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# ESTIMATION OF RAINFALL RECHARGE IN A COASTAL AREA THROUGH INVERSE GROUNDWATER MODELLING


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# PREFACE

Rainfall is the principal natural source of recharge to the aquifers. Quantification of the rate of this natural recharge is a basic pre-requisite for efficient management of the groundwater resource. It is particularly vital in a coastal aquifer where the draft in excess of this natural recharge rate may result in withdrawal from storage, leading to declining water table levels and adverse effects including adverse salt balance and sea water intrusion. However, the natural phenomena of rainfall recharge which is a function of a number of hydrological, meteorological and geological factors is very complex to study and analyze. The recharge from rainfall varies both in space and time. A number of methods for estimation of rainfall recharge are available which mainly include the empirical formulae under specific field conditions, experimental studies, water table fluctuation method, water balance approach, and inverse modelling (determination of the recharge necessary to maintain the ground water levels).

Among the above methods, the inverse modelling is comparatively a new technique of estimation of recharge. It is called an 'Inverse Modelling' because, contrary to the 'Forward or Direct Modelling', where recharge is postulated known and hydraulic heads computed, it is the recharge estimate which is computed from field measurements of hydraulic heads. Though, a few standard software like MODINV, MODFLOWP, PMWIN etc. are available now a days for tackling the inverse problem, the recharge estimation by this method has not yet achieved the share of attention it deserves.

The present study brings out the application of MODINV (MODular INVerse model) for estimation of distributed rainfall recharge in different zones of a coastal aquifer of Central Godavari Delta in Andhra Pradesh. The study has been carried out by Sri J.V. Tyagi, Scientist 'C' as a part of the annual work programme of the 'Groundwater Modelling and Conjunctive Use Division' for the year 1999-2000. During the course of the study, Dr. Sudhir Kumar, Scientist 'C' also provided the logistic and the technical support.

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# ABSTRACT

The present study focusses attention on the use of inverse modelling technique for estimation of rainfall recharge in a coastal aquifer. MODINV (MODular INVerse model) which is a software for parameter optimisation of 3D Ground Water Flow Model, MODFLOW, is applied to Central Godavari Delta in Andhra Pradesh to estimate the distributed rainfall recharge during monsoon season.

The model is formulated based on the available information on the physical and hydrogeological framework of the study area. A number of recharge zones are defined in the model to take care of the spatial variation of recharge. The model is calibrated for the transient conditions during non-monsoon period of 1985 when the recharge takes place solely due to return flow of applied irrigation, the quantity of which is estimated before hand with the available data. The calibrated model is then used to optimize the recharge during different stress periods of monsoon season of 1985. The recharge from rainfall is computed by subtracting the estimated quantities of recharge due to return flow from the total model recharge.

On a distributed basis, the rainfall-recharge coefficient in the lower, middle and upper reaches of the study delta is found to vary from 0.11 to 0.25. The recharge coefficient as calculated on lumped basis works out to 0.1717 for the study area.

# 1.0 INTRODUCTION

Groundwater is one of the most important and widely distributed resources of the earth. In view of the rapidly expanding urban, industrial and agricultural water requirements in many areas and the normally associated critical unreliability of surface water supplies, ground water exploration and its use has assumed a fundamental importance for logical economic development. Groundwater development forms the bulk of irrigation development programmes in most of the states of India. For planned development of groundwater resources, however, it becomes essential to quantify the groundwater resources of different administrative units/basins on a realistic basis. Since ground water is a dynamic and replenishable resource, its potential is generally estimated from the component of annual recharge which could be developed by means of suitable groundwater structures. Since rainfall is the principal natural source of groundwater recharge, quantification of the rate of this natural recharge is a basic prerequisite for efficient groundwater resource management. It becomes particularly vital in a coastal aquifer where the draft in excess of this natural rate of recharge can induce the sea water intrusion into the fresh water aquifers.

Rainfall recharge which is a small fraction of total rainfall primarily depends upon a number of factors, e.g. soil moisture characteristics, topography, vegetal cover, soil moisture deficiency, thickness of soil at surface, depth to water table, intensity and duration of rainfall, and other meteorological factors. Quantification of rainfall recharge is, thus, one of the most difficult task in the evaluation of ground water resource. Estimates, by whatever method, are normally, and almost inevitably, subject to large error.

Basically, the principal methods of recharge estimation can be grouped into following two categories :

(a) 'from above' – by analysis of water moving downwards through the unsaturated zone of soil, e.g. lysimeter measurements, tracers, soil moisture budget models and one dimensional soil water flow models.

(b) 'from below'- by inferring the recharge from water table changes, e.g. water level fluctuation method, ground water balance approach, inverse modelling (determination of the recharge necessary to maintain the groundwater levels).

The approach of inverse modelling was considered by Freeze (1983) to be the most "straightforward way of estimating ground water recharge". It is called an 'Inverse Modelling' because, contrary to the 'Forward or Direct Modelling', where recharge is postulated known and hydraulic heads computed, it is the recharge estimate which is computed from field measurements of hydraulic heads.

Though, a few standard software like MODINV, MODFLOWP (Parameter estimation), PMWIN etc. are available now a days for tackling the inverse problem, the recharge estimation by this method has not yet achieved the share of attention it deserves. The present study is, therefore, carried out to estimate the rainfall recharge through the technique of inverse modelling. The study is taken up in a coastal aquifer of Central Godavari Delta of Andhra Pradesh. MODINV (MODular INVerse model) which is a software for parameter optimisation of MODFLOW has been used to optimize the recharge values in different zones of the study area. The model is calibrated during non-monsoon season when only field irrigation is expected to recharge the ground water system. Using the calibrated model, the rainfall recharge is estimated during the monsoon season for different zones of the study area

## 2.0 PROBLEM DEFINITION

As stated in previous section, it is proposed to estimate the groundwater recharge from rainfall and its spatial variation over the Central Godavari delta in Andhra Pradesh using the technique of inverse ground water modelling. MODINV, a parameter optimization software for MODFLOW, is proposed to be used for optimization of recharge in the study area. Based on the available groundwater level data, the model would be calibrated during non-monsoon season. The calibrated model will then be used to estimate the zone-wise recharge during the monsoon season.

# 3.0 STUDY AREA

## 3.1 General

The area selected for the study constitutes a part of the delta system of river Godavari in Andhra Pradesh. The River Godavari is one of the largest perennial rivers of India and flows from west to east across the peninsula. Towards the end of its course, it pierces the Eastern Ghats and flows into the plains between the Ghats and the sea. Upto Dowleswaram in East Godavari Dist. of Andhra Pradesh, the river is known as Akhanda Godavari. Below this point the river bifurcates into two branches, the Gowthami Godavari being the eastern and the Vasista Godavari being the western branch. In between the two branches lies the rich alluvial deposits and is known as the Central Godavari Delta. As the western branch of the river i.e. Vasista again bifurcates in its lower reach at Gannavaram into two branches, the Central Godavari delta is also divided into two parts. Several small islands are also formed due to a number of streamlets of rivers Gowthami and Vasishta.

## 3.2 Location and Areal Extent

The study area lies in East Godavari Dist. of Andhra Pradesh State and forms a part of Central Godavari Delta with its hydrological boundaries as river Gowthami Godavari in the east, river Vasistha Godavari and its branch Vainateya in the west and the Bay of Bengal in the south. With a view to having fairly clear boundaries, the clear area between the nearest streams of river Gowthami and Vasista has been selected for the study and as such the island Polavaram and other small islands have been omitted. Geographically, the study area is located between 16°25' to 16°55' N latitude and 81°44' to 82°15' E longitude and is shown in Fig.3.1. The total geographical area under study measures to 825 sq.kms, covering fully or partly the following revenue mandals.

(1).Amalapuram, (2) Ambajipeta, (3) Allavaram, (4) Atreyapuram, (5) Inavilli, (6) Katrenikona, (7) Kothapeta, (8) Mummidivaram, (9) Ravulapalem, (10) Uppalaguptam, (11) P.Gannavaram.

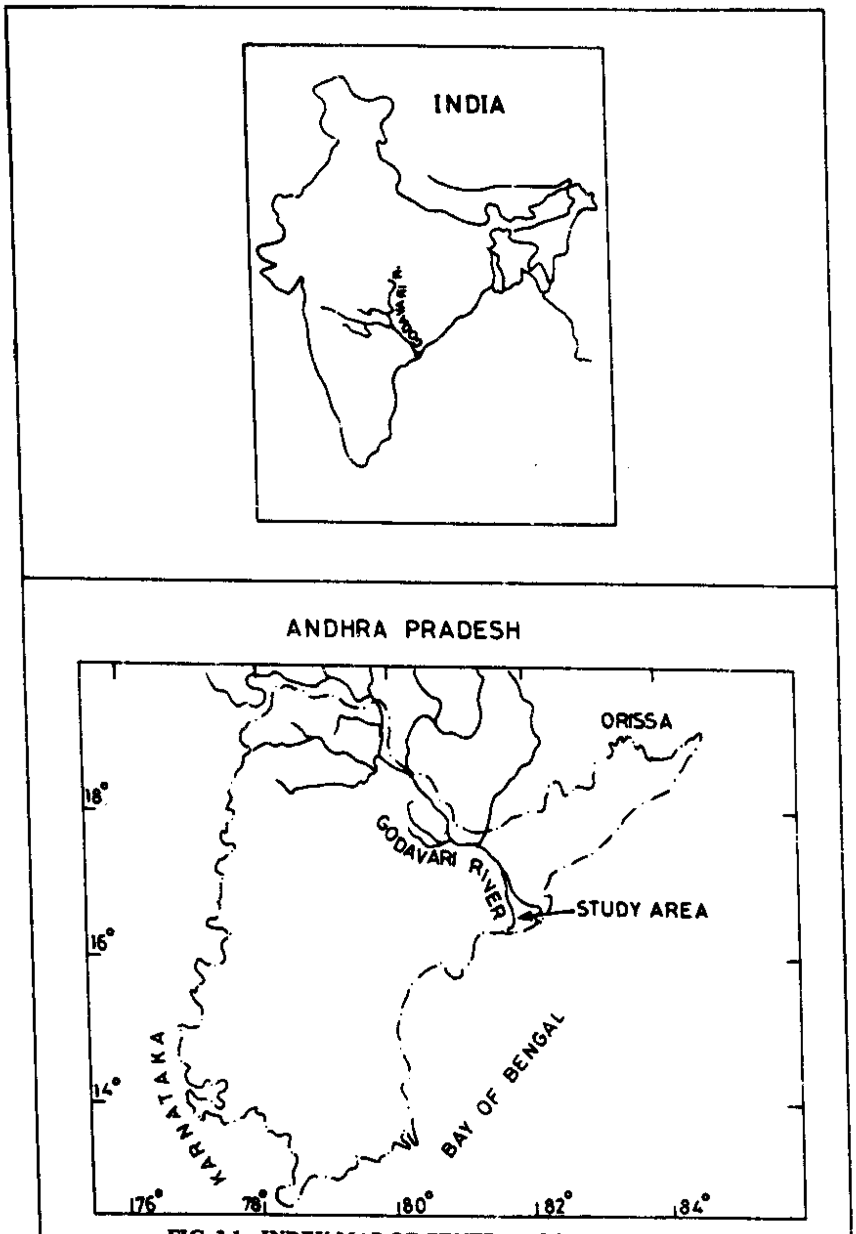


**FIG. 3.1 : INDEX MAP OF CENTRAL GODAVARI DELTA**

## 3.3 Topography and Soils

The study area consists of alluvial plain formed by river Godavari. It has a very gentle land slope towards the sea with elevations varying from about 10 m at upper reaches to about 1 m near the coast. The coast line along the study area measures to about 40 km. The topographical map of the study area is given in Fig.3.2. Texturally, a major part of the study area consists of sandy loams and sandy clay loams. The silty soils which are very deep, medium textured with fine loamy sub soils are located all along the river Godavari as a recent river deposits. The very deep, coarse textured soils with sandy sub soils representing the coastal sand are also found along the sea.

## 3.4 Climate and Rainfall

Being the coastal region, the climate of the study area is comparatively equitable. Though it is very warm in April and June with a maximum temperature of about 39°C, it is never oppressive during the rest of the year. The mean minimum temperature during the two coldest months i.e. December and January for the E. G. Dist. varies from 19°C to 21°C, while the mean maximum temperature varies from 27° C to 29°C. The mean minimum temperature during April to June varies from 26°C to 29°C with mean maximum temperature ranging from 35°C to 37°C. The study area has three distinctive monsoon seasons i.e. South-West monsoon period from June to Sept., North-East monsoon period from oct. to Feb., and West monsoon period from March to May. More than half of the annual rainfall is brought by the South-West monsoon, while the large portion of the rest occur in October and November. The normal annual rainfall of the E.G. Dist is 1142 mm. The rain gauge stations in and around the study area and their Thiessen polygons as used in the study are shown in Fig. 3.3.

## 3.5 Irrigation and Drainage

The Godavari delta Irrigation System is one of the oldest and most important irrigation systems in the state of Andhra Pradesh playing a vital role in the rice economy of India for over a century. The entire study area is under the command of Godavari

Central Canal system and is served by main canal, three branch canals, one distributary and a large number of irrigation channels. The canal network in the study area is shown in Fig. 3.4. The canal system remains operational for 11 months with one month closure period during April-May. Besides, a good number of shallow tube-wells and filter points also exist in the study area. The entire area under paddy which is the major crop in both the seasons is irrigated by canal water. Other important crops like sugarcane, banana and vegetables are partly irrigated by tube-wells and partly by canal water. The study area is served by a number of major, medium and minor drains to remove the surplus water from the fields that gets accumulated especially during the south-west monsoon when the area is subject to incidence of heavy and wide spread rainfall. The ground fall on an average is about 1/5000 in head reaches and 1/7500 in the lower reaches of the study area. Consequently, the out-fall of the drainage from the irrigated area, through the network of surface drains, is rather slow especially in the lower reaches resulting in drainage problems of severe nature.

## 3.6 Agriculture

From agriculture point of view, the alluvial soils are considered to be the most fertile lands and paddy being the major crop of the Godavari delta system, it is known as the rice bowl of Andhra Pradesh. The study area (i.e. part of the delta system) is also predominantly a rice growing area in both *kharif* and *rabi* seasons. Other important crops in the study area include coconut, sugarcane, banana, turmeric and vegetables. The crops like maize, *jowar, bajra* etc. are found in patches only. There are mainly two cropping seasons namely *kharif* and *rabi*. The *kharif* season commences from $1^{st}$ June when irrigation water is released through the canal system and extends upto November. The *rabi* season is from December to April of the succeeding year. The net area sown forms about 68% of total geographical area under study. The gross cropped area is about 95000 ha of which about 75% is occupied by paddy, 15% by coconut trees and the remaining by other crops. The area under other miscellaneous deep-rooted trees is almost negligible as there are no forests in the study area.

### 3.7 Sea Water Intrusion

The chemical analysis of ground water samples collected from observation wells spread over the study area was carried out. The analysis indicates that the T.D.S. values as high as 2000 are found in the wells located near the two arms of river Godavari, while a reducing trend in T.D.S. values is observed in the wells located away from the rivers. Comparatively low values of T.D.S. are also observed in the wells located near the sea. The groundwater table data analysis reveals that the flow takes place from the aquifer towards the sea. This indicates that there is no sea water intrusion directly from the sea into the aquifers due to reversal of gradients. However, in the case of rivers, the groundwater table contours indicate that the flow takes place from the rivers into the aquifer. During high tides in pre-monsoon as well as post-monsoon period, sea water rushes through the mouth of the river, upstream for distances upto 40 to 50 km. It takes a few days for this sea bore to recede back to the sea. During this period salt water infiltrates into the groundwater aquifer from the river banks and bed.

### 3.8 Data Availability

A variety of data on hydrological, meteorological and geological aspects of the study area were collected from different Depts. Field visits were also undertaken to gather the necessary information. These data were processed and made use of in the study. A summary of the available data and information is presented below:

1. Topographical map of the study area (Fig.3.2)
2. Map showing rain gauge stations and their Thiessen polygons(Fig.3.3)
3. Map showing canal network in the study area (Fig.3.4)
4. Map showing location of observation wells (Fig.3.5)
5. Monthly rainfall data of rain gauge stations
6. Monthly ground water levels in observations wells
7. Canal discharge data
8. Length, cross-sections and other design details of main canal, branch canal and distributaries and their command areas.
9. Number of wells/tube-wells in the study area
10. Stage of the rivers
11. Land use pattern data
12. Cropping pattern data
13. Data on geology and aquifer characteristics
14. Data on potential evapotranspiration, and other climatic data
15. Groundwater quality data for the study area.

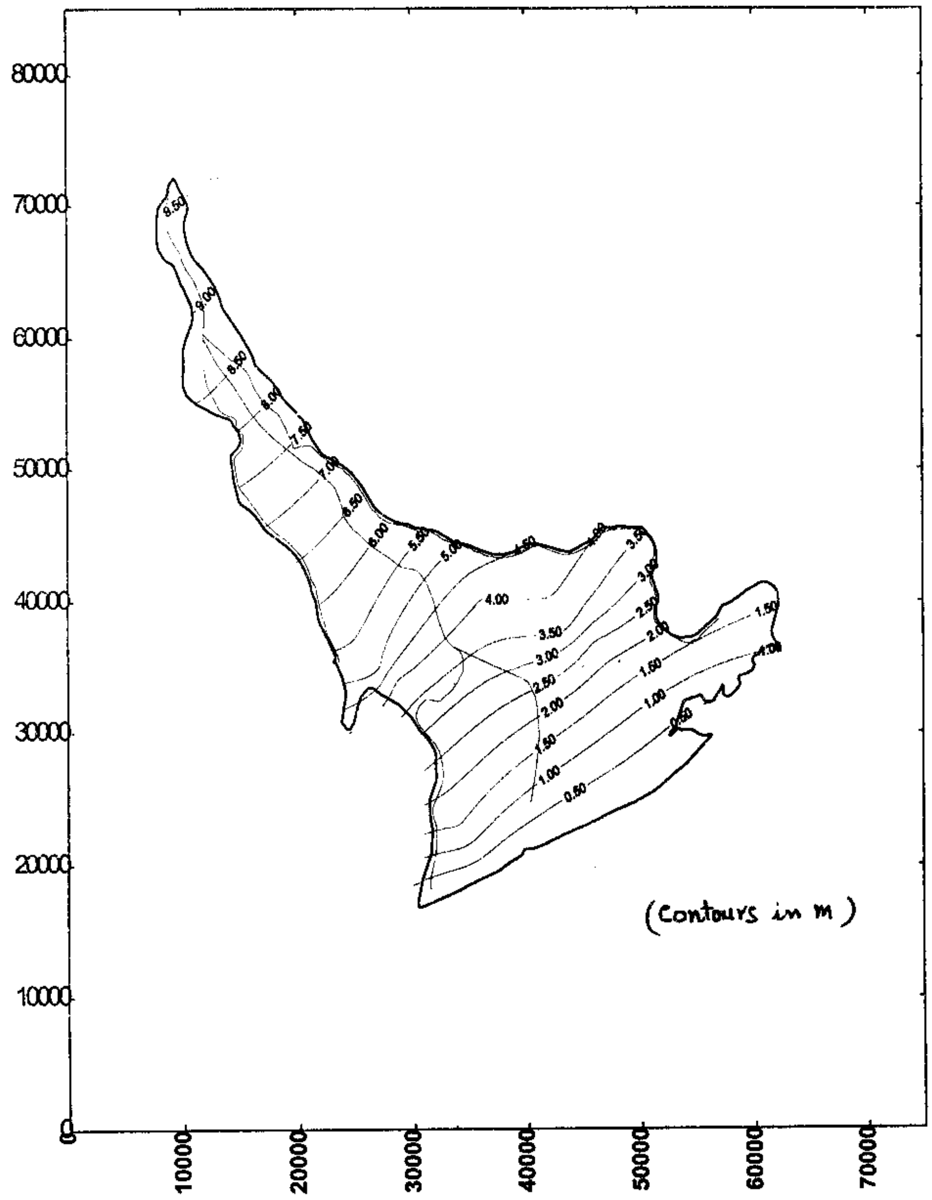


FIG. 3.2 : TOPOGRAPHICAL MAP OF STUDY AREA

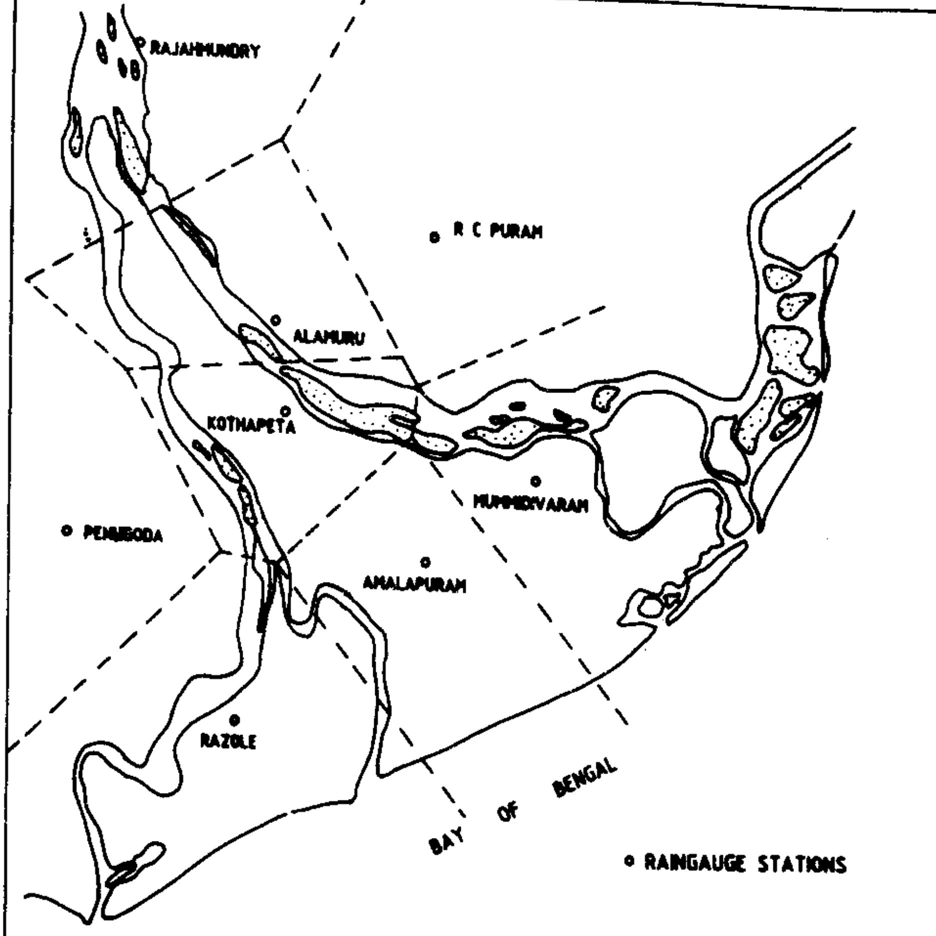


FIG. 3.3 : RAINGAUGE STATIONS AND THEIR THIESSEN POLYGONS IN THE STUDY AREA

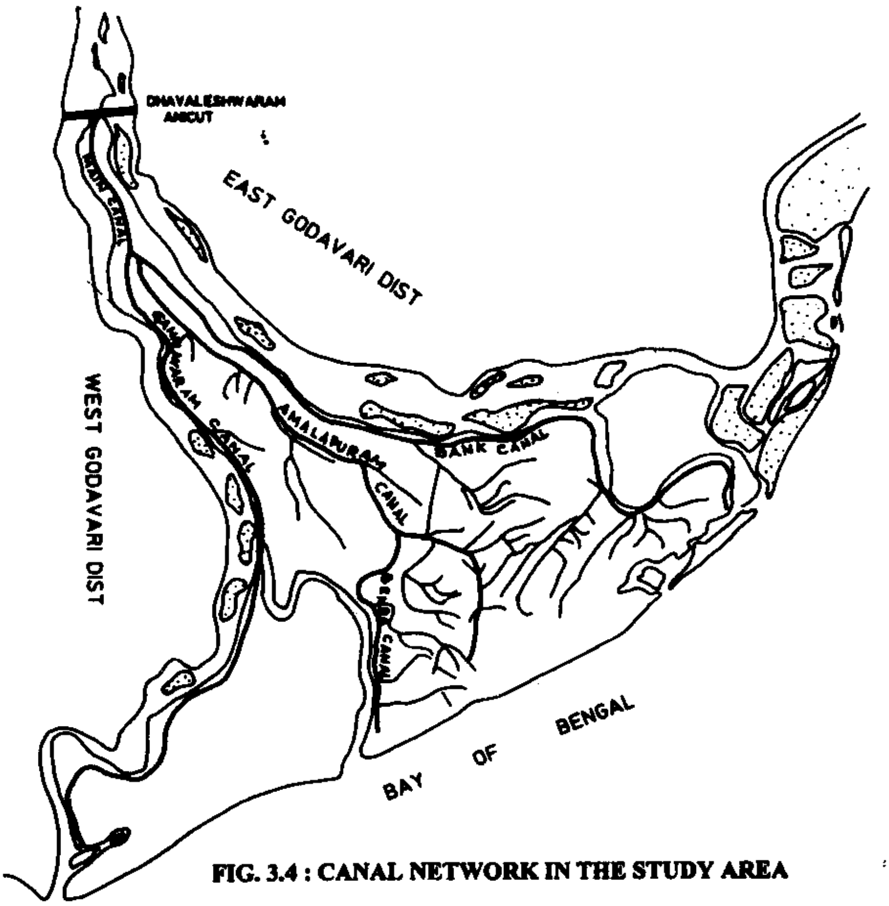


**FIG. 3.4 : CANAL NETWORK IN THE STUDY AREA**

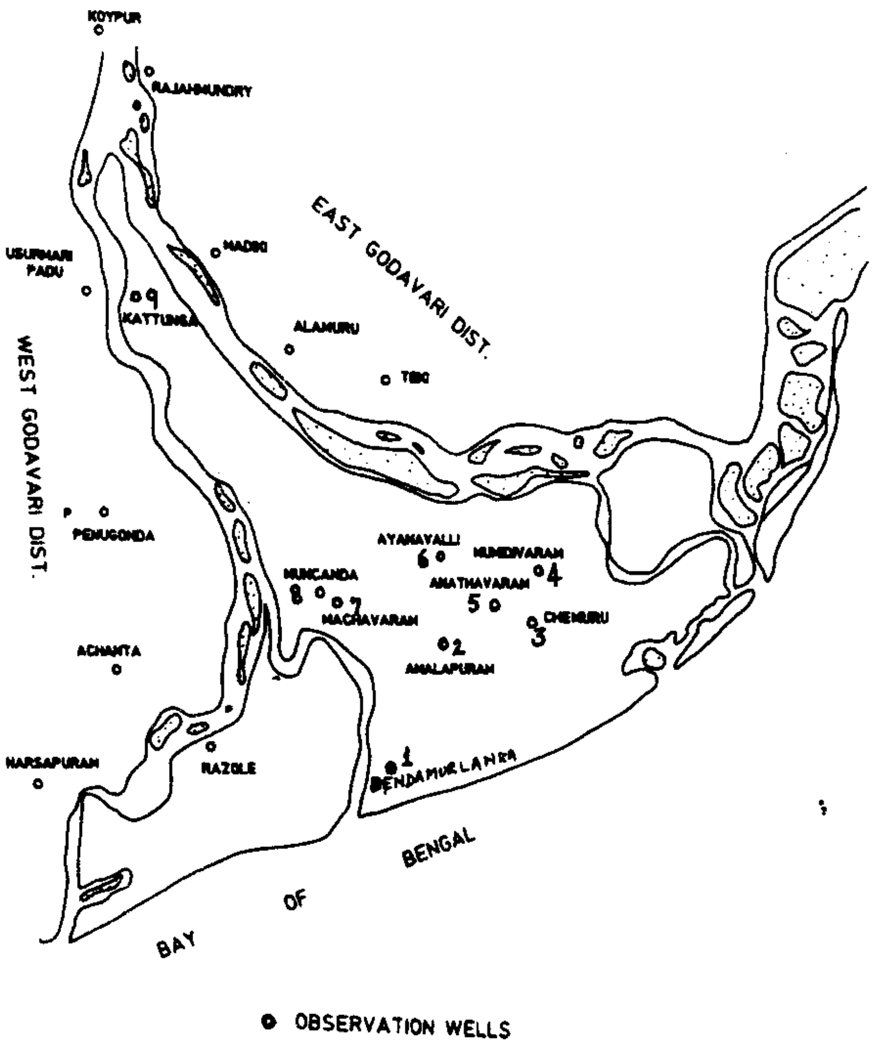


FIG. 3.5 : OBSERVATION WELLS IN THE STUDY AREA

## 4.0 DESCRIPTION OF MODINV

### 4.1 An Overview

The MODINV suite of software comprises a number of programs, built to enhance the usefulness of the popular USGS finite difference flow model, MODFLOW. The MODINV is basically a parameter optimisation program for MODFLOW. Using this software, the specific values taken by any parameter type that MODFLOW can read as a 2-dimensional data array can be optimised such that model-generated heads are as well matched as possible to those observed in the field. Steady state and transient, single and multi-layer, confined and unconfined models can all be calibrated in this manner. As well as providing optimised parameter values, MODINV indicates the reliability of these aquifer parameter value estimates, given the observed head data that is used in calibration. Parameter values can be fixed, grouped or transformed to enhance optimisation stability and efficiency.

### 4.2 MODINV Software

Model parameter (or input) estimation is often referred to as the "inverse problem" to distinguish it from the "forward problem". The latter refers simply to the process of mathematical modelling and the means by which this is achieved for different physical systems. For groundwater modelling, MODFLOW is a program that carries out forward modelling using the finite-difference method; aquifer parameters and inputs are provided by the user and model outputs (heads or drawdowns) are calculated. On the other hand, when model outputs are known and an attempt is made to solve for one or a number of the model parameters or inputs, then inverse modelling is being attempted.

Here, both aquifer input (eg. recharge) and physical properties (eg. transmissivity) will together be referred to as model "parameters" for the sake of simplicity of expression. Table-4.1 provides a list of such parameter types. As discussed latter, on the assumption that some of the model parameters are known, we are attempting to ascertain the values of the other parameters on the basis of a set of water level or head measurements taken at a number of boresites at one or a number of times.

**Table 4.1 : Two Dimensional Real Arrays That Can Serve as Parameter Types.**

| MODFLOW package | Array | Remarks |
|---|---|---|
| BCF | Primary storage capacity | (all layers) |
| | Secondary storage capacity | (all layers) |
| | Transmissivity | (all layers) |
| | Hydraulic conductivity | (all layers) |
| | Layer bottom elevation | (all layers) |
| | Layer top elevation | (all layers) |
| | Vertical hydraulic conductivity/thickness | (all layers but bottom) |
| RCH | Recharge rate | (all stress periods ) |
| EVT | Surface elevation | (all stress periods ) |
| | Maximum EVT rate | (all stress periods ) |
| | EVT extinction depth | (all stress periods ) |

Fundamental to the operation of most inverse modelling algorithms is an ability to calculate model outputs using current estimates of model parameters, i.e., to carry out routine solutions of the forward problem. These model outputs are compared with measurements (in the present case through a weighted sum of squared differences criterion) and the parameters are then adjusted to obtain a more favourable comparison. MODINV uses MODFLOW as its forward processor. However as field-observed head data exist at only a discrete number of bores, the two-dimensional head arrays constituting MODFLOW's output are interpolated to yield MODFLOW-predicted heads at these boresites; it is these interpolated head values that are compared with historical or steady-state head data, and it is the weighted squared sum of the differences between these two sets of heads which is minimized. Hence the total forward model can be considered to be MODFLOW plus the two-dimensional head array interpolation procedure; the forward model outputs are then the heads at boresites whose positions and layer numbers are nominated by the user.

### 4.2.1 The Mathematical Model

The purpose of a mathematical model is to predict the behaviour of a system as it responds to changing conditions. If we consider these conditions as inputs to the model, then once we know the parameters of the model, it is a simple matter to obtain model outputs for as many different inputs as we like. For a groundwater model the inputs are normally considered to be the sources (or sinks) of water (for example recharge, EVT rate, well pumping rate, etc) while the parameters are aquifer physical properties such as transmissivity and storage capacity. Boundary conditions such as lateral model inflow rate or constant head levels are often considered to be part of the model itself, being neither an input nor a parameter, the latter term normally referring to something which can be adjusted at the model calibration stage. The model outputs are the heads or water levels in the aquifer

The partial differential equation describing groundwater flow is

$$\frac{\partial}{\partial x}(K_{xx}\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(K_{yy}\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(K_{zz}\frac{\partial h}{\partial z})-W=S_s\frac{\partial h}{\partial t} \quad ...(1)$$

where,

| | |
|---|---|
| $K_{xx}, K_{yy}, K_{zz}$ | : hydraulic conductivities along the major axes |
| h | : potentiometric head |
| W | : volumetric flux per unit volume and represents sources and or sinks |
| $S_s$ | : is the specific storage of the aquifer |
| t | : time |

Considering the above equation together with the given boundary conditions as the model, then the parameters ($K_{xx}$, $K_{yy}$, $K_{zz}$ and $S_s$) are the model, "distributed", parameters. The fact that they are distributed means that they require for their complete description a knowledge of their value at every point within the three-dimensional space occupied by the aquifer. That is, as functions of position, they should be represented as $K_{xx}(x,y,z)$, $K_{yy}(x,y,z)$, etc. Model inputs are functions of time as well as location, and hence can be written as W(x,y,z,t). Similarly, the model output, h, is both time and space dependent and can likewise be represented as h(x,y,z,t). Hence equation 1 can be represented conceptually by the following equation:

$$M(K_{xx}, K_{yy}, K_{zz}, S_s; W) = h \qquad ...(2)$$

where the semi-colon in the bracketed term above separates the model parameters (time-independent), from the inputs (time-dependent). Again boundary conditions, (though they may be time-dependent) are assumed to be part of the model, and hence the "M" term.

To model a natural system, simplifying assumptions are made to all terms of equation (2). Spatial discretization of the M operator, through which a differential equation is converted to a matrix equation, is fully described in the MODFLOW manual. To construct a parameter estimation algorithm on which to base MODINV, some further simplifications are made. In particular, it is assumed that all distributed parameters are "piecewise constant", ie. they are constant within each of a number of zones which, when put together, cover the area of the model. These zones of constant parameter value do not necessarily coincide for each different parameter type; however each parameter type now requires for its complete description only a few numbers, these representing the values that the parameter takes within each zone of constancy for that parameter.

Model inputs, W, are subdivided by MODFLOW into two types, viz. those that are distributed across the mesh (recharge and EVT) and those that take on "point" or "line" distributions (well recharge, drainage, river, general head boundary). For the first group MODFLOW requires either a two-dimensional matrix of inputs (recharge) or a number of two-dimensional matrices by which the source or sink two-dimensional matrix can be calculated (EVT); for the second group MODFLOW requires either the model input, or the means by which it can be calculated, at each pertinent cell, the latter being nominated specifically by row, column and layer number. MODFLOW assumes that each input is piecewise constant in the time domain, the period of constancy being referred to as a "stress period". For recharge and EVT, let us also assume that the recharge rate and the properties that determine the EVT rate can be represented, like the model parameters, by a limited number of spatial zones within each of which the recharge or a particular EVT characteristic is constant for a given stress period. These zones do not need to coincide for different stress periods or for each of the three different EVT characteristics and recharge.

Let us now introduce a further simplification by acknowledging that, when calibrating the aquifer, we only have field measurements to compare with the model output (head or water level) at a finite, relatively small, number of points, these being the locations of bores within the model area. Furthermore these borehole heads are known at only one or a finite number of times, corresponding to field sampling episodes. Assuming that we have a means by which a MODFLOW head array output can be interpolated onto these same boresites at specific times corresponding to the measurement times, our model output can be considered as a number of sequences of real numbers, each sequence providing a time series of water levels or heads at a particular bore. Forcing these model heads to coincide as closely as possible with the measured heads at the measurement times is the basis for model calibration. Of course in the steady-state case, there is only one observed head and one model head for each bore.

After making all these assumptions, equation (2) can be rewritten

$$\mathbf{M}(\mathbf{p}) = \mathbf{h} \qquad \text{...(3)}$$

where **p** is finite-dimensional vector of numbers representing all of

i. the values taken by each of the model parameters within each of their respective constant-parameter zones.

ii. the values taken by recharge rate within each of its constant-recharge zones for each stress period,

iii. the values taken by each of the three parameters determining EVT in each of their respective constant-value zones for each stress period, and

iv. the values taken by the coefficients that determine single cell model inputs, (ie. rivers, drains, wells and general head boundaries) in each of the model cells that are subject to such inputs for each stress period.

h is also a finite dimensional vector. It consists of head (ie. model output) values at specific boresites at times for which a model output has been requested. If **p** contains N elements and **h** contains M elements, then M is a continuous vector function from N-dimensional space to M-dimensional space.

M as written in equation (3) does not have an inverse; i.e., if **h** is known it does not follow that **p** can be determined. This is easily demonstrated by considering a model that is subject to both recharge and EVT. If the former is increased everywhere while the latter is decreased by the same amount, the model's output heads will be unchanged. Hence, given h, it is impossible to determine both recharge and the EVT coefficients. With other model parameters, such as the transmissivity distribution, also having a strong effect on h, it is easy to see that there are many different p's which will produce the same, or almost the same, h. Some of these different p's can be obtained from any given **p** which satisfies (3) by varying some of its elements in such a way that the effect of changing one or a number of these elements is balanced exactly (or almost exactly) by simultaneously changing one or a number of its other elements in a certain manner. This is an example of parameter correlation, of which more will be said later.

So it is obvious that if we are going to use measured aquifer heads as the basis for model calibration, it will be necessary to assume that some elements of **p** are known. We will then be left with the problem of estimating the remaining elements, for which we may or may not be able to obtain a solution, depending on the degree of parameter correlation that remains. If parameter correlation is still too high, a numerical inversion algorithm will not converge to a solution or, at best, will show signs of instability. This is not the fault of the algorithm, for it cannot answer an impossible question. In general, the fewer the parameter types for which you require estimates, and.the fewer constant-parameter-value sub-areas for those parameter types, the more likely is the algorithm to perform well. Of course, with fewer parameter sub-areas the degree of fit between model and observed heads may not be as good (see later); however you may not be able to escape the fact that your borehole head data is insufficient for any finer detail of aquifer property determination.

Returning to equation (3), then, we can rewrite it as follows:

m (p) = **h** ...(4)

where those parameters and inputs of the model which are assumed known are now included in the revised model function, m, and **p** is of reduced dimension. In fact it must be of smaller dimension than **h** because when we derive a set of simultaneous equations to solve for the elements of **p** in the next section, there must be fewer unknowns than equations (the number elements of h) in order for the system of equations to be capable of solution.

In the MODINV algorithm, the elements of **p** can be the parameter values taken by up to three different types of parameter within their respective constant-parameter sub-areas. Parameter types can be anything that MODFLOW can read as a two-dimensional array of real numbers. This includes transmissivity, storage capacity, recharge for any (or up to five) stress periods, etc.; one "parameter type" corresponds to each such two-dimensional array. Though recharge is strictly a model input rather than a parameter, and though quantities such as maximum EVT rate likewise govern another model input, we will refer to anything that MODFLOW can accept as a two-dimensional real array (with the exception of initial heads) as a "parameter type" in the discussion that follows. The values taken by any such "parameter type" within its (unique) zones of piecewise-constancy are thus admissible elements of the vector p.

### 4.2.2 The Inverse Problem

m in equation (4) is a continuous function of p, mapping N-dimensional space into M- dimensional space where N is the number of elements of **p** (equal to the number of individual parameter values requiring estimation) and M is the number of elements of **h** (equal to the number of observation times multiplied by the number of observation bores). Let J be the Jacobian matrix of m. This is the matrix whose ith row is the derivative of the ith element of h with respect to each of the elements (in order) of p. Hence J has M rows (same as h) and N columns (same as p). Let $h_0$ and $p_0$ satisfy equation 4. If we now change each of the elements of $p_0$ by a small amount to obtain the vector p, the resultant change to the head vector can be approximately calculated as

$$\Delta h = h - h_0 \approx J(p - p_0) = J\Delta p \qquad ...(5)$$

where the approximation improves as $\Delta p$, and hence $\Delta h$, are reduced. Let us assume that we presently have an estimate for each of the values of each of our unknown parameter types within their respective constant-value sub-areas; let this vector of estimates be $p_0$. Using our model (ie. MODFLOW), $h_0$ is readily calculated using equation 4. Let h in equation 5 be the vector of heads observed from bores in the aquifer; of course the elements of h must pertain to the same bores at the same measurement times as do the corresponding elements of $h_0$. Then in equation 5, there is only one unknown, viz. p, the vector the model is calibrated provided, of course, that the constant-parameter sub-area boundaries are well chosen.

However, there is a problem. If M, the number of heads, is less than N, the number of unknowns, there are less equations than unknowns and we cannot solve for p. If M is greater than N, then we could select a set of N of the M equations represented by (5) and obtain a solution for p; however, selecting another set of N equation may give us a different solution and we are left with the question for which solution is best. If N and M are equal we obtain a unique solution for p, but the lack of redundancy in our observations gives us no protection against the effects of head measurement errors or of an inappropriate parameter sub-area zonation scheme. Without such redundancy, parameter value estimates may be erroneous, and the calibrated model may thus provide a poor basis for predicting future aquifer behaviour.

To solve the inverse problem, then, we must formulate it slightly differently. Again, let us assume that we have a current set of parameter estimates, $p_0$, and hence a corresponding set of model-generated heads, $h_0$, calculated on the basis of $p_0$ for a number of bores at a number of times. Corresponding to the elements of $h_o$ we have a vector, $h_m$, of head measurements. We wish to improve our current estimate, $p_o$, to a new parameter vector $p_l$, generating a head vector $h_l$ through equation (4) that is "closer" to $h_m$ than $h_o$. As it is foolish to expect that we can choose our parameters such that all the corresponding elements of $h_l$ and $h_m$ are exactly equal, and as we wish to make use of all the measured heads in establishing $p_l$, we choose as our criterion for determining $p_l$ that

$$\phi = (h_l - h_m)^t \, W(h_l - h_m) = \text{minimum} \qquad \ldots(6a)$$

where the superscript "t" refers to the transpose of the vector. $\phi$ is often referred to as the "objective function". In this equation, W is a "weighting matrix" which, in MODINV, is assumed to contain diagonal elements only. Hence equation (6a) can also be written as

$$\phi = \sum_{i=1}^{M} (h_{li} - h_{mi})^2 \, w_i = \text{minimum} \qquad \ldots(6b)$$

where $w_i$ is the i'th diagonal element of W. In other words, the weighted sum of the squares of the differences between model and observed heads must be a minimum. The use of weights allows us to give measured head values which we "trust" a greater say in the determination of parameter values than those which we do not. Alternatively it provides a means by which we can enforce a condition that heads calculated in a particular model sub-area, or at a particular time, be better matched to reality than those elsewhere or at other times, if they cannot all be simultaneously matched as well as we would like. The diagonal elements of the matrix W, then, can also be considered as a weighting vector of dimension M, each element of which determines the importance of the corresponding element of $h_m$ in governing the estimation process; it is good practice to select these weights from the interval [0, 1]. If measurements are missing from some bores at certain times, you can use "dummy" measurements for the corresponding elements of $h_m$ , and set the corresponding measurement weights to zero; in this way, such elements have no effect on the estimation process.

Defining

$$\mathbf{\Delta h = h_l - h_o \ ; \ \ \Delta p = p_l - p_o} \qquad ...(7)$$

where $h_l$, $P_l$ and $h_o$ , $p_o$ jointly satisfy (4), then $\Delta h$ and $\Delta p$ approximately satisfy (5), with the approximation improving with proximity of $h_o$ to $h_l$ and $p_o$ to $p_l$ , ie.

$$\mathbf{\Delta h = h_l - h_o \cong J(p_l - p_o) = J\Delta p} \qquad ...(8)$$

Substituting (8) into (6)

$$(\Delta h + h_o - h_m)^t W(\Delta h + h_o - h_m) = \text{minimum}$$

which is equivalent to:

$$\Delta h^t W\Delta h + \Delta h^t W(h_o - h_m) + (h_o - h_m)^t W\Delta h + (h_o - h_m)^t W(h_o - h_m) = \text{minimum}$$

ie.

$$\mathbf{\Delta h^t W\Delta h + 2\Delta h^t W(h_o - h_m) = minimum} \qquad ...(9)$$

where constant terms have been ignored because they cannot be minimized and we have made use of the fact that W is a symmetric matrix. Substituting (8) into (9):

$$(J\Delta p)^t W(J\Delta p) + 2(J\Delta p)^t W(h_o - h_m) = \text{minimum}$$

ie.

$$\mathbf{\Delta p^t(J^tWJ)\Delta p + 2\Delta p^tJ^tW(h_o - h_m) = minimum} \qquad ...(10)$$

Now if both terms on the left of (10) are differentiated with respect to each element of p and the right hand side is equated to zero in each case (because of the minimum), we obtain

$$\mathbf{(J^tWJ)\Delta p = -J^tW(h_o - h_m)} \qquad ...(11)$$

$J^tWJ$ is a N x N matrix (often referred to as the "normal" matrix); hence equation (11) represents N equations in N unknowns which can be solved for the elements of $\Delta p$ provided $J^tWJ$ is not singular. A singular matrix implies that, even though there may have been more borehole head observations than there are unknown parameter values, there is still insufficient information for unique parameter value determination. For example if, in a steady-state model, you ask that both recharge and transmissivity be determined everywhere in the model, you will obtain a singular normal matrix because the values taken by one parameter type (eg. transmissivity) for a particular head distribution, depend on the values taken by the other parameter type (recharge). However if you assume that recharge is known everywhere you can then estimate the transmissivity distribution, and vice versa.

Problems can also arise if the normal matrix is nearly singular; if this occurs MODINV may have trouble minimizing $\phi$ of equation (6). Considering the steady-state problem again, this can occur if you attempt to estimate the transmissivity distribution with too great a spatial precision in an area where there are too few borehole head observations. If there are many model sub-areas in a zone of sparse measurement, it will be possible to simultaneously vary the transmissivities of these zones in such a way as to maintain the model heads in the observation bores relatively unchanged. This means that the observed borehole heads do not have the power to tell you what the individual transmissivities are; this is the phenomenon of high parameter value correlation again. The higher the degree of such correlation, the closer will the normal matrix approach to singularity, and the greater will be the possibility of numerical instability.

From equation (11)

$$\Delta \mathbf{p} = - (\mathbf{J}^t\mathbf{W}\mathbf{J})^{-1}\mathbf{J}^t\mathbf{W}(\mathbf{h}_o - \mathbf{h}_m) \quad ...(12)$$

while from (7)

$$\mathbf{p}_1 = \Delta \mathbf{p} + \mathbf{p}_o \quad ...(13)$$

and an improved set of parameter values has been obtained. Because (8) is only approximately correct (especially if $p_0$ is a poor estimate of the aquifer parameters so that $\Delta p$ needs to be large), the process outlined above needs to be repeated to obtain another estimate $p_2$, then another, $p_3$ etc. until further improvement is impossible, or until $\phi$) of equation (6) is low enough to indicate an acceptably good fit between model and field data.

MODINV does not use equation (12) for parameter improvement; rather, it uses a slight modification of it. Defining ($\phi_0$ as the value of the objective function (weighted sum of squared head differences between model and observed heads) with model heads calculated on the basis of the parameter set $p_0$, it can be shown that

$$\nabla\phi_0 = 2\mathbf{J}^t\mathbf{W}(\mathbf{h}_o - \mathbf{h}_m) \quad ...(14)$$

where $\nabla\phi_0$ is the gradient of $\phi_0$ with respect to the elements of $p_0$. Substituting (14) into (12) and using (13) we obtain:

$$p_1 = P_0 - \frac{(J^tWJ)\ \nabla\phi_0}{2}$$

Generalizing this to the i+lth iteration:

$$p_{i+1} = P_i - \frac{(J^tWJ)\ \nabla\phi_0}{2} \quad ...(15)$$

The hardest part of using equation (15) to improve parameter value estimates is calculating the Jacobian matrix, J. In MODINV, finite differences are used. For $p_i$, the parameter set at the beginning of the i+lth iteration, MODFLOW is run to obtain the corresponding $h_i$ vector. Then a single element of $p_i$ is increased by a small amount and

MODFLOW is run again to determine a new set of heads, $h_i^j$, where the superscript "j" indicates that the jth parameter value, ie. the jth element of $p_i$, was varied. The jth column of the Jacobian is then calculated as the vector

$$\frac{h_i^j - h_i}{\delta p_i^j}$$

where $\delta p_i^j$ is the change to the jth element of the parameter value vector $p_i$. When convergence has nearly been obtained, MODINV uses central differences for greater accuracy in derivative calculations. In this case each $p_i^j$ is first increased, and then decreased, by $p_i^j$ to obtain, respectively, the set of heads $h_{i(1)}^j$ and $h_{i(2)}^j$. The jth column of the Jacobian is then approximated as the vector

$$\frac{h_{i(1)}^j - h_{i(2)}^j}{2\delta p_i^j}$$

The gradient vector is calculated in similar fashion.

Calculation of head derivatives by finite differences is very time consuming. In fact for most MODINV runs, this accounts for over 70% of the computing time. There are more efficient methods of derivatives calculation that can be used under confined aquifer conditions (ie. when equation (1) is linear); there are also other optimization methods available that do not require the calculation of explicit head derivatives with respect to individual parameter values for their implementation. However the calculation of derivatives of head with respect to parameter values by finite differences is perfectly general, being useable for any distributed parameter type that MODFLOW can read, under both confined and unconfined conditions. Also the Gauss-Marquardt method (see next section) which makes use of these derivatives for optimization, converges to a solution in fewer steps by far than most other methods. Note that the reason why it does not completely converge in a single step is that the relationship between $\Delta h$ and $\Delta p$ upon which all of the above theory is based (equation 8) is only approximately correct.

MODINV provides you with the choice of optimizing either parameter values themselves or the logarithms or "logistic transformations" of parameter values, the last being defined by the relationship

$$P_t = \log [p/(p-1)] \qquad ...(16)$$

where **p** is the parameter value and $p_t$ is its logistic transformation. In the latter two cases, everything said so far about parameter estimation and derivatives calculation applies just as well, provided the transformed parameter value, rather than the parameter value itself, is considered as the parameter value to be estimated. There are two advantages that accompany log or logistic parameter value transformation in certain cases:

1. There is strong evidence that the probability distribution satisfied by some aquifer parameters (transmissivity, hydraulic conductivity) is log-normal, rather than directly normal. For estimating such parameter types it is better to optimist the logarithms of the parameter values than the parameter values themselves because parameter stochastic property inferences drawn from a least squares inversion (discussed later), assume that parameter values possess a multidimensional normal probability distribution. Also, in such cases, optimization convergence appears to be faster and more stable.

2. Some parameters must take values within a certain range for them to have any meaning. For example, transmissivity must never be negative and storage capacity must be between 0 and 1. If a parameter is left at the mercy of an iterative adjustment procedure that pays no attention to whether it is given sensible values or not, errors could result. By optimizing the log of the parameter, the parameter itself can never become negative. Similarly, no matter what value is given to the logistic transformation of a parameter, the parameter itself will never be outside the interval (0,1).

### 4.2.3 The Gauss-Marquardt-Levenberg Method

Equation 15 describes the Gauss method of solution of the inverse problem. Defining

$$N = J^t W J \qquad ...(17)$$

$$f_i = \nabla \phi_i / 2 \qquad ...(18)$$

(15) can be rewritten:

$$\Delta p_i = -N^{-1} f_i \qquad ...(19)$$

While the method often converges rapidly (the more rapidly it converges, the fewer optimization iterations are required), its performance is not perfect, especially in cases where parameter correlations are high. To make the method more robust and reliable, it is usually modified in a manner similar to that originally outlined by Levenberg in 1944 and then by Marquardt in 1963, though the method is normally named after the latter author.

To implement the method, N. the normal matrix, is modified by increasing all of its diagonal elements. In the MODINV algorithm N is modified to $N_m$ by adding a fixed amount, , to all diagonal elements, ie.

$$N_m = (N + \lambda I) \qquad ...(20)$$

where $\lambda$ is a positive constant and I is the N x N identity matrix. If $N_m$ now replaces N in (19) a $\Delta p$ is obtained which has been found to be more reliable in many cases than that obtained solely using the Gauss method (ie. a $\lambda$ of zero). (It should be noted that when $\lambda$ is high, the resulting $\Delta p$ is the same as that obtained using the so-called "gradient" method of parameter value adjustment.) However we are still left with two problems when using the Marquardt enhancement of the Gauss method. These are

(i) how should k be determined at each optimization iteration, and

(ii) once a $\Delta p$ is obtained using $N_m$ in place of N in (19), what fraction of this $\Delta p$ should actually be added to $p_i$ to determine the $p_{i+1}$ which yields the minimum weighted sum of squared differences between model and observed heads.

The first problem, the value of $\lambda$, is not formally solved; instead, experience dictates the best choice. In the initial stages of an estimation process $\lambda$ should normally be high (otherwise the solution may not converge), especially if the initial parameter value estimates (ie. the elements of $p_0$) are poor. As the process progresses and the upgraded estimates become better and better, $\lambda$ is normally reduced because the Gauss method has a superior performance over the gradient method for parameter values which are close to optimum.

PREINV, the MODINV preprocessor, asks you for an initial $\lambda$. This is the $\lambda$ value used in the very first attempt at reduction of $\phi$, the objective function, of equation (6). For each MODINV optimization iteration, one or a number of $\lambda$'s are tried. For the first optimization iteration the initial $\lambda$ is used first; for later optimization iterations a $\lambda$ is first tried which is reduced by a certain factor (supplied by you to PREINV) below that which worked best for the previous iteration. Unless the objective function is drastically reduced with this first $\lambda$, a second $\lambda$, reduced from the first by this same factor, is tested. If the use of the second $\lambda$ achieves a **p** vector that lowers $\phi$ by a significantly greater amount than that achieved through the use of the first one, then $\lambda$ is lowered again and the process is repeated. When it is judged that $\phi$ cannot be further significantly reduced by lowering $\lambda$, or a maximum of five it's have been tested, the best **p** is accepted as the updated parameter set. Sometimes, however, $\lambda$ must be increased to obtain an improved parameter set. While experimenting with different values of $\lambda$ in this fashion is a little cumbersome, and certainly consumes computer processing time, it is worth the effort because it is important, for each optimization iteration, to achieve a good parameter improvement. With each new optimization iteration the Jacobian matrix must be recalculated, and this is the most time-consumptive part of the whole inversion exercise; hence the best or nearly the best, parameter improvement possible must be achieved for each optimization iteration in order that fewer overall iterations are required in the whole inversion process. If a few values of $\lambda$ must be tested to maximize the improvement realised for each iteration, then it is worth the effort. Fortunately it has been found that, if the initial $\lambda$ and its adjustment factor are well chosen, most iterations require the use of only one or two $\lambda$'s so that little time needs to be spent in this kind of experimentation.

The second problem, that of the step size, is solved in the following manner. Once a $\lambda$ has been chosen and an $N_m$ calculated and substituted into (19), the latter equation is solved for $\Delta p$. However because we have used $N_m$ rather than N. $\Delta p$ now indicates only the direction of parameter change, the actual size of the change being $\beta \Delta p$, where ,$\beta$ is a factor which must be calculated such that $\beta \Delta p$ provides the maximum possible reduction in o of any parameter changes that take place in the direction of $\Delta p$. It has been shown by Carrerra and Neuman (1986b) that ,$\beta$ can be calculated as

$$\beta_i = \frac{(hm - h_i)^t W \gamma_i}{\gamma_i^t W \gamma_i} \qquad ...(21)$$

for the i+1th iteration, where

$$\gamma_i = \partial \mathbf{h}_{i+1} / \partial \beta_i \qquad ...(22)$$

In MODINV, the $\gamma$ vector is calculated using finite (forward or central, as appropriate) differences.

### 4.2.4 Measured Head Standard Deviations and the Reference Variance

In the above discussion, no assumptions were made concerning the probability distributions of head observations or (transformed) parameter values for model sub-areas. Our sole criterion for deciding on a set of model parameter values was that the sum of the weighted squared differences between model and measured heads be minimized using this set. Hence we arrived at a set of parameter values for which the fit between model and observed heads is optimum in the weighted least squares sense.

If we make the assumption that both the head measurements and the individual parameter values (those values taken by parameter types within their constant-parameter zones) are normally distributed, then we can say something quantitative about the level of uncertainty pertaining to these estimated parameters values. It is to this topic that we now turn. Note that, in the discussion that follows, if a parameter value has been mathematically transformed so that its log or logistic transformation is estimated, then the following discussion is applicable to the transformed parameter value rather than the parameter itself; in what follows, "parameter value" will refer to whichever of these is being estimated.

At first it may seem that the idea of observed heads being subject to a probability distribution is fallacious because they can normally be determined to the nearest centimetre at least. This is certainly correct, but when you come to calibrate your model it is likely that you will have to accept discrepancies between model and observed heads that are much greater than this. These discrepancies are attributable to the fact that head levels, as actually measured, are subject to small-scale random spatial variations superimposed on regional head variations because of the presence of aquifer spatial inhomogeneity. The actual aquifer recharge, transmissivity etc. distribution is far more complex than our model has the power to replicate, and probably far more complex than we have the ability, or inclination, to measure. Our model seeks to reproduce the first-order or major determinators of groundwater flow as they are expressed in the definition of constant parameter value sub-areas within the model. Second-order earth physical property variations which are superimposed on these major earth property subdivisions are not modelled; rather we are content to acknowledge their existence by noting that every head measurement is subject to both a deterministic effect (which we attempt to predict using the model), and a random effect (attributable to the fact that the model is a simplification of reality). of course if it becomes apparent, through running MODINV with a given aquifer physical property zonation scheme, that the random head variations are excessively large, we may be inclined to add additional sub-areas to our model, thus needing to estimate a greater number of parameter values. Or we may adjust sub-area boundaries. But we acknowledge that we will never remove these random head variations entirely, because we will never have a perfect fit between model and reality.

Because it is thus a stochastic variable, a complete representation of each head measurement must include both the measured value itself, and a quantitative description of the probability function from which this measurement was taken. This description is simplified if we assume a normal probability distribution for the heads. In this case, our vector of observed head values, $h_m$, can be considered as a collection of sample values of random variables whose mean is estimated at each sample point and observation time as the best-fit-model head at that point and time. The weight matrix is proportional to the inverse of the measured heads covariance matrix, ie.

$$W = \sigma_o^2 V_h^{-1} \qquad ...(23)$$

where $V_h$ is the measured head covariance matrix and $\sigma_o^2$ is a proportionality constant, referred to as the reference variance. When head measurements are made in the field the latter's value is unknown. However it is determined as part of the least squares estimation process; see below.

Because W is a diagonal matrix, so too is $V_h$. This means that we assume that the head measurements are uncorrelated both in space and time, ie. head measurement uncertainties at any one bore at any one time have nothing to do with those at another bore at another time. This assumption is not as obviously true as it first sounds, given the origin of measured head random variations as discussed above. If an unaccounted-for transmissivity inhomogeneity, for example, places a lower limit on the squared model minus measured head sum, then its effects may be apparent on any bores that are in, or close to, the heterogeneity; head "errors" are thus correlated for all bores affected by the some inhomogeneity. Also, if we are carrying out a parameter estimation procedure using head measurements taken at a number of times, then it is likely that heads at a particular bore will be over- or under-estimated by the model not in a completely random fashion, but for a number of sampling times in a row (see Carrerra and Neuman, 1986a). However the MODINV algorithm does not try to incorporate such spatial or temporal measured head correlation into the inversion process, both for reasons of simplicity and because the degree of spatial and temporal correlation is difficult to estimate and depends on the final model, which it is the estimation algorithm's purpose to determine. The weight matrix, then, being proportional to the inverse of the measured head covariance matrix for a set of uncorrelated heads, is a diagonal matrix. A diagonal element is large if the uncertainty level pertaining to the corresponding head measurement is considered to be small and vice versa. If, for an evenly distributed set of measurement bores, you assign smaller weights to head measurements in certain parts of your model, you are asking MODINV to give greater importance to the matching of model and observed heads over other parts of the model, presumably because the heads in the area with low weights are subject to greater uncertainty, probably because of greater aquifer heterogeneity there.

Any diagonal term of a covariance matrix expresses the variance of the pertinent parameter value; the variance is the square of the standard deviation. By allocating relative weights to your measured heads, you are, in reality, allocating relative variances and hence relative standard deviations. The absolute variances for these heads depends on how good a fit you end up achieving between model and measured heads. On the assumption that your model (including its constant-parameter sub-areas as defined by you) is correct (an assumption which you should always treat with suspicion), a good overall fit between modelled and measured heads indicates that the head measurement standard derivations must be small. It can be shown (eg. Mikhail (1976, p288) that an unbiased estimate for $_o^2$ is given by

$$\sigma_o^2 = [(\mathbf{h} - \mathbf{h}_m)^t \, \mathbf{W}(\mathbf{h} - \mathbf{h}_m)] \, / \mathbf{r} = \phi \, / \, \mathbf{r} \qquad \text{...(24)}$$

where **h** is the vector of optimized model heads, $h_m$ is the vector of measured heads and r is the redundancy. The latter is defined as the number of observations minus the number of parameter values for which estimates are required, ie.

$$\mathbf{r} = \mathbf{M} - \mathbf{N} \qquad \text{...(25)}$$

with M and N defined earlier in Section 3.2.1.

Once parameter values have been optimized, given your set of parameter zonation sub-area boundaries, $\sigma_o^2$ can be calculated from (24); you can then calculate the standard deviation of individual head measurements by multiplying the inverse of each head measurement weight by the newly-determined reference variance (equation 23) and taking the square root. If you arrive at a figure that you consider too high, you can change the model by, for example, maintaining the same number of parameters and shifting 9 parameter zonation boundaries, or by adding some extra parameter constant-value zones (so that "random" variations responsible for the unsatisfactorily high head measurement variances now become incorporated into the model). However you should beware of trying to use a model with too many parameters as computing times for MODINV rise linearly with the number of parameter values for which an estimate is required. Also, the more parameter values there are, the more likely are some combinations of values to be highly correlated. This means that you may not end up with a model that is any better (in terms of its ability to predict water levels over the model area) than one parameterized with fewer variables because this high degree of parameter value correlation will be reflected in high parameter value variances (see next section). Also, convergence problems and numerical instability may raise their ugly heads. Carrerra and Neuman (1986c) provide a good discussion on complexity in aquifer parameterization, to which you are referred for more details; in general, simpler is better.

In the MODINV algorithm, the reference variance is calculated after each parameter upgrade. In PREINV you are asked to provide a reference variance which, if achieved, will cause optimization to be terminated. If you indicate, using this reference variance, an overall measured minus model head discrepancy that you can tolerate, further optimization can be forestalled once (and if) this tolerable discrepancy has been achieved.

### 4.2.5 The Parameter Value Covariance Matrix

It can be shown that the parameter value covariance matrix can be estimated by

$$V_p = \sigma_o^2 (J^t W J)^{-1} \qquad ...(26)$$

The diagonal elements of this matrix are the variances (id the squared standard deviations) of the individual parameter values while the off-diagonals elements are the covariances between respective parameter value pairs; these latter are indicative of how highly correlated two different parameter values are. It is important to note that the derivation of (26) relies on two assumptions, neither of which are strictly correct in the groundwater modelling context.

The first assumption is that heads and parameter values are normally distributed. While this assumption may be more closely adhered to if parameters are transformed, it will never be completely correct. As with measured heads, the concept of parameter values as random variables relates to the effects of aquifer inhomogeneities superimposed on the simplifications inherent in the process of model construction.

The second assumption is that model head and parameter value variations are linearly related in the manner described by equation (8). As previously discussed, this linear relationship is only approximately correct, with the approximation worsening for larger parameter and head variations about specific head and parameter values, the latter being related to each other through the nonlinear relationship of equation (4). Hence if the covariance matrix indicates that a parameter standard deviation is large, which is the same as saying that the parameter value could vary widely and still be used in the calibrated model, then the exact value of that standard deviation, as provided by the covariance matrix, cannot be correct because such a wide parameter variation will put it outside the range of the linearity assumption.

Nevertheless, the parameter covariance matrix is one of the most useful pieces of information to come out of the inversion process. Its principle role is that of an indicator of how well your borehole head measurements are able to define aquifer properties (including recharge or EVT if you are estimating either). For while your model heads may be well matched to the measured heads, (the reference variance may be satisfactory), some parameter value standard deviations may still be large. This indicates that, as mentioned above, these parameter values can be made to vary by large amounts with little effect on the model heads at the boresites. If this applies to a single parameter value, it will have a high variance and will be uncorrelated with other parameter values. If, however, two or more parameter values can be simultaneously varied in a certain relationship to each other while causing minimal change to the model heads at the observation bores over time, then these parameter values will each have a high standard deviation and the covariance between pairs of such parameters, as indicated by the pertinent off-diagonal elements of the covariance matrix, will also be large. This indicates high parameter value correlation or, to put it another way, a high degree of stochastic dependence between the pertinent parameter values. If you were to run MODINV again while holding one (or more) of a set of highly- correlated parameter values fixed, the standard deviations of the other members of the set may then be small because the definition of the model now includes the first member(s) of the set as fixed. As the first parameter now has no opportunity to vary in harmony with the others, for minimal resultant head variation at the observation boresites, the standard deviations of the others cannot be as large.

Thus the parameter covariance matrix tells you something about your model that the goodness of fit between model and observed heads cannot tell you. For example, if the density of observation bores is low or zero over a certain part of the aquifer, parameter values estimated in that area may not be well defined, and this will be indicated in the covariance matrix. While your model may appear to be well calibrated because the model replicates observed heads at the existing observation bores with a good degree of accuracy, its capacity to predict water levels over other parts of the aquifer may be highly suspect if the calculation of these latter heads relies on parameter values which are, locally, ill defined. By varying highly correlated parameters in directions defined by the parameter covariance matrix eigen vectors, you can test the effect of simultaneous parameter variation on modelled heads at both observation bores and elsewhere. If the head change at the observation bores is small, but is great at other places that may be of interest to you, then you may not have enough information to parameterize your aquifer if one of the model's tasks is to predict water levels in this other area with any accuracy.

Correlation between pairs of parameters can be displayed as a correlation coefficient matrix. If $_{ij}$ is an element of the parameter value covariance matrix, then the corresponding element of the correlation coefficient matrix is given by

$$\rho_{ij} = \sigma_{ij} / [\sigma_i^2 \sigma_j^2]^{1/2} \quad ...(27)$$

where $\sigma_i^2$ and $\sigma_j^2$ are the ith and jth diagonal elements of the covariance matrix; obviously the correlation coefficient matrix has diagonal elements of unity. The correlation between different pairs of parameter values is then readily apparent from the pertinent off-diagonal elements, a high degree of correlation between parameter value pairs being indicated by a correlation coefficient close to 1 or -1 (a correlation coefficient cannot be higher than 1 or less than -1).

Another method of displaying the wealth of information that is available in the parameter value covariance matrix, is to display its eigenvalues and eigenvectors. The latter define the directions of the axes of the parameter confidence "ellipse" (actually, it is only an ellipse in two dimensions, ie. if only two parameter values are estimated), whereas the square roots of the eigenvalues are the magnitudes of the semiaxes of the parameter confidence ellipse. If all eigenvectors have only one component, then the axes of the confidence ellipse will lie along the parameter value axes, and parameter values are thus all uncorrelated. In the more usual case, the degree of correlation between different parameter value estimates can be obtained by examining the components of the eigenvectors. If, for example, the i'th, j'th and k'th components of a particular normalized eigenvector are much larger than the other components of that vector, and the eigenvalue corresponding to that eigenvector is larger than most of the eigenvalues corresponding to the other covariance matrix eigenvectors, then this indicates that the *linear combination* of the i'th, j'th and k'th parameter values is better determined than are the individual values; the coefficients of this linear combination correspond to the respective eigenvector elements. See Carrerra and Neuman (1986c) for a further discussion of how the covariance matrix eigenvectors and eigenvalues can be used to understand the power and limitations that your measurement set possesses in parameterizing your model.

### 4.2.6 Final Points

As described above, MODINV employs the Gauss-Marquardt method to minimize the sum of the weighted squared head differences between measured and model heads. This "objective function" is the same function that is minimized in the Maximum Likelihood method of parameter estimation. In fact, in the present case, the only difference between the two methods is in the estimated value of the reference variance; in the Maximum Likelihood method, the denominator in equation (24) is M, the number of observations, rather than M-N, the redundancy. When M is much higher than N the two estimates are close; however $\sigma_o^2$ as provided by (24) has the advantage that it is an unbiased estimate.

Some worrying questions are (i) whether $\phi$, as defined by (6) has a single minimum, and (ii) if so, whether MODINV will always find it. Unfortunately there is no single answer to both these questions that applies to all modelling situations and all parameter types and combinations of parameter types for which estimated values are being sought. Experience in using MODINV has demonstrated that $\phi$ can converge to a local minimum in some cases, for which parameter values are far from optimum. In other cases it will not converge at all. However both of these phenomena are more likely to occur when many parameter values of different types are being simultaneously estimated; in such cases you can often dramatically improve MODINV's performance simply by holding a few key parameter values fixed, or by using fewer parameter values in a less complex areal distribution. Failure to converge to a global minimum is often a signal that parameter value correlations are high and that you are consequently asking too much of your data in trying to resolve individual values. Hence, not only will a simpler model improve MODINV's performance, but it may be a more realistic representation of the true information content of your measurements. As such, predictions made with the calibrated model will tend to be "conservative" in that the possibility of predicting spurious local head variations, resulting from the presence of local, poorly defined parameter values, will be reduced.

### 4.3 MODINV Processing Steps

Fig.4.1 shows a simplified flow chart of the MODINV algorithm; see Table 4.2 for a list of symbols used in Fig.4.1. You can tell what part of its algorithm MODINV is executing at any time by inspecting its continually-updated run record which is written to file MODINV.PRN. If you are running MODINV from the terminal and have requested a screen display of computation progress, then additions to file MODINV.PRN are also sent to the screen, allowing you to monitor the progress of the optimization process. If you are running MODINV as a batch job, screen display is not available. However some systems will allow you to read MODINV.PRN, even though it is concurrently held open by MODINV; other systems allow you to read a batch job log file (while the batch job is executing) containing information that would have been sent to the screen if the job were run from the terminal. In either case, periodic inspection of the MODINV output allows you to monitor MODINV run progress.

**Table 4.2 : Symbols Used in Fig.4.1**

| Symbols | Description |
|---|---|
| $\phi$ | Objective function |
| $\nabla\phi$ | Gradient of the objective function |
| J | Jacobian matrix |
| N | Normal matrix |
| I | Identity matrix |
| $\Sigma$ | Covariance matrix |
| $\lambda$ | Marquardt lambda |
| $p_i$ | Parameter value estimates for I'th .iteration |
| $\Delta p_i$ | Parameter optimization direction vector for I'th iteration |
| $\beta$ | Fraction of $\Delta p$ by which to obtain $p_{i+1}$ $p_i$ |
| $\gamma$ | Derivative vector of model heads w.r.t. $\beta$ |
| m | Number of parameter values requiring optimization |
| i | Optimization iteration number |

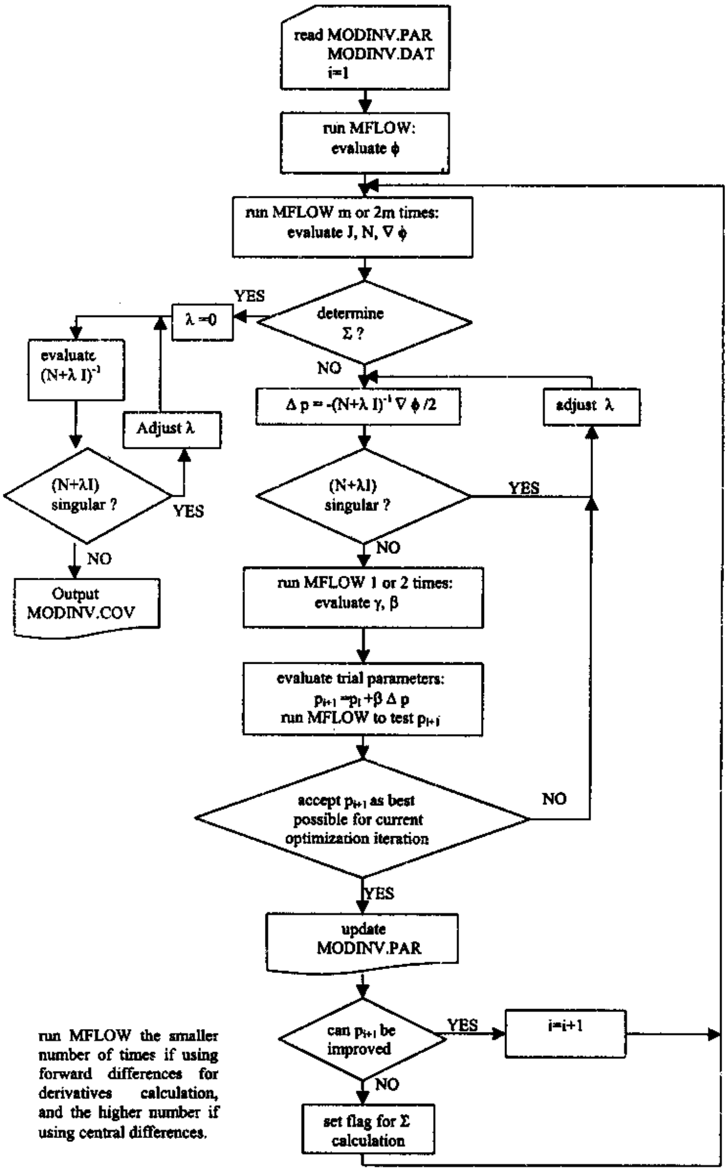


**FIG. 4.1 SIMPLIFLIED FLOW CHART OF MODINV ALGORITHM**

## 5.0 MODEL FORMULATION

### 5.1 Site Conceptualisation

Based upon the available information, a conceptual model is postulated to provide a framework that describes flow system geometry and the physical processes to be simulated by the numerical model. The study area in Central Godavari Delta measures to about 825 sq.km bounded by rivers Gowthami Godavari in the east, Vasistha Godavari and its branch Vainataya in the west, and Bay of Bengal in the south. The geology of the area is interpreted from the exploration borehole information at Mandapeta. The study area and its environs is underlaid by coastal alluvium. The alluvial deposits of the area are essentially contributed by Godavari river. The subsurface geology existing at Mandapeta and approximate depth of units are given in Table 5.1.

**Table 5.1 : Lithology at Mandapeta**

| Depth range (m) | | Description |
|---|---|---|
| From | To | |
| 0.0 | 1.5 | Top Soil |
| 1.5 | 18.0 | Sand, fine to medium |
| 18.0 | 19.5 | Sand, Coarse to very coarse |
| 19.5 | 31.0 | Clay |
| 31.0 | 39.0 | Sand, medium to very coarse |
| 39.0 | 49.5 | Sand, medium to coarse |
| 49.5 | 55.5 | Sand, fine to medium |
| 55.5 | 61.5 | Sand, fine |
| 61.5 | 67.5 | Sand, fine to medium |
| 67.5 | 73.5 | Sand, medium |
| Below | 73.5 | Sand, coarse to very coarse |

The thickness and lateral continuity of individual layer as given in Table 5.1 does not vary much throughout the flow domain in the study area. The study area consists of mainly two aquifers for groundwater development – the unconfined or phreatic aquifer and the confined aquifer. Hydrogeological investigations carried out in the study area reveal that a large number of shallow tubewells and filter points, mainly used for irrigation purpose, have been sunk into the phreatic aquifer. This aquifer is recharged mainly through the direct infiltration of rain water, besides some recharge taking place due to irrigation return flow. Therefore instead of incorporating two aquifers, only one aquifer system of unconfined aquifer is conceptualised in the present model. The clay bed below this aquifer would act as the bottom boundary for the conceptual model.

The study area has a gentle land slope from the upper reaches towards the sea and so follows the ground water flow direction. The average depth of water table below ground level during pre-monsoon period varies from about 7 m in the upper reach to about 1.75 m near the coast. The average seasonal water table fluctuation (i.e. pre to post-monsoon) in these reaches is observed to vary from about 4 m to about 1 m respectively.

## 5.2 Spatial Domain

The spatial domain is descretized into 75x85 grids, each grid having a dimension of 1 km x 1 km. The aquifer is represented by a single model layer having a uniform thickness of 18 m. Spatial extent of rivers, canals and coastal line would be accomodated through river package and Head boundary package as available in MODFLOW/MODINV. The active cells over the spatial domain are shown in Fig.5.1.

## 5.3 Boundary Conditions

As stated earlier, the study delta is hydrologically bounded by rivers on two sides and Bay of Bengal on the third side. The rivers which are the distributaries of river Godavari are very deep to act as the hydrological barriers. So, in the conceptual model these two rivers are bounded by inactive cells to indicate the basin boundaries showing no sub-surface inflow or outflow from this basin to other adjacent basins or sub-basins. The Bay of Bengal is considered by the constant head boundary with the constant head taken as 0 m.

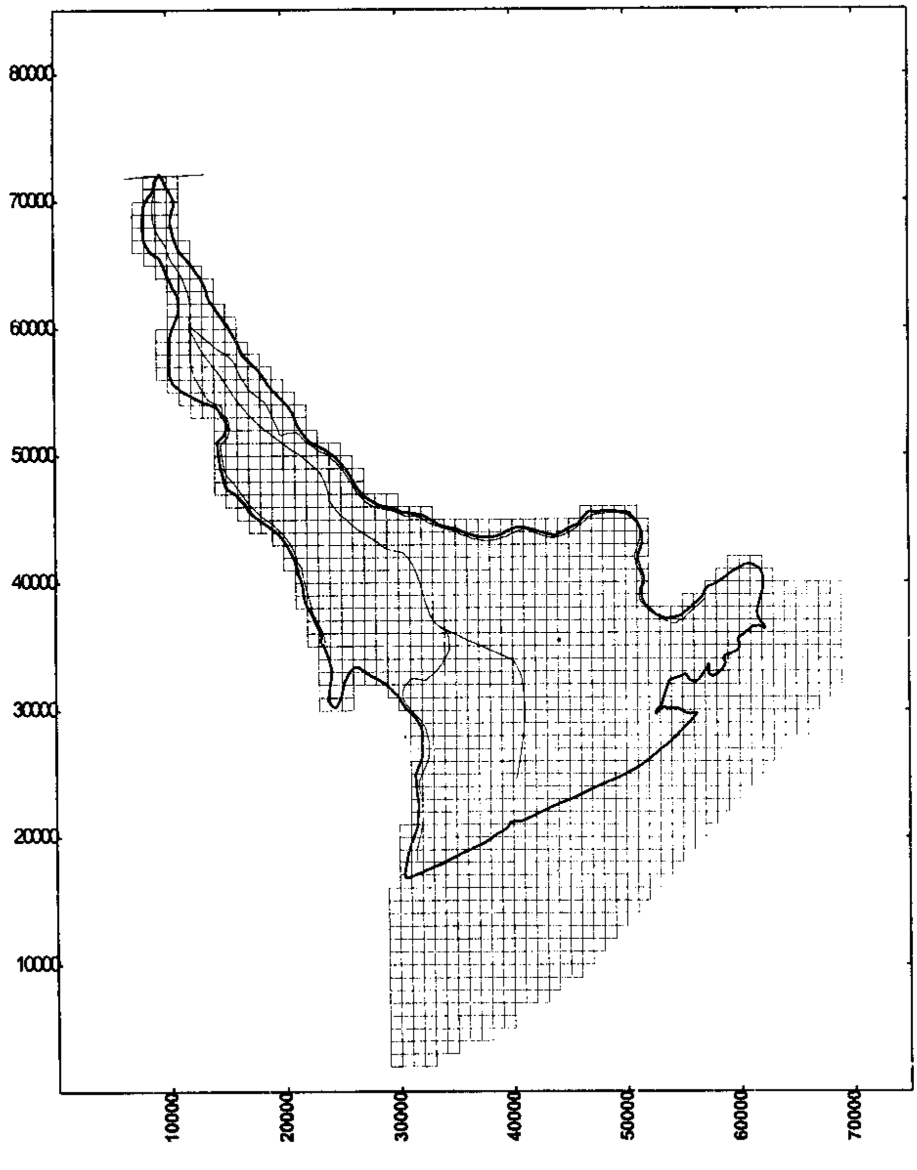


FIG. 5.1 ACTIVE CELLS IN MODEL

## 5.4 Temporal Consideration

The model is proposed to be calibrated for transient conditions for the monthly water table levels of non-monsoon period of 1985. The calibrated model would be used to provide the estimates of monthly recharge values corresponding to the monthly water table levels during monsoon period of 1985. The number of stress periods for the above cases would be taken as 4 and 5 (5 being the maximum limit of MODINV) respectively, each representing 30 days.

## 5.5 River and Canal Network

The two natural rivers namely, Gowthami and Vasistha are running on either side of the study delta. Besides, the followings are also present in the study area.
(1) Gannavaram canal, (2) Amlapuram canal, (3) Benda canal, (4) Bank canal

These rivers and canals are expected to interact with the aquifer system depending upon their stages and the water table levels in the aquifer. Therefore, these natural rivers and canals would be represented in the model through the river package, as provided in the numerical model. The rivers and canals would be divided into a number of reaches depending upon the variation in their geometry. Physical and hydrologic characteristics of these reaches would be transferred to the model by three parameters i.e., river bottom elevation, conductance and the stage. The conductance values for each reach is estimated by the actual stream width, length of cell, hydraulic conductivity of the bed material and thickness of the transmitting layer.

## 5.6 Discharge Wells

As per the information collected from the Chief Planning Office, E.G. Dist., nearly 3950 shallow tubewells and filter points existed in the study area. These structures are well spread over the basin and their yield varies between 8500 to 1200 gph. These discharge wells are incorporated in the model through well package. Depending upon the actual area irrigated by these wells in *kharif* and *rabi* seasons, the monthly draft through these wells is worked out. This draft is uniformly distributed over the study basin through 396 numbers of pumping wells in the model (the limit on maximum no. of wells in MODINV is 400).

## 5.7 Evapotranspiration

The EVT package in the model accomodates the evapotranspiration process. The EVT surface elevation, maximum EVT rate and EVT extinction depth would be defined for each stress period of simulation. The EVT surface elevation is taken as the elevation of the land surface. Since the study area has a large number of coconut trees which are expected to draw water directly from the ground water reservoir, the EVT extinction depth is taken as 3 m. The maximum EVT rates for different stress periods as applicable to the area would be provided to the model.

## 5.8 Recharge Rate

Recharge refers to the infiltrated water that crosses the water table and becomes part of the ground water flow system. The rainfall and the irrigation return flow are the two major sources of recharge in the study area and are considered in the conceptual model. As the recharge rates generally vary in space, recent modelling studies incorporate spatial variation in recharge by defining recharge zones. Typically, there is little hydrogeologic information to use in defining recharge zones and in assigning recharge rates to each zone. Instead, recharge zonation is usually justified on the basis of a successful calibration. In the present case too, the spatial variability in recharge rates would be put into the conceptual model through recharge zones considering the factors such as rainfall, unsaturated thickness and location of rivers and canals. The further discussion on estimation of recharge rates would be presented in the forthcoming chapters.

## 5.9 Hydraulic Conductivity and Storage Property

In the study area, the soils in the zone of water table fluctuation are assumed to be homogeneous and isotropic. As seen from the well log data, mainly fine to medium sand is encountered in this zone. The hydraulic conductivity and specific yield for this soil medium generally vary from 6 to 15 m/day, and 9 to 18% respectively. The representative value of these parameters would be decided during calibration process.

# 6.0 MODEL CALIBRATION AND SENSITIVITY ANALYSIS

## 6.1 Model Calibration

Parameter estimation of a groundwater model is essentially synonymous with model calibration, which is synonymous with solving the inverse problem. Calibration of a flow model refers to a demonstration that the model is capable of producing field measured heads and flows which are the calibration targets (or the calibraticn values). This is accomplished by finding a set of parameters, boundary conditions, and stresses that produce simulated heads and fluxes that match field measured values within a pre-established range of error. The calibration process can be performed either by trial and error method or by automated technique. The MODINV as used in the present study employs the automated calibration technique to optimize the parameter values by comparing the simulated heads with the observed ones for each iteration.

After formulation of the conceptual model, the input data and the initial estimates of parameter values are transferred to the model through PREMOD and PREINV, which are the preprocessors for MODFLOW and MODINV respectively. To input the initial head conditions, the heads at the nodes of each finite difference grid are interpolated from the set of randomly observed water table levels using Kriging technique. The node values are then transferred to the cell centres through a programme specially written for this purpose. A total of 8 recharge zones are defined in the model to take care of the spatial variation in the recharge rates. The model is calibrated for transient conditions of the monthly water levels of non-monsoon period of 1985 by taking 4 stress periods (Feb. to May 1985), each representing 30 days, with initial conditions taken as of January. The reason behind the selection of non-monsoon period for calibration purpose is that the recharge during this period takes place mainly from irrigation return flow (which can be estimated from irrigation water quantities), and little or no recharge from rainfall.

The mean areal rainfall over the study area from Feb. to May 85 is computed as little as 15 mm and is therefore neglected for recharge purpose. Based on the G.W.Estimation Committee norms, the recharge volume due to return flow, derived both from canal and well irrigation, over the calibration period of Feb. to May 85 is estimated as 107 MCM (NIH Report No.CS-117). Though, it is a lumped estimate of recharge over the study area it would set the target for recharge stress over the calibration period.

A uniform rate of recharge which is computed from the lumped recharge estimate is assigned to each of the recharge zones as an initial estimate. These recharge rates, however, have been defined in the model as parameters for estimation. This would allow the recharge rates to be optimized for each stress period and for each zone depending upon the flow conditions in the respective zones. The model is initially calibrated for the steady state conditions taken as the average of 10 years January water levels. This is followed by the second calibration for the transient conditions by observing the following steps :

1. Run MODINV with steady state calibrated values of the physical flow system parameters and optimize the recharge rates. MODINV will perform optimization of the recharge rates to have a best possible match between the simulated and observed heads under the given conditions.

2. Run MODFLOW with the optimized recharge rates and the corresponding parameter set. Output the flow budget.

3. Compare the sum total of recharge quantities of all the stress periods (as obtained from the flow budget) with the lumped estimate of the return flow recharge (as computed earlier).

4. If the quantities in the step 3 are not comparable, repeat the steps 1 to 3 by systematically varying, in successive runs, the hydraulic conductivity, specific yield, recharge zone boundaries and the boundaries conditions.

5. The calibration is treated as complete when a parameter set and the physical flow system boundaries are achieved which are capable of reproducing the target values of the recharge stress and the heads.

Numerous model runs were carried out using the above procedure and the calibration was achieved. The objective function, which is the sum of the weighted squared head difference between observed and model heads, was obtained as low as 0.233. The calibrated values of hydraulic conductivity and specific yield are obtained as 10.3

m/day and 10% respectively. The observed and simulated heads are given in Table 6.1. While the final recharge zones are shown in Fig. 6.1, the optimized rates of recharge for each of these zones and the total simulated recharge volumes during each stress period are presented in Table 6.2.

**Table 6.1 : Observed and Simulated Heads (m) for Calibration Period**

| Stress Period (month) | | Well Nos. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 (Feb.,85) | Obs. | 0.31 | 2.67 | 1.52 | 2.63 | 3.46 | 2.00 | 2.02 | 2.81 | 3.42 |
| | Sim. | 0.31 | 2.73 | 1.52 | 2.63 | 3.41 | 2.00 | 2.01 | 2.81 | 3.41 |
| 2 (Mar.,85) | Obs. | 0.31 | 2.33 | 1.44 | 2.58 | 3.36 | 1.67 | 1.70 | 2.58 | 3.03 |
| | Sim. | 0.31 | 2.50 | 1.44 | 2.58 | 3.20 | 1.67 | 1.70 | 2.58 | 3.03 |
| 3 (Apr.,85) | Obs. | 0.14 | 2.00 | 1.24 | 2.11 | 2.98 | 1.42 | 1.46 | 2.30 | 2.64 |
| | Sim. | 0.14 | 2.13 | 1.24 | 2.11 | 2.85 | 1.42 | 1.46 | 2.30 | 2.64 |
| 4 (May,85) | Obs. | -0.39 | 1.68 | 1.06 | 1.63 | 2.79 | 1.24 | 1.29 | 2.11 | 2.32 |
| | Sim. | -0.39 | 1.88 | 1.06 | 1.63 | 2.60 | 1.24 | 1.29 | 2.11 | 2.32 |

The observed and simulated heads as presented in Table 6.1 are found to be matching very well. The scatter plots of observed and simulated heads for all 4 stress periods are given in Fig.6.2 (a to d). The points in all the plots are observed to fall almost along a straight line. The correlation coefficients between observed and simulated heads are found as high as 0.999, 0.992, 0.994, and 0.989 for stress periods 1 to 4 respectively.

From Table 6.2, the recharge rates for stress period 4 are observed to be very low and the simulated recharge volume is also found to be 9.59 MCM. This is due to the reason that the canals were closed from 15$^{th}$ of April and whatever simulated recharge is obtained in this period might be probably due to the time lag between the infiltration and the water actually joining the water table. Also, some recharge might be caused due to well irrigation return flow. The total simulated recharge and the lumped estimates of recharge are found to be very close showing a difference of about 2.8 MCM only. In view of the large study area, the discrepancy of 2.8 MCM can be considered to be very reasonable and within acceptable limits.

With the above performance, it was felt that calibration was in an acceptable stage and the calibrated model could be used for estimation of recharge in monsoon season.

## 6.2 Sensitivity analysis

The sensitivity analysis is performed for specific yield and hydraulic conductivity by changing one parameter value at a time. The calibrated value of each parameter was systematically changed within the previously established plausible range. The resulting change in average water level from the calibrated solution with respect to specific yield and hydraulic conductivity for stress period 4 are plotted in Fig. 6.3 and 6.4 respectively

**Table 6.2 : Optimised Rates of Recharge and the Simulated Recharge volumes for Calibration Period.**

| Stress period (month) | Recharge zones | Recharge rates (m) | Total simulated. Recharge vol. in the stress period (MCM) | Remarks |
|---|---|---|---|---|
| 1 (Feb.,85) | 1 | 0.00943 | | Canals running |
| | 2 | 0.04168 | | |
| | 3 | 0.00245 | | |
| | 4 | 0.00116 | | |
| | 5 | 0.11455 | | |
| | 6 | 0.08678 | | |
| | 7 | 0.04888 | | |
| | 8 | 0.06265 | 33.77176 | |
| 2 (March, 85) | 1 | 0.01286 | | Canals running |
| | 2 | 0.01925 | | |
| | 3 | 0.00180 | | |
| | 4 | 0.00056 | | |
| | 5 | 0.08871 | | |
| | 6 | 0.09552 | | |
| | 7 | 0.07666 | | |
| | 8 | 0.08516 | 39.27406 | |
| 3 (April, 85) | 1 | 0.00130 | | Canals closed from 15th April |
| | 2 | 0.00054 | | |
| | 3 | 0.00183 | | |
| | 4 | 0.00086 | | |
| | 5 | 0.07055 | | |
| | 6 | 0.03404 | | |
| | 7 | 0.06430 | | |
| | 8 | 0.06927 | 27.16137 | |
| 4 (May, 85) | 1 | 0.00025 | | Canals closed |
| | 2 | 0.00087 | | |
| | 3 | 0.00048 | | |
| | 4 | 0.00044 | | |
| | 5 | 0.06134 | | |
| | 6 | 0.00279 | | |
| | 7 | 0.05209 | | |
| | 8 | 0.00522 | 9.59281 | |

(1) Total simulated recharge volume in all the stress periods = 109.80 MCM

(2) Lumped estimate of recharge from return flow. (canals + wells) = 107.00 MCM

(3) Difference = 2.80 MCM

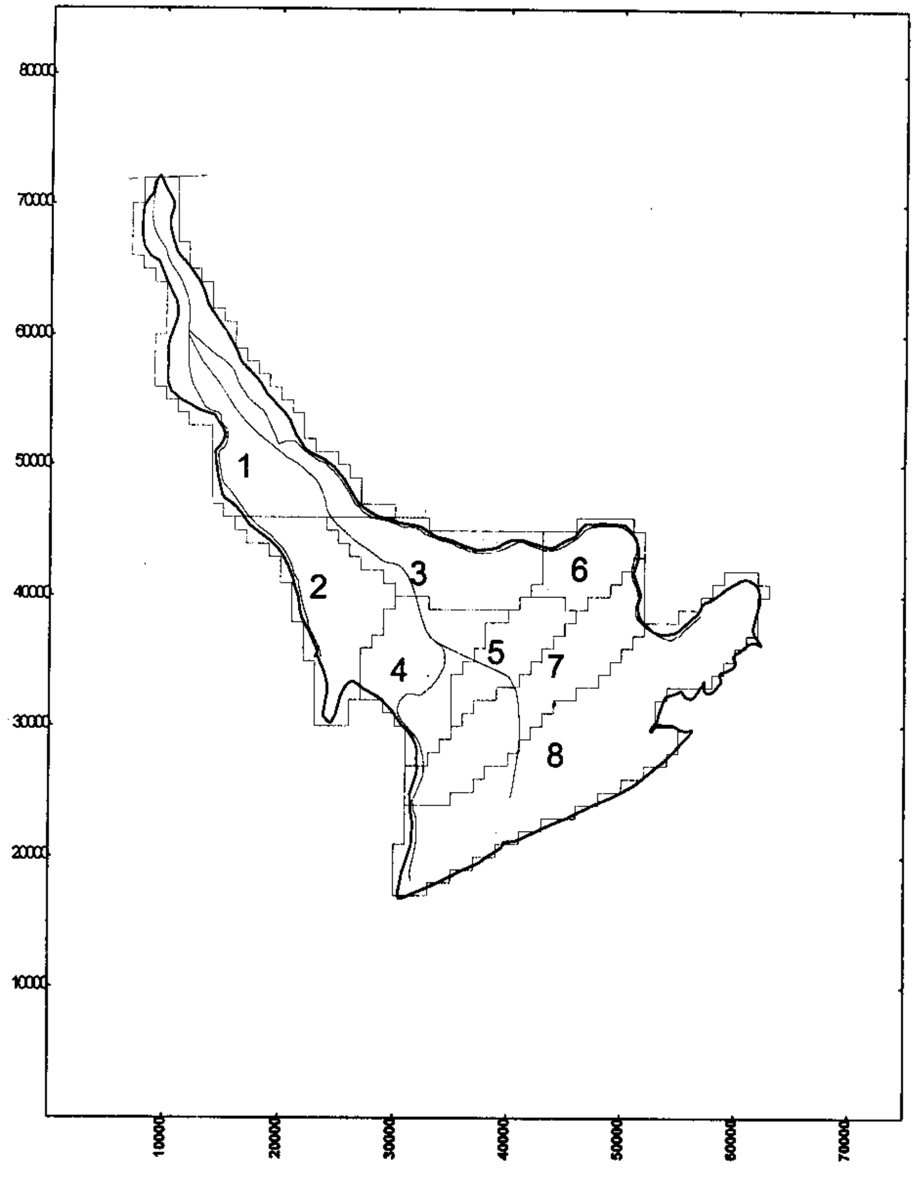


FIG. 6.1 :.RECHARGE ZONES IN MODEL

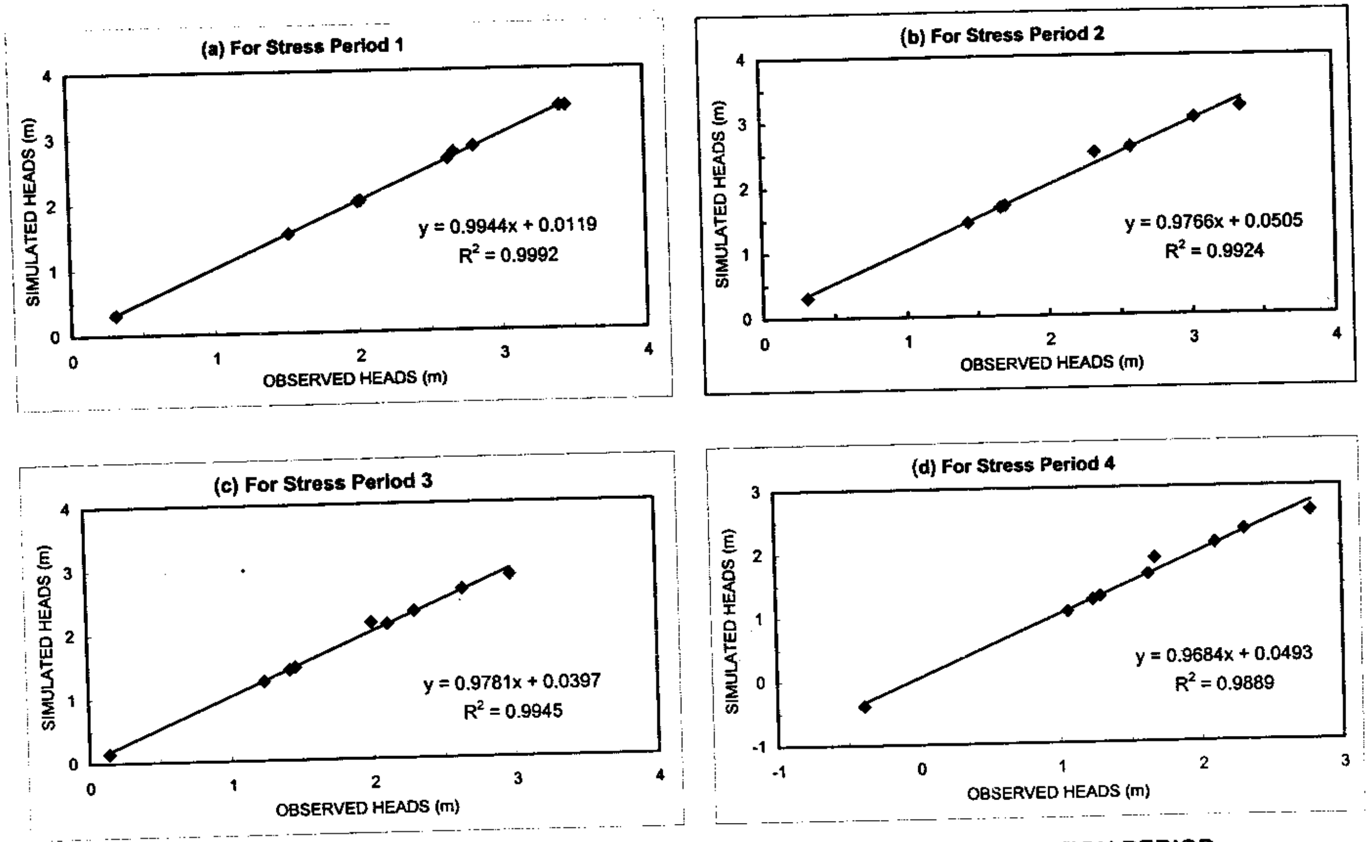


**FIG. 6.2 : PLOT BETWEEN OBSERVED AND SIMULATED HEADS FOR CALIBRATION PERIOD**

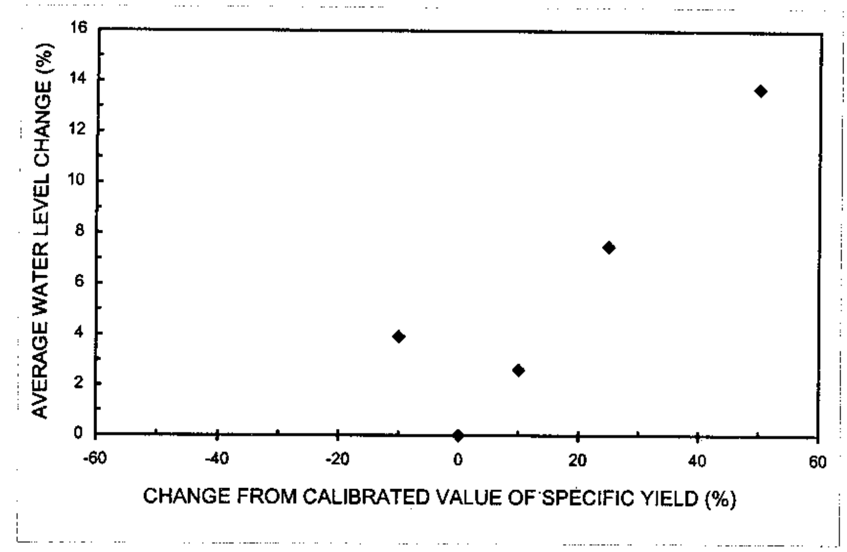


**FIG. 6.3 : CHANGE IN WATER LEVEL W.R.T. SPECIFIC YIELD**

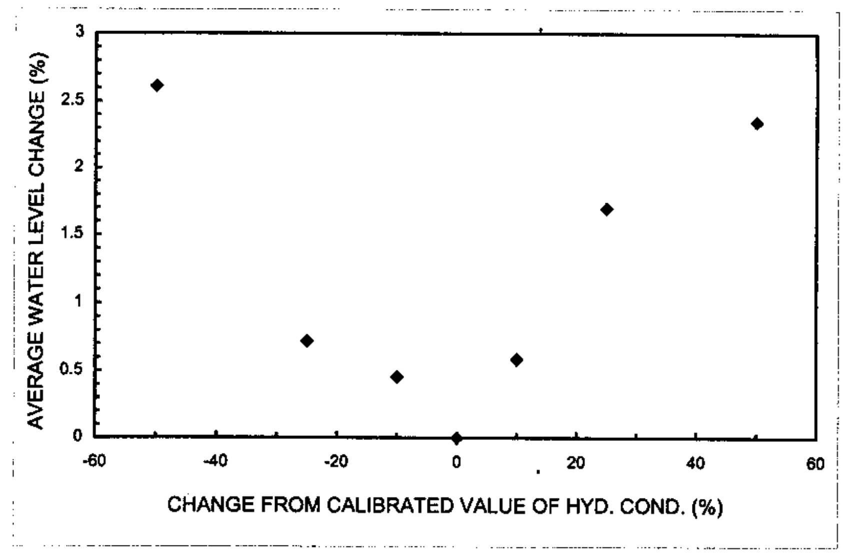


**FIG. 6.4 : CHANGE IN WATER LEVEL W.R.T. HYDRAULIC CONDUCTIVITY**

## 7.0 RESULTS AND DISCUSSIONS

### 7.1 Estimation of Recharge

Recharge to the aquifer is estimated during monsoon season since the major portion of annual recharge takes place during this period. The calibrated MODINV was run under transient conditions with 5 stress periods i.e. July to Nov.85, to optimize the recharge rates for the corresponding observed heads. Only river stages and evapotranspiration values were modified in the input data and all other parameters were kept as per the calibrated model. The objective function for optimization process during this period is obtained as 0.1715. The observed heads and the corresponding simulated heads are presented in Table 7.1. The contour maps of observed and simulated heads for all 5 stress periods are given in Figs. 7.1 to 7.5. It can be observed that the simulated and observed heads are very much comparable and hence acceptable. The optimized recharge rates for each of the recharge zones and the total simulated recharge volume during each stress period are given in Table 7.2.

**Table 7.1 : Observed and Simulated Heads (m) for Monsoon Period**

| Stress Period (month) | | Well Nos. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | Obs. | -0.40 | 2.30 | 0.90 | 2.78 | 3.11 | 1.47 | 1.89 | 2.06 | 2.09 |
| (July,85) | Sim. | -0.40 | 2.32 | 0.89 | 2.76 | 3.15 | 1.51 | 1.90 | 2.07 | 2.07 |
| 2 | Obs. | -0.07 | 2.69 | 1.36 | 3.48 | 3.72 | 2.33 | 2.54 | 3.16 | 6.09 |
| (Aug.,85) | Sim. | -0.06 | 2.80 | 1.35 | 3.51 | 3.61 | 2.33 | 2.55 | 3.15 | 6.13 |
| 3 | Obs. | 0.46 | 2.64 | 1.52 | 2.88 | 3.56 | 2.53 | 2.50 | 3.11 | 5.53 |
| (Sep.,85) | Sim. | 0.48 | 2.74 | 1.52 | 2.96 | 3.53 | 2.55 | 2.52 | 3.13 | 5.57 |
| 4 | Obs. | 0.62 | 3.03 | 2.51 | 3.08 | 4.09 | 2.74 | 3.06 | 3.73 | 6.49 |
| (Oct.,85) | Sim. | 0.64 | 3.21 | 2.52 | 3.14 | 3.98 | 2.76 | 3.07 | 3.76 | 6.54 |
| 5 (Nov.,85) | Obs. | 0.46 | 2.43 | 2.1 | 2.68 | 3.54 | 3.11 | 2.78 | 3.41 | 6.44 |
| | Sim. | 0.45 | 2.58 | 2.05 | 2.69 | 3.35 | 3.12 | 2.78 | 3.40 | 6.50 |

Table 7.2 : **Optimized Rates of Recharge and the Simulated Recharge Volumes for Monsoon Season**

| Stress periods (Months) | Recharge zones | Recharge rates (m) | Total sim. Recharge volume in the stress period (MCM) | Remarks |
|---|---|---|---|---|
| 1 (July, 85) | 1 | 0.04478 | | Canals running |
| | 2 | 0.05883 | | |
| | 3 | 0.05818 | | |
| | 4 | 0.11166 | | |
| | 5 | 0.13331 | | |
| | 6 | 0.20990 | | |
| | 7 | 0.03543 | | |
| | 8 | 0.04191 | 53.71920 | |
| 2 (Aug.,85) | 1 | 0.30875 | | Canals running |
| | 2 | 0,15068 | | |
| | 3 | 0.10820 | | |
| | 4 | 0.08574 | | |
| | 5 | 0.09470 | | |
| | 6 | 0.12780 | | |
| | 7 | 0.07765 | | |
| | 8 | 0.06060 | 113.17762 | |
| 3 (Sept.,85) | 1 | 0.00382 | | Canals running |
| | 2 | 0.03983 | | |
| | 3 | 0.07007 | | |
| | 4 | 0.04043 | | |
| | 5 | 0.07777 | | |
| | 6 | 0.01013 | | |
| | 7 | 0.07689 | | |
| | 8 | 0.11629 | 49.49640 | |
| 4 (Oct.,85) | 1 | 0.08514 | | Canals running |
| | 2 | 0.11131 | | |
| | 3 | 0.06183 | | |
| | 4 | 0.09543 | | |
| | 5 | 0.11133 | | |
| | 6 | 0.05783 | | |
| | 7 | 0.16255 | | |
| | 8 | 0.06477 | 74.65700 | |
| 5 (Nov.,85) | 1 | 0.02681 | | Canals running |
| | 2 | 0.00885 | | |
| | 3 | 0.09579 | | |
| | 4 | 0.01803 | | |
| | 5 | 0.01185 | | |
| | 6 | 0.01797 | | |
| | 7 | 0.02241 | | |
| | 8 | 0.03824 | 26.5601 | |
| Total | | | 317.61032 | |

(1) Total simulated recharge in all the Stress periods = 317.61032 MCM

(2) Estimated recharge due to return flow from June to Oct.85= 180.00 MCM (canals + wells)

(3) Recharge due to rainfall = 317.61032 – 180
= 137.61032 MCM

(4) Mean areal rainfall over the basin = 971 mm = 801.07 MCM

(5) Rainfall – Recharge Coefficient (Lumped basis) =137.61032/801.075
= 0.1717

## 7.2 Analysis of Rainfall Recharge

The recharge rates as obtained above (Table 7.2) are comprised of recharges occuring from two sources viz., recharge from rainfall, and recharge from return flow. Since the study aims at establishing the recharge from rainfall alone, it is accomplished by subtracting the recharge due to return flow from the total simulated recharge. The recharge due to return flow of irrigation water over 5 months of simulation period is estimated as 180 MCM (NIH Report CS -117). This lumped estimate is worked out using G.W.Estimation Committee norms for the average quantity of irrigation water delivered at the outlets.

The recharge from rainfall thus computed, which is again a lumped estimate over the study area, is obtained as 137.61032 MCM (Table 7.2). Based on the mean areal rainfall during the period, the rainfall-recharge coefficient (recharge due to rainfall/rainfall) is also calculated for the study area. The lumped recharge coefficient calculated as 0.1717 is found well within the prescribed range for the alluvial soils.

## 7.3 Spatial Variation of Rainfall Recharge

The above analysis gives a lumped estimate of rainfall-recharge coefficient over the study area. The recharge rates as observed from Table 7.2 are, however, found to vary in different zones. An attempt is, therefore, made in the present section to quantify the spatial variation in recharge coefficient across the study area.

The distribution of rainfall-recharge coefficient in different recharge zones is worked out and presented in Table 7.3. The total depth of simulated recharge in each zone is calculated by summing up the optimized rates of recharge of all the stress periods in that zone. As the entire study area is covered by a well distributed canal and distribution network system, it is assumed for the analysis purpose that the total recharge due to return flow (which is estimated on lumped basis as 180 MCM) is uniformly distributed over the study area of 825 sq. km., and thus contributes to a tune of 0.21818 m everywhere. With this assumption, the recharge due to rainfall in each zone is calculated and the rainfall-recharge coefficient established for each of the zones.

The rainfall-recharge coefficient is found to vary from 0.11 to 0.25 in different recharge zones. A study of location of these recharge zones (Fig.6.1) gives a definite trend of spatial variation of recharge coefficient across the study delta. Based upon the coefficient values, all the recharge zones are regrouped into following three reaches:

1. Upper reach    Zone 1
2. Middle reach    Zone 2,3,4,5,6 and 7
3. Lower reach    Zone 8

The lower reach comprising an area of 225 sq.km. (27%) along the coast has a lowest recharge coefficient of 0.11. The highest recharge coefficient of 0.25 is obtained in upper reach which accounts for an area of 176 sq.km (21%). While the recharge coefficient in most part of the middle reach (335 sq.km.; 41%) varies in the range of 0.145 to 0.176, an area of about 89 sq.km. (11%) in this zone is found to have a higher coefficient which ranges between 0.206 to 0.22. This higher value might be the effect of some local phenomenon which needs to be investigated in the field. One of the possible reasons for variation in recharge coefficient from 0.11 to 0.25 might be the effect of water table depth in these zones. As stated elsewhere in the report, the average depth of water table below ground level during pre-monsoon period varies from about 1.75 m in the lower reach to about 7 m in Upper reach. The average seasonal water table fluctuations (i.e pre to post monsoon) in these zones are observed to vary from about 1 m to about 4 m respectively. This indicates that the aquifer in the lower reach can get fully recharged with smaller amount of rainfall as compared to that in the upper reach. Once the water table in the lower reach rises to its highest position (i.e. close to the ground surface), the rainfall in excess of aquifer recharge capacity goes as runoff and thereby reduces the recharge-coefficient in this reach..

**Table 7.3 : Distribution of Rainfall-Recharge Coefficient in Different Recharge Zones**

| Recharge zones | Total depth of simulated recharge in the zone over five stress periods (m) | Depth of recharge due to return flow over five stress periods (m) | Depth of rainfall recharge (m) | Mean areal rainfall over the recharge zone during five stress periods(mm) | Rainfall-recharge coefficient |
|---|---|---|---|---|---|
| *(1)* | *(2)* | *(3)* | *(4)=(2)-(3)* | *(5)* | *(6)=(4)*1000/(5)* |
| 1 | 0.46930 | 0.21818 | 0.25112 | 1005.5 | 0.2497 |
| 2 | 0.36950 | 0.21818 | 0.15132 | 1042.9 | 0.1450 |
| 3 | 0.39407 | 0.21818 | 0.17589 | 999.3 | 0.1760 |
| 4 | 0.35129 | 0.21818 | 0.13311 | 916.0 | 0.1453 |
| 5 | 0.42896 | 0.21818 | 0.21078 | 958.7 | 0.2198 |
| 6 | 0.42363 | 0.21818 | 0.20545 | 998.0 | 0.2058 |
| 7 | 0.37493 | 0.21818 | 0.15675 | 947.2 | 0.1654 |
| 8 | 0.32181 | 0.21818 | 0.10363 | 942.0 | 0.1100 |

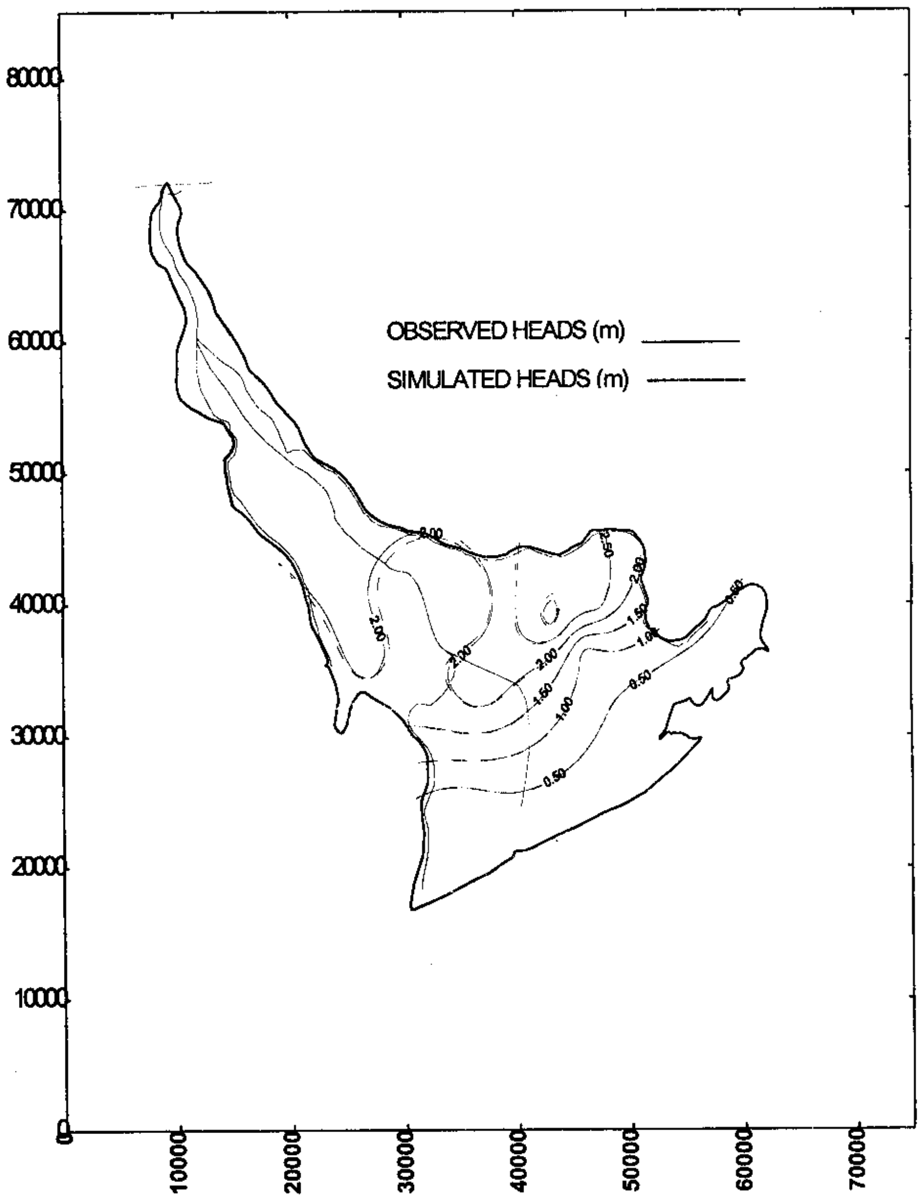


FIG. 7.1 : CONTOURS OF OBSERVED AND SIMULATED HEADS FOR STRESS PERIOD 1

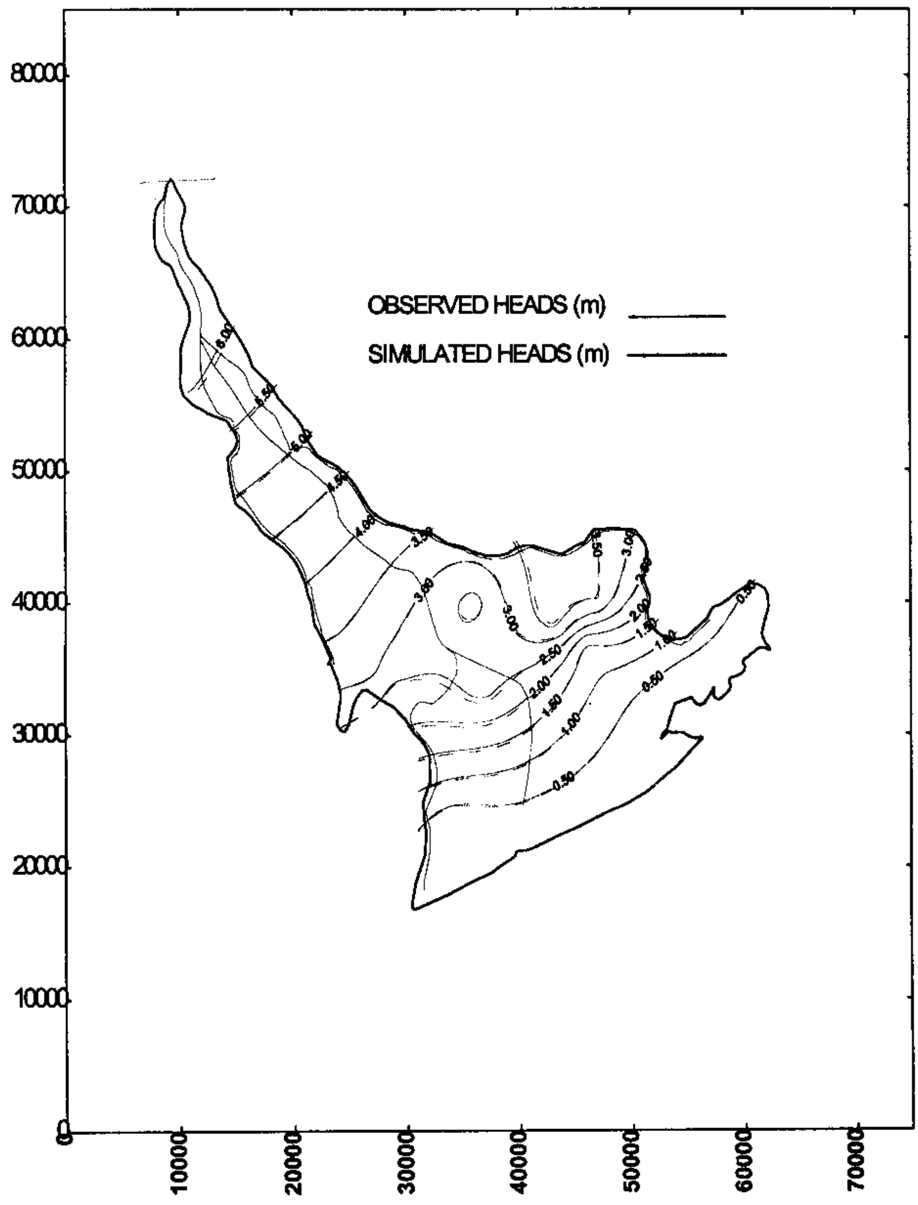


FIG. 7.2 : CONTOURS OF OBSERVED AND SIMULATED HEADS
FOR STRESS PERIOD 2

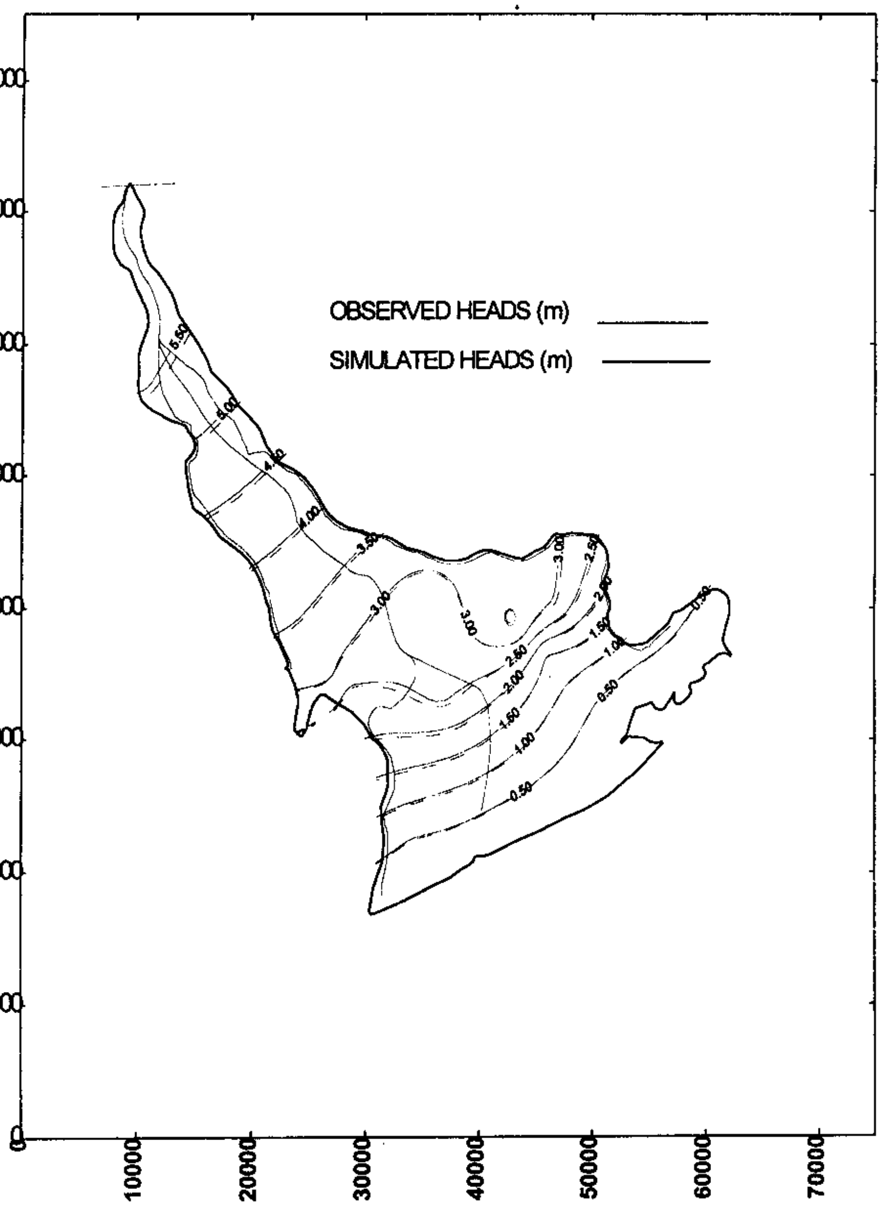


FIG. 7.3 : CONTOURS OF OBSERVED AND SIMULATED HEADS
FOR STRESS PERIOD 3

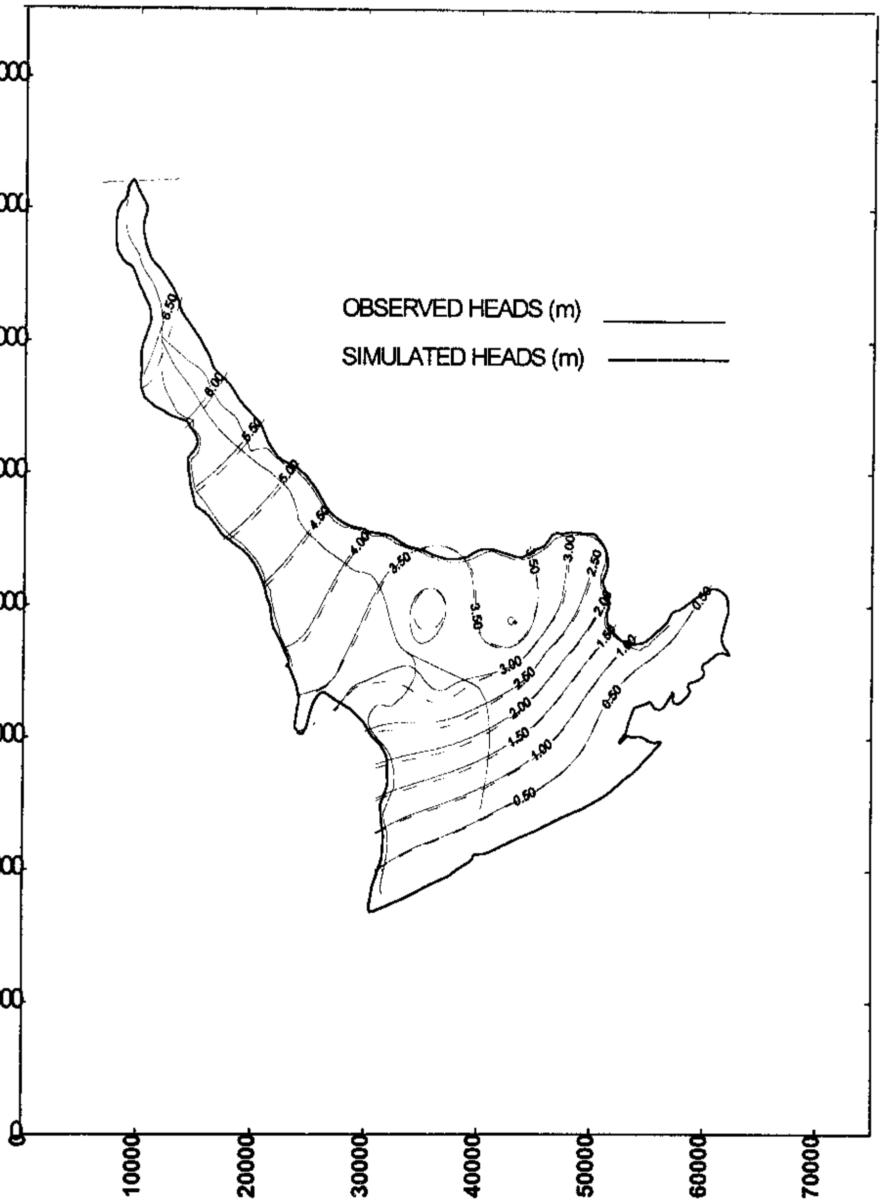


FIG. 7.4 : CONTOURS OF OBSERVED AND SIMULATED HEADS FOR STRESS PERIOD 4

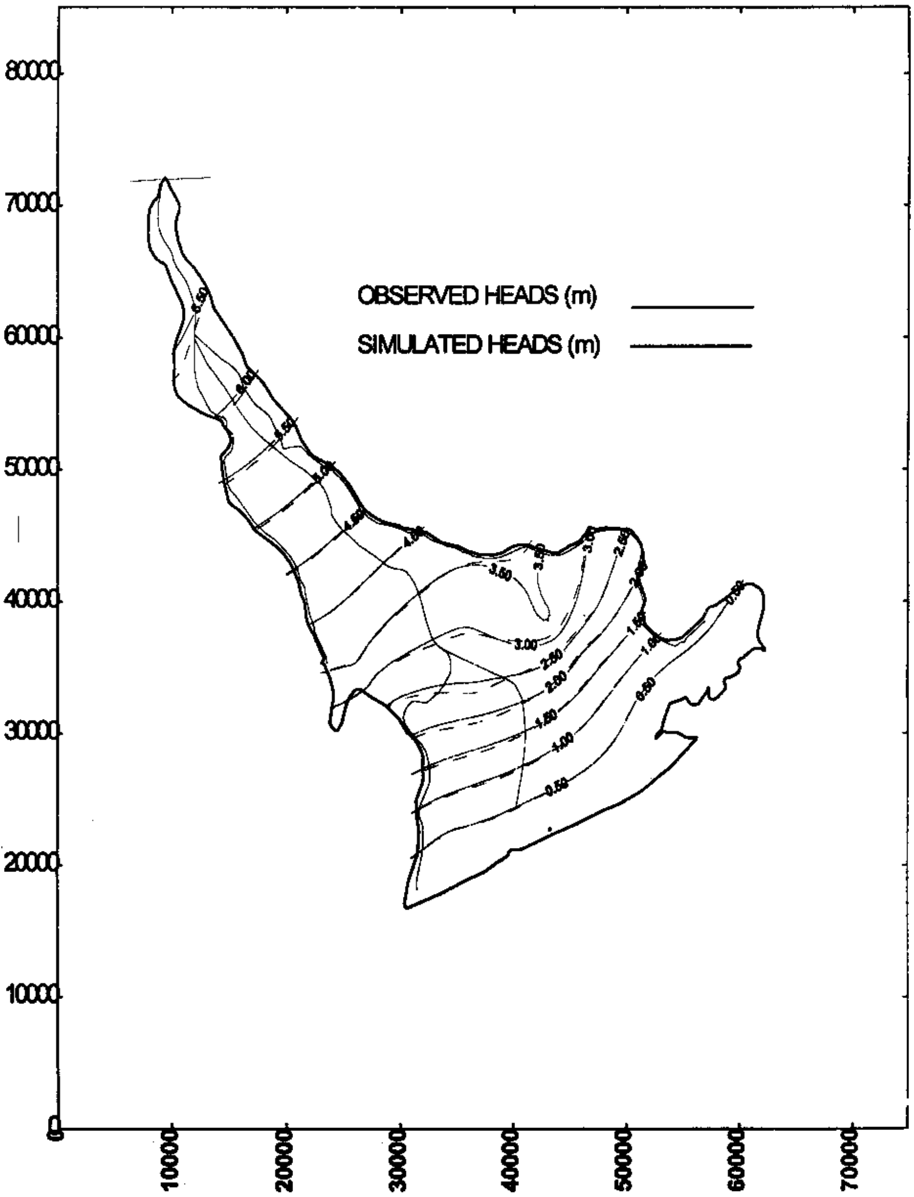


FIG. 7.5 : CONTOURS OF OBSERVED AND SIMULATED HEADS FOR STRESS PERIOD 5

## 8.0 CONCLUSIONS

The MODINV was used to estimate the rainfall recharge during monsoon season in the Central Godavari Delta of Andhra Pradesh. Based on the results of the study, the following conclusions are drawn :

(1) The inverse modelling technique is a viable distributed approach for estimating the rainfall recharge.

(2) The distributed values of rainfall-recharge coefficient in the lower, middle and upper reaches of the study delta are found to vary from 0.11 to 0.25 during the monsoon period of 1985.

(3) The results of the study were also analysed to compute the recharge coefficient on a lumped basis. The recharge coefficient for the study area as a whole is found to be 0.1717.

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## REFERENCES:


American Society of Civil Engineers, Groundwater Management, Third Edition, ASCE Manuals and Reports on Engineering Practice, No.40.

Anderson, P. Marry, and William W. Woessner, (1992). Applied Groundwater Modelling: Simulation of Flow and Advective Transport, Academic Press, Inc., San Diego, California.

Australian Centre for Tropical Freshwater Research, (1990). Manual for the MODINV, James Cook University, Townsville, Australia.

Boonstra, J., N.A. de Ridder, Numerical Modelling of Groundwater Basins.

Groundwater Estimation Methodology, (1984). Report of the Groundwater Estimation Committee, Ministry of Irrigation, Govt. of India, New Delhi.

Simmers, J., (1988). Estimation of Natural Groundwater Recharge, NATO ASI Series C: V.222, D. Reidel Publishing Company, Dordrecht, Holland.

Tyagi, J.V. et.al., (1992). Groundwater Balance Study in Central Godavari Delta of Andhra Pradesh, Part I, NIH Report, TR-166.

Tyagi, J.V. et al. (1992). Seasonal Groundwater Balance Study in Central Godavari Delta of Andhra Pradesh, Part-II, NIH Report, CS-117.

Weir, G.J., (1989). The Direct Inverse Problem in Aquifers, Water Resourc. Res., 5(4),749-753.

Yeh, W.W-G., (1986). Review of Parameter Identification Procedures in Groundwater Hydrology: The Inverse Problem,. Water Resourc. Res., 22(2), 95-108.

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