSION

A uniform square grid net work having 49 nodes as shown in Figure 5 has been adopted for computing the well function.

Assuming a set of aquifer parameters T, S_y and S, the discrete kernel coefficients $\delta_1(\xi,i,n)$ and $\delta_2(\xi,i,n)$ are generated for i=1,49 ; and ξ=1,49 for different values of n. A grid size of 100m is adopted for obtaining the result. The computer programme developed for computation of the well function is given in Apprndix I.

The well function has been defined as $W(u)=s(r,t)/[Q_p/(4\pi T)]$, where s(r,t) is the drawdown at time t at a radial distance r from the pumping well, and Q_p is the uniform pumping rate. The non-pdimensional time parameter, 1/u, has been assumed as, $1/u=(4Tt)/r^2(S_y+S)$. The variation of well function with time is shown in Figure 6 for different ratios of S_y/S. A plot of time draw down graph plotted in the same scale as that of the type curve can be matched with any of the curves from which the parameters T, S_y, and S can be evaluated.

5.0 CONCLUSIONS

A ground water flow model using discrete kernel approach has been developed and type curves have been prepared using which the parameters T, S , S_y of an unconfined aquifer without delayed yield can be evaluated from aquifer test data. The initial part of the well function has steep slope as that of the Theis' curve. The middle part is flat due to recharge from the dewatered zone owing to specific yield. The later part tends to converge with Theis' type curve.

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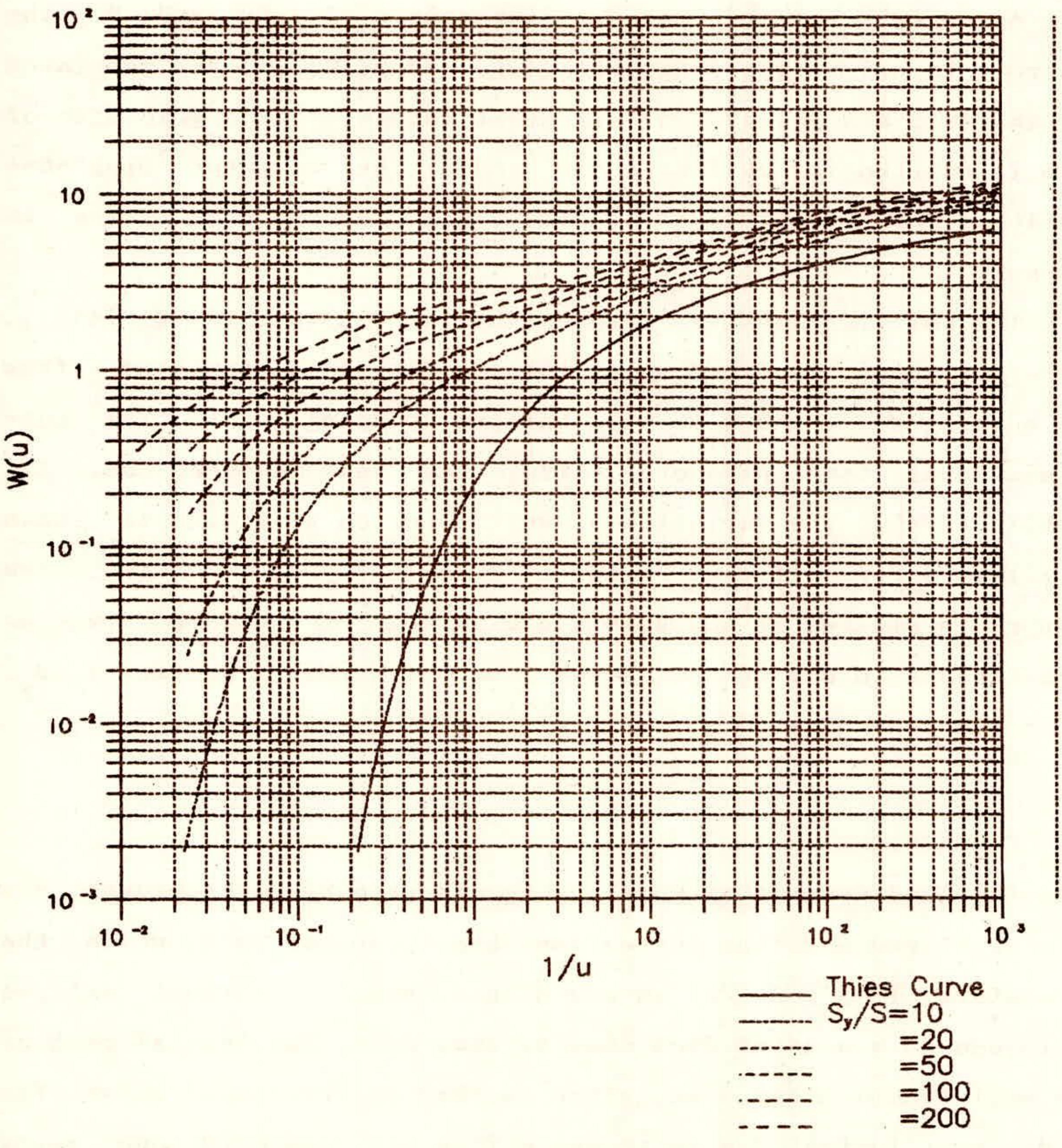

FIG. 6 - Family of type curves: $W(u)$ vs $1/u$ for different values of S_y/S.

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```
C     RUSTON.FOR IS THE PROGRAMME TO COMPUTE SPECIFIC YIELD
      DIMENSION DRPQU(49,49,10),DRPQC(49,49,10),QR(49,10),DDU(49,10)
     1AAA(10,10),B(10),A(10),BBB(10),QP(10),DRWC(5,10),TIMEF(25,10),
     2WELLF(25,10),THIESW(10),TDPQ(10)
C     GENERATING DRPQ AND DRPQC FOR NODES IN LOWER TRIANGLE
C     OF THE SQUARE GRID SYSTEM.
      OPEN(UNIT=1,FILE='RUSTON.DAT',STATUS='OLD')
      OPEN(UNIT=2,FILE='RUSTON.OUT',STATUS='NEW')
C
      READ(1,201)T1,SY,S,DELX,RW,QP(1)
201   FORMAT(6F10.5)
      READ (1,204)NTIME
204   FORMAT(I3)
C     DRPQU -  DISCRETE KERNEL GENERATED USING SY
C     DRPQC -  DISCRETE KERNEL GENERATED USINF S
C     QR    -  RATE OF EXCHANGE OF FLOW BETWEEN THE TWO LAYERS
C     DDU   -  DRAWDOWN IN THE UPPER LAYER
C     QP    -  UNIFORM PUMPING RATE
C     DELX  -  LENGTH OF SQUARE GRID
C     T1    -  TRANSMISSIBILITY OF THE AQUIFER
C     SY    -  SPECIFIC YIELD
C     S     -  STORAGE CO-EFFICIENT
C
      DO 1000 NNN=1,25
      DO 90 I=2,NTIME
90    QP(I)=QP(1)
      PAI=3.14159265
      J1=1
      J=1
      DO 206 N=1,NTIME
      AN=N
      CALL DRIJ(AN,DELX,T1,SY,DMRR)
      DRPQU(J,J1,N)=DMRR
      CALL DRIJ(AN,DELX,T1,S,DMRR)
      DRPQC(J,J1,N)=DMRR
206   CONTINUE
      DO 215 J2=8,9
      AJ2=J2
      R=(((AJ2-8.)*DELX)**2+DELX**2)**0.5
      DO 216 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,SY,R,DN)
      DRPQU(J,J2,N)=DN
      CALL DPQ(AN,T1,S,R,DN)
      DRPQC(J,J2,N)=DN
216   CONTINUE
215   CONTINUE
      DO 225 J3=15,17
      AJ3=J3
      R=(((AJ3-15.)*DELX)**2+(DELX*2.)**2)**0.5
      DO 226 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,SY,R,DN)
      DRPQU(J,J3,N)=DN
      CALL DPQ(AN,T1,S,R,DN)
      DRPQC(J,J3,N)=DN
226   CONTINUE
225   CONTINUE
```

```
      DO 235 J4=22,25
      AJ4=J4
      R=(((AJ4-22.)*DELX)**2+(DELX*3.)**2)**0.5
      DO 236 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,SY,R,DN)
      DRPQU(J,J4,N)=DN
      CALL DPQ(AN,T1,S,R,DN)
      DRPQC(J,J4,N)=DN
236   CONTINUE
235   CONTINUE
      DO 245 J5=29,33
      AJ5=J5
      R=(((AJ5-29.)*DELX)**2+(DELX*4.)**2)**0.5
      DO 246 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,SY,R,DN)
      DRPQU(J,J5,N)=DN
      CALL DPQ(AN,T1,S,R,DN)
      DRPQC(J,J5,N)=DN
246   CONTINUE
245   CONTINUE
      DO 255 J6=36,41
      AJ6=J6
      R=(((AJ6-36.)*DELX)**2+(DELX*5.)**2)**0.5
      DO 256 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,SY,R,DN)
      DRPQU(J,J6,N)=DN
      CALL DPQ(AN,T1,S,R,DN)
      DRPQC(J,J6,N)=DN
256   CONTINUE
255   CONTINUE
      DO 265 J7=43,49
      AJ7=J7
      R=(((AJ7-43.)*DELX)**2+(DELX*6.)**2)**0.5
      DO 266 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,SY,R,DN)
      DRPQU(J,J7,N)=DN
      CALL DPQ(AN,T1,S,R,DN)
      DRPQC(J,J7,N)=DN
266   CONTINUE
265   CONTINUE
      DO 380 N=1,NTIME
      AN=N
      TIMEF(1,N)= 4.*(T1/(SY+S))*AN/((3.*DELX)**2+(3.*DELX)**2)
      TIMEF(8,N)= 4.*(T1/(SY+S))*AN/((2.*DELX)**2+(3.*DELX)**2)
      TIMEF(15,N)=4.*(T1/(SY+S))*AN/((1.*DELX)**2+(3.*DELX)**2)
      TIMEF(22,N)=4.*(T1/(SY+S))*AN/((0.*DELX)**2+(3.*DELX)**2)
      TIMEF(9,N)= 4.*(T1/(SY+S))*AN/((2.*DELX)**2+(2.*DELX)**2)
      TIMEF(16,N)=4.*(T1/(SY+S))*AN/((1.*DELX)**2+(2.*DELX)**2)
      TIMEF(23,N)=4.*(T1/(SY+S))*AN/((0.*DELX)**2+(2.*DELX)**2)
      TIMEF(17,N)=4.*(T1/(SY+S))*AN/((1.*DELX)**2+(1.*DELX)**2)
      TIMEF(24,N)=4.*(T1/(SY+S))*AN/((0.*DELX)**2+(1.*DELX)**2)
      TIMEF(25,N)=4.*(T1/(SY+S))*AN/RW**2
380   CONTINUE
C
```

```
C     EQUATING DRPQU AND DRPQC FOR NODES IN UPPER TRIANGLE OF THE
C     SQUARE GRID SYSTEM.
      I=1
      IM=43
 16   M=0
      DO 17 JJ=I,IM,7
      IK=JJ-M*6
      DO 18 N=1,NTIME
      DRPQC(J,IK,N)=DRPQC(J,JJ,N)
 18   DRPQU(J,IK,N)=DRPQU(J,JJ,N)
      M=M+1
 17   CONTINUE
      I=I+8
      IM=IM+1
      IF(IM-48)16,16,19
 19   DO 20 J=8,22,7
      I=J
      JP=J-7
      IM=43
 21   DO 22 JN=I,IM,7
      JNP=JN-7
      DO 23 N=1,NTIME
      DRPQC(J,JN,N)=DRPQC(JP,JNP,N)
 23   DRPQU(J,JN,N)=DRPQU(JP,JNP,N)
 22   CONTINUE
      I=I+1
      IM=IM+1
      IF(IM-49)21,21,24
 24   LI=J-7
      LIM=J-1
      M=1
 25   MM=M*14
      DO 26 LJN=LI,LIM
      LJNP=LJN+MM
      DO 27 N=1,NTIME
      DRPQC(J,LJN,N)=DRPQC(J,LJNP,N)
 27   DRPQU(J,LJN,N)=DRPQU(J,LJNP,N)
 26   CONTINUE
      M=M+1
      LI=LI-7
      LIM=LIM-7
      IF(LIM-7)28,25,25
 28   CONTINUE
 20   CONTINUE
      DO 29 J=9,23,7
      I=J
      JP=J-1
      IM=44
 30   DO 31 JN=I,IM,7
      JNP=JN-1
      DO 32 N=1,NTIME
      DRPQC(J,JN,N)=DRPQC(JP,JNP,N)
 32   DRPQU(J,JN,N)=DRPQU(JP,JNP,N)
 31   CONTINUE
      I=I+1
      IM=IM+1
      IF(IM-49)30,30,33
 33   DO 34 KJN=1,43,7
```

```
      KJNP=KJN+1
      DO 35 N=1,NTIME
      DRPQC(J,KJN,N)=DRPQC(JP,KJNP,N)
35    DRPQU(J,KJN,N)=DRPQU(JP,KJNP,N)
34    CONTINUE
      LI=J-7
      LIM=J-2
      M=1
36    MM=M*14
      DO 37 LJN=LI,LIM
      LJNP=LJN+MM
      DO 38 N=1,NTIME
      DRPQC(J,LJN,N)=DRPQC(J,LJNP,N)
38    DRPQU(J,LJN,N)=DRPQU(J,LJNP,N)
37    CONTINUE
      M=M+1
      LI=LI-7
      LIM=LIM-7
      IF(LIM-7)39,36,36
39    CONTINUE
29    CONTINUE
      DO 40 J=17,24,7
      I=J
      JP=J-1
      IM=45
41    DO 42 JN=I,IM,7
      JNP=JN-1
      DO 43 N=1,NTIME
      DRPQC(J,JN,N)=DRPQC(JP,JNP,N)
43    DRPQU(J,JN,N)=DRPQU(JP,JNP,N)
42    CONTINUE
      I=I+1
      IM=IM+1
      IF(IM-49)41,41,44
44    KI=2
      KIM=44
      M=1
45    DO 46 KJN=KI,KIM,7
      JM=J-M
      KJNM=KJN+M
      DO 47 N=1,NTIME
      DRPQC(J,KJN,N)=DRPQC(JM,KJNM,N)
47    DRPQU(J,KJN,N)=DRPQU(JM,KJNM,N)
46    CONTINUE
      KI=KI-1
      KIM=KIM-1
      M=M+1
      IF(KIM-43)48,45,45
48    LI=J-7
      LIM=J-3
      M=1
49    MM=M*14
      DO 50 LJN=LI,LIM
      LJNP=LJN+MM
      DO 51 N=1,NTIME
      DRPQC(J,LJN,N)=DRPQC(J,LJNP,N)
51    DRPQU(J,LJN,N)=DRPQU(J,LJNP,N)
50    CONTINUE
```

```
      M=M+1
      LI=LI-7
      LIM=LIM-7
      IF(LIM-7)52,49,49
52    CONTINUE
40    CONTINUE
      J=25
      I=J
      JP=J-1
      IM=46
53    DO 54 JN=I,IM,7
      JNP=JN-1
      DO 55 N=1,NTIME
      DRPQC(J,JN,N)=DRPQC(JP,JNP,N)
55    DRPQU(J,JN,N)=DRPQU(JP,JNP,N)
54    CONTINUE
      I=I+1
      IM=IM+1
      IF(IM-49)53,53,56
56    KI=3
      KIM=45
      M=1
57    DO 58 KJN=KI,KIM,7
      JM=J-M
      KJNM=KJN+M
      DO 59 N=1,NTIME
      DRPQC(J,KJN,N)=DRPQC(JM,KJNM,N)
59    DRPQU(J,KJN,N)=DRPQU(JM,KJNM,N)
58    CONTINUE
      KI=KI-1
      KIM=KIM-1
      M=M+1
      IF(KIM-43)60,57,57
60    LI=J-7
      LIM=J-4
      M=1
61    MM=M*14
      DO 62 LJN=LI,LIM
      LJNP=LJN+MM
      DO 63 N=1,NTIME
      DRPQC(J,LJN,N)=DRPQC(J,LJNP,N)
63    DRPQU(J,LJN,N)=DRPQU(J,LJNP,N)
62    CONTINUE
      M=M+1
      LI=LI-7
      LIM=LIM-7
      IF(LIM-7)64,61,61
64    CONTINUE
      DO 65 N=1,NTIME
      AN=N
      CALL DPQ(AN,T1,S,RW,DN)
      DRWC(5,N)=DN
      R=DELX/4.
      CALL DPQ(AN,T1,S,R,DN)
      DRWC(1,N)=DN
      R=DELX/2.
      CALL DPQ(AN,T1,S,R,DN)
      DRWC(2,N)=DN
```

```
      R=(2.0*(DELX/4.)**2)**0.5
      CALL DPQ(AN,T1,S,R,DN)
      DRWC(3,N)=DN
      R=(2.*(DELX/2.)**2)**0.5
      CALL DPQ(AN,T1,S,R,DN)
      DRWC(4,N)=DN
      DRPQC(25,25,N)=(DRWC(5,N)+4.*(DRWC(1,N)+DRWC(2,N)+DRWC(3,N)+
     1DRWC(4,N)))/17.0
65    CONTINUE
C
C     CALCULATING QR FOR  10 COMMON NODES FOR THE FIRST TIME STEP
      C=1./(SY*DELX**2)
      NN=1
      I=1
      KMIN=1
      KMAX=22
300   DO 301 K=KMIN,KMAX,7
      A(1)=DRPQU(K,1,NN)+DRPQU(K,7,NN)+DRPQU(K,43,NN)+DRPQU(K,49,NN)
      A(2)=DRPQU(K,2,NN)+DRPQU(K,6,NN)+DRPQU(K,8,NN)+DRPQU(K,14,NN)+
     1DRPQU(K,36,NN)+DRPQU(K,42,NN)+DRPQU(K,44,NN)+DRPQU(K,48,NN)
      A(3)=DRPQU(K,3,NN)+DRPQU(K,5,NN)+DRPQU(K,15,NN)+DRPQU(K,21,NN)+
     1DRPQU(K,29,NN)+DRPQU(K,35,NN)+DRPQU(K,45,NN)+DRPQU(K,47,NN)
      A(4)=DRPQU(K,4,NN)+DRPQU(K,22,NN)+DRPQU(K,28,NN)+DRPQU(K,46,NN)
      A(5)=DRPQU(K,9,NN)+DRPQU(K,13,NN)+DRPQU(K,37,NN)+DRPQU(K,41,NN)
      A(6)=DRPQU(K,10,NN)+DRPQU(K,12,NN)+DRPQU(K,16,NN)+DRPQU(K,20,NN)+
     1DRPQU(K,30,NN)+DRPQU(K,34,NN)+DRPQU(K,38,NN)+DRPQU(K,40,NN)
      A(7)=DRPQU(K,11,NN)+DRPQU(K,23,NN)+DRPQU(K,27,NN)+DRPQU(K,39,NN)
      A(8)=DRPQU(K,17,NN)+DRPQU(K,19,NN)+DRPQU(K,31,NN)+DRPQU(K,33,NN)
      A(9)=DRPQU(K,18,NN)+DRPQU(K,24,NN)+DRPQU(K,26,NN)+DRPQU(K,32,NN)
      A(10)=DRPQU(K,25,NN)
      DO 302 J=1,10
      IF(J.EQ.I) GO TO 303
      AAA(I,J)=A(J)
      GO TO 302
303   AAA(I,J)=A(J)+C
302   CONTINUE
      I=I+1
301   CONTINUE
      KMIN=KMIN+8
      KMAX=KMAX+1
      IF(KMAX-25)300,300,304
304   CONTINUE
      B(1)=DRPQC(25,1,1)*QP(1)
      B(2)=DRPQC(25,8,1)*QP(1)
      B(3)=DRPQC(25,15,1)*QP(1)
      B(4)=DRPQC(25,22,1)*QP(1)
      B(5)=DRPQC(25,9,1)*QP(1)
      B(6)=DRPQC(25,16,1)*QP(1)
      B(7)=DRPQC(25,23,1)*QP(1)
      B(8)=DRPQC(25,17,1)*QP(1)
      B(9)=DRPQC(25,24,1)*QP(1)
      B(10)=DRPQC(25,25,1)*QP(1)
      MMM=10
      CALL MATIN(AAA,MMM)
      DO 310 MM=1,10
      BBB(MM)=0.
      DO 311 JK=1,10
311   BBB(MM)=BBB(MM)+AAA(MM,JK)*B(JK)
```

```
310    CONTINUE
       KMIN=1
       KMAX=22
       J=1
305    DO 306 K=KMIN,KMAX,7
       QR(K,NN)=BBB(J)
       J=J+1
306    CONTINUE
       KMIN=KMIN+8
       KMAX=KMAX+1
       IF(KMAX-25)305,305,309
309    CONTINUE
C
C      EQUATING QR FOR OTHER NODES FOR THE FIRST TIME STEP
       N=1
110    QR(49,N)=QR(1,N)
       JMIN=29
       JMAX=43
       IMIN=1
       IMAX=7
111    M=1
       DO 112 J=JMIN,JMAX,7
       JMM=J-M*14  .
       QR(J,N)=QR(JMM,N)
       M=M+1
112    CONTINUE
       M=0
       DO 113 I=IMIN,IMAX
       IMM=I+M*6
       QR(I,N)=QR(IMM,N)
       M=M+1
113    CONTINUE
       JMIN=JMIN+1
       JMAX=JMAX+1
       IMIN=IMIN+8
       IMAX=IMAX+7
       IF(IMAX-42)111,111,114
114    CONTINUE
C
C      CALCULATION OF DRAWDOWN FOR 10 COMMON NODES FOR THE FIRST TIME
C      STEP.
       N=1
       KMIN=1
       KMAX=22
120    DO 121 K=KMIN,KMAX,7
       SUMU=0.
       DO 122 IP=1,49
       IF(IP.EQ.K) GO TO 123
       SUMU=SUMU+QR(IP,N)*DRPQU(K,IP,N)
123    CONTINUE
122    CONTINUE
       DDU(K,N)=-QR(K,N)*DRPQU(K,K,N)-SUMU+DRPQC(K,25,1)*QP(1)
       WELLF(K,N)=DDU(K,N)*(4.*PAI*T1)/QP(1)
124    FORMAT(10X,2I5,3E16.5)
121    CONTINUE
       KMIN=KMIN+8
       KMAX=KMAX+1
       IF(KMAX-25)120,120,125
```

```
125   CONTINUE
C
C     CALCULATING QR FOR 10 COMMON NODES FOR THE SECOND AND ONWARD
C     TIME STEPS.
      DO 128 N=2,NTIME
      I=1
      KMIN=1
      KMAX=22
131   DO 132 K=KMIN,KMAX,7
      BB=QP(N)*DRPQC(K,25,1)
      BB1=0.
      BB2=0.
      BB3=0.
      BB4=0.
      BB5=0.
      BB6=0.
      BB7=0.
      BB8=0.
      BB9=0.
      BB10=0.
      DO 133 NG=1,N-1
      NN=N-NG+1
      BB=BB+QP(NG)*DRPQC(K,25,NN)
      A(1)=DRPQU(K,1,NN)+DRPQU(K,7,NN)+DRPQU(K,43,NN)+DRPQU(K,49,NN)
      A(2)=DRPQU(K,2,NN)+DRPQU(K,6,NN)+DRPQU(K,8,NN)+DRPQU(K,14,NN)+
     1DRPQU(K,36,NN)+DRPQU(K,42,NN)+DRPQU(K,44,NN)+DRPQU(K,48,NN)
      A(3)=DRPQU(K,3,NN)+DRPQU(K,5,NN)+DRPQU(K,15,NN)+DRPQU(K,21,NN)+
     1DRPQU(K,29,NN)+DRPQU(K,35,NN)+DRPQU(K,45,NN)+DRPQU(K,47,NN)
      A(4)=DRPQU(K,4,NN)+DRPQU(K,22,NN)+DRPQU(K,28,NN)+DRPQU(K,46,NN)
      A(5)=DRPQU(K,9,NN)+DRPQU(K,13,NN)+DRPQU(K,37,NN)+DRPQU(K,41,NN)
      A(6)=DRPQU(K,10,NN)+DRPQU(K,12,NN)+DRPQU(K,16,NN)+DRPQU(K,20,NN)+
     1DRPQU(K,30,NN)+DRPQU(K,34,NN)+DRPQU(K,38,NN)+DRPQU(K,40,NN)
      A(7)=DRPQU(K,11,NN)+DRPQU(K,23,NN)+DRPQU(K,27,NN)+DRPQU(K,39,NN)
      A(8)=DRPQU(K,17,NN)+DRPQU(K,19,NN)+DRPQU(K,31,NN)+DRPQU(K,33,NN)
      A(9)=DRPQU(K,18,NN)+DRPQU(K,24,NN)+DRPQU(K,26,NN)+DRPQU(K,32,NN)
      A(10)=DRPQU(K,25,NN)
      BB1=BB1+QR(1,NG)*A(1)
      BB2=BB2+QR(8,NG)*A(2)
      BB3=BB3+QR(15,NG)*A(3)
      BB4=BB4+QR(22,NG)*A(4)
      BB5=BB5+QR(9,NG)*A(5)
      BB6=BB6+QR(16,NG)*A(6)
      BB7=BB7+QR(23,NG)*A(7)
      BB8=BB8+QR(17,NG)*A(8)
      BB9=BB9+QR(24,NG)*A(9)
      BB10=BB10+QR(25,NG)*A(10)
133   CONTINUE
      B(I)=BB-(BB1+BB2+BB3+BB4+BB5+BB6+BB7+BB8+BB9+BB10)
136   CONTINUE
135   CONTINUE
      I = I+1
132   CONTINUE
      KMIN = KMIN + 8
      KMAX = KMAX +1
      IF(KMAX-25) 131,131,137
137   CONTINUE
      B(1)=B(1)-DDU(1,N-1)
      B(2)=B(2)-DDU(8,N-1)
```

```
      B(3)=B(3)-DDU(15,N-1)
      B(4)=B(4)-DDU(22,N-1)
      B(5)=B(5)-DDU(9,N-1)
      B(6)=B(6)-DDU(16,N-1)
      B(7)=B(7)-DDU(23,N-1)
      B(8)=B(8)-DDU(17,N-1)
      B(9)=B(9)-DDU(24,N-1)
      B(10)=B(10)-DDU(25,N-1)
      DO 312 MM=1,10
      BBB (MM) =0.
      DO 313 JK =1,10
313   BBB(MM) =BBB(MM)+AAA(MM,JK)*B(JK)
312   CONTINUE
      KMIN = 1
      KMAX = 22
      J = 1
400   DO 401 K = KMIN,KMAX,7
      QR(K,N) = BBB(J)
      J = J+1
401   CONTINUE
      KMIN = KMIN + 8
      KMAX = KMAX +1
      IF(KMAX-25) 400,400,141
C
C     EQUATING QR FOR OTHER NODES FOR THE SECOND AND ONWARD TIME STEPS
141   QR(49,N)=QR(1,N)
      JMIN=29
      JMAX=43
      IMIN=1
      IMAX=7
142   M=1
      DO 143 J=JMIN,JMAX,7
      JMM=J-M*14
      QR(J,N)=QR(JMM,N)
      M=M+1
143   CONTINUE
      M=0
      DO 144 I=IMIN,IMAX
      IMM=I+M*6
      QR(I,N)=QR(IMM,N)
      M=M+1
144   CONTINUE
      JMIN=JMIN+1
      JMAX=JMAX+1
      IMIN=IMIN+8
      IMAX=IMAX+7
      IF(IMAX-42)142,142,145
145   CONTINUE
146   CONTINUE
C
C     CALCULATING DDU AND DDC FOR 10 COMMON NODES FOR THE SECOND AND
C     ONWARD TIME STEPS.
      KMIN=1
      KMAX=22
150   DO 151 K=KMIN,KMAX,7
      SUMU1=0.
      DO 152 IP=1,49
      TERMU1=0.
```

```
      DO 153 NG=1,N
      NP=N-NG+1
      TERMU1=TERMU1+QR(IP,NG)*DRPQU(K,IP,NP)
153   CONTINUE
      SUMU1=SUMU1+TERMU1
152   CONTINUE
      SUMU2=0.
      DO 155 NG=1,N
      NP=N-NG+1
      SUMU2=SUMU2+QP(NG)*DRPQC(K,25,NP)
155   CONTINUE
      DDU(K,N)=SUMU2-SUMU1
      WELLF(K,N)=DDU(K,N)*(4.*PAI*T1)/QP(1)
151   CONTINUE
      KMIN=KMIN+8
      KMAX=KMAX+1
      IF(KMAX-25)150,150,157
157   CONTINUE
128   CONTINUE
      INDEX=111
      IF (INDEX.EQ.111) GO TO 999
      WRITE(2,385)
385   FORMAT(3x,'TIMEF',5X,'WELLF,NODE 1',3X,'TIMEF',5X,'WELLF,NODE8',
     13X,'TIMEF',4X,'WELLF,NODE 15',3X,'TIMEF',4X,'WELLF,NODE22')
      DO 381 N=1,NTIME
      WRITE(2,382)TIMEF(1,N),WELLF(1,N),TIMEF(8,N),WELLF(8,N),
     1TIMEF(15,N),WELLF(15,N),TIMEF(22,N),WELLF(22,N)
381   CONTINUE
      WRITE(2,386)
386   FORMAT(3x,'TIMEF',5X,'WELLF,NODE 9',3X,'TIMEF',5X,'WELLF,NODE16',
     13X,'TIMEF',3X,'WELLF,NODE 23',3X,'TIMEF',4X,'WELLF,NODE17')
      DO 383 N=1,NTIME
      WRITE(2,382)TIMEF(9,N),WELLF(9,N),TIMEF(16,N),WELLF(16,N),
     1TIMEF(23,N),WELLF(23,N),TIMEF(17,N),WELLF(17,N)
383   CONTINUE
999   CONTINUE
      R=DELX
      DO 1001 N=1,NTIME
      AN=N
      PHAI=SY+S
      CALL DPQ(AN,T1,PHAI,DELX,DN)
1001  TDPQ(N)=DN
      DO 1003 N=1,NTIME
      SUM=0.
      DO 1002 NGAMA=1,N
1002  SUM=SUM+TDPQ(N-NGAMA+1)*QP(NGAMA)
1003  THIESW(N)=SUM*4.*PAI*T1/QP(1)
      DO 384 N=5,NTIME
      WRITE(2,382)TIMEF(24,N),WELLF(24,N),THIESW(N)
384   CONTINUE
382   FORMAT(3E12.4)
      T1=T1*2.0
      QP(1)=QP(1)*2.0
1000  CONTINUE
      STOP
      END
C
      SUBROUTINE DRIJ(AN,DELX,T2,PHI2,DMRR)
```

```
C       implicit real*8 (a-h,o-z)
      DIMENSION F(101)
      SUM=0.
      TAI=0.
      DO1I=1,101
      IF(ABS(AN-TAI)-.0001)2,2,5
    5 X=DELX/(4.*((AN-TAI)*T2/PHI2)**0.5)
      CALL ERF(X,ERFX)
      F(I)=ERFX**2
      TAI=TAI+0.01
      GO TO 1
    2 F(101)=1.0
    1 CONTINUE
      DO 4 I=1,100
    4 SUM=SUM+(F(I)+F(I+1))/2.*0.01
      DMRR=SUM/(PHI2*DELX*DELX)
      RETURN
      END
C
      SUBROUTINE ERF(X,ERFX)
      IF(X-9.)1,2,2
 1    CONTINUE
      T=1.0/(1.0+.3275911*X)
      ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.42141374*T**3-1.45315202*
     1T**4+1.06140542*T**5)*EXP(-X**2)
      GO TO 3
 2    ERFX=1.
 3    CONTINUE
      RETURN
      END
C
      SUBROUTINE DPQ(AN,T2,PHI2,R,DN)
      PAI=3.14159265
      CAPA=T2/PHI2
      X=R*R/(4.0*CAPA*AN)
      CALL EXI(X,EXFN)
      AA=EXFN
      IF(ABS(AN-1.0)-.001)1,1,2
    2 X=R*R/(4.0*CAPA*(AN-1.))
      CALL EXI(X,EXFN)
      DN=(AA-EXFN)/(4.0*PAI*T2)
      GO TO 3
    1 EXFN=0.
      DN=AA/(4.0*T2*PAI)
    3 CONTINUE
      RETURN
      END
C
      SUBROUTINE EXI(X,EXFN)
      IF(X-1.0)1,1,22
1     EXFN=-ALOG(X)-0.57721566+0.99999193*X-0.24991055*X**2+0.05519968*
     1X**3-0.00976004*X**4+0.00107857*X**5
      GO TO 3
22    CONTINUE
      IF(X- 80.)5,4,4
5     CONTINUE
2     EXFN=((X**4+8.5733287*X**3+18.059017*X**2+8.6347608*X+0.26777373)/
     1(X**4+9.5733223*X**3+25.632956*X**2+21.099653*X+3.9584969))/
```

```
     2(X*EXP(X))
      GO TO 3
4     EXFN=0.
3     CONTINUE
      RETURN
      END
C
C     PROGRAAAME FOR MATRIX INVERSE BY BIFURCATION METHOD
      SUBROUTINE MATIN (AAA,MMM)
C      implicit real*8 (a-h,o-z)
      DIMENSION AAA(10,10),B(10),C(10)
      NN=MMM-1
      AAA(1,1)=1./AAA(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
  3   B(I)=B(I)+AAA(I,J)*AAA(J,K)
      D=0.0
      DO 4 I=1,M
  4   D=D+AAA(K,I)*B(I)
      D=-D+AAA(K,K)
      AAA(K,K)=1./D
      DO 5 I=1,M
  5   AAA(I,K)=-B(I)*AAA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
  6   C(J)=C(J)+AAA(K,I)*AAA(I,J)
      DO 7 J=1,M
  7   AAA(K,J)=-C(J)*AAA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
  8   AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
      RETURN
      END
```

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