TR-141

MATHEMATICAL MODELLING OF FLOW FROM A GROUP OF SPRINGS

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PREFACE

Springs provide a viable and ready source of clean water for the remote rural community especially in the hilly area where logistic difficulty prevails in creating storage of water. There are many springs in the Himalayas, in the Western Ghats and elsewhere some of which occur in groups. The existing hydrological models are based on the the assumption that the spring discharge is linearly proportional to the dynamic storage in the springflow domain. The validity of such assumption is yet to be verified.

In the present study, a mathematical model has been developed to analyse unsteady flow from a group of springs. The basic solution for rise in piezometric surface due to recharge from a rectangular basin given by Hantush has been used in the analysis. Duhamal's integration has been used to account for time variant recharge to the spring flow domain. The expression for spring discharge has been obtained in terms of response function coefficients. Any of the springs gets activated when the piezometric surface tends to rise above its threshold. The analysis assumes that once the piezometric surface touches the spring's threshold there is no further rise in the piezometric surface at the location of the spring.

The report is part of the work programme of Groundwater Assessment Division for the year 1992-93. The study has been carried out by Shri A.K.Bhar, Scientist E and by Dr. G.C. Mishra, Scientist F of the Division.

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(S.M.Seth)

Director

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Abstract

The existing hydrological springflow models are based on the assumption that the spring discharge is linearly proportional to the dynamic storage in the springflow domain. The validity of such assumption is yet to be verified. In the present study, a mathematical model has been developed to analyse unsteady flow from a group of springs using the basic solution for rise in piezometric surface due to recharge from a rectangular basin given by Hantush. It is found that the relationship between the spring discharge during the recession period and the dynamic storage of the spring is not linear even for a linear aquifer system.

1.0 Introduction

1.1 General

Spring is a ready source of water, a place of natural beauty, and a recreational spot. Springs generally provide clean water. They are found in the Himalayas, in the Western Ghats and in other places in India where it is logistically difficult to create storage for water. As such, study of springflow has relevance to the water supply to rural areas, specially in the hilly region.

Springs are part of the groundwater system and are natural exits through which groundwater emerges at the earth surface as concentrated discharge from aquifers. Springs occur in various sizes from small trickle to large stream in both in water table and artesian conditions. Necessary conditions to produce springs are many and are related to different combination of geologic, hydrologic, hydraulic, pedologic, climatic and even biological controls. Due to single or combined influence of these controlling factors, springs at times occur as a group. A few large springs may indicate the existence of thick transmissive aquifers whereas frequent small springs tend to indicate thin aquifers of low transmissivity. Hydrologic study of springs aids in the evaluation of groundwater potential of an area.

1.2 Occurrence of springs in group:

There could be various natural situations for springs to occur in groups depending upon the geomorphology and hydrogeological conditions. Some of these situations for giving rise to springs in groups are:

1.2.1 Springs issuing from thin permeable formation:

Blocking of downward movement of groundwater by an aquiclude forces water to move laterally and at the outcrop of the permeable layer in a valley a line of springs results. Outcrop of bedrock or nearness of bedrock to surface is the controlling factor. Well known thousand springs along the Snake river in southern Idaho, U.S.A is an example (Tarbuck and Lutgens,1990). Various other type of springs which can emerge in group due to similar condition are contact, gravity, perched and barrier springs (Fig.1). The discharge of such springs are small and could considerably vary periodically.

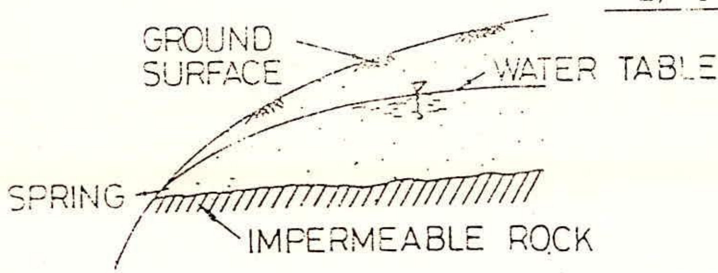
1.2.2 Springs issuing from alluvial fan deposits:

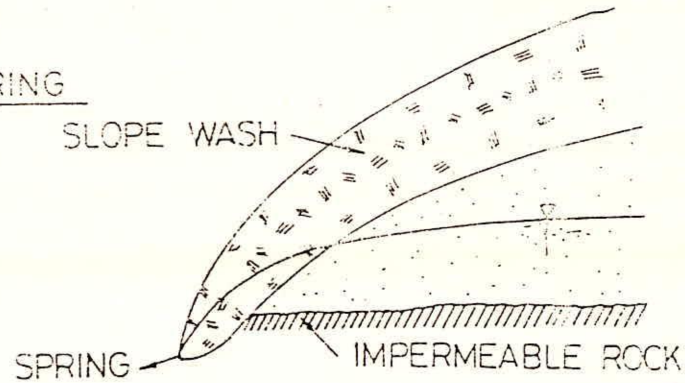
Boulders and pebbles rolling down a river are dropped down as soon as the river enters into the plains due to an abrupt change in the gradient of the valley floor and consequent reduction in the transporting capacity of the river. Such accumulations at a place where running water enters into the plains, are known as alluvial fans as the rock fragments are arranged in a radiating fan like pattern in these deposits. The fan in the vicinity of the hill will have steep slope, higher porosity to receive recharge and higher transmissivity compared to the area at the end of the fan. Groups of springs emerge from the outer boundary of the fan. Intersection of sloping water table with the land surface is the controlling feature in these springs. Deep underlying bed rock does not have any control over the flow of the springs. Discharge of such springs are usually small. Such situation occurs at the foothills of the Siwalik hills in Nainital district at the boundary between Bhabar and Tarai. The northern boundary of Bhabar belt is in contact with the Siwalik hill range and the southern limit of the Bhabar is the spring line which defines the northern limit of the Tarai sediments (Pandey,Rao and Raju,1968) (Fig.2).

1.2.3 Springs issuing from karstic rock:

Carbonate rocks springs represent constricted discharges at widely separated outlets. At the start, lines of week discharging

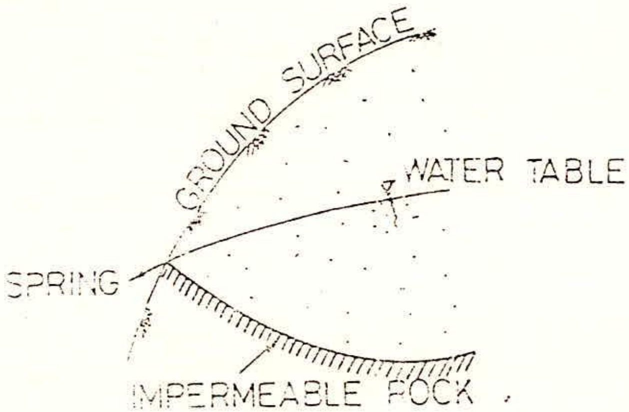
a). GRAVITY SPRING

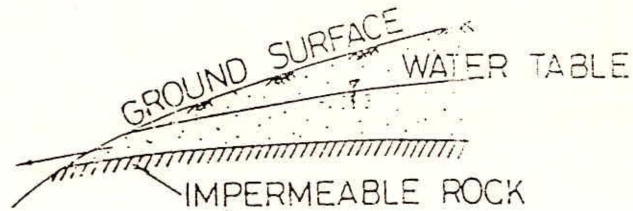




GRAVITY SPRING IN SLOPE WASH

b). PERCHED SPRING





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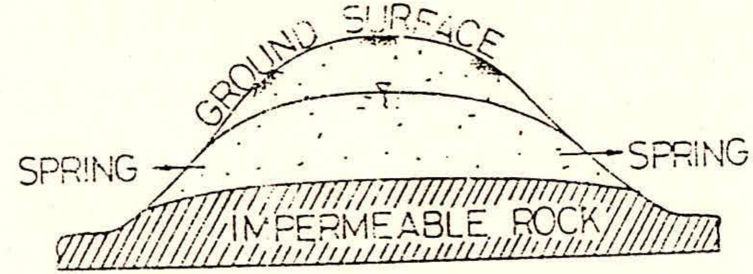
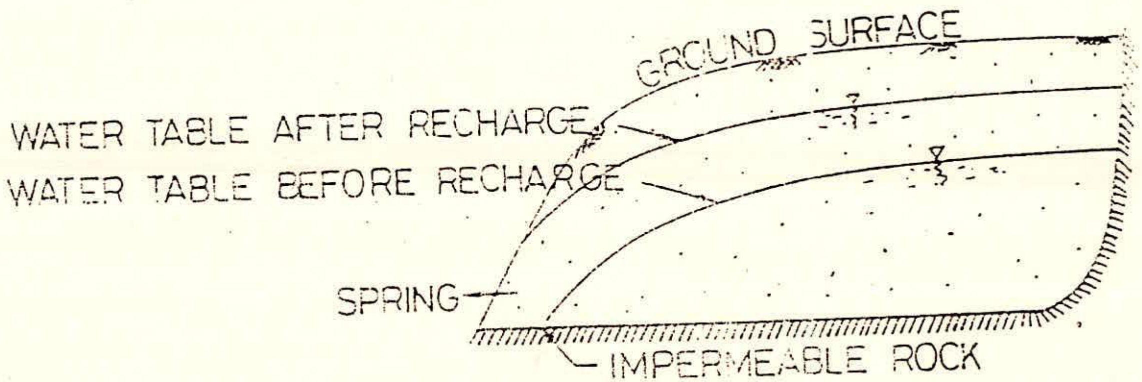
c). CONTACT SPRING



d). OVER FLOWING OR OVER SPILLING SPRING



Contd.,

e). BARRIER SPRINGS

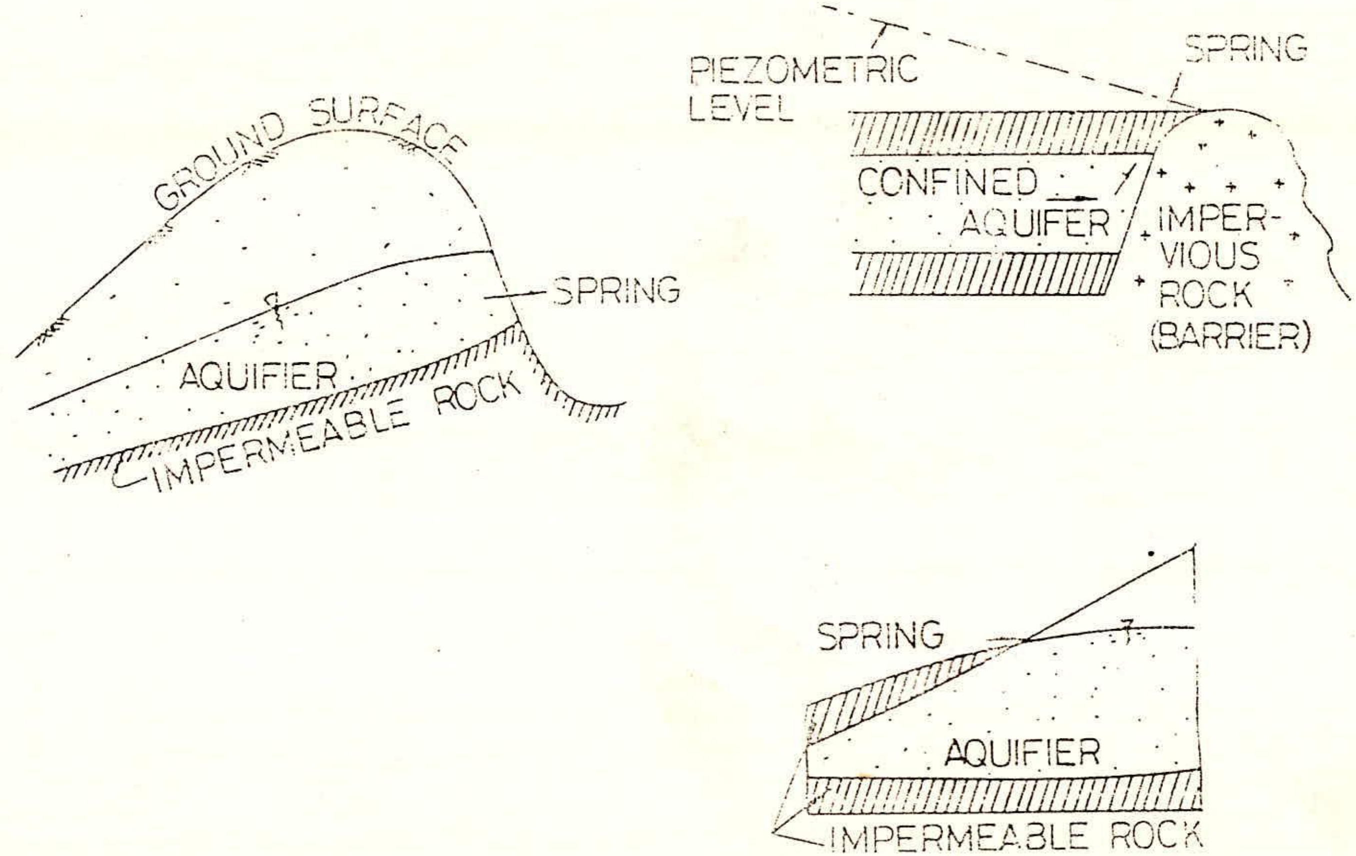


Fig.1 Various types of springs issuing from veener formations overlying relatively impermeable bedrock

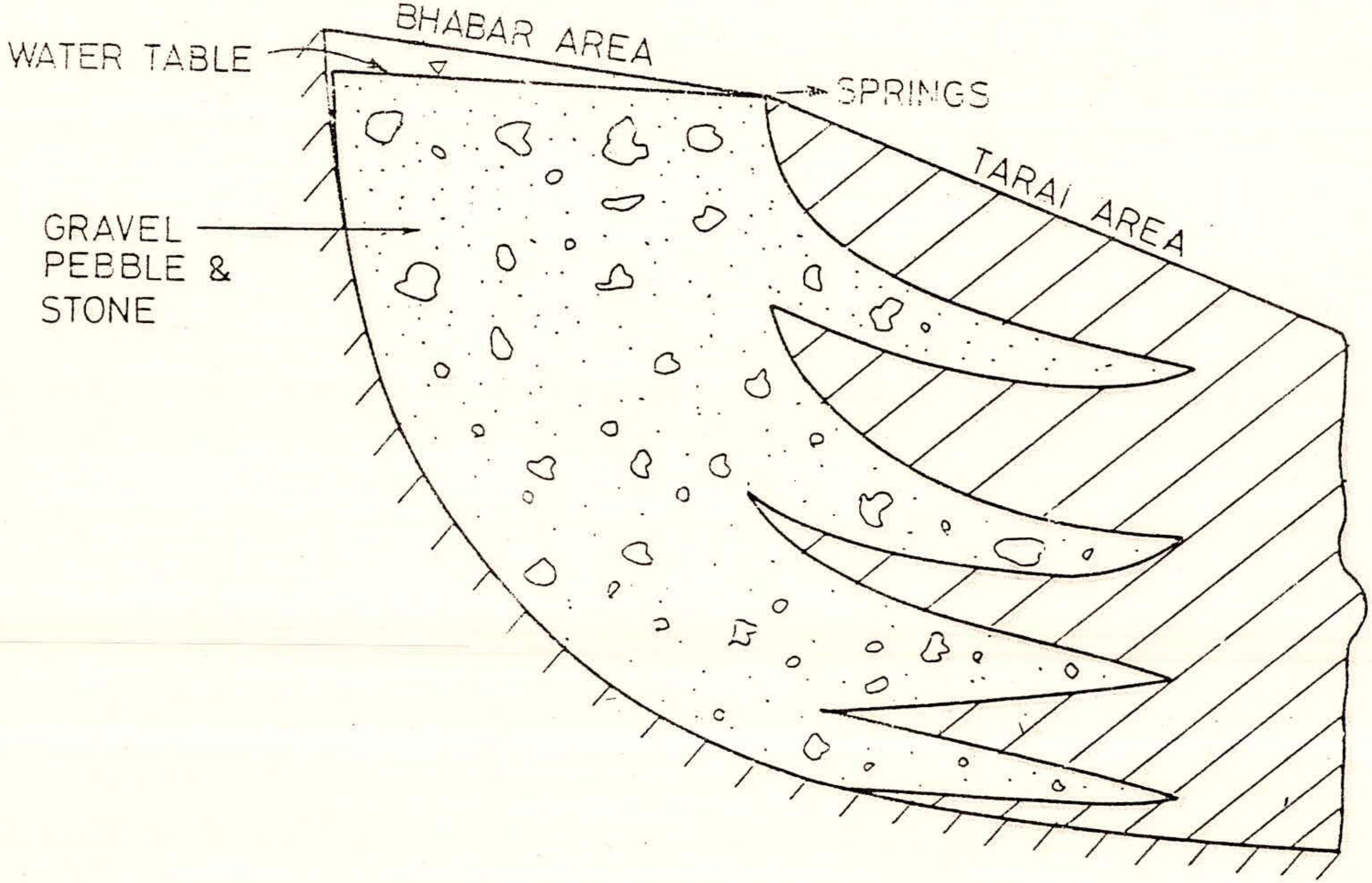


FIG. 2–SPRINGS EMERGING FROM ALLUVIAL FAN DEPOSIT

springs may develop. The largest spring progressively captures the water feeding its neighbours and the smaller springs become dry. This is specially so in a strongly developed karstic rock. A karst indicates the development of chemically enlarged openings in a carbonate rock and in other non carbonate rocks like gypsum (Fig.3). Erosion of a karstic valley may reach so deep that it reaches the groundwater level. Lines of groundwater flow converge on the valley and dissolve bedrock at a faster rate than for adjacent uplands. Less permeable residual soils and floodplain sediments promote the formation of line of springs at the contact of bedrock and valley fill sediments (Parizek,1975)(Fig.4). The majority of important springs in karsts are located along the perimeter of the erosion base. A common characteristic of these springs, whether permanent or temporary, is their direct dependence between precipitation and their outflows. It is possible to have two closely spaced springs, one having a high capacity and the other a low capacity. The two springs of the Pliva river near Jajce (Yugoslavia) are good examples (Milanovic, 1981).

Submarine springs and springs at the sea surface were formed during continental phase when the base of erosion was lower than at present. These springs are active only during the wet season. At this time they discharge substantial quantities of fresh water into the sea changing the salt content and temperature of the sea water in the coastal belt. Continuous submarine springs are rare. Their main characteristic is their considerable variation in flow. More than 50 localities with submarine springs have been discovered along the Adriatic coast in Yugoslavia. The submarine shelf, between Florida and the Bahama Islands, is composed of karstified limestone and is covered with thousands of sinkholes.

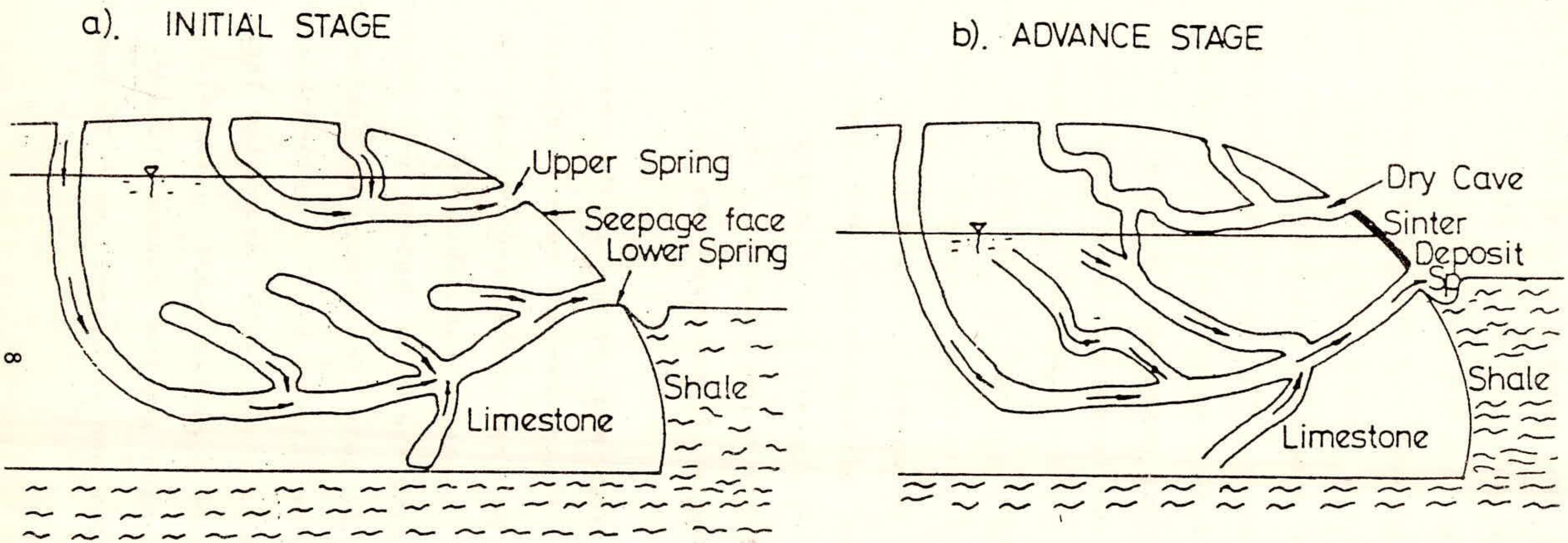


Fig. 3 Development of solution channel

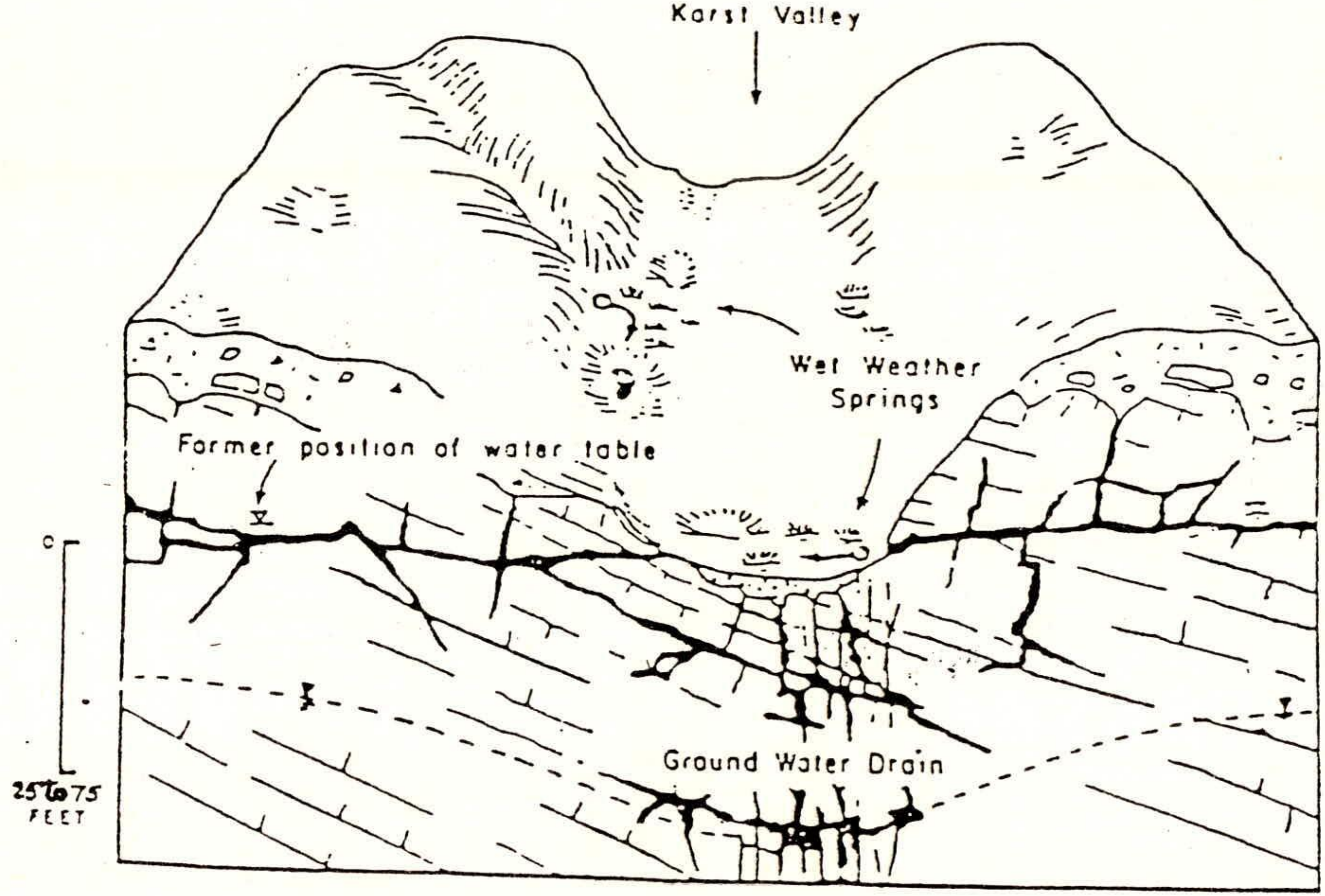


Fig.4 Eroded Karst Valley upto Water Table

Submarine springs are active in many of them. Similar cases of coastal springs also have been identified at many locations along the Adriatic coast. At particular sites, it was observed that water from the coastal aquifer discharges during winter through a long line of springs (Milanovic,1981).

2.0 Hydrologic Modelling of Springflow:

A spring emerges at ground surface through a threshold point where the drawdown is constant till the spring is active. The spring serves as a boundary condition for the regional groundwater flow analysis and the threshold point or outlet of a spring is considered as a fixed head boundary. But when the water table in the vicinity of the spring drops below the spring outlet, it ceases to be a boundary of the flow domain.

Many existing mathematical models of the springflow have been developed on the assumption that the spring outflow is linearly proportional to the dynamic storage in the spring flow domain. The validity of this assumption needs to be checked. For this purpose a rigourous mathematical model has been presented for analysing flow from a spring and a group of springs.

3.0 Mathematical Models for Springflow:

Statement of the Problem:

A schematic configuration of a spring flow domain is shown in Fig.5(a). The corresponding idealised flow domain adopted for the analysis is shown in Fig.5(b). The basic saturated flow equation describing flow to a spring is the Boussinesq's equation:

$$\phi \frac{\partial s}{\partial t} - T \frac{\partial^2 s}{\partial x^2} - T \frac{\partial^2 s}{\partial y^2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} q_r(\xi, \vartheta, \tau)\, \delta(\xi - x, \vartheta - y, \tau - t)\, d\xi \; d\vartheta \; d\tau \qquad \ldots(1)$$

where ϕ is the drainable (or effective) porosity, s is the rise in

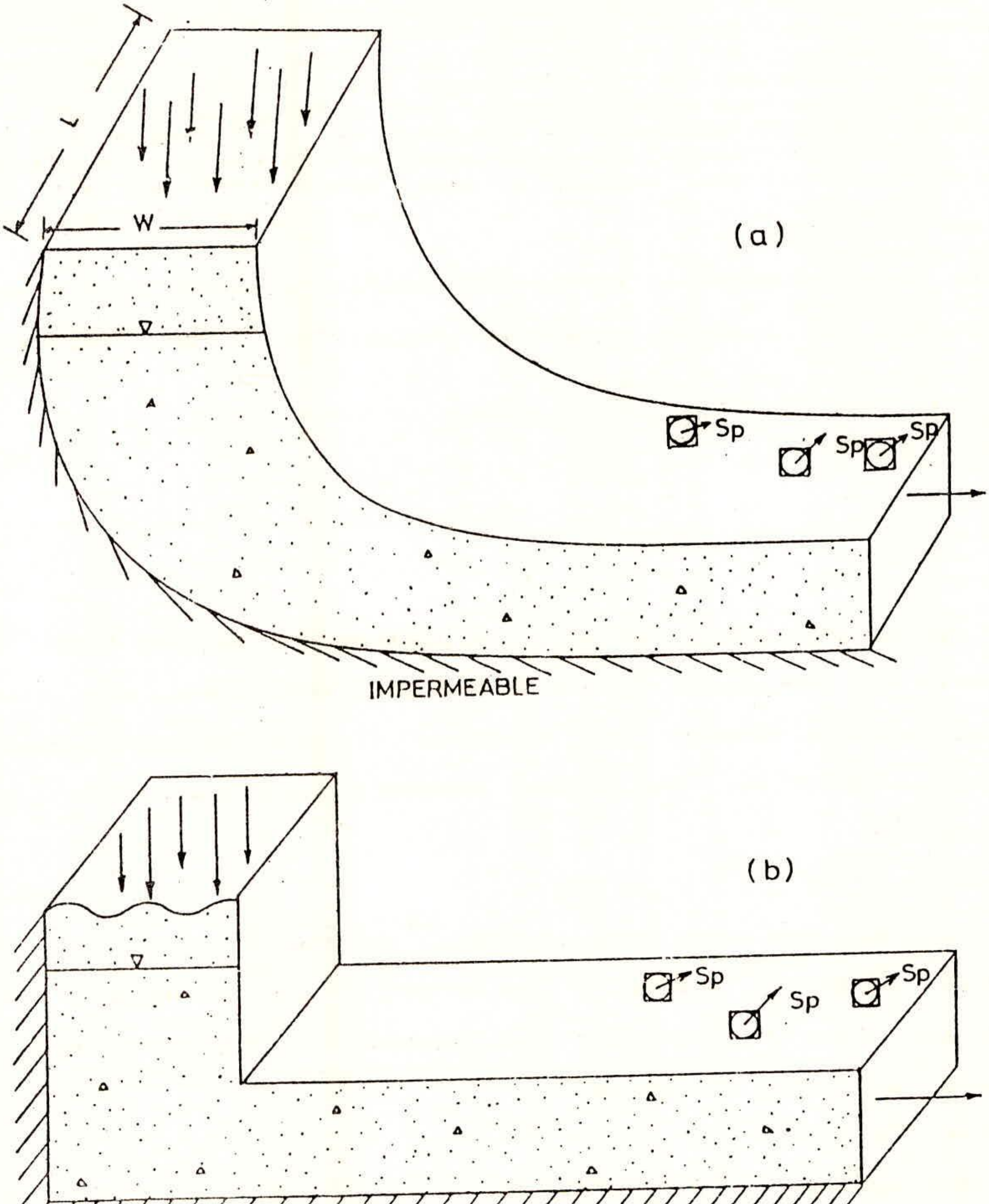


FIG. 5 a & b – SCHEMATIC AND IDEALISED FLOW DOMAIN OF A SPRING piezometric surface, t is time, x and y are the horizontal Cartesian coordinates, T is the transmissivity, $q_r(\xi,\vartheta,\tau)$ is the instantaneous recharge or discharge rate per unit area (positive for recharge and negative for discharge) and $\delta(\xi-x,\vartheta-y,\tau-t)$ is a Dirac delta function singular at the point of coordinates x, y, and t. The level of the initially rest piezometric surface is taken as the datum.

The required solution to the differential equation (1) needs to satisfy the initial condition $s(x,y,0)=0$. The boundary conditions to be satisfied are:

$$\frac{\partial s}{\partial x}\Big|_{x=0}=0 \qquad \ldots(2)$$

$$s(x,\pm\infty,t)=0\ ; \qquad \ldots(3)$$

$$s(\infty,y,t)=0; \qquad \ldots(4)$$

A spring gets activated when the piezometric surface tends to rise above its threshold. Once a spring gets activated, the rise in piezometric surface at the location of the spring remains invariant. Therefore, the other boundary conditions to be satisfied are:

$$s(x_i,y_i,t)=z_i,\ t > t_i, i=1,N \qquad \ldots(5)$$

where x_i, y_i are the coordinate of the i^{th} spring, t_i is time of activation of the i^{th} spring, z_i is height of the threshold of the i^{th} spring above the initially rest piezometric surface, and N are the total number of springs.

METHOD OF SOLUTION:

3.1 Single Spring

The method of image can be applied to convert the finite flow domain into an infinite flow domain. The boundary condition stated in equation(2) is thereby satisfied. The system of image and real springs is shown in Fig.6.

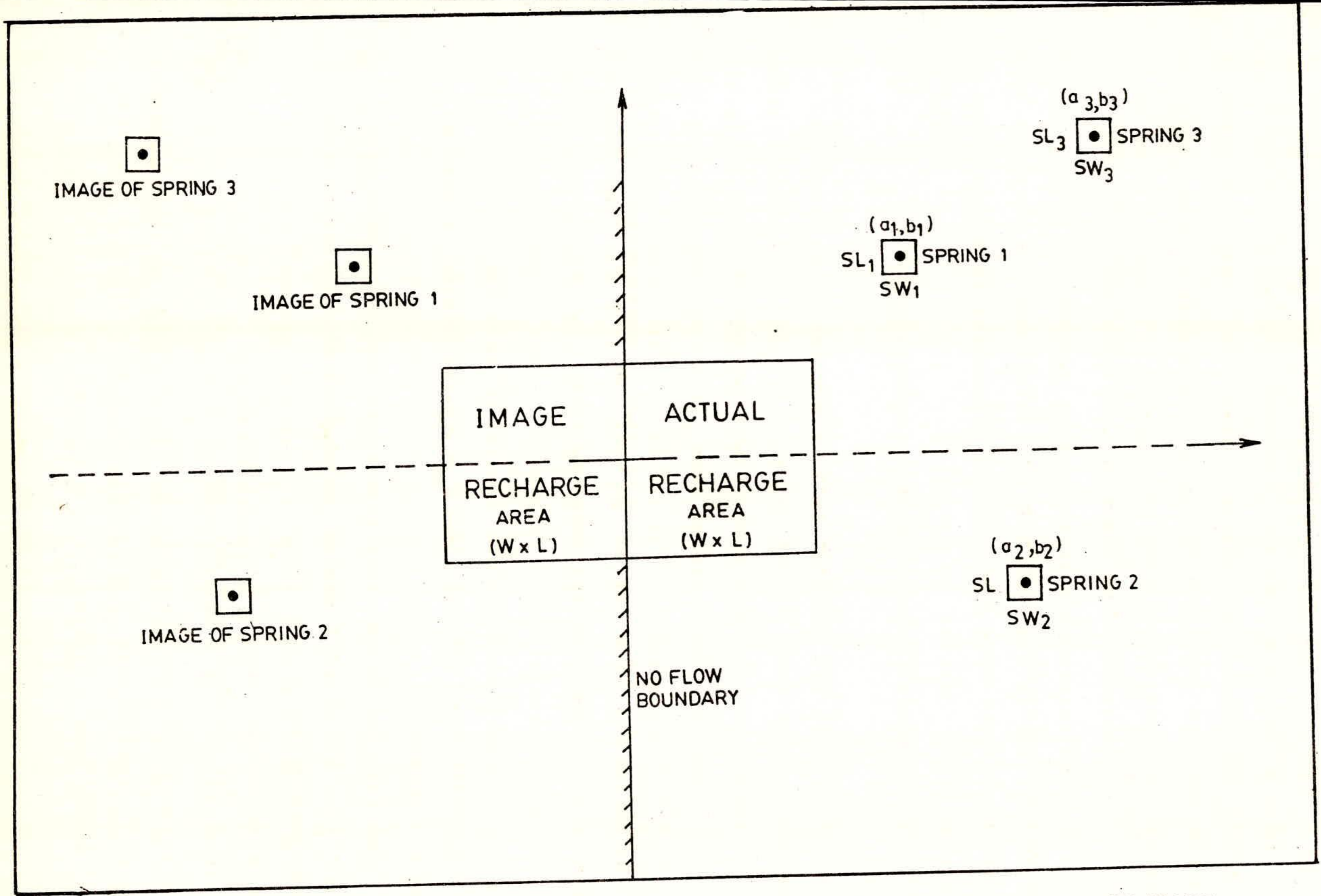


FIG. 6 FLOW DOMAIN OF THE PROPOSED MODEL FOR MULTIPLE SPRINGS BASED ON THEORY OF IMAGE.

Let the origin of the X,Y co ordinate system be chosen at the centre of the equivalent recharge zone. Let the time span be discretised into uniform time steps. Let during a time step the recharge rate and the spring discharge be seperate constants; however they may vary from one time step to next.

The rise in piezometric surface at spring 1, $s(x_1,y_1,n\Delta t)$, due to the time variant recharge, $R(\gamma)$, through the recharge zone in the equivalent flow domain until the spring gets activated is given by:

$$s(x_1,y_1,n\Delta t)=\sum_{\gamma=1}^{n} R(\gamma)\ \delta(2W,L;x_1,y_1;\Delta t;n-\gamma+1) \qquad \ldots(6)$$

in which $R(\gamma)$ is the recharge rate (m^3 per unit time step) per unit area during time step γ ; $\delta(A,B;X,Y;\Delta t;m)$ is a discrete kernel coefficient; A and B are the width and length of the excitation zone; X,Y are the coordinate of the point of observation, the coordinate being measured from a local origin chosen at the centre of the excitation zone; Δt is the time step size. The discrete kernel coefficients are the response at an observation point due to unit excitation per unit area given to the system during the first unit time period. In the present problem the excitation zone is either the area through which recharge takes place or the spring's opening.

Let the spring get activated at $t=N_1\Delta t$ and the rising piezometric surface touches the spring's threshold at $t=(N_1-1)\Delta t$. Since just prior to activation $s(x_1,y_1,n\Delta t) = z_i$, therefore,

$$\sum_{\gamma=1}^{N_1-1} R(\gamma)\ \delta(2W,L;x_1,y_1;\Delta t;\ N_1-\gamma\ ) = z_1 \qquad \ldots(7)$$

The time of activation of the first spring can be predicted from equation(7) using an iteration procedure. As the spring gets activated at $n=N_1$, therefore, $q_1(\gamma)=0$ for $\gamma = 1,2,\ldots N_1-1$.

The expression for rise in piezometric surface at location of the first spring after its activation is given by:

$$s(x_1,y_1,n\Delta t)$$
$$=\sum_{\gamma=1}^{n}[\ R(\gamma)\ \delta(2W,L;x_1,y_1;\Delta t;n-\gamma+1)\ ]$$
$$-\sum_{\gamma=1}^{n}[q_1(\gamma)\{\delta(a_1,b_1;0,0;\Delta t;n-\gamma+1)+\delta(a_1,b_1;2x_1,0;\Delta t;n-\gamma+1)\}] \quad ...(8)$$

Since after activation of the spring, $s(x_1,y_1,n\Delta t)=z_1$, therefore,

$$\sum_{\gamma=1}^{n}[\ R(\gamma)\ \delta(2W,L;x_1,y_1;\Delta t;n-\gamma+1)\ ]$$
$$-\sum_{\gamma=1}^{n}[q_1(\gamma)\{\delta(a_1,b_1;0,0;\Delta t;n-\gamma+1)+\delta(a_1,b_1;2x_1,0;\Delta t;n-\gamma+1)\}]=z_1 \quad ...(9)$$

Splitting the second temporal summation into two parts, one part containing the summation up to (n-1)th terms, and the other part the n th term, equation(9) is simplified to:

$$\sum_{\gamma=1}^{n}[R(\gamma)\ \delta(2W,L;x_1,y_1;\Delta t;n-\gamma+1)]$$
$$-\sum_{\gamma=1}^{n-1}[q_1(\gamma)\{\delta(a_1,b_1;0,0;\Delta t;n-\gamma+1)+\delta(a_1,b_1;2x_1,0;\Delta t;n-\gamma+1)\}]$$
$$-\ q_1(n)\ \{\ \delta(a_1,b_1;0,0;\Delta t;1)+\ \delta(a_1,b_1;2x_1,0;\Delta t;1)\}w=\ z_1 \quad ...(10)$$

Solving for $q_1(n)$, we obtain

$$q_1(n)\ =\ [\ \sum_{\gamma=1}^{n}\{R(\gamma)\delta(2W,L;x_1,y_1;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n-1}\{q_1(\gamma)(\delta(a_1,b_1;0,0;\Delta t;n-\gamma+1)$$
$$+\delta(a_1,b_1;2x_1,0;n-\gamma+1))\}-z_1\ ]/$$
$$[\delta(a_1,b_1;0,0;\Delta t;1)+\delta(a_1,b_1;2x_1,0;\Delta t;1)] \quad ...(11)$$

Since the spring gets activated during N_1^{th} time step, $q_1(\gamma)=0$ for $\gamma=1,2,..,\ N_1-1$. $q_1(n),n\ \geq\ N_1$, can be solved in succession starting from time step N_1. For time step N_1 equation (11) reduces to

$$q_1(N_1) = \left[\sum_{\gamma=1}^{N_1} \{R(\gamma)\delta(2W,L;x_1,y_1;\Delta t;N_1-\gamma+1)\} - z_1 \right] / \left[\delta(a_1,b_1;0,0;\Delta t;1)+\delta(a_1,b_1;2x_1,0;\Delta t;1)\right] \quad ...(12)$$

The $\delta(.)$ coefficients can be obtained from Hantush's solution (1967 vide Bouwer, 1978) for the rise of piezometric surface due to uniform recharge at a constant rate from a rectangular basin (fig.7). The expression for $\delta(A,B;X,Y:\Delta t;m)$ is:

$$\begin{aligned}\delta(A,B;X,Y;\Delta t;m) = \frac{m}{4\phi} & [F\{(A/2+X)\eta_1,(B/2+Y)\eta_1\}+F\{(A/2+X)\eta_1,(B/2-Y)\eta_1\} \\ & +F\{(A/2-X)\eta_1,(B/2+Y)\eta_1\}+F\{(A/2-X)\eta_1,(B/2-Y)\eta_1\}]\Delta t \\ -\frac{(m-1)}{4\phi} & [F\{(A/2+X)\eta_2,(B/2+Y)\eta_2\}+F\{(A/2+X)\eta_2,(B/2-Y)\eta_2\} \\ & + F\{(A/2-X)\eta_2,(B/2+Y)\eta_2\}+F\{(A/2-X)\eta_2,(B/2-Y)\eta_2\}]\Delta t \quad ...(13)\end{aligned}$$

where

m = time step ,

ϕ = coefficient of storage,

T = trasmissivity,

A,B = width and length of the recharge basin, or spring opening

X,Y = coordinates of observation point measured from the centre of the local origin (recharge basin, or spring)

$\eta_1 = (4Tm\Delta t\ /\phi)^{-0.5}$

$\eta_2 = \{4T(m-1)\Delta t/\phi\}^{-0.5}$

$F(\Phi,\Psi) = \int_0^1 \mathrm{erf}(\Phi\ \tau^{-0.5}).\mathrm{erf}(\Psi\ \tau^{-0.5})\ d\tau$

$F(\Phi,\Psi)$ values have been tabulated by Hantush.

3.2 Multiple Springs

The multiple springs problem has been solved considering three number of springs. The solution can be obtained for any number of springs.

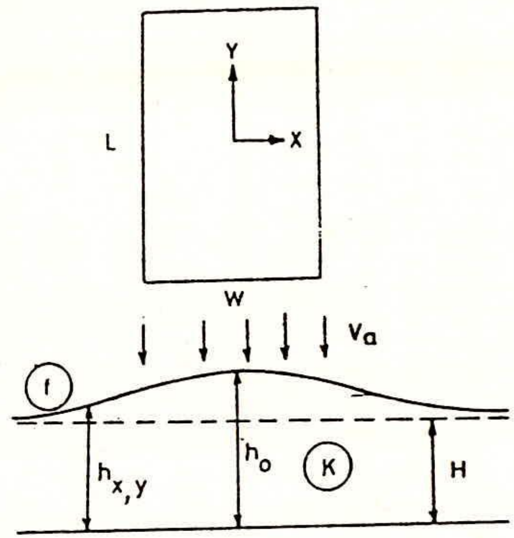


FIG. 7 · INFILTRATION AREA AND GROUNDWATER MOUND

Statement of the problem:

Locations of a group of springs and an outcrop through which recharge takes place to the aquifer are shown in Fig.5. Let the spring aquifer system be initially relaxed. A time variant recharge occures through the recharge zone. The time is reckoned since the onset of recharge. It is required to find the time of activation and discharge of each spring.

Analysis:

Introducing the image springs, the no-flow boundary condition along x=0 is satisfied. By introducing the image spring the finite aquifer is converted to an infinite aquifer (Fig.6). The flow equations for multiple springs have been derived using the Hantush basic solution for rise in piezometric surface in an infinite aquifer due to recharge from a rectangular basin.

The piezometric surface in the aquifer rises due to the recharge. The spring nearest to the recharge zone gets first activated when the piezometric surface tends to rise above the spring's threshold. Once a spring gets activated, the rise in piezometric surface under combined action of the recharge and the spring discharge remains invariant at the spring's threshold.

Up to the time any other spring gets activated, the discharge from the first spring is given by equation (11).

Let the second spring get activated at $n=N_2$. Just prior to activation the rise in piezometric surface at the second spring is

$$\sum_{\gamma=1}^{N_2-1}\{R(\gamma)\ \delta(2W,L;\ x_2,y_2;\Delta t;N_2-\gamma)\}$$
$$-\sum_{\gamma=1}^{N_2-1}[q_1(\gamma)\{\delta(a_1,b_1,x_2-x_1,y_2-y_1,\Delta t;N_2-\gamma)+$$
$$\delta(a_1,b_1;x_2+x_1,y_2-y_1;\Delta t;N_2-\gamma)\}]=z_2 \qquad \ldots(14)$$

N_2 can be solved from equation (14) by an iteration procedure.

Till the second spring gets activated the discharge from the second spring, $q_2(\gamma)=0$ for $\gamma = 1,2, \ldots N_2-1$.

The expressions for rise in piezometric surface at the first and the second spring locations after activation of the second spring are given by:

$$s(x_1,y_1,n\Delta t)= \sum_{\gamma=1}^{n} \{R(\gamma)\ \delta(2W,L;\ x_1,y_1;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n} [q_1(\gamma)\{\ \delta(a_1,b_1;0,0;\ \Delta t;\ n-\gamma+1)+$$
$$\delta(a_1,b_1;2x_1,0;\Delta t;\ n-\gamma+1)\}]$$
$$-\sum_{\gamma=1}^{n} [q_2(\gamma)\ \{\delta(a_2,b_2;x_1-x_2,\ y_1-y_2;\Delta t;\ n-\gamma+1)+$$
$$\delta(a_2,b_2;x_1+x_2,\ y_1-y_2;\Delta t;n-\gamma+1)\}] \quad \ldots(15)$$

$$s(x_2,y_2,n\Delta t)=\sum_{\gamma=1}^{n} \{R(\gamma)\ \delta(2W,L;\ x_2,y_2;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n} [q_1(\gamma)\{\ \delta(a_1,b_1,x_2-x_1,y_2-\ y_1,\ \Delta t;\ n-\gamma+1)+$$
$$\delta(a_1,b_1;x_2+x_1,y_2-y_1;\Delta t;\ n-\gamma+1)\}]$$
$$-\sum_{\gamma=1}^{n}[q_2(\gamma)\ \{\delta(a_2,b_2;0,0;\Delta t;\ n-\gamma+1)+$$
$$\delta(a_2,b_2;2x_2,0;\Delta t;n-\gamma+1)\}] \quad \ldots(16)$$

After activation of the first and the second spring at all time $s(x_1,y_1,n\Delta t)=z_1$ and $s(x_2,y_2,n\Delta t)= z_2$. Equating equations (15) and (16) to z_1 and z_2 respectively and rewriting the following equations in terms of the unknown $q_1(n)$, and $q_2(n)$ are obtained:

$$q_1(n)\{\delta(a_1,b_1;0,0;\Delta t;1)+\delta(a_1,b_1;2x_1,0;\Delta t;1)\}$$
$$+q_2(n)\{\delta(a_2,b_2;x_1-x_2,y_1-y_2;\Delta t;1)+\delta(a_2,b_2;x_1+x_2,y_1-y_2;\Delta t;1)\}$$
$$=\sum_{\gamma=1}^{n}\{R(\gamma)\,\delta(2W,L;x_1,y_1;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n-1}[q_1(\gamma)\{\delta(a_1,b_1,0,0;\Delta t;n-\gamma+1)+$$
$$\delta(a_1,b_1;2x_1,0;\Delta t;n-\gamma+1)\}]$$
$$-\sum_{\gamma=1}^{n-1}[q_2(\gamma)\{\delta(a_2,b_2;x_1-x_2,y_1-y_2;\Delta t;n-\gamma+1)+$$
$$\delta(a_2,b_2;x_1+x_2,y_1-y_2;\Delta t;n-\gamma+1)\}]\quad -z_1$$

...(17)

$$q_1(n)\{\delta(a_1,b_1,x_2-x_1,y_2-y_1,\Delta t;1)+\delta(a_1,b_1;x_2+x_1,y_2-y_1;\Delta t;1)\}$$
$$+q_2(n)\{\delta(a_2,b_2;0,0;\Delta t;1)+\delta(a_2,b_2;2x_2,0;\Delta t;1)\}$$
$$=\sum_{\gamma=1}^{n}\{R(\gamma)\,\delta(2W,L;x_2,y_2;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n-1}[q_1(\gamma)\{\delta(a_1,b_1,x_2-x_1,y_2-y_1,\Delta t;n-\gamma+1)+$$
$$\delta(a_1,b_1;x_2+x_1,y_2-y_1;\Delta t;n-\gamma+1)\}]$$
$$-\sum_{\gamma=1}^{n-1}[q_2(\gamma)\{\delta(a_2,b_2;0,0;\Delta t;n-\gamma+1)+$$
$$\delta(a_2,b_2;2x_2,0;\Delta t;n-\gamma+1)\}]$$
$$-z_2$$

...(18)

In matrix notation equations 17 and 18 can be written as

$$[\,a\,].[\,b\,]=[\,c\,] \qquad ...(19)$$

in which

$a(1,1)=\delta(a_1,b_1;0,0;\Delta t;1)+\delta(a_1,b_1;2x_1,0;\Delta t;1);$

$a(1,2)=\delta(a_2,b_2;x_1-x_2,y_1-y_2;\Delta t;1)+\delta(a_2,b_2;x_1+x_2,y_1-y_2;\Delta t;1)$

$a(2,1)=\delta(a_1,b_1;x_2-x_1,y_2-y_1;\Delta t;1)+\delta(a_1,b_1;x_2+x_1,y_2-y_1;\Delta t;1);$

$$a(2,2)=\delta(a_2,b_2;0,0;\ \Delta t;\ 1) + \delta(a_2,b_2;2x_2,0;\Delta t;\ 1);$$

$$b(1) = q_1(n)$$

$$b(2) = q_2(n)$$

$$c(1)= \sum_{\gamma=1}^{n} \{R(\gamma)\ \delta(2W,L;\ x_1,y_1;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n-1} [q_1(\gamma)\{\ \delta(a_1,b_1\ ,0,0;\ \Delta t;\ n-\gamma\ +1)+$$
$$\delta(a_1,b_1;2x_1,0;\Delta t;\ n-\gamma+1)\}]$$
$$-\sum_{\gamma=1}^{n-1} [q_2(\gamma)\ \{\delta(a_2,b_2;x_1-x_2,\ y_1-y_2;\Delta t;\ n-\gamma\ +1)+$$
$$\delta(a_2,b_2;x_1+x_2,\ y_1-y_2;\Delta t;n-\gamma+1)\}]$$

$$-z_1$$

$$c(2)= \sum_{\gamma=1}^{n} \{R(\gamma)\ \delta(2W,L;\ x_2,y_2;\Delta t;n-\gamma+1)\}$$
$$-\sum_{\gamma=1}^{n-1} [q_1(\gamma)\ \{\ \delta(a_1,b_1;x_2-x_1,y_2-y_1;\ \Delta t;\ n-\gamma\ +1)+$$
$$\delta(a_1,b_1;x_2+x_1,y_2-y_1;\Delta t;\ n-\gamma+1)\}]$$
$$-\sum_{\gamma=1}^{n-1} [q_2(\gamma)\ \ \{\delta(a_2,b_2;0,0;\Delta t;\ \ n-\gamma\ \ +1)+$$
$$\delta(a_2,b_2;2x_2,0;\Delta t;n-\gamma+1)\}]$$

$$-z_2$$

Thus $[b]=[a]^{-1}[c]$ ...(20)

Let the third spring become active during time $N_3\Delta t$. At the end of $(N_3-1)^{th}$ time step the rise in piezometric surface at the location of the third spring is equal to z_3. Hence,

$$s(x_3,y_3;(N_3-1)\Delta t)=\sum_{\gamma=1}^{N_3-1}\{R(\gamma)\ \delta(2W,L;\ x_3,y_3;\Delta t;N_3-\gamma\ )\}$$

$$-\sum_{\gamma=1}^{N_3-1}[q_1(\gamma)\{\delta(a_1,b_1;x_3-x_1,y_3-y_1;\Delta t;N_3-\gamma\ )+$$

$$\delta(a_1,b_1;x_3+x_1,y_3-y_1;\Delta t;N_3-\gamma\ )\}]$$

$$-\sum_{\gamma=1}^{N_3-1}[q_2(\gamma)\{\delta(a_2,b_2;x_3-x_2,y_3-y_2;\Delta t;N_3-\gamma\ )+$$

$$\delta(a_2,b_2;x_3+x_2,y_3-y_2;\Delta t;N_3-\gamma\ )\}]$$

$$=z_3$$

...(21)

N_3 can be known from equation (21) by an iteration procedure.

When all the springs are active the expression for drawdown at spring locations 1, 2 and 3 are given by:

$$s(x_1,y_1,n\Delta t)=\sum_{\gamma=1}^{n}\{R(\gamma)\ \delta(2W,L;\ x_1,y_1;\Delta t;n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n}[q_1(\gamma)\{\ \delta(a_1,b_1;0,0;\ \Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_1,b_1;2x_1,0;\Delta t;\ n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n}[q_2(\gamma)\ \{\delta(a_2,b_2;x_1-x_2,\ y_1-y_2;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_2,b_2;x_1+x_2,\ y_1-y_2;\Delta t;n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n}[q_3(\gamma)\ \{\delta(a_3,b_3;x_1-x_3,\ y_1-y_3;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_3,b_3;x_1+x_3,\ y_1-y_3;\Delta t;n-\gamma+1)\}]$$

$$=\ z_1$$

...(22)

$$s(x_2,y_2,n\Delta t)=\sum_{\gamma=1}^{n}\{R(\gamma)\ \delta(2W,L;\ x_2,y_2;\Delta t;n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n}[q_1(\gamma)\{\ \delta(a_1,b_1;x_2-x_1,\ y_2-y_1;\ \Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_1,b_1;x_2+x_1,\ y_2-y_1;\Delta t;\ n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n} [q_2(\gamma) \{\delta(a_2, b_2; 0, 0; \Delta t; n-\gamma+1) +$$

$$\delta(a_2, b_2; 2x_2, 0; \Delta t; n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n} [q_3(\gamma) \{\delta(a_3, b_3; x_2-x_3, y_2-y_3; \Delta t; n-\gamma+1) +$$

$$\delta(a_3, b_3; x_2+x_3, y_2-y_3; \Delta t; n-\gamma+1)\}]$$

$$= z_2 \qquad \ldots(23)$$

$$s(x_3, y_3, n\Delta t) = \sum_{\gamma=1}^{n} \{R(\gamma)\ \delta(2W, L; x_3, y_3; \Delta t; n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n} [q_1(\gamma) \{\delta(a_1, b_1; x_3-x_1, y_3-y_1; \Delta t; n-\gamma+1) +$$

$$\delta(a_1, b_1; x_3+x_1, y_3-y_1; \Delta t; n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n} [q_2(\gamma) \{\delta(a_2, b_2; x_3-x_2, y_3-y_2; \Delta t; n-\gamma+1) +$$

$$\delta(a_2, b_2; x_3+x_2, y_3-y_2; \Delta t; n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n} [q_3(\gamma) \{\delta(a_3, b_3; 0, 0; \Delta t; n-\gamma+1) +$$

$$\delta(a_3, b_3; 2x_3, 0; \Delta t; n-\gamma+1)\}]$$

$$= z_3 \qquad \ldots(24)$$

The three unknowns $q_1(n)$ $q_2(n)$ $q_3(n)$ can be solved from equations(22), (23) and (24). In matrix notation,

$$[b] = [a]^{-1}[c] \qquad \ldots(25)$$

in which b(1), and b(2); a(1,1), a(1,2), a(2,1), and a(2,2) are as defined earlier. Other elements of the matrix are:

$b(3) = q_3(n)$

$$a(1,3) = \{\delta(a_3, b_3; x_1-x_3, y_1-y_3; \Delta t; n-\gamma+1) +$$

$$\delta(a_3, b_3; x_1+x_3, y_1-y_3; \Delta t; n-\gamma+1)\}$$

$$a(2,3) = \{\delta(a_3, b_3; x_2-x_3, y_2-y_3; \Delta t; n-\gamma+1) +$$

$$\delta(a_3,b_3;x_2+x_3,\ y_2-y_3;\Delta t;n-\gamma+1)\}.$$

$$a(3,1)=\{\ \delta(a_1,b_1;x_3-x_1,\ y_3-y_1;\ \Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_1,b_1;x_3+x_1,\ y_3-y_1;\Delta t;\ n-\gamma+1)\}$$

$$a(3,2)=\ \{\delta(a_2,b_2;x_3-x_2,\ y_3-y_2;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_2,b_2;x_3+x_2,\ y_3-y_2;\Delta t;n-\gamma+1)\}$$

$$a(3,3)=\{\delta(a_3,b_3;0,0;\Delta t;\ n-\gamma\ +1)+\delta(a_3,b_3;2x_3,0\ ;\Delta t;n-\gamma+1)\}$$

$$c(1)=\sum_{\gamma=1}^{n}\ \{R(\gamma)\ \delta(2W,L;\ x_1,y_1;\Delta t;n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n-1}[q_1(\gamma)\{\ \delta(a_1,b_1;0,0;\ \Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_1,b_1;2x_1,0;\Delta t;\ n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n-1}[q_2(\gamma)\ \{\delta(a_2,b_2;x_1-x_2,\ y_1-y_2;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_2,b_2;x_1+x_2,\ y_1-y_2;\Delta t;n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n-1}[q_3(\gamma)\ \{\delta(a_3,b_3;x_1-x_3,\ y_1-y_3;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_3,b_3;x_1+x_3,\ y_1-y_3;\Delta t;n-\gamma+1)\}]$$

$$-\ z_1$$

$$c(2)=\sum_{\gamma=1}^{n}\ \{R(\gamma)\ \delta(2W,L;\ x_2,y_2;\Delta t;n-\gamma+1)\}$$

$$-\sum_{\gamma=1}^{n-1}[q_1(\gamma)\{\ \delta(a_1,b_1;x_2-x_1,\ y_2-y_1;\ \Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_1,b_1;x_2+x_1,\ y_2-y_1;\Delta t;\ n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n-1}[q_2(\gamma)\ \{\delta(a_2,b_2;0,0\ ;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_2,b_2;2x_2,0\ ;\Delta t;n-\gamma+1)\}]$$

$$-\sum_{\gamma=1}^{n-1}[q_3(\gamma)\ \{\delta(a_3,b_3;x_2-x_3,\ y_2-y_3;\Delta t;\ n-\gamma\ +1)+$$

$$\delta(a_3,b_3;x_2+x_3,\ y_2-y_3;\Delta t;n-\gamma+1)\}]$$

$$-\ z_2$$

$$
\begin{aligned}
c(3) = \sum_{\gamma=1}^{n} & \{R(\gamma)\ \delta(2W, L;\ x_3, y_3; \Delta t; n-\gamma+1)\} \\
& -\sum_{\gamma=1}^{n-1} [q_1(\gamma)\{\ \delta(a_1, b_1; x_3-x_1,\ y_3-y_1;\ \Delta t;\ n-\gamma\ +1)+ \\
& \qquad \delta(a_1, b_1; x_3+x_1,\ y_3-y_1; \Delta t;\ n-\gamma+1)\}] \\
& -\sum_{\gamma=1}^{n-1} [q_2(\gamma)\ \{\delta(a_2, b_2; x_3-x_2,\ y_3-y_2; \Delta t;\ n-\gamma\ +1)+ \\
& \qquad \delta(a_2, b_2; x_3+x_2,\ y_3-y_2; \Delta t; n-\gamma+1)\}] \\
& -\sum_{\gamma=1}^{n-1} [q_3(\gamma)\ \{\delta(a_3, b_3; 0, 0; \Delta t;\ n-\gamma\ +1)+ \\
& \qquad \delta(a_3, b_3; 2x_3, 0\ ; \Delta t; n-\gamma+1)\}] \\
& -z_3
\end{aligned}
$$

3.3 Results and discussion

For single Spring

A recharge of 20 cm is assumed to occur in a span of 120 days continuously at a uniform rate of (1/600) m/day through a recharging area having W=250m and L=2000m. Variation of spring flow with time in response to this recharge has been computed by the model and plotted on a linear scale in Fig.8. Out of $1\text{x}10^{5}$ cu.m of total recharge to the aquifer, only $0.25\ \text{x}10^{5}$ cu.m appears as springflow during 240 days after the commencement of recharge. It is also found that out of the total recharge, only $0.40\text{x}10^{5}$ cu.m of water appears as springflow and the remaining $0.60\text{x}10^{5}$ cu.m never appears as springflow.

The springflow during recession has been expressed as (Mandel and Shiftan, 1981): $q(t+\Delta t) = q(t) \exp(-\Delta t/t_o)$, where t_o is known as depletion time. The depletion time is a parameter of the spring and is the time that will be required to empty the live storage of the spring at the present flow rate, i.e.,the dynamic storage at any time t is equal to $q(t).t_o$.

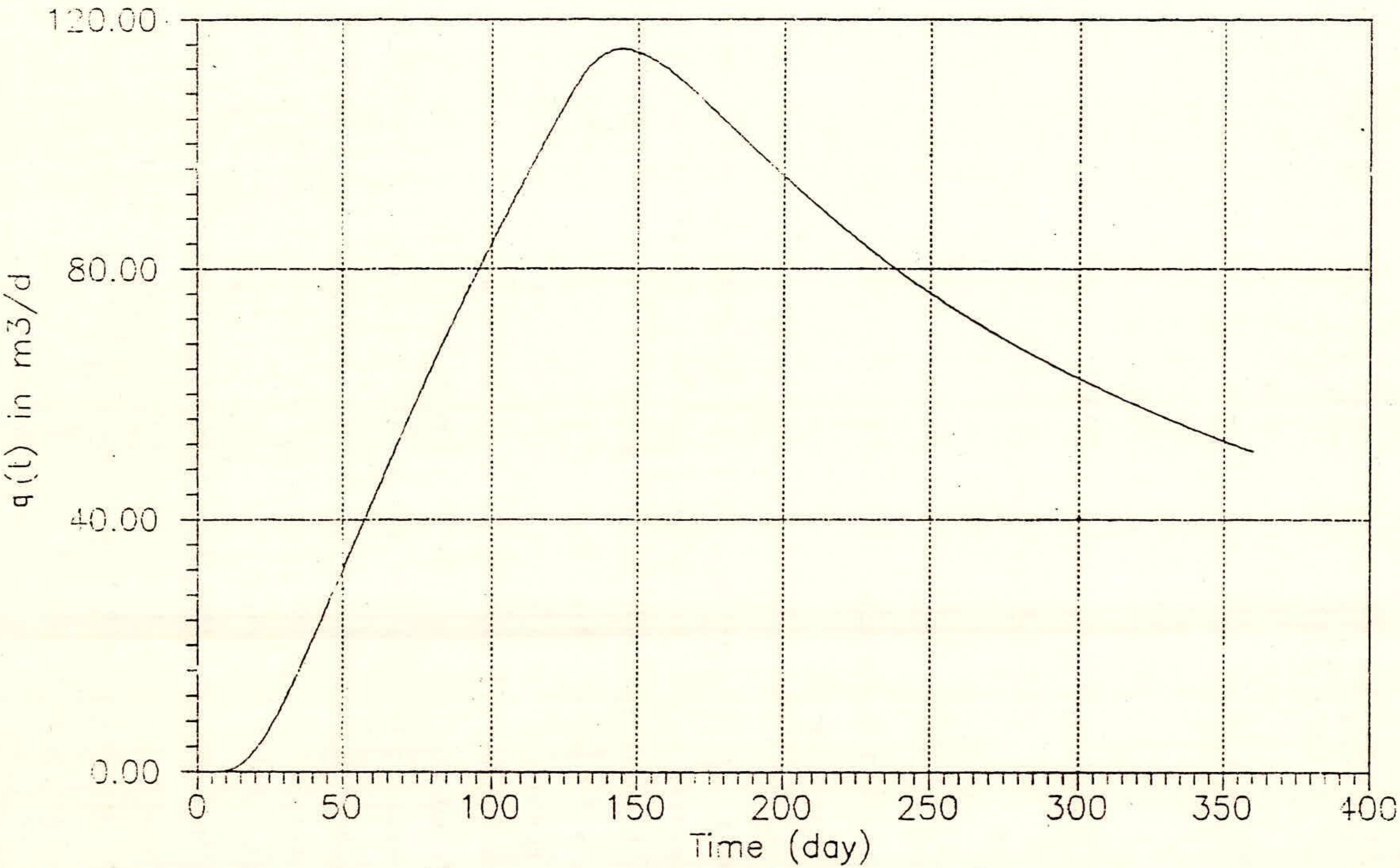


FIG.8 VARIATION OF SPRING DISCHARGE WITH TIME

(T = 200 m^2/day, ϕ = 0.01, l = 2000 m, Size of the spring = 1 m x 1 m, Duration of recharge 120 days, Recharge rate = 0.0016 m/day, Size of the recharge area = 2000 m x 250 m)

Let S_o is the part of the total recharge which never appears as springflow. S_o has been ascertained from the plot of cumulative recharge and cumulative discharge. The dynamic storage at any time which will subsequently appear as springflow is equal to the difference between the total recharge and the summation of S_o and cumulative spring discharge up to that time. Hence,

$$t_o = [\ R - \{ \sum_{\gamma=1}^{n} q(\gamma) + S_o \ \}]/q(n) \qquad \ldots(25)$$

The values of depletion time at various time steps (days) during recession have been evaluated and are given in Table 1.

Table 1: Variation of depletion time

Days after the cessation of recharge (days)	Depletion time (days)
80	220
90	221
100	224
110	229
120	234
130	240
140	245
150	251
160	258
170	264
180	271
190	278
200	285
210	288
220	294
230	302

Perusal of the values of depletion time in Table 1 shows that the depletion time which has been assumed as constant varies with time and as such the springflow during the recession period does not follow strictly the exponential decay curve and the springflow is not truly linearly proportional to the dynamic storage which will subsequently appear as springflow. From the semilog plot (Fig.9) of recession portion of springflow the value of depletion time is 274. So, using this value of depletion time, the dynamic storage of the spring will be overestimated.

The variation between springflow and the total storage in the aquifer due to recharge is depicted in Fig.10. After the cessation of recharge after 120 days, the springflow continue to rise for another 26 days and then discharge declines. This is depicted by the falling line after the loop in the plotting. Fig.11 gives the combined plotting of cumulative discharge (firm line), storage (dashed line) and cumulative springflow. The cumulative recharge increases and becomes constant after the cessation of recharge (120 days). The storage also follows similar trend and decreases gradually after the cessation of recharge due to spring flow. The storage will become more or less constant and will be equal to storage which will never appear as springflow. It is estimated that the portion of storage which will never appear as spring flow is around 60,000 cu.m . The discharge from the spring will be zero or negligible when the storage will be constant.

For Multiple Springs

Two illustrative configuration of three springs were taken for this model. In the first case, all the three springs are in one line along X-asis with coordinates (2000m,0), 6000m,0) and 8000m,0). For the second case, the three springs are on a along Y-axis with (2000m,1000),(2000m,0m)and (2000m,-1000m).

In the first case, the spring 1,2 and 3 get activated after 1 day, 14 days and 24 days respectively after the onset of recharge.

In the second case, the central spring(2) which is closer to the recharge area is activated after a day and the other two springs(1,3) get activated after 2 days after the onset of recharge. The discharges from the spring-1 and spring-2 are presented in Fig. 12. The discharge of spring-1 is lower than the discharge of spring-2. The difference between the two springs goes on increasing and is maximum at the time of cessation of recharge. It maintains this difference for another two weeks and the difference goes on decreasing thereafter the springs hydrogram start to fall downwards. These are so as the spring-2 being nearer to recharge zone starts with a higher discharge and also responds early after the cessation of the recharge in its discharge. Eventually the discharges of the spring-1 and 2 become equal after the 220 days and the discharge from spring 1 supersedes the discharge from the spring-2 marginally till the onset of next recharge.

Fig. 13 shows the variation of cumulative recharge and cumulative spring discharge with time. Cumulative recharge has become constant and plotted as a straight line after the cessation of recharge after 120 days. The rate of increase of cumulative discharge from springs shows decreasing trend after 165 days i.e. after almost one and half months after the cessation of discharge.

The Computer Program developed for computing the springflow, recharge and storage is given in the Appendix.

The mathematical models developed appears to perform reasonably for predicting springflow, but their efficacy should be checked with field data.

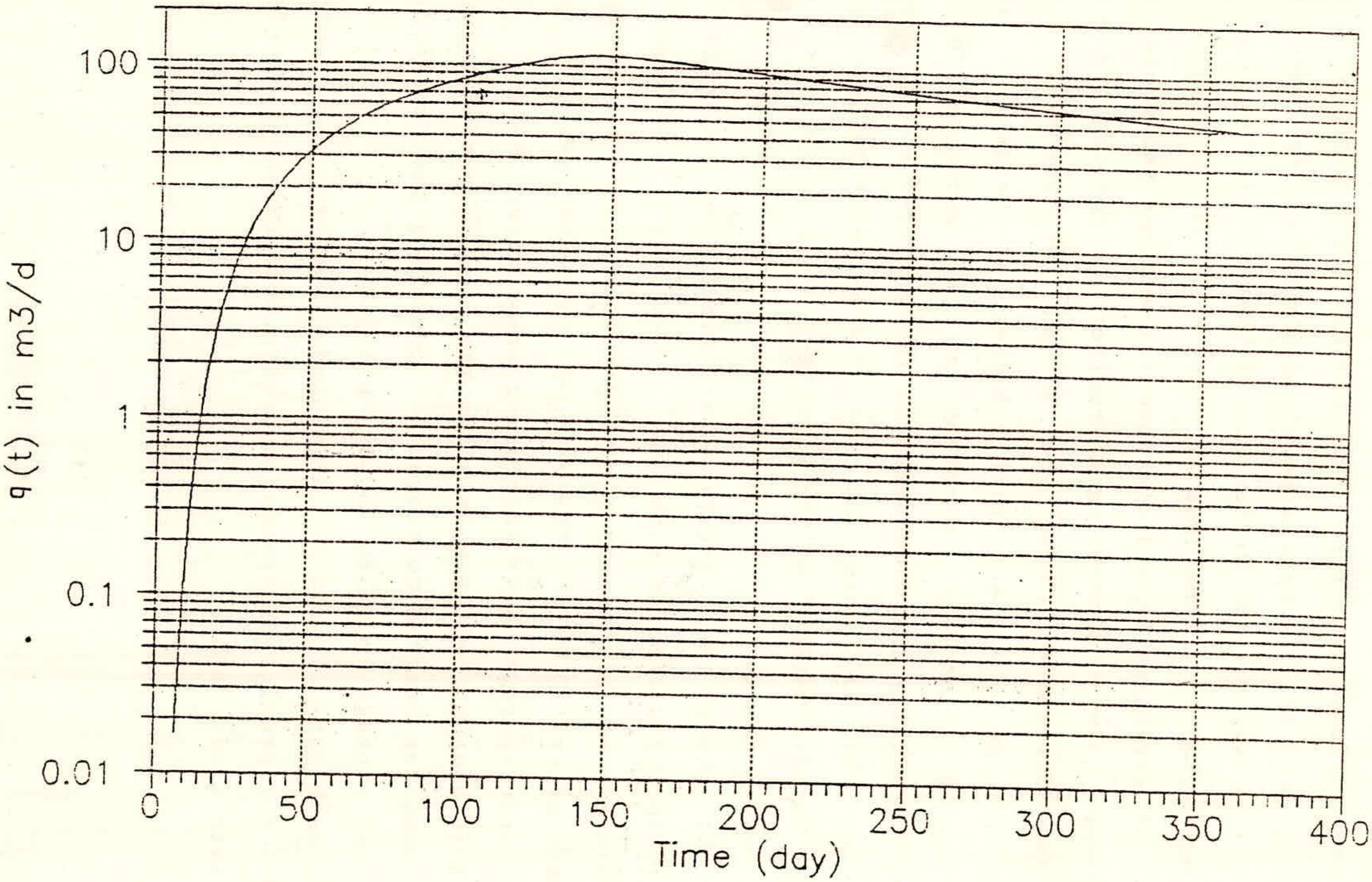


FIG.9 VARIATION OF SPRING DISCHARGE WITH TIME

(T = 200 m^2/day, ϕ = 0.01, Distance of the spring 2000 m, Size of the spring 1 mx 1 m Recharge rate 0.00166 m/day, Duration of recharge 120 days, Size of the recharge area 2000 m x 250 m)

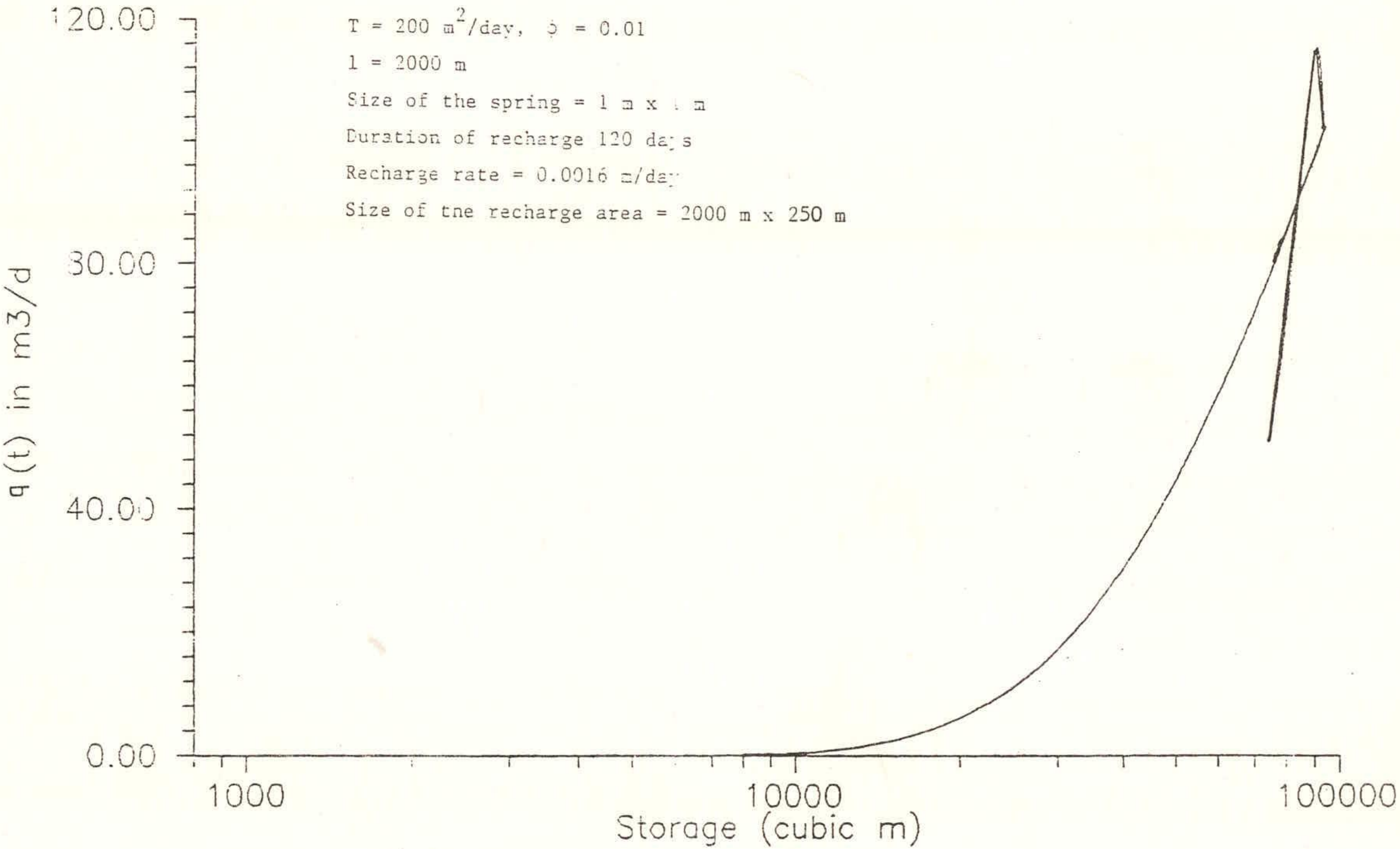


FIG.10 VARIATION OF SPRING DISCHARGE WITH STORAGE (SEMI LOG PLOT)

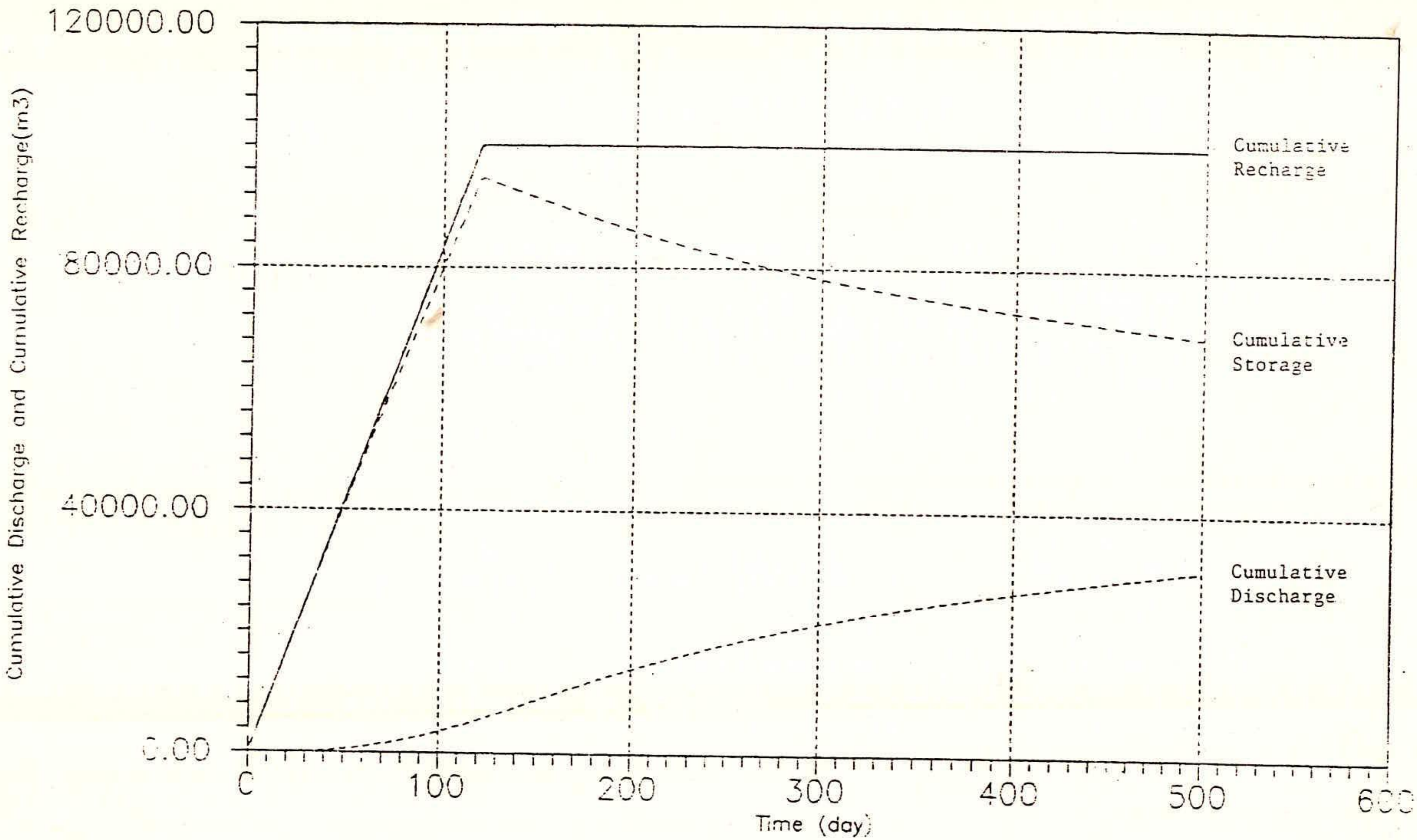


FIG.11 Variation of cumulative recharge, storage and cumulative discharge for a spring
(T = 200 sq.m/day, Φ = 0.01, l = 2000 m, size of the spring = 1m x 1m,
Duration of recharge = 120 days, Recharge rate = 0.0016 m/day,
Size of the recharge area = 2000 m x 250 m).

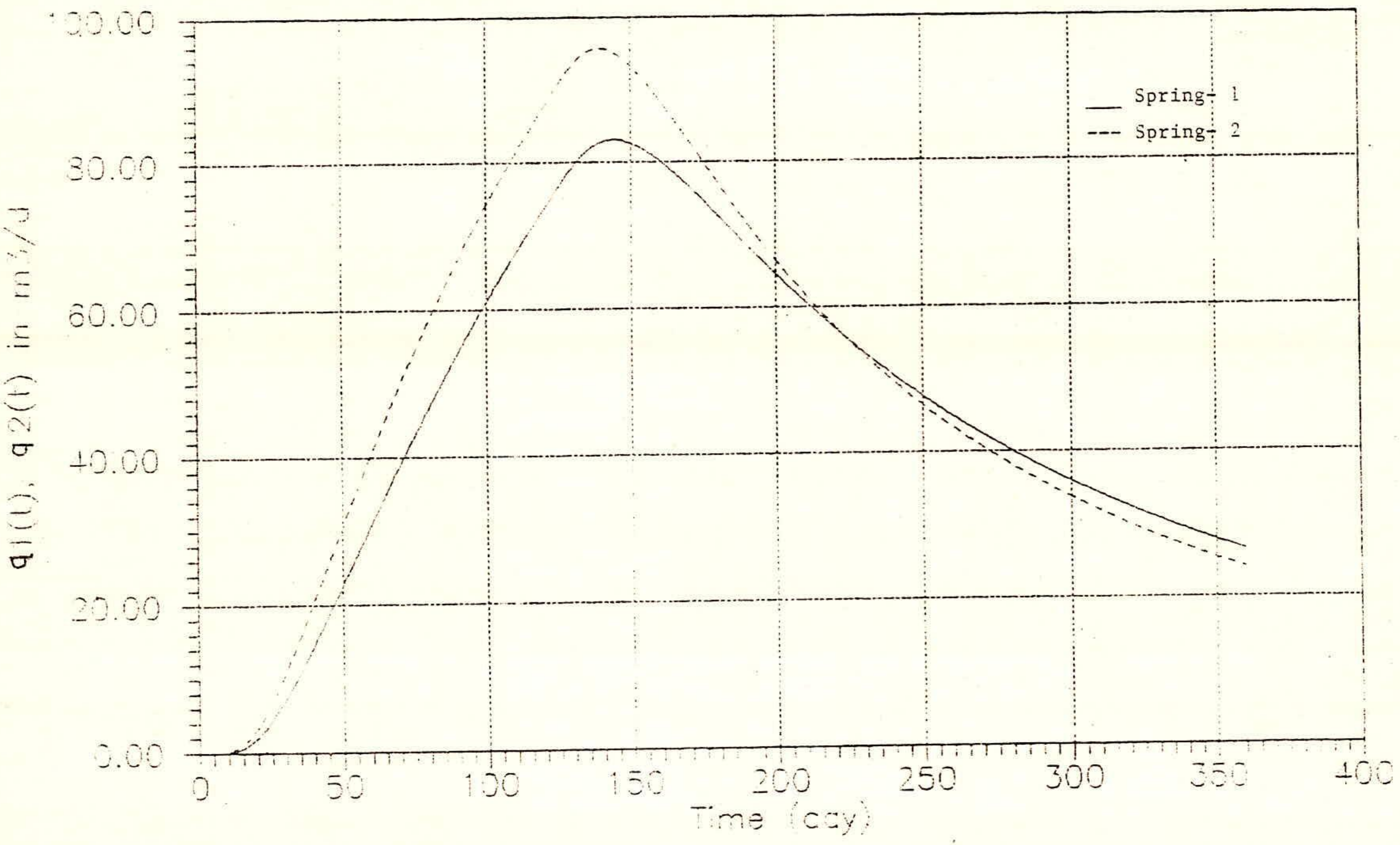


FIG. 12 VARIATION OF SPRING DISCHARGE WITH TIME

(T = 200 sq.m/day, ϕ = 0.01, size of the springs = 1m x 1m, Duration of recharge = 120 days, Recharge rate = 0.0016 m/day, Size of the recharge area = 2000 m x 250 m)

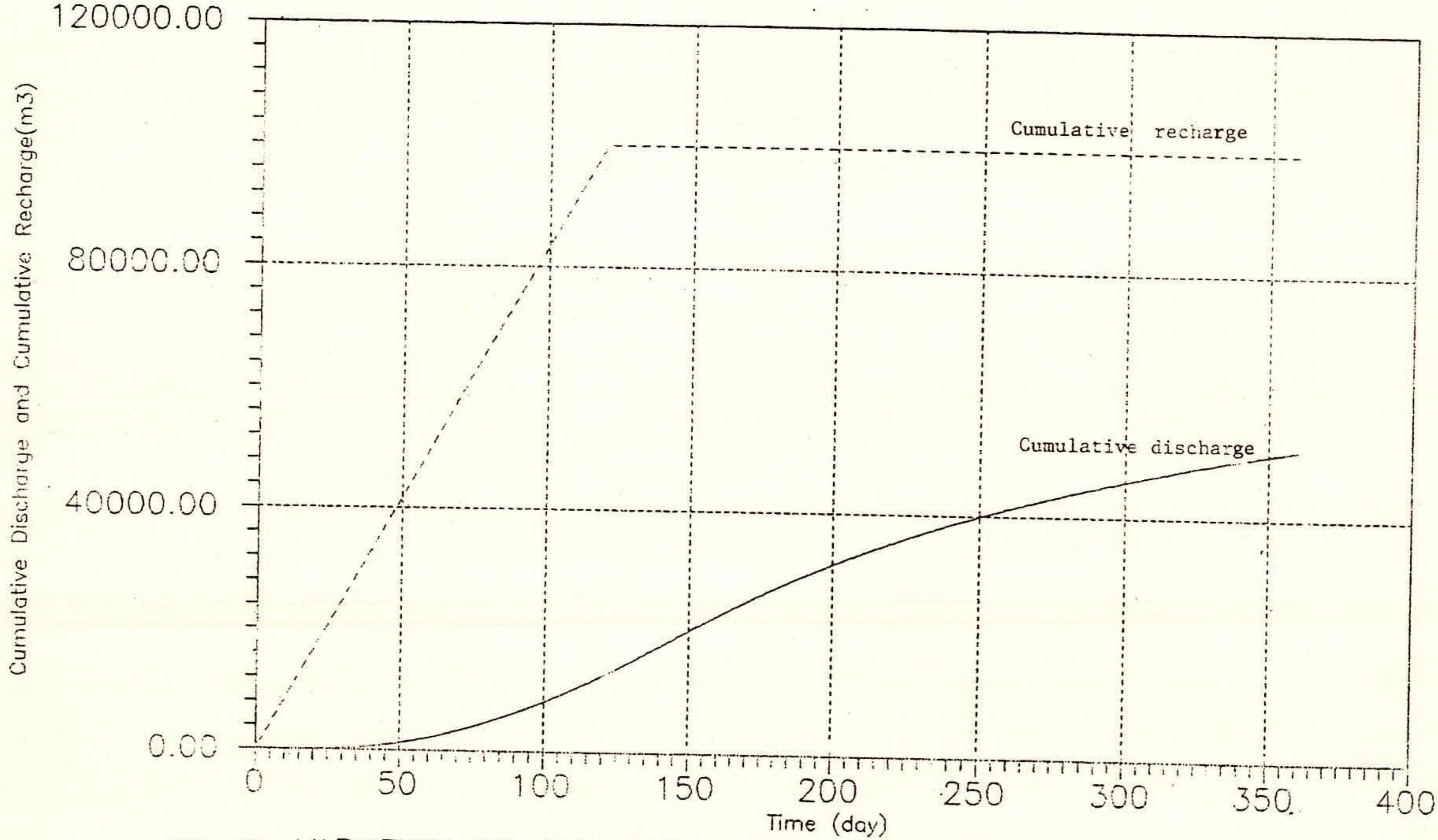


FIG. 13 VARIATION OF CUMULATIVE DISHARGE AND CUMULATIVE RECHARGE WITH TIME

(T = 200 sq.m/day, ϕ = 0.01, Size of the springs = 1m x 1m, Duration of **recharge** = 120 days, Recharge rate = 0.0016 m/day, Size of the recharge area = 2000 m x 250 m)

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```
c:        CASE OF 3 SPRINGS
c:        ==============
c:        31-03-93
c:C       THIS PROGRAMME IS TO STUDY DISCHARGE OF A GROUP OF SPRINGS
          IMPLICIT REAL*8(A-H,O-Z)
          DIMENSION DELTA(7,3,500),RECH(500),XCORD(7),YCORD(7)
          DIMENSION GW(100),GX(100),AAA(3,3),BBB(3),DISCH(3,500)
          DIMENSION AA(2,2),BB(2),SAL(7),SW(7),PREV(7)
          OPEN(UNIT=1,FILE='SPR3L.DAT',STATUS='OLD')
          OPEN(UNIT=2,FILE='SPR3L.OUT',STATUS='NEW')
          READ (1,*)SAL(7),SW(7),XCORD(7),YCORD(7)
          READ (1,*)SAL(1),SW(1),XCORD(1),YCORD(1)
          READ (1,*)SAL(2),SW(2),XCORD(2),YCORD(2)
          READ (1,*)SAL(3),SW(3),XCORD(3),YCORD(3)
          READ (1,*)T,PHI,RECHR
          READ (1,*)NTIME,NRECH
          READ(1,*)(GW(I),I=1,96)
          READ(1,*)(GX(I),I=1,96)
C         SAL(7),SW(7)=DIMENSION OF RECHARGE AREA ALONG Y AND X DIRECTION
C         SAL(I),SW(I);I=1,3  ARE DIMENSIONS OF THE SPRING OPENING
C         DELTA(EXCITATION,RESPONSE,TIME)
C         1,2,3 =REAL SPRING LOCATIONS
C         7=RECHARGE LOCATION
C         4,5,6=IMAGE SPRING LOCATIONS
          SAL(4)=SAL(1)
          SW(4)=SW(1)
          SAL(5)=SAL(2)
          SW(5)=SW(2)
          SAL(6)=SAL(3)
          SW(6)=SW(3)
          XCORD(4)=-XCORD(1)
          XCORD(5)=-XCORD(2)
          XCORD(6)=-XCORD(3)
          YCORD(4)=YCORD(1)
          YCORD(5)=YCORD(2)
          YCORD(6)=YCORD(3)
          DO 53 I=1,NTIME
53        RECH(I)=0.0
          DO 52 I=1,NRECH
52        RECH(I)=RECHR
          WRITE(2,78)
78        FORMAT(3X,'DIMENSION OF RECHARGE AREA'3X,'SAL(7)',9X,'SW(7)')
          WRITE(2,75)SAL(7),SW(7)
75        FORMAT(27X,2F10.2)
          WRITE(2,68)
68        FORMAT(2X,'DIMENSION OF SPRING AND THEIR LOCATION')
          WRITE(2,71)
71        FORMAT(6X,'SAL(1)',7X,'SW(1)',3X,'XCORD(1)',3X,'YCORD(1)')
          WRITE(2,76)SAL(1),SW(1),XCORD(1),YCORD(1)
76        FORMAT(4F10.2)
          WRITE(2,72)
72        FORMAT(6X,'SAL(2)',7X,'SW(2)',3X,'XCORD(2)',3X,'YCORD(2)')
          WRITE(2,76)SAL(2),SW(2),XCORD(2),YCORD(2)
          WRITE(2,73)
73        FORMAT(6X,'SAL(3)',7X,'SW(3)',3X,'XCORD(3)',3X,'YCORD(3)')
          WRITE(2,76)SAL(3),SW(3),XCORD(3),YCORD(3)
          WRITE(2,74)
74        FORMAT(2X,'T',7X,'PHI',5X,'RECHR',2X,'NRECH')
          WRITE(2,59)T,PHI,RECHR,NRECH
59        FORMAT(2X,2F10.2,F10.5,I3)
```

```
      DO 513 J=1,7
      DO 514 I=1,3
      PREV(J)=0.
      DO 500 N=1,NTIME
      AN=N
      CALL HGAUS(GW,GX,T,PHI,AN,SAL(J),SW(J),XCORD(J),YCORD(J),
     1 XCORD(I),YCORD(I),RES)
      DELTA(J,I,N)=RES-PREV(J)
      PREV(J)=RES
500   CONTINUE
514   CONTINUE
513   CONTINUE
C
      AAA(1,1)=DELTA(1,1,1)+DELTA(4,1,1)
      AAA(1,2)=DELTA(2,1,1)+DELTA(5,1,1)
      AAA(1,3)=DELTA(3,1,1)+DELTA(6,1,1)
      AAA(2,1)=DELTA(1,2,1)+DELTA(4,2,1)
      AAA(2,2)=DELTA(2,2,1)+DELTA(5,2,1)
      AAA(2,3)=DELTA(3,2,1)+DELTA(6,2,1)
      AAA(3,1)=DELTA(1,3,1)+DELTA(4,3,1)
      AAA(3,2)=DELTA(2,3,1)+DELTA(5,3,1)
      AAA(3,3)=DELTA(3,3,1)+DELTA(6,3,1)
C
      AA(1,1)=DELTA(1,1,1)+DELTA(4,1,1)
      AA(1,2)=DELTA(2,1,1)+DELTA(5,1,1)
      AA(2,1)=DELTA(1,2,1)+DELTA(4,2,1)
      AA(2,2)=DELTA(2,2,1)+DELTA(5,2,1)
C
C
      DO 307 J=1,NTIME
      DO 308 I=1,3
      DISCH(I,J)=0.0
308   CONTINUE
307   CONTINUE
C
C
C
      DISCH(1,1)=RECH(1)*DELTA(7,1,1)/(DELTA(1,1,1)+DELTA(4,1,1))
C
      N=2
850   CONTINUE
      SUM1=0.
      SUM2=0.
      SUM11=0.
      DO 802 NGAMA=1,N
802   SUM1=SUM1+RECH(NGAMA)*DELTA(7,1,N-NGAMA+1)
      DO 803 NGAMA=1,N-1
      SUM2=SUM2+DISCH(1,NGAMA)*(DELTA(1,1,N-NGAMA+1)
     1 +DELTA(4,1,N-NGAMA+1))
803   CONTINUE
      DISCH(1,N)=(SUM1-SUM2)/(DELTA(1,1,1)+DELTA(4,1,1))
      DO 852 NGAMA=1,N
852   SUM11=SUM11+RECH(NGAMA)*DELTA(7,2,N-NGAMA+1)-DISCH(1,NGAMA)*
     1 (DELTA(1,2,N-NGAMA+1)+DELTA(4,2,N-NGAMA+1))
      IF(SUM11.LE.0.0)GO TO 853
      GO TO 854
853   N=N+1
      GO TO 850
```

```
 854      CONTINUE
          M=2
          CALL MATIN1(AA,M)
          M=N+1
 865      CONTINUE
          SUM22=0.
          SUM1=0.
          SUM2=0.
          DO 805  NGAMA=1,M
          SUM1=SUM1+RECH(NGAMA)*DELTA(7,1,M-NGAMA+1)
 805      SUM2=SUM2+RECH(NGAMA)*DELTA(7,2,M-NGAMA+1)
          SUM4=0.
          SUM5=0.
          DO 806 NGAMA=1,M-1
          SUM4=SUM4+
     1    DISCH(1,NGAMA)*(DELTA(1,1,M-NGAMA+1)+DELTA(4,1,M-NGAMA+1))+
     2    DISCH(2,NGAMA)*(DELTA(2,1,M-NGAMA+1)+DELTA(5,1,M-NGAMA+1))
          SUM5=SUM5+
     1    DISCH(1,NGAMA)*(DELTA(1,2,M-NGAMA+1)+DELTA(4,2,M-NGAMA+1))+
     2    DISCH(2,NGAMA)*(DELTA(2,2,M-NGAMA+1)+DELTA(5,2,M-NGAMA+1))
 806      CONTINUE
          BB(1)=SUM1-SUM4
          BB(2)=SUM2-SUM5
          DISCH(1,M)=AA(1,1)*BB(1)+AA(1,2)*BB(2)
          DISCH(2,M)=AA(2,1)*BB(1)+AA(2,2)*BB(2)
          DO 866 NGAMA=1,M
          SUM22=SUM22+RECH(NGAMA)*DELTA(7,3,M-NGAMA+1)-
     1    DISCH(1,NGAMA)*(DELTA(1,3,M-NGAMA+1)+DELTA(4,3,M-NGAMA+1))-
     2    DISCH(2,NGAMA)*(DELTA(2,3,M-NGAMA+1)+DELTA(5,3,M-NGAMA+1))
 866      CONTINUE
          IF(SUM22.LE.0.0)GO TO 867
          GO TO  868
 867      M=M+1
          GO TO  865
 868      CONTINUE
          N=3
          CALL MATIN (AAA,N)
          DO 555 N=m+1,NTIME
          SUM1=0.
          SUM2=0.
          SUM3=0.
          DO 666 NGAMA=1,N
          SUM1=SUM1+RECH(NGAMA)*DELTA(7,1,N-NGAMA+1)
          SUM2=SUM2+RECH(NGAMA)*DELTA(7,2,N-NGAMA+1)
 666      SUM3=SUM3+RECH(NGAMA)*DELTA(7,3,N-NGAMA+1)
          SUM4=0.
          SUM5=0.
          SUM6=0.
          DO 777 NGAMA=1,N-1
          SUM4=SUM4+
     1    DISCH(1,NGAMA)*(DELTA(1,1,N-NGAMA+1)+DELTA(4,1,N-NGAMA+1))+
     2    DISCH(2,NGAMA)*(DELTA(2,1,N-NGAMA+1)+DELTA(5,1,N-NGAMA+1))+
     3    DISCH(3,NGAMA)*(DELTA(3,1,N-NGAMA+1)+DELTA(6,1,N-NGAMA+1))
          SUM5=SUM5+
     1    DISCH(1,NGAMA)*(DELTA(1,2,N-NGAMA+1)+DELTA(4,2,N-NGAMA+1))+
     2    DISCH(2,NGAMA)*(DELTA(2,2,N-NGAMA+1)+DELTA(5,2,N-NGAMA+1))+
     3    DISCH(3,NGAMA)*(DELTA(3,2,N-NGAMA+1)+DELTA(6,2,N-NGAMA+1))
          SUM6=SUM6+
     1    DISCH(1,NGAMA)*(DELTA(1,3,N-NGAMA+1)+DELTA(4,3,N-NGAMA+1))+
     2    DISCH(2,NGAMA)*(DELTA(2,3,N-NGAMA+1)+DELTA(5,3,N-NGAMA+1))+
```

```
     3  DISCH(3,NGAMA)*(DELTA(3,3,N-NGAMA+1)+DELTA(6,3,N-NGAMA+1))
777     CONTINUE
        BBB(1)=SUM1-SUM4
        BBB(2)=SUM2-SUM5
        BBB(3)=SUM3-SUM6
        DISCH(1,N)=AAA(1,1)*BBB(1)+AAA(1,2)*BBB(2)+AAA(1,3)*BBB(3)
        DISCH(2,N)=AAA(2,1)*BBB(1)+AAA(2,2)*BBB(2)+AAA(2,3)*BBB(3)
        DISCH(3,N)=AAA(3,1)*BBB(1)+AAA(3,2)*BBB(2)+AAA(3,3)*BBB(3)
555     CONTINUE
        CUMDIS=0.0
        CUMREC=0.0
        WRITE(2,51)
51      FORMAT(5X,'TIME',3X,'FLOW1',2X,'FLOW2',2X,'FLOW3',
     1  2X,'CFLOW',5X,'CUMDIS',5X,'STORAGE')
        DO 888 N=1,NTIME
        CUMDIS=CUMDIS+DISCH(1,N)+DISCH(2,N)+DISCH(3,N)
        CUMREC=CUMREC+RECH(N)*SAL(7)*SW(7)*0.50
        STOR=CUMREC-CUMDIS
        WRITE(2,13)N,DISCH(1,N),DISCH(2,N),DISCH(3,N),CUMDIS,CUMREC,STOR
13      FORMAT(I5,6E11.3)
888     CONTINUE
        STOP
        END
C
C
C
        SUBROUTINE ERF(X,ERFX)
        IMPLICIT REAL*8(A-H,O-Z)
        XINDEX=X
        IF(X)4,5,5
4       X=-X
5       CONTINUE
        IF(X-9.)1,2,2
1       CONTINUE
        T=1.0/(1.0+0.3275911*X)
        ERFX=1.0-(0.25482959*T-0.28449673*T**2+1.4214137*T**3
     1  -1.453152*T**4+1.061405*T**5)*DEXP(-X**2)
        GO TO 3
2       ERFX=1.
3       CONTINUE
        IF(XINDEX)6,7,7
6       ERFX=-ERFX
7       CONTINUE
        RETURN
        END
C
C
        SUBROUTINE HGAUS(GW,GX,T,PHI,AN,AL,W,X1,Y1,X2,Y2,RES)
        IMPLICIT REAL*8(A-H,O-Z)
        DIMENSION GW(100),GX(100)
        ALPHA=T/PHI
        X=X2-X1
        Y=Y2-Y1
        C1=(ALPHA*AN)**0.5
        TERM1=(W*.5+X)*.5/C1
        TERM2=(W*.5-X)*.5/C1
        TERM3=(AL*.5+Y)*.5/C1
        TERM4=(AL*.5-Y)*.5/C1
        IF(DABS(Y).LE.0.001)GO TO 200
        CALL GAUSSQ(GW,GX,TERM1,TERM3,RES1)
```

```
      SUM 1 = RES 1
      CALL GAUSSQ(GW,GX,TERM2,TERM3,RES1)
      SUM2=RES1
      CALL GAUSSQ(GW,GX,TERM1,TERM4,RES1)
      SUM3=RES1
      CALL GAUSSQ(GW,GX,TERM2,TERM4,RES1)
      SUM4=RES1
      RES=0.25/PHI*AN*(SUM1+SUM2+SUM3+SUM4)
      RETURN
200   CONTINUE
      IF(DABS(X).LE.0.001) GO TO 100
      CALL GAUSSQ(GW,GX,TERM1,TERM3,RES1)
      SUM1=RES1
      CALL GAUSSQ(GW,GX,TERM2,TERM3,RES1)
      SUM2=RES1
      RES=0.25/PHI*AN*2.0*(SUM1+SUM2)
      RETURN
100   CONTINUE
      CALL GAUSSQ(GW,GX,TERM1,TERM3,RES1)
      RES=AN*RES1/PHI
      RETURN
      END
      SUBROUTINE GAUSSQ(GW,GX,A,B,RES1)
      implicit real*8 (a-h,o-z)
      DIMENSION GW(100),GX(100)
      RES1=0.0
      DO 100 I=1,96
      Y=GX(I)
      X=A/(0.5+0.5*Y)**0.5
      CALL ERF(X,ERFX)
      F1=ERFX
      X=B/(0.5+0.5*Y)**0.5
      CALL ERF(X,ERFX)
      F2=ERFX
      RES1=RES1+(F1*F2)*GW(I)
100   CONTINUE
      RES1=RES1*0.50
      RETURN
      END
      SUBROUTINE MATIN (AAA,MMM)
      implicit real*8 (a-h,o-z)
      DIMENSION AAA(3,3),B(3),C(3)
      NN=MMM-1
      AAA(1,1)=1./AAA(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
    3 B(I)=B(I)+AAA(I,J)*AAA(J,K)
      D=0.0
      DO 4 I=1,M
    4 D=D+AAA(K,I)*B(I)
      D=-D+AAA(K,K)
      AAA(K,K)=1./D
      DO 5 I=1,M
    5 AAA(I,K)=-B(I)*AAA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
```

```
6     C(J)=C(J)+AAA(K,I)*AAA(I,J)
      DO 7 J=1,M
7     AAA(K,J)=-C(J)*AAA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AAA(I,J)=AAA(I,J)-B(I)*AAA(K,J)
      RETURN
      END
      SUBROUTINE MATIN1 (AA,MMM)
      implicit real*8 (a-h,o-z)
      DIMENSION AA(2,2),B(2),C(2)
      NN=MMM-1
      AA(1,1)=1./AA(1,1)
      DO 8 M=1,NN
      K=M+1
      DO 3 I=1,M
      B(I)=0.0
      DO 3 J=1,M
3     B(I)=B(I)+AA(I,J)*AA(J,K)
      D=0.0
      DO 4 I=1,M
4     D=D+AA(K,I)*B(I)
      D=-D+AA(K,K)
      AA(K,K)=1./D
      DO 5 I=1,M
5     AA(I,K)=-B(I)*AA(K,K)
      DO 6 J=1,M
      C(J)=0.0
      DO 6 I=1,M
6     C(J)=C(J)+AA(K,I)*AA(I,J)
      DO 7 J=1,M
7     AA(K,J)=-C(J)*AA(K,K)
      DO 8 I=1,M
      DO 8 J=1,M
8     AA(I,J)=AA(I,J)-B(I)*AA(K,J)
      RETURN
      END
```

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