TR-137

GUIDELINE FOR APPLICATION OF MUSKINGUM-CUNGE METHOD OF FLOOD ROUTING

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PREFACE

The simplified methods of channel routing enjoy a position of pre-eminence within the gamut of available routing methods. These methods are simple to formulate and use and may well account for most of the flood wave phenomena when practical applications are considered. The more refined versions simulate the diffusion wave model and therein lies their inherent strength.

A simplified method which has received increasing attention is Muskingum-Cunge method in which the routing parameters are calculated based on channel and flow characteristics. On the other hand in the conventional Muskingum method, the parameters are determined by calibration using measured inflow and outflow hydrographs only. Muskingum-Cunge method, therefore is a physically based and more realistic than the others, as the former takes into account the characteristics of the channel as well. An improved version of the methodology put forward by Ponce and Yevjevich(1978) is documented along with nine illustrative examples in the form of TESTS for its efficient use in field. Attempt has been made to simplify the use of technique up to the extent possible so as to enhance its practical applicability. However, the suggestions from the user would of course be of great help in improving the preparation of Guide Lines in future.

Some of the important features of the improved version of the methodology include the uniqueness in parameter estimation for a given flow at a site, different options for various operations like inclusion of lateral inflow and triangular shaped inflow hydrograph, wave celerity computation based on the data availability, explicit data requirement for its use etc. Incorporation of the idea of working table preparation in field would be of great help in real world applications.

The whole text is divided into two parts; the first contains the description of the teschnique including basic concepts and the second part exclusively deals with the application of the technique in field. To make it convenient to the users, an elaborative description of the use of technique in different way with its procedure for application, flow chart for data input etc., is included.

The effort in preparation of the Guide Line has been put by Sh. S.K. Mishra, Scientist C in coordination with Dr. S.M. Seth, Scientist F of the Institute.

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ABSTRACT

Muskingum-Cunge method is a simplified method for flood routing involving use for physical characteristics of channel and also flow characteristics for estimation of parameters. It provides a linear kinematic wave solution and shows flood wave attenuation due to numerical diffusion of the scheme. A unique feature of the Muskingum-Cunge method is the grid independence of the calculated outflow hydrograph, which sets it apart from other linear kinematic wave solutions. Ponce and Yevjevich(1978) have presented an improved version of the Muskingum-Cunge method. This guide line provides an interactive as well as conventional (optional) computer program for use on PCs for calibration as well as simulation using Muskingum-Cunge method, alongwith provision for incorporation of lateral inflow. Both The modes i.e. interactive and conventional, of the program include sufficient checks for obtaining best possible results. It also provides option for generating the triangular shaped inflow hydrograph for use in routing, for solutions when only information available is peak discharge, time to peak and time base. Wherever rating curve is available, it is used for wave celerity computation, otherwise, the information about the average flow of the site corresponding to the inflow peak is used for this purpose. Input data organization is illustrated for various nine tests with the help of Flow Chart and in conventional way for guidance of users.

INTRODUCTION

The modelling of the hydrologic cycle has been of great concern to the hydrologists, scientists and other research professionals since long. Flood/flows routing, one of the paramount components, is a phenomenon which comes into picture soon after the precipitation on the watershed converts into runoff and starts flowing, over the uplands or in the streams, and becomes further important when dealt with the distributed watershed modelling at a micro level.

Several techniques have come up with the age and especially the evolution of computers and rapid advancement in their memory and computational capabilities, have further helped in the development of more and more complicated theories. Routing techniques are basically classified as (i) hydrologic and (ii) hydraulic; the former being simpler while the latter more complex. Simplified routing techniques, falling under the intermediate category of the two, are based upon the suitable approximations applied to the St. Venant's equations which are the representation of full dynamic wave and these are still considered effective engineering tools in watershed hydrology. Muskingum-Cunge method of flood routing, a physically based technique, is one of the most popular simplified routing techniques world over.

This report, basically aimed at to present the guide lines for proper utilization of Muskingum-Cunge method in field, also covers an extensive review of literature, its advancements, limitations and its suitability to Indian rivers/basins. Attempts have been made to prepare in such a way that it suits not only to the researchers in general but also to the field engineers in particular.

ROUTING METHODOLOGIES

Before providing a full description of the methodlogy, it would rather be of great help to the user that some of the terms, frequently used in the text, are explained so that to familiarize them with these terms at a glance. These are:

Translation : During routing of a flood wave through the channel the origin of the outflow hydrograph, when plotted with the inflow on a grph paper, is shifted in respect of time. This shifting of the origin in respect of time is known as the translation of the hydrograph.

Diffusion: Routing phenomena in practice results into the reduction of outflow peak (for the case when no lateral inflow joins the stream reach). The reduction of the peak relative to the inflow peak is termed as the diffusion.

Dispersion: During the travel of flood wave, in practice, the base of the outflow hydrograph comes out to be larger than that of the inflow hydrograph. The spreading of the base of the outflow is termed as the dispersion.

These three can be best understood through Fig. 1.

Numerical Diffusion: The kinematic wave solution if solved comletely and with highest degree of accuracy must result in no diffusion but only in translation. However, in practice the available solution techniques don't provide the solution of required degree of accuracy. Therefore, the error involved in the application of the solution tecnique to the kinamatic wave equation, gives rise to the diffusion also known as the numerical diffusion (for more details see Appendix-A)

Amplification: the term amplification means as usually it means. i.e to raise or to increase. Usually, the routing phenomena results in diffusion i.e. reduction in peak but in some cases like when the lateral inflow joins the stream reach or the value of weighting parameter taken more than 0.5 (explained subsequently) the outflow peak comes out to be more than the inflow peak and sometimes occur earlier than the inflow peak. This behaviour is known as the amplification.

This much input to the user would help user understand the following text comfortably. It is planned to follow the said procedure from basic Muskingum method onwards.

Basic Muskingum Method:

Equations used are:

(i) Continuity equation

$$I - Q = \frac{dS}{dt} \qquad .. (i)$$

(ii) Storage-outflow equation:

$$S = K [XI + (1-X)Q] \qquad .. (ii)$$

The finite difference form of each and then their combination gives:

$$Q2 = CO.I2 + C1.I1 + C2.Q1 \qquad ..(iii)$$

where,

Q1 = outflow discharge at time step 1
Q2 = outflow discharge at time step 2
I1 = inflow discharge at time step 1
i2 = inflow discharge at time step 2

$$CO = \frac{(\Delta t/K) - 2.X}{2(1-X) + (\Delta t/ K)} \qquad ..(iva)$$

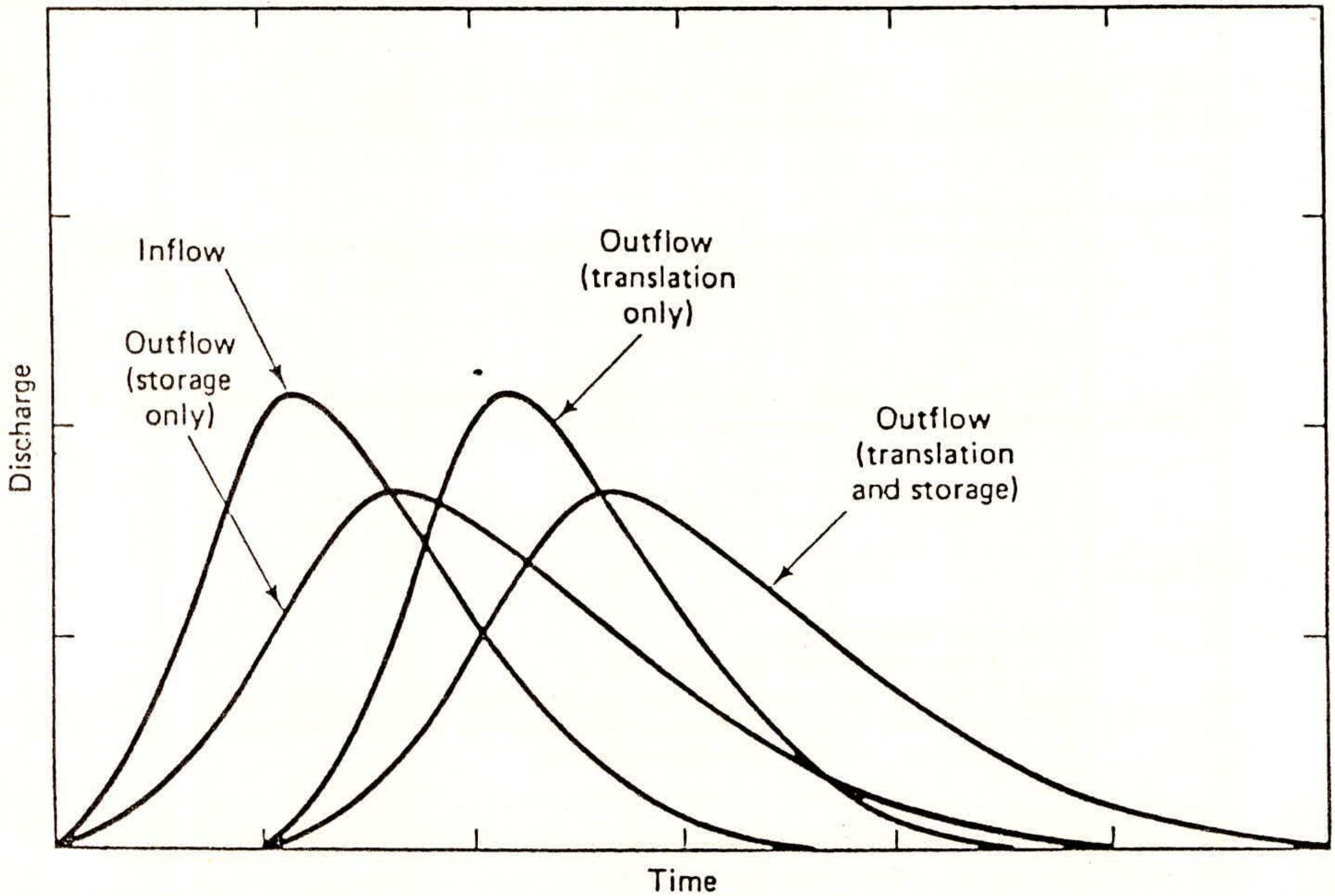

Figure ·1 Translation and storage processes in stream channel routing.

$$C1 = \frac{(\Delta t/K) + 2.X}{2(1-X) + (\Delta t/K)} \qquad ..(ivb)$$

$$C2 = \frac{2(1-X) - (\Delta t/K)}{2(1-X) + (\Delta t/K)} \qquad ..(ivc)$$

Since (CO+C1+C2) = 1 (for no lateral inflow case), the routing coefficients can be interpreted as weighting coefficients.

Kinematic Wave Method:

In differential form the continuity equation can be written as :

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad ..(v)$$

Uniform flow in the open channels is described by the Manning's or Chezy formulae. The manning equation is :

$$Q = \frac{1}{n} A.R^{2/3} S_f^{1/2} \qquad ..(vi)$$

where,

Q = the discharge
t = time step at discharge Q
A = flow area corresponding to the discharge Q
R = hydraulic radius corresponding to discharge Q
S_f = energy slope at dischrge Q

The Manning's equation can be written in the form of discharge-area rating curve, defined as follows:

$$Q = \alpha A^{\beta} \qquad ..(vii)$$

where,

$$\alpha = \frac{1}{n} \frac{S_f^{1/2}}{P^{2/3}} \qquad (viia)$$

$$\beta = \frac{5}{3}$$

P is the wetted perimeter corresponding to discharge Q, it is A/R

Differentiation of Eq. (vi) gives

$$\frac{dQ}{dA} = \beta \frac{Q}{A} = \beta V \qquad ..(viii)$$

Where, V is the mean velocity of discharge Q, and β is the rating exponent.

The multiplication of Eq.(v) and (viii) and applying the chain rule the kinematic wave equation is obtained:

$$\frac{\partial Q}{\partial t} + \left(\frac{dQ}{dA}\right)\frac{\partial Q}{\partial x} = 0 \qquad ..(ix)$$

or, alternatively

$$\frac{\partial Q}{\partial t} + (\beta \cdot V)\frac{\partial Q}{\partial x} = 0 \qquad ..(ixa)$$

Eqs.(ix) & (ixa) describes the movement of waves which are kinematic in nature. These are referred to as kinematic waves, i.e. waves for which the inertia and pressure (flow depth) gradient have been neglected. Eq. (ix) is a first order partial differential eqution. Therfore, kinematic waves travel with wave celerity dQ/dA (βV) and do not attenuate. Wave attenuation can only be described by a second order differential equation.

The absence of wave attenuation can be further explained by restoring to a mathematical arguement. Since dQ/dA is the celerity of the unsteady (i.e. wave like) Q, it can be replaced by dx/dt. Therefore, this makes Eq.(ix) as:

$$\frac{\partial Q}{\partial t} + \left(\frac{dx}{dt}\right)\frac{\partial Q}{\partial x} = 0 \qquad ..(ixb)$$

which is equal to the total derivative dQ/dt. Since the right side of the Eq.(ixb) is zero, it follows that Q remains constant in time for waves travelling with wave celerity dQ/dA.

This technique can be applied if the following inequality is satisfied.

$$\frac{trSoVo}{do} \geq 85 \qquad ..(x)$$

where,

tr = time of rise of inflow hydrograph
Vo = average velocity of the flow
do = average flow depth
So = channel bottom slope

Diffusion Waves:

Earlier, the kinematic wave equation is derived using a statement of unsteady uniform flow(i.e. friction slope is equal to bottom slope) in lieu of momentum conservation. In deriving the diffusion wave, a statement of steady non-uniform flow(i.e. friction slope is equal to water surface slope) is used instead (Fig. 2). This leads to

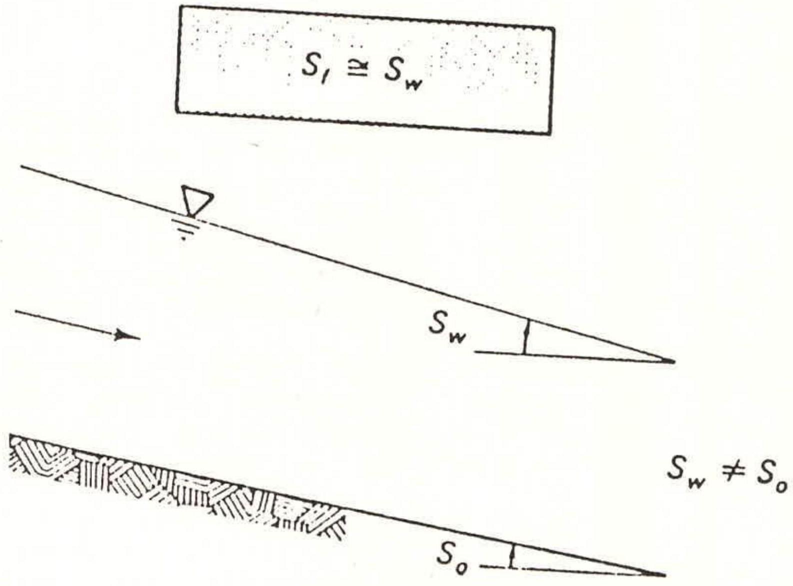

Fig. 2 : Diffusion Wave Assumption

$$Q = \frac{1}{n} AR^{2/3} (So - \frac{dy}{dx})^{1/2} \qquad ..(via)$$

in which the term So - (dy/dx) is the water surface slope. The difference between the kinematic and diffusion waves is in the term dy/dx. From a physical stand point, the term dy/dx accounts for the natural diffusion processes present in unsteady open channel flow phenomena. For the drivation of the diffusion wave theory, the user may refer to (Ponce, 1989). Here the final form of the equation is given and that is

$$\frac{\partial Q}{\partial t} + (\frac{\partial Q}{\partial A}) \frac{\partial Q}{\partial x} = (\frac{Qo}{2TSo}) \frac{\partial^2 Q}{\partial x^2} \qquad ..(xi)$$

The left side of the equation is recognised as the kinematic wave equation, with Q/ A as the wave celerity. the right side is a second order (partial differential) term that accounts for the physical diffusion effect. The coefficient of the second-order term has the units of diffusivity, being referred to as the hydraulic diffusivity, or channel diffusivity. The hydraulic diffusivity is a characteristic of the flow and channel, defined as :

$$\nu_h = \frac{Qo}{2TSo} = \frac{qo}{2So} \qquad ..(xii)$$

in which qo = Qo/T is the reference flow per unit width. From Eq. it is concluded that hydraulic diffusivity is small for steep bottom slopes (e.g., those of small mountain streams), and large for mild bottom slopes e.g., tidal rivers. The diffusion wave equation describes the movement of flood waves in a better way than Eq.(v). It falls short from describing the full momentum effects, but it does physically account for peak flow attenuation.

Eq.(xi) is a second-order parabolic partial differential equation. It can be solved analytically, leading to Hayami's diffusion analogy solution for flood waves (Hayami, 1951) or numerically with the aid of a numerical scheme for parabolic equations such as the Crank-Nicolson scheme (Crandall, 1956). An alternate approach is to match the hydraulic diffusivity with the numerical diffusion coefficient of the Muskingum scheme. This approach is the basis of the Muskingum-Cunge scheme.

Most flood waves have a small amount of physical diffusion; therefore, they are better approximated by the diffusion wave rather than the kinematic wave. for this reason, diffusion waves apply to a much wider range of practical problems than kinematic waves. Where the diffusion wave fails, only the dynamic wave can properly describe the translation and diffusion of flood waves. The dynamic wave, however, is strongly diffusive, especially for flows well in the subcritical regime(Ponce et al., 1978). In practice, most flood flows are mildly diffusive and therefore are subject to modelling with the diffusion wave.

To determine if a wave is a diffusion wave, it should satisfy the following dimensionless ineuality:

$$tr\ So\ (g/do)^{1/2} \geq 15 \quad ..(xiii)$$

where, g is the gravitational acceleration. The greater the left side of this inequality, the more likely it is that the wave is a diffusion wave.

Muskingum-Cunge Method:

Eq.(v) is a non-linear first-order partial differential equation describing the change of discharge Q in time and space. It is non-linear because the wave celerity V (or dQ/dA) varies with discharge. The non-linearity, however, is usually mild, and therefore, Eq.(v) can also be solved in a linear mode by considering the wave celerity to be constant.

The following discussion would enable to understand that there lies a striking similarity between the Muskingum and kinematic wave routing equations. Muskingum method can calculate runoff diffusion, ostensibly by varying the parameter X while certain numerical solution techniques applied to the linear kinematic wave equation produce a certain amount of numerical diffusion and/or dispersion. Furthermore, unlike the kinematic wave equation, the diffusion wave equation does have the capability to describe physical diffusion.

From these properties, Cunge(1969) concluded that the Muskingum method is a linear kinematic wave solution and that the flood wave attenuation shown by the calculation is due to the numerical diffusion of the scheme itself. The assertion can be proved by discretising the kinematic wave equation (Eq.(v)) on the x-t plane (Fig 3) in a way that parallels the Muskingum method as follows:

$$\frac{X.(Q_j^{n+1} - Q_j^{n}) + (1-X).(Q_{j+1}^{n+1} - Q_{j+1}^{n})}{\Delta t} + c\,\frac{(Q_{j+1}^{n} - Q_j^{n}) + (Q_{j+1}^{n+1} - Q_j^{n+1})}{2.\Delta x} = 0 \quad ..(1)$$

where c is the wave celerity.

Solving equation (1) for the unknown discharge leads to the following routing equation:

$$Q_{j+1}^{n+1} = C0\ Q_j^{n+1} + C1\ Q_j^{n} + C2\ Q_{j+1}^{n} \quad ..(2)$$

the routing coefficients are:

$$C0 = \frac{c.(\Delta t/\Delta x) - 2.X}{2(1-X) + c(\Delta t/\Delta x)} \quad ..(3)$$

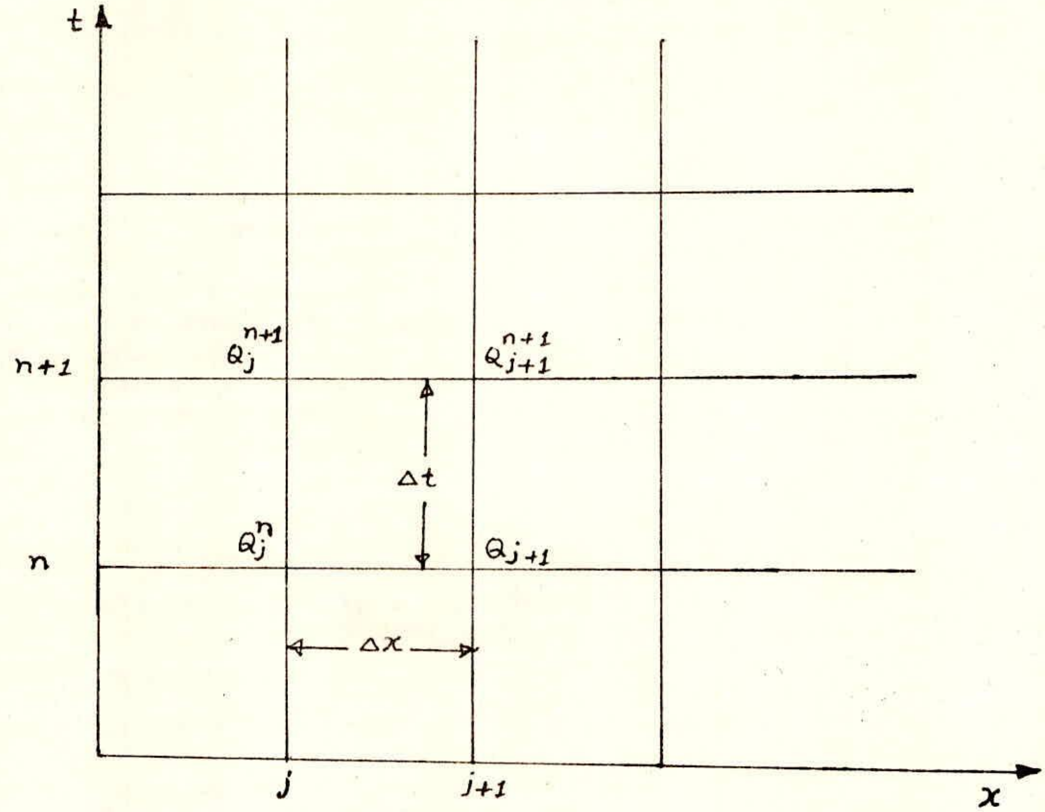

Fig. 3 : Spatial and Temporal Discretization in Muskingum Diffusion Model

$$C1 = \frac{c.(\Delta t/\Delta x) + 2.X}{2(1-X) + c(\Delta t/\Delta x)} \qquad .. (4)$$

$$C2 = \frac{2(1-X) - c.(\Delta t/\Delta x)}{2(1-X) + c(\Delta t/\Delta x)} \qquad .. (5)$$

By defining : $K = \Delta x / c$ .. (6)

it is seen that the two sets of equations (3) to (5) are comparable to Muskingum routing coefficients.

Eq. (6) confirms that K is in fact the flood wave travel time. In a linear mode 'c' is constant and equal to a reference value; in a non-linear mode, it varies with discharge. For practical applications, if the stage-discharge rating and cross-sectional geometry are available(i.e. stage-discharge-top width tables), it can calculated by using the equation.

$$C = \frac{1}{T} . \frac{\partial Q}{\partial y} \qquad .. (7)$$

In the absence of stage-discharge rating and cross-sectional geometry data, it can also be computed by

$$C = \beta V \qquad .. (7a)$$

where, V is the average velocity of the flood wave and β is the rating exponent.

In practice, the numerical diffusion can be used to simulate the physical diffusion of the channel flood wave. By expanding the discharge function $Q(J \cdot \Delta X, n \cdot \Delta t)$, the numerical diffusion coefficient of the Muskingum scheme is derived (see Appendix- A)

$$\nu_n = c. \Delta x \ (0.5 - X) \qquad .. (8)$$

in which ν_n is the numerical diffusion coefficient of the Muskingum scheme. The equations reveals the following:

(i) for X = 0.5 there is no numerical diffusion although there is numerical dispersion.

(ii) for X > 0.5 the numerical diffusion is negative, i.e. numerical amplification which explains the behaviour of the Muskingum method for this range of X-values.

(iii) for X = 0 the numerical diffusion coefficient is zero, clearly the trivial case.

A predictive equation for X can be obtained by matching the hydraulic diffusivity $\nu_h = (2qo/So)$ (where, qo is the reference discharge per unit width and So is water surface slope; for practical purposes the bed slope of the channel), with the numerical diffusion coefficient. This leads to the following:

$$X = \frac{1}{2} \left(1 - \frac{qo}{So.c.\Delta x}\right) \quad .. (9)$$

With X calculated by .(9), the Muskingum method is referred to as Muskingum-Cunge method. Using this equation, the routing parameter X can be calculated as a function of the following numerical and physical properties: (1) reach length, x; (2) reference discharge per unit width, qo; (3) kinematic wave celerity c, and (4) bottom slope, So.

The choice of reference flow has a bearing on the calculated results, although the overall effect is likely to be small. For practical applications, either an average or peak flow value can be used as reference flow. The peak flow value has the advantage that it can be readily ascertained, although a better approximation may be obtained by using the average value. The linear mode of computation is referred to as the constant parameter Muskingum-Cunge method to distinguish it from the variable parameter Muskingum-Cunge method, in which the routing parameters are allowed to vary with the flow. In linear mode, it resembles with the Muskingum method, with the difference that the routing parameters are based on measurable flow and channel characteristics.

The Muskingum-Cunge method can be applied to the situations where kinematic or diffusion waves can be applied. Therefore, the inequality criteria given for either of these methods would be applicable for its use in the field(Eq. (x) or (xiii)).

LIMITATIONS

As reported earlier the Muskingum-Cunge method of flood routing has been of great interest to the professionals. The method, named after Cunge(1969) provided the derivation of the model based upon the equality concept of hydraulic diffusion and numerical diffusion. Its numerical behaviour, however is not completely understood. For instance, there is no generally accepted spatial resolution criteria to preserve accuracy. Evidence of continuing controversy on the subject is the number of contributions (Koussis, 1976; Ponce and Theurer, 1980; and Weinmann and Laurenson, 1979), but the matter still remains to be clarified.

A direct consequence of insufficient spatial resolution is the notorious negative outflows which have been documented in certain cases (Kundzewicz, 1980 and Weinmann and Laurenson, 1979). Until the reason for the anomaly is identified, the simplified methods will suffer from a lack of credibility. This can only hamper their wide acceptance for practical channel routing applications.

Koussis(1976) was among the first to recognize the effect of insufficient spatial resolution on the hydrograph calculated by the Muskingum-Cunge method and to tie this effect to the grid size. He proposed, " in order to obtain reasonable results", the following restriction

$$\Delta t \geq \frac{2X.\Delta x}{c} \quad .. (10)$$

in which Δt is the time step; Δx is the space step; X is the Muskingum parameter an c is the flood wave celerity for the reference flow. He did not elaborate on how Eq. (10) was developed.

Weinmann and Laurenson (1981) focused on the method and its variations. They pointed out the possibility of unrealistic negative outflows but suggested that for practical purposes these negative outflows were small enough and sufficiently short lined if the following inequality is satisfied:

$$\frac{\Delta x}{c} < \frac{Tr}{2X} \quad .. (11)$$

where, Tr is the period of rise of inflow hydrograph.

The similarity between Koussis' and Weinmann and Laurenson's Criteria is apparent. A basis for comparison can be implemented if Tr is expresses in terms of Δt. Assuming that in a given case $(Tr/\Delta t) = n$, where n is an integer greater than 5, equation (11) reduces to :

$$\frac{\Delta x}{c} < \frac{n.\Delta t}{2X} \quad .. (12)$$

which indicates the Koussis' criterion is more conservative in the estimation of Δx than Weinmann and Laurenson's.

Ponce and Theurer(1980) reported the results of preliminary findings that seemed to indicate that if accuracy is to be preserved (i.e. avoidance of negative outflows), the following inequality should hold:

$$CD \geq \xi \quad .. (13)$$

where,C and D are Courant and Cell Reynold's numbers, respectively, defined as follows:

$$C = c \frac{\Delta t}{\Delta x} \quad .. (14)$$

$$D = \frac{qo}{So.c.\Delta x} \quad .. (15)$$

where, qo = reference unit-width discharge and So = equilibrium water surface slope. They recommended the value of $\xi = 0.25$ which provide satisfactory accuracy in most cases.

Kundzewicz(1980) provided the convolution method of analysis to the Muskingum diffusion model and concluded that Eq. (11) does not hold for the general case. He further stated that a search for a general condition to suppress negative outflows may prove to be useless.

CHOICE OF SPACE AND TIME STEPS

Accuracy criteria such as Eq. 10, 11 or 13 provide guide lines for the selection of space and time steps if negative outflows are to be eliminated. Following Ponce and Yeyjevich(1978), space and time steps are used together with channel and flood wave descriptors to calculate the routing parameters C and D, Eqs. 14 and 15. In turn, these, parameters are used in the routing equations (Fig. 1).

$$Q_{j+1}^{n+1} = C0\ Q_{j}^{n+1} + C1\ Q_{j}^{n} + C2\ Q_{j+1}^{n} \qquad ..\ (2)$$

Where, C0, C1 and C2 are defined as follows:

$$C0 = \frac{1 + C - D}{1 + C + D} \qquad ..\ (16a)$$

$$C1 = \frac{-1 + C + D}{1 + C + D} \qquad ..\ (16b)$$

$$C2 = \frac{1 - C + D}{1 + C + D} \qquad ..\ (16c)$$

Choice of Space step:

Given Eqs. 10, 11 and 13, the choice of space step is not altogether clear. An upper limit on Δx is certainly a necessity in any numerical procedure and the present method is no exception. Eq.(10) would seem to indicate that the upper limit should be

$$x \leq \frac{c.\Delta t}{2X} \qquad ..\ (17)$$

However, the condition suffer from a significant draw-back; it is undefined for X = 0. In Muskingum-Cunge method, X = 0 implies that Δx is fixed and Eq. 17 would no longer be applicable.

The same type of criticism holds for Eq. 11 and 13; on the other hand, is expressed in terms of D rather than X and the expression for the upper limit on Δx based on this equation is:

$$\Delta x \leq \left(\frac{qo.\Delta t}{So}\right)^{0.5} \qquad ..\ (18)$$

The question of a lower limit on Δx is also one that has led to some confusion, primarily due to the existence of a functional relationship between X and Δx as indicated by Eq. (17). Historically, parameter X has been interpreted as a weighting factor and this led Miller and Cunge (1975), among others, to suggest a lower bound for X: $X \geq 0$. The criterion has been echoed by Weinmann and Laurenson's review(1979). The lower bound can be calculated by setting from which

$$\Delta x \geq \left(\frac{qo}{So.c}\right) \qquad ..\ (19)$$

The RHS of Eq.(19) is recognized as the 'characteristic reach length' first identified by Kalinin and Milyukov in connection with the routing method bearing their name.

Further analysis and experience with the method has shown, however, that in fact there is no theoretical lower limit on Δx. Ponce and Theurer(1980) as well as others have reported experience with Δx values violating Eq. (19). However, practical computations using such a condition help dispel the myth of X being a weighting factor to be restricted in the range $0.0 \leq X \leq 0.5$.

Choice of Time Step

The criterion for selecting the time step has also led to some confusion. An upper bound on Δt is already a necessity if the solution is to remain within a reasonable numerical framework. Unlike spatial resolution, temporal resolution is a concept more readily grasped in hydrologic applications, since hydrographs are depicted in the temporal domain. Established practice is to set an upper limit on Δt such that

$$\Delta t \leq \frac{Tp}{5} \qquad ..\ (20)$$

defined earlier also. In practice, hydrographs are skewed to the right; therefore, Eq. (20) guarantees a satisfactory level of temporal resolution.

In addition to the previously given requirement of temporal resolution, there is a question as to whether the Muskingum-Cunge model is bound by the Courant numerical stability criterion usually applicable to hydraulic problems. If this is the case, then an additional constraint on Δt should be imposed.

$$\Delta t \leq \frac{\Delta x}{c} \qquad ..\ (21)$$

which follows from

$$C \leq 1 \qquad ..\ (22)$$

Practical computations, however, have shown that violating Eqs. (21) and (22) do not lead to numerical instability. This behaviour is explained by recalling that the method is essentially a finite difference formulation of the kinematic wave equation. In theory (Ponce et al., 1979), this feature provides unconditional stability with respect to the Courant condition, thereby invalidating Eq. (22). A lower limit on Δt is mainly a question of computational resources. A decision on this matter left to the individual modeller.

REMEDIES

From the preceding discussion, it follows that there is no theoretical justification for a lower limit on either Δt or Δx.

Furthermore, while there is an upper limit on Δt given by the temporal resolution requirement, a similar upper limit on Δx is not clearly discernible. Given Eq. 14, 15 and 13, Ponce and Theurer(1982) modified the Koussis' criterion in terms of C and D:

$$C + D \geq 1 \qquad .. (23)$$

In this form, the overdeterminancy of Eq. (11) is removed. In effect, for Δx=0, D = 1 and Eq.(23) reduces to $C \geq 0$. Since X=0 implies that Δx is finite (Eq.15), the condition $C \geq 0$ reduces to the trivial one.

Weinmann and Laurenson's criterion, Eq. (12), leads to (Ponce and Theurer, 1982):

$$nC + D \geq 1 \qquad .. (24)$$

and similar comments to that Koussis' apply. Ponce and Theurer's remains

$$CD \geq 0.25. \qquad .. (25)$$

MERITS

A unique feature of the Muskingum-Cunge method is the grid independence of the calculated outflow hydrograph, which sets it apart from the linear kinematic wave solutions featuring uncontrolled numerical diffusion and dispersion. If numerical dispersion is minimized(by keeping the value of Courant number, described below, as close to 1.00 as practicable), calculated outflow at downstream end of the channel reach will essentially be the same, regardless of how-many sub-reaches are used in computation. This is because X is a function of Δx and the routing coefficients C0, C1, and C2 vary with reach length.

SCOPE FOR IMPROVEMENT:

A contemplation of the past works reported in the literature reveals that there is enough scope for improving the methodology. An improved methodology would be that which contains the following capabilities in reproducing outflows:

(i) non-reproducibility of negative outflows
(ii) minimum dispersion producing ability
(iii) representability of physical diffusion
(iv) absence of amplification
(v) adaptability to the minimum set of physically measurable data
(vi) availability of explicit criteria for indirect measurements
(vii) consistency; ability to produce the same results independent of grid size.

Although it is very difficult to find all the features described above in one technique. However, an improved version of Muskingum-Cunge method (Ponce and Yevjevich,1978), described in the earlier section, up to the large extent, does have these capabilities. The improved version has therfore been adapted for use.

SALIENT FEATURES OF THE METHOD AT A GLANCE

1) This is a physically based method. It uses flow and channel characteristics i.e. channel slope, reach length, top width, peak discharge and wave celerity, in parameters' computation.
2) It is an improved version developed by Ponce and Yeyjevich (1978)
3) Data preparation for the method is simpler than that for the earlier version.
4) Explicit and unique data requirement of the method for an event and a reach under consideration is a special feature of the method.
5) The working table for different peak discharges and corresponding parameters would serve a useful guide line for the users in the field.
6) The software included in guideline ia PC based and the program can be used interactively as well as in usual way of first preparing the input data file and then run the program.
7) Uniqueness in making choices for time and space steps is further advantage over the earlier version.
8) The program includes sufficient checks for obtaining the best possible results without negative outflow.
9) For simulation of an event in the field, the working table prepared for the reach can be utilized for selecting the parameters of the method for use.
10) The program provides option for generating the triangular shape of the inflow and its use in routing, provided some preliminary information on the peak discharge, time to peak and time base is available.
11) It also includes the option for wave celerity computation by using either the rating curve of the inflow gauging site or the average flow of the site.

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PROCEDURE FOR APPLICATION

The application of the technique is described in steps as below:

(A) FOR CALIBRATION

1) Inflow and Outflow Hydrographs:

Obtain the inflow and outflow hydrographs.

2) Check for applicability of Muskingum-Cunge method

The methodology would be applicable subject to the satisfaction of any of the following critera:

$$\frac{TrSoVo}{do} \geqslant 85 \qquad .. (x)$$

or

$$Tr.So(g/do)^{1/2} \geqslant 15 \qquad .. (xiii)$$

where Tr= time of rise of inflow hydrograph; So= water surface slope (can be replaced by bed slope); do= average flow depth at inflow gauge site; g= gravitational acceleration.
If any of the condition satisfies, the method can be applied, else one may go for dynamic routing.

2) Computation of reference flow :

From the inflow hydrograph, calculate the reference flow, qo as below:

$$qo = Qp/T \qquad .. (27)$$

where, Qp = peak flow; and T = top width corresponding to the peak flow (at the inflow gauging station)

3) Computation of Wave Celerity:

(i) If Rating Curve is Available:

Calculate wave celerity for the two sites (inflow and outflow gauging sites) separately as:

$$c = \frac{1}{T}\frac{\partial Q}{\partial y} \qquad .. (28)$$

NOTE: The slope of the rating curve should be calculated in the vicinity of peak discharge of inflow or outflow.

The wave celerities, so calculated for the inflow and outflow gauging sites, are then averaged to calculate the average wave celerity of the flood in the reach.

(ii) If Rating Curve is not Available:

In this case, it is imperative to have either the cross-sectional area of the upstream site corresponding to the peak discharge of inflow or the average velocity corresponding to the inflow peak. The average velocity is calculated as:

$$V = \frac{Qp}{Ap} \qquad ..\ (19)$$

where, Qp=inflow peak discharge;Ap=flow area corresponding to Qp.

The average velocity may be measured by current meter too i.e. the velocity of flow at 0.6do from the top of water surface. Here, do is the depth of flow.

To estimate the wave celerity of the flood, the average velocity is multiplied by a factor β called rating exponent. This factor lies in the proximity of 1.67.

4) Choice of Time and space Step (Δt & Δx)

If the length of the reach =L; take $\Delta x = L/N$, N=1,2,3,4... The value of N should be chosen such that the following criteria are satisfied:

$$\Delta t \leq tr/5 \quad \text{and} \quad \Delta t \leq \Delta x/c \qquad ..\ (30)$$

where, tr = time of rise of inflow hydrograph and

$$\frac{qo \cdot \Delta t}{0.25.\ So} \geq \Delta x \geq \frac{qo}{So\ c} \qquad ..\ (31)$$

Now, calculate C (Courant Number) and D (Cell Reynold's Number) as follows:

$$C = c\ \frac{\Delta t}{\Delta x}$$

$$D = \frac{qo}{So\ c \cdot \Delta x}$$

Again check for

$$C\ D \geq 0.25$$

and

$$C + D \geq 1$$

If the conditions are not satisfied, iterate step 4 for other value of Δx and Δt until the inequalities are met with.

Obtain the value of Δt and Δx and corresponding values of C and D.

5) Optional Step

The computation made in this step are not to be utilized in further computations but to have an overall idea of the parameter values. These (K and X) or (C and D) values can be used interchangeably without any loss of accuracy. But if used (K and X) Eqs. 3, 4 and 5 should be used for coefficients' computation.

Storage coefficient

$$K = \Delta x / c$$

and the weighting parameter

$$X = \frac{1}{2}(1 - D)$$

6) Routing :

Apply the regressive form of equation for routing:

$$Q_{j+1}^{n+1} = C0.Q_{j}^{n+1} + C1.Q_{j}^{n} + C2.Q_{j+1}^{n}$$

in which,

Q_{j}^{n} & Q_{j}^{n+1} = Inflow ordinates at $n \cdot \Delta t$ and $(n+1) \cdot \Delta t$ intervals, respectively.

Q_{j+1}^{n} & Q_{j+1}^{n+1} = Outflow ordinates at $n \cdot \Delta t$ and $(n+1) \cdot \Delta t$ intervals, respectively.

$$C0 = \frac{-1+C+D}{1+C+D}$$

$$C1 = \frac{1+C-D}{1+C+D}$$

$$C2 = \frac{1-C+D}{1+C+D}$$

Calculate the outflow hydrographs regressively for one Δx.

LATERAL INFLOW INCORPORATION

For lateral inflow incorporation in the procedure, shape of the lateral inflow and its joining location, in case a major tributary joins. the reach under consideration, are to be supplied externally. Here, three cases have been implemented (1) it joins the sub-reach at the upstream with the inflow, (2) it joins the reach at the downstream of the sub-reach with the outflow, and (3) it joins the reach at the middle of the sub-reach. A routine is nested for taking this into account suitably on volumetric difference on inflow and outflow basis. The lateral inflow is distributed in the supplied form and is conveniently added with the inflow (Case 1), or with the outflow (Case 3) or divided equally among the inflow and outflow (Case 2). TESTS 7,8,9 particularly illustrate these applications.

Now, formulate the strategy in such a way so that for the next sub-reach the resulting outflow from the previous sub-reach becomes inflow to the former sub-reach. To get the final outflow hydrograph, Step 6 is to be iterated N times.

7) Check for reasonableness of the Results:
(This is left to the user to try.)
For comparing observed values Qi with computed values Qi, calculate

$$\text{Efficiency } \eta(\%) = [\text{sqrt}(1 - Fo/F1)] \times 100.0 \qquad ..\ (32)$$

where, $F0 = \sum_{i=1}^{n} (Qi - \bar{Q}i)^2$, $\quad \bar{Q}i$ = Mean of observed valuses

$F1 = \sum_{i=1}^{n} (Qi - \hat{Q}i)^2$, and $\quad \hat{Q}_i$ = *Calculated outflow*

n = Total no. of values.

8) Checks

If the resulting outflow hydrograph does not seem to be reasonable, check for

(i) the applicability criterion of the method
(ii) computation of wave celerity
(iii) reference flow; if necessary, alter reference flow
(iv) the space and time step x and t. But all changes should satisfy the necessary checks

8) Preparation of Working Table:

If the methodology works out, it cab be applied to a series of events covering a wide range of peak discharges. Prepare a working table for inflow peak discharge and corresponding values of C and D.

9) Later Applications:

Apply the method to route a flood adapting the values of C and D corresponding to the peak of inflow hydrograph. However, care should be taken that all the checks are applied before use.

NOTE: Frequent updating of the working table is desirable; as the characteristics of the channel may vary with time.

(B) FOR SIMULATION:

1) If the working table is available

(i) Divide the total length of the channel/river according to the Step 4 (described in section A)

(i) Obtain the inflow hydrograph and corresponding to the peak of the inflow hydrograph, select the parameter values of C and D.

(ii) Apply the checks, mentioned in Step 4(section A) and if

(a) the checks are not violated,

- route the flood as per Step 6(section A)
- apply the checks mentioned in Step 6 and 7(section A)
- if the methodology doesn't work out, alter the values of C and D in the vicinity of peak flood

(b) the checks are violated,

- alter the values of C and D in the vicinity of peak discharge so that the checks are reasonably satisfied

DESCRIPTION OF THE FLOW CHART

NOTE: THe flow chart may not be confused with flow chart for the program agorithm rather it is for showing how the data has been organised for various TESTs i.e T1,T2,T3,T4,T5,T6,T7,T8,T9.

Further, it is for the option when the data is to be supplied through the input files which are MCDF1.DAT and MCDF2.DAT.

Symbols Used : Symbols used in alphabatic form are: F,T,S and D defined as below:

****

F1 : For the file MCDF1.DAT from which the data is to be read.

F2 : For the file MCDF2.DAT from which the additional data is to be read.

****

S1 & S2 are the data to be supplied on Screen only for lateral inflow.

NOTE: DATA FILE IF F1=1, DATA WILL BE READ FROM FILE 'MCDF1.DAT' (following is the total data which should be read from MCDF1.DAT)

****

DATA NO.	DESCRIPTION	VALUES SUPPLIED
D1 :	PEAK DISCHARGE	1000.00
D2 :	TIME TO PEAK	5.00
D3 :	CHANNEL SLOPE	0.000868
D4 :	OPTION FOR READING DATA (1:if wave celerity computed using rating curve 2: if wave celerity computed using average velocity)	1 OR 2
D5 :	FLOW AREA	400.00
D6 :	BETA	1.60
D7 :	inverse SLOPE OF RATING CURVE	300.00
D8 :	TOP WIDTH	100.00
D9 :	LENGTH OF THE CHANNEL	14.40
D10:	OPTION FOR FLOW (1: for triangular shaped flow 2: for regular shaped flow)	1 OR 2
D11:	BASE FLOW	0.00
D12:	TIME BASE OF TRIANGULAR HYD.	10.00
D13:	NUMBER OF REACHES (The no. of reaches will depend upon the min. & max. reach length criterion Eq.)	1
D14:	TIME INTERVAL	1.0

****

end of file MCDF1.DAT

```
NOTE: DATA FILE  IF F2=1, DATA WILL BE READ FROM FILE 'MCDF2.DAT'
(The following is the data which should be read from MCDF2.DAT)
*****************************************************************
DATA     DESCRIPTION                                   VALUES
NO.                                                    SUPPLIED
*****************************************************************
D15: NP                                                   9
     (Total no. of discharge ordinates
      excluding zero hour ordinate)

D16:  QIN                                  0. 200. 400. 600. 800.
     (inflow ordinates including zero   1000. 800. 600. 400. 200.
      hour ordinate)

D17: (QOUT(I),I=0,NP)                      0. 250. 500. 700. 900.
     (Outflow ordinates including zero  1200. 1000. 700. 520. 220.
      hour ordinate)

D18: DTHL                                                1.0
     (Time interval of the lateral
      inflow ordinates; preferably same
      as of inflow & outflow)

D19: NORDL                                               8
     (Total no. of ordinates excluding
      that of zero hour)

D20: XLAT                                   0. 100. 300. 500. 400.
     (Hypothetical ordinates of lateral        300. 200. 100. 0.
      inflow for defining its shape)

D21: LLN                                            1 or 2 or 3
     (It may have one value out of 1,2 & 3)
      1 : if lateral inflow joins the sub-
          reach at its upstream
      2 : if lateral inflow joins the sub-
          reach in the middle
      3 : if lateral inflow joins the sub-
          reach at the end of subreach
*****************************************************************
                                             end of file MCDF2.DAT
```

SEQUENTIAL DATA INPUT FOR VARIOUS TESTS
****

The name of the program as given is MCDF.FOR. This is to be first loaded on an IBM compatible PC. With the help of a FORTRAN compiler, the file MCDF.FOR is to be compiled so as to prepare an executable version of the program. Now, how to work with the program is as below. Here, it is important to note that the description given for help will not be displayed by the program.

(the following to be done to start the program)

C:\>MCDF <return>

(Description: type MCDF and then press return)

(soon after the following will be displayed on the screen:)

```
THANK YOU FOR RUNNING muskingum-cunge PROGRAM
THIS PROGRAM SOLVES THE STREAM CHANNEL ROUTING PROBLEM
BY MUSKINGUM-CUNGE METHOD
DO  YOU WANT TO SUPPLY THE DATA ON THE SCREEN(1) OR WANT TO  READ
FROM INPUT FILE(1/2)?:
2
```

(this is to be supplied by the user for tests T1 through T9 illustrated in this guide line)

(Now, data sequence (to be supplied through MCDF1.DAT) for TESTS.
Note: This Table will not be displayed on screen.it is only for guide)

T1	T2	T3	T4	T5	T6	T7	T8	T9
D1	D1	D1	D1	D1	D1	D1	D1	D1
D2	D2	D2	D2	D2	D2	D2	D2	D2
D3	D3	D3	D3	D3	D3	D3	D3	D3
D4	D4	D4	D4	D4	D4	D4	D4	D4
D5	D5	*	*	D5	*	D5	D5	D5
D6	D6	*	*	D6	*	D6	D6	D6
*	*	D7	D7	*	D7	*	*	*
D8	D8	D8	D8	D8	D8	D8	D8	D8
D9	D9	D9	D9	D9	D9	D9	D9	D9
D10	D10	D10	D10	D10	D10	D10	D10	D10
D11	*	D11	*	*	*	*	*	*
D12	*	D12	*	*	*	*	*	*
D13	D13	D13	D13	D13	D13	D13	D13	D13
D14	D14	D14	D14	D14	D14	D14	D14	D14

* indicates that the particular data is not required

(Now, after reading the above data for a particular test (only one test is permissible at a time), the following will be displayed on the screen:)

DO YOU WANT TO STORE THE DATA IN A FILE ? (1/2)

2
(Description: if the user types 1 on the screen, the data will be stored in file MCDF3.DAT or if typed 2 then no action will be taken by the prgram)

(Here afterwards, the data from the file MCDF2.DAT will be taken as input to the program. The program will move for this additional data only if the value supplied for data no. D10, described above, is 2 otherwise it will jump to the print option for output, given below. It is again to note that the following Table will not be displayed, it is given only for illustration)

T1	T2	T3	T4	T5	T6	T7	T8	T9
*	D15	*	D15	D15	D15	D15	D15	D15
*	D16	*	D16	D16	D16	D16	D16	D16

* indicates that the particular data is not required

(After reading this much data, the following will be displayed asking for the incorporation of lateral inflow)

DO YOU WANT TO COMPUTE AND INCORPORATE LATERAL INFLOW (1) OR NOT; (1/2)

1
(If the user wishes to incorporate lateral inflow, then 1 should be typed else 2 for no. If 2 is supplied, the program would move to the print option for output. Here 1 is supplied as an exmaple)

```
*****************************************************************
T1      T2       T3       T4       T5       T6       T7       T8       T9
*****************************************************************

*       *        *        *        D17      D17      D17      D17      D17
*****************************************************************
```

(Now, the program will analyse for volumetric difference of inflow and outflow. If it is less than 5%, the option for lateral inflow will no more be valid and the program would move for print option for output else it would search for further data)

```
*****************************************************************
T1      T2       T3       T4       T5       T6       T7       T8       T9
*****************************************************************

*       *        *        *        D17      D17      D17      D17      D17
*       *        *        *        D18      D18      D18      D18      D18
*       *        *        *        D19      D19      D19      D19      D19
*       *        *        *        D20      D20      D20      D20      D20
*       *        *        *        D21      D21      D21      D21      D21
*****************************************************************
```

* indicates that the particular data is not required

(after reading the data as above, the following will be displayed. Earlier at some places it is mentioned that the program will move/jump to the print option for output. This is the step to which the program would move.)

(Note: if for the option DO YOU WANT TO STORE THE DATA, 1 has been supplied, the data of MCDF2.DAT will be stored in file MCDF4.DAT)

DO YOU WANT YOUR OUTPUT TO THE SCREEN (Y/N)?:
N
(If for this option, the user types Y, then the result will be displayed on the screen else if N, the result will be stored in the file MCDF.OUT)

(Afterwards, the following will be displayed indicating that the program has succesfully worked. If it is not displayed, indicates some error either with data or with the loading of the program)

THANK YOU FOR RUNNING THE PROGRAM. PLEASE CALL AGAIN

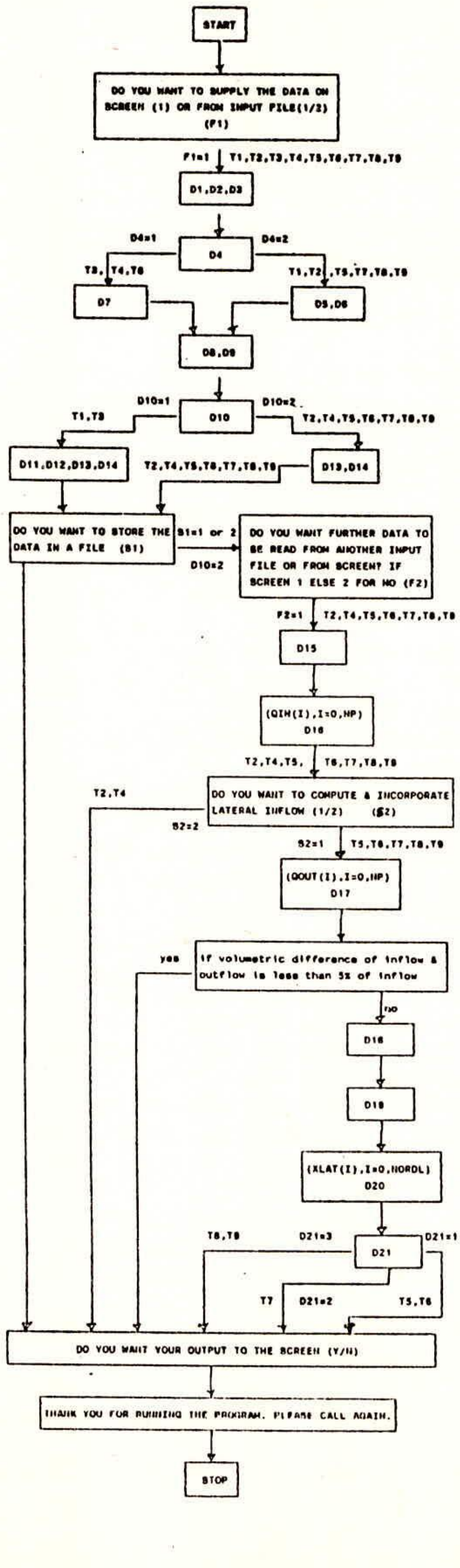
(Here is the end of the execution. Now the curser will return to its original mode and that is)

```
C:\>
*****************************************************************
```

NOTE: HOW TO USE INTERACTIVE MODE OF THE PROGRAM?

Further to note and very important in a sense is that the above does not describe how to utilise the interactive mode of the program. For this, the sequence of the input data for various TESTS remains unchanged but differs in the way that all the input data are to be supplied interactively as done in some cases, mentioned above for inputing the data say for example: DO YOU WANT TO STORE THE DATA(1/2)? and the typed no. was 2 in the second line. In the same way, the input data are to be supplied on the screen itself rather in the input file(s). This is quite simple to understand and hence left to the user to try.

FLOW CHART FOR DATA INPUT TO VARIOUS TESTS



```
C               ***********************************************
C               *PROGRAM FOR ROUTING BY MUSKINGUM-CUNGE METHOD*
C               *                                             *
C       ***     *                                             *     ***
C               *         NATIONAL INSTITUTE OF HYDROLOGY     *
C               *            ROORKEE-247 667 U P, INDIA       *
C               ***********************************************
C
C       SOURCE: PONCE, V.M.(1989),'ENGINEERING HYDROLOGY',PRENTICE HALL,
C               ENGLEWOOD CLIFFS, NEW JERSEY 07632.
C
C This is a modified version including the following above and over the
C source:
C       1. conventional mode of data input (optional)
C       2. incorporation of regular shaped inflow (optional)
C       3. incorporation of lateral inflow (optional
C       4. options for the joining of lateral inflow with the sub-reach
C       5. application of two more tests for better output
C       6. provision for input storage on hard disk (optional)
C
C       DEFINITIONS OF COMMONLY USED VARIABLES
C       **************************************
C
C       INPUT = OPTION FOR THE MODE OF SUPPLYING INPUT DATA(1 OR 2)
C       QIN   = OBSERVED INFLOW HYDROGRAPH ORDINATES (CUMECS)(OR COMPUTED
C               USING TRIANGULAR SHAPE)
C       QOUT  = OUTFLOW HYDROGRAPH ORDINATES (CUMECS)
C       QPEK  = PEAK DISCHARGE (CUMECS)
C       TPEK  = TIME TO PEAK IN HOURS
C       BSLO  = BASE FLOW (CUMECS)
C       NOPT  = OPTION NO. FOR WAVE CELERITY COMPUTATION(1 OR 2)
C       FARE  = FLOW AREA IN Sq. m (CORRESPONDING TO PEAK FLOW)
C       BETA  = RATING EXPONENT FOR WAVE CELERITY COMPUTATION(DIMENSIONLESS)
C       SLART = inverse SLOPE OF THE RATING CURVE (FOR WAVE CELERITY
C               COMPUTATION)
C       TWQP  = TOP WIDTH OF THE CHANNEL CORRESPONG TO PEAK FLOW)
C       DXKM  = TOTAL LENGTH OF THE REACH (Km)
C       MOPT  = OPTION NO. FOR TRIANGULAR INFLOW HYDROGRAPH(1 OR 2)
C       QBAS  = BASE FLOW IN CUMECS
C       TBAS  = TIME BASE OF THE TRIANGULAR SHAPED INFLOW HYDROGRAPH( HOURS)
C       ANAME = AUXILIARY NAME FOR DEFINITIONS OF THE INPUT DATA
C       NRECH = NUMER OF REACHES(TO BREAK THE TOTAL REACH LENGTH
C               TO SATISFY CRITERIA)
C       DTH   = TIME INTERVAL OF INFLOW ORDINATES
C       DTHL  = TIME INTERVAL OF LATERAL FLOW ORDINATES
C       LATOPT= OPTION FOR LATERAL INFLOW INCORPORATION(1 OR 2)
C       LLN   = OPTION FOR THE LOCATION WHERE LATERAL FLOW MEETS STREAM(1,2,3)
C       NUL   = VARIABLE, IF %VOLUMETRIC LATERAL INFLOW < 5, MAKES THE
C               LATERAL INFLOW OPTION INVALID
C       NADDI = VARIABLE TO CHANGE THE 'INPUT' VALUES FOR SUPPLYING LATER DATA
C       IPRINT= OPTION FOR SAVING THE INPUT INFORMATION ON HARD DISK(1 OR 2)
C
C***********************************************************************
C       NOTE: INFORMATION ON FILES:
C
C       MCDF1.DAT : INPUT DATA FILE REQUIRED BY THE PROGRAM NECESSARILY
```

```
C       MCDF2.DAT : INPUT DATA FILE CONTAINING OPTIONAL DATA REQUIREMENT
C       MCDF3.DAT : COPY OF MCDF1.DAT BUT ALSO STORES THE DATA SUPPLIED
C                   ON THE SCREEN
C       MCDF4.DAT : COPY OF MCDF2.DAT BUT ALSO STORES THE DATA SUPPLIED
C                   ON THE SCREEN
C       MCDF.OUT  : MAIN OUTPUT FILE CONTAINING THE RESULTS
C*****************************************************************************
        PARAMETER (NZ=200)
        DIMENSION TIME(0:NZ),QIN(0:NZ),QINO(0:NZ)
        DIMENSION QOUT(0:NZ),QOUTO(0:NZ),YLAT(0:NZ)
        DOUBLE PRECISION QIN,QINO,QOUT,QOUTO
        CHARACTER*6 A
        CHARACTER*80 ANAME
C**************************************************
        OPEN(UNIT=7,FILE='MCDF1.DAT',STATUS='UNKNOWN')
        OPEN(UNIT=8,FILE='MCDF2.DAT',STATUS='UNKNOWN')
        OPEN(UNIT=9,FILE='MCDF3.DAT',STATUS='UNKNOWN')
        OPEN(UNIT=11,FILE='MCDF4.DAT',STATUS='UNKNOWN')
        OPEN(UNIT=10,FILE='MCDF.OUT',STATUS='UNKNOWN')
C**************************************************
        WRITE(*,*)'THANK YOU FOR RUNNING muskingum-cunge PROGRAM'
        WRITE(*,*)'THIS PROGRAM SOLVES THE STREAM CHANNEL ROUTING PROBLEM'
        WRITE(*,*)'BY THE MUSKINGUM-CUNGE METHOD'
        WRITE(*,*)'DO YOU WANT TO SUPPLY THE DATA ON THE SCREEN (1)',
     1' OR WANT TO READ FROM INPUT FILE (1/2)?:'
        READ(*,*) INPUT
        WRITE(10,888)
        WRITE(10,*)'*****************************************************'
        WRITE(10,*)'          ROUTING BY *MUSKINGUM-CUNGE* METHOD        '
        WRITE(10,*)'*****************************************************'
        WRITE(10,888)
888     FORMAT(//)
        IF(INPUT.EQ.1)THEN
10      WRITE(*,*)'ENTER THE PEAK DISCHARGE (M3/S):  '
        READ(*,*) QPEK
        IF(QPEK.LE.0.) THEN
        WRITE(*,*)'PEAK DISCHARGE CANNOT BE ZERO OR NEGATIVE.',
     1  ' PLEASE TRY AGAIN.'
        GO TO 10
        ENDIF
12      WRITE(*,*)'ENTER THE TIME-TO-PEAK (H):  '
        READ(*,*) TPEK
        IF(TPEK.LE.0.)THEN
        WRITE(*,*)'TIME-TO-PEAK CANNOT BE ZERO OR NEGATIVE.',
     1  ' PLEASE TRY AGAIN.'
        GO TO 12
        ENDIF
14      WRITE(*,*)'ENTER THE CHANNEL BED SLOPE (M/M):  '
        READ(*,*) BSLO
        IF(BSLO.LE.0)THEN
        WRITE(*,*)'BED SLOPE CANNOT BE ZERO OR NEGATIVE.',
     1' PLEASE TRY AGAIN.'
        GOTO 14
        ENDIF
72      WRITE(*,*)'OPTIONS: ENTER NOPT=1 IF WAVE CELERITY BASED ON',
     1' RATING CURVE; ENTER NOPT=2, IF CELERITY FROM AVERAGE VELOCITY'
        READ(*,*)NOPT
```

```
      IF(NOPT.LE.0)THEN
      WRITE(*,*)'NOPT  CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
      GOTO 72
      ENDIF
      IF(NOPT.EQ.2) THEN
15    WRITE(*,*)'ENTER THE FLOW AREA CORRESPONDING TO THE PEAK',
     1 ' DISCHARGE'
      READ(*,*) FARE
      IF(FARE.LE.0)THEN
      WRITE(*,*)'FLOW AREA CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
      GOTO 15
      ENDIF
17    WRITE(*,*)'ENTER THE RATING EXPONENT (BETA):  '
      READ(*,*) BETA
      IF(BETA.LE.0)THEN
      WRITE(*,*)'EXPONENT BETA CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
      GOTO 17
      ENDIF
      ELSEIF(NOPT.EQ.1)THEN
73     WRITE(*,*)'inverse SLOPE OF THE RATING CURVE IN THE',
     1 ' VICINITY OF PEAK DISCHARGE:  '
      READ(*,*) SLRAT
      IF(SLRAT.LE.0)THEN
      WRITE(*,*)'SLOPE OF RATING CURVE CANNOT BE ZERO OR NEGATIVE.',
     1  ' PLEASE TRY AGAIN.'
      GOTO 73
      ENDIF
      ENDIF
16    WRITE(*,*)'ENTER THE CHANNEL TOP WIDTH CORRESPONDING TO',
     1' THE PEAK DISCHARGE (M):  '
      READ(*,*) TWQP
      IF(TWQP.LE.0)THEN
      WRITE(*,*)'CHANNEL TOP WIDTH CANNOT BE ZERO OR NEGATIVE.',
     1' PLEASE TRY AGAIN.'
      GOTO 16
      ENDIF
18    WRITE(*,*)'ENTER THE TOTAL REACH LENGTH (KM):  '
      READ(*,*) DXKM
      IF(DXKM.LE.0)THEN
      WRITE(*,*)'TOTAL REACH LENGTH CANNOT BE ZERO OR NEGATIVE.',
     1' PLEASE TRY AGAIN.'
      GOTO 18
      ENDIF
71    WRITE(*,*)'FOR THE OPTIONS (1) IF TRIANGULAR; MOPT=1',
     1 ' (2) IF REGULAR SHAPED; MOPT=2 :  '
      READ(*,*) MOPT
      IF(MOPT.LE.0.) THEN
      WRITE(*,*)' MOPT CANNOT BE ZERO OR NEGATIVE.',
     1  ' PLEASE TRY AGAIN.'
      GO TO 71
      ENDIF
      IF(MOPT.EQ.1)THEN
```

```
11      WRITE(*,*)'ENTER THE BASEFLOW (M3/S):  '
        READ(*,*) QBAS
        IF(QBAS.LT.0.) THEN
        WRITE(*,*)'BASE FLOW CANNOT BE NEGATIVE.  PLEASE TRY AGAIN.'
        GO TO 11
        ENDIF
13      WRITE(*,*)'ENTER THE TIME BASE (H):  '
        READ(*,*) TBAS
        IF(TBAS.LE.TPEK)THEN
        WRITE(*,*)'TIME BASE CANNOT BE LESS THAN OR EQUAL TO',
     1 ' TIME-TO-PEAK.  PLEASE TRY AGAIN.'
        GOTO 13
        ENDIF
        ENDIF
75      WRITE(*,*)'ENTER THE NUMBER OF REACHES :  '
        READ(*,*) NRECH
        IF(NRECH.LE.0)THEN
        WRITE(*,*)'REACH LENGTH CANNOT BE ZERO OR NEGATIVE.',
     1' PLEASE TRY AGAIN.'
        GOTO 75
        ENDIF
19      WRITE(*,*)'ENTER THE TIME INTERVAL (HR):  '
        READ(*,*) DTH
        IF(DTH.LE.0)THEN
        WRITE(*,*)'TIME INTERVAL CANNOT BE ZERO OR NEGATIVE.',
     1' PLEASE TRY AGAIN.'
        GO TO 19
        ELSEIF((TPEK/DTH).LT.5.)THEN
        WRITE(*,*)'CAUTION.  RATIO OF TIME-TO-PEAK TO TIME',
     1' INTERVAL IS LESS THAN 5.','THE RESULTS MAY NOT BE ACCURATE.'
        ENDIF
        ELSEIF(INPUT.EQ.2)THEN
        READ(7,169)QPEK,ANAME
        READ(7,169)TPEK,ANAME
        READ(7,167)BSLO,ANAME
        READ(7,168)NOPT,ANAME
                IF(NOPT.EQ.2)THEN
        READ(7,169)FARE,ANAME
        READ(7,169)BETA,ANAME
                ELSEIF(NOPT.EQ.1)THEN
        READ(7,169)SLRAT,ANAME
                ENDIF
        READ(7,169)TWQP,ANAME
        READ(7,169)DXKM,ANAME
        READ(7,168)MOPT,ANAME
                IF(MOPT.EQ.1)THEN
        READ(7,169)QBAS,ANAME
        READ(7,169)TBAS,ANAME
                ENDIF
        READ(7,168)NRECH,ANAME
        READ(7,169)DTH,ANAME
167     FORMAT(F12.6,A67)
168     FORMAT(I2,A77)
169     FORMAT(F8.2,A71)
        ENDIF
        WRITE(*,*)'DO YOU WANT TO STORE THE DATA IN A FILE? (1/2)'
        READ(*,*) IPRINT
```

```
        IF(IPRINT.EQ.1)THEN
        WRITE(9,170)QPEK
        WRITE(9,171)TPEK
        WRITE(9,172)BSLO
        WRITE(9,173)NOPT
                IF(NOPT.EQ.2)THEN
        WRITE(9,174)FARE
        WRITE(9,175)BETA
                ELSEIF(NOPT.EQ.1)THEN
        WRITE(9,176)SLRAT
                ENDIF
        WRITE(9,177)TWQP
        WRITE(9,178)DXKM
        WRITE(9,179)MOPT
                IF(MOPT.EQ.1)THEN
        WRITE(9,180)QBAS
        WRITE(9,181)TBAS
                ENDIF
        WRITE(9,182)NRECH
        WRITE(9,183)DTH
170     FORMAT(F8.2,57X,'PEAK DISCHARGE')
171     FORMAT(F8.2,59X,'TIME TO PEAK')
172     FORMAT(F12.6,54X,'CHANNEL SLOPE')
173     FORMAT(I2,54X,'OPTION FOR READING DATA')
174     FORMAT(F8.2,62X,'FLOW AREA')
175     FORMAT(F8.2,67X,'BETA')
176     FORMAT(F8.2,42X,'INVERSE SLOPE OF RATING CURVE')
177     FORMAT(F8.2,62X,'TOP WIDTH')
178     FORMAT(F8.2,50X,'LENGTH OF THE CHANNEL')
179     FORMAT(I2,62X,'OPTION FOR FLOW')
180     FORMAT(F8.2,62X,'BASE FLOW')
181     FORMAT(F8.2,43X,'TIME BASE OF TRIANGULAR HYD.')
182     FORMAT(I2,60X,'NUMBER OF REACHES')
183     FORMAT(F8.2,58X,'TIME INTERVAL')
        ENDIF
C
        IF(MOPT.EQ.2)THEN
        WRITE(*,*)'DO YOU WANT FURTHER DATA TO BE READ FROM ANOTHER',
     1' INPUT FILE OR FROM SCREEN? IF FROM FILE (1) ELSE (2) FOR NO:'
        READ(*,*) NADDI
        IF(NADDI.EQ.1)THEN
        INPUT=2
        ELSEIF(NADDI.EQ.2)THEN
        INPUT=1
        ENDIF
        ENDIF
C***************
C   OPTIONS
C***************
        IF(MOPT.EQ.1) THEN
        CALL OPT1(QIN,QOUT,NZ,QPEK,QBAS,TPEK,TBAS,DTH)
        ELSEIF(MOPT.EQ.2)THEN
        CALL OPT2(QIN,QOUTO,DTH,NP,NZ,LLN,YLAT,INPUT,IPRINT)
        ENDIF
C****************************************
```

```
C  CALCULATION OF ROUTING PARAMETERS
C*************************************
        DTS= DTH*3600.
        DXM= DXKM*1000./FLOAT(NRECH)
        IF(NOPT.EQ.2)THEN
        CPEK= BETA*QPEK/FARE
        ELSEIF(NOPT.EQ.1)THEN
        CPEK= SLRAT/TWQP
        ENDIF
        COU= CPEK*DTS/DXM
        REYO=TWQP*BSLO*CPEK*DXM
        REY= QPEK/REYO
C***************************
C    CHECKS TO BE APPLIED
C***************************
        WRITE(10,*)'******************'
        WRITE(10,*)'CHECKS APPLICATION'
        WRITE(10,*)'******************'
        RR=COU+REY
        IF(RR.GT.1.000)THEN
        WRITE(10,*)'(1)RESOLUTION CRITERIA OF C+D>1 CHECKED'
        ELSE
        WRITE(10,*)'(1)RESOLUTION CRITERIA OF C+D>1 NOT FULFILLED'
        ENDIF
        DXMN=QPEK/(BSLO*CPEK*TWQP)
        DXMX=(QPEK*DTS)/(0.25*BSLO*TWQP)
        IF(DXM.LT.DXMN.OR.DXM.GT.DXMX)THEN
        WRITE(10,*)'(2) SUB-REACH LENGTH CRITERIA NOT SATISFIED'
        WRITE(10,*)'try to ALTER THE NUMER OF REACHES IF POSSIBLE'
        WRITE(10,366)DXMN,DXMX,DXM
366     FORMAT(/1X,'MIN. REACH LENGTH=',F8.1,2X,'MAX. REACH LENGTH'
     1' LENGTH =',F8.1,' GIVEN LENGTH= 1'/)
        ELSE
        WRITE(10,*)'(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED'
        ENDIF
        DTSM=DXM/CPEK
        IF(DTS.GT.DTSM)THEN
        WRITE(10,*)'(3) DELTA t CRITERIA NOT SATISFIED'
        WRITE(*,*)'RESULTS MAY NOT BE ACCURATE FOR THIS TIME INTERVAL'
        ELSE
        WRITE(10,*)'(3) DELTA t CRITERIA SATISFIED'
        ENDIF
        WRITE(10,*)
C************************************************************************
C                STREAM CHANNEL ROUTING CALCULATIONS
C************************************************************************
        C= 1./(1.+COU+REY)
        CO= C*(-1+COU+REY)
        C1= C*(1.+COU-REY)
        C2= C*(1.-COU+REY)
        DO N=1,NRECH
        DO 50 J=1,NZ
        IF(LLN.EQ.1.OR.LLN.EQ.2.OR.LLN.EQ.3)THEN
        IF(LLN.EQ.1)QOUT(J)= CO*(QIN(J)+YLAT(J)) +
     1C1*(QIN(J-1)+YLAT(J-1)) + C2*QOUT(J-1)
        IF(LLN.EQ.2)QOUT(J)= CO*(QIN(J)+0.5*YLAT(J)) +
```

```
     1C1*(QIN(J-1)+0.5*YLAT(J-1)) + C2*QOUT(J-1)
        ELSE
        QOUT(J)= CO*QIN(J) + C1*QIN(J-1) + C2*QOUT(J-1)
        ENDIF
50      CONTINUE
        DO  J=1,NZ
        IF(LLN.EQ.2)QOUT(J)=QOUT(J)+0.5*YLAT(J)
        IF(LLN.EQ.3)QOUT(J)=QOUT(J)+YLAT(J)
        ENDDO
C***********************************
C    PROPER ORDERING OF THE OUTPUT
C***********************************
        IF(MOPT.EQ.1)THEN
        NP=0
        DO 60 J=1,NZ
        NP= NP+1
        IF((QOUT(J)-QBAS).LT.0.001.AND.J.GT.15)THEN
        GOTO 70
        ENDIF
60      CONTINUE
        ELSEIF(MOPT.EQ.2)THEN
        QBAS=QIN(0)
        KK=0
        DO J=NP,NZ,1
        IF((QOUT(J)-QBAS).LT.0.001.AND.J.GT.15)THEN
        NP=NP+KK
        GOTO 70
        ENDIF
        KK=KK+1
        ENDDO
        ENDIF
70      CONTINUE
        IF(N.LT.NRECH)THEN
        DO J=0,NZ
        IF(N.EQ.1)QINO(J)=QIN(J)
        QIN(J)=QOUT(J)
        ENDDO
        ENDIF
        ENDDO
        IF(NRECH.GT.1.AND.MOPT.EQ.2)THEN
        DO J=0,NZ
        QIN(J)=QINO(J)
        ENDDO
        ENDIF
C**********************
C   PRINTING RESULTS
C**********************
        DO 80 J=1,NP
80      TIME(J)= TIME (J-1) + DTH
        WRITE(*,*)'DO YOU WANT YOUR OUTPUT TO THE SCREEN (Y/N)?:'
        READ(*,'(A)') A
        IF(A.EQ.'YES'.OR.A.EQ.'yes'.OR.A.EQ.'YE'.OR.A.EQ.'ye'.
     1OR.A.EQ.'Y'.OR.A.EQ.'y')THEN
        WRITE(*,*)
        WRITE(*,*)'ROUTING ANALYSIS'
        WRITE(*,888)
```

```
      WRITE(*,*)'----------------------------------------------------'
      WRITE(*,*)'     TIME                INFLOW              OUTFLOW '
      WRITE(*,*)'      (H)                (M3/S)              (M3/S) '
      WRITE(*,*)'----------------------------------------------------'
      DO 901 J=0,NP
901   WRITE(*,900) TIME(J),QIN(J),QOUT(J)
      WRITE(*,*)'----------------------------------------------------'
      ENDIF
      WRITE(10,*)
      WRITE(10,*)'ROUTING ANALYSIS'
      WRITE(10,888)
      WRITE(10,*)'----------------------------------------------------'
      WRITE(10,*)'     TIME                INFLOW              OUTFLOW '
      WRITE(10,*)'      (H)                (M3/S)              (M3/S) '
      WRITE(10,*)'----------------------------------------------------'
      DO 90 J=0,NP
90    WRITE(10,900) TIME(J),QIN(J),QOUT(J)
900   FORMAT(1X,F9.2,2F20.3)
      WRITE(10,*)'----------------------------------------------------'
      WRITE(*,*)'THANK YOU FOR RUNNING THE PROGRAM.  PLEASE',
     1 ' CALL AGAIN.'
      STOP
      END
C*****************************************************************************
C    SUBROUTINE FOR TRIAGULAR FLOW GENERATION
CC*****************************************************************************
      SUBROUTINE OPT1(QIN,QOUT,NZ,QPEK,QBAS,TPEK,TBAS,DTH)
      DIMENSION QIN(0:NZ),QOUT(0:NZ)
      DOUBLE PRECISION QIN,QOUT
      DO 20 J=0,NZ
      QIN(J)= QBAS
20    QOUT(J)= QBAS
      NU= TPEK/DTH + 0.01
      ND= (TBAS-TPEK)/DTH + 0.01
      NT= NU+ND
      DO 30 J=1,NU
30    QIN(J)= QBAS + (QPEK-QBAS)*FLOAT(J)/NU
      DO 40 J=NT-1,NU+1,-1
40    QIN(J)= QBAS + (QPEK-QBAS)*FLOAT(NT-J)/ND
      RETURN
      END
C
C*****************************************************************************
C    SUBROUTINE FOR THE REGULAR SHAPED FLOW
C*****************************************************************************
      SUBROUTINE OPT2(QIN,QOUTO,DTH,NP,NZ,LLN,YLAT,INPUT,IPRINT)
      DIMENSION QIN(0:NZ),YLAT(0:NZ),QOUTO(0:NZ)
      DOUBLE PRECISION QIN,QOUTO
      COMMON NUL
      IF(INPUT.EQ.1)THEN
1     WRITE(*,*)'ENTER THE TOTAL NO. OF DISCHARGE ORDINATES:  '
      READ(*,*) NP
      IF(NP.LE.0.) THEN
      WRITE(*,*)'THE NO. CANNOT BE ZERO OR NEGATIVE.
     1 PLEASE TRY AGAIN.'
      GO TO 1
```

```
      ENDIF
      DO  L1=0,NP
10    WRITE(*,*)'ENTER THE INFLOW ORDINATES (M3/S):  '
      READ(*,*) QIN(L1)
      IF(QIN(L1).LT.0.)THEN
      WRITE(*,*)'PEAK DISCHARGE CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
      GO TO 10
      ENDIF
      ENDDO
      ELSEIF(INPUT.EQ.2)THEN
      READ(8,*)NP
      READ(8,*)(QIN(I),I=0,NP)
      IF(IPRINT.EQ.1)THEN
      WRITE(11,21)NP
      WRITE(11,22)(QIN(I),I=0,NP)
      ENDIF
      ENDIF
C*****************************************************************
C     OPTION FOR LATERAL INFLOW COMPUTATION AND INCORPORATION
C*****************************************************************
      WRITE(*,*)'DO YOU WANT TO COMPUTE AND INCORPORATE',
     1' LATERAL INFLOW (1) OR NOT; (1/2)'
      READ(*,*) LATOPT
      IF(LATOPT.EQ.1)THEN
      IF(INPUT.EQ.1)THEN
      DO  L=0,NP
111   WRITE(*,*)'ENTER THE OUTFLOW ORDINATES (M3/S):  '
      READ(*,*) QOUTO(L)
      IF(QOUTO(L).LT.0.)THEN
      WRITE(*,*)'PEAK DISCHARGE CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
      GO TO 111
      ENDIF
      ENDDO
      ELSEIF(INPUT.EQ.2)THEN
      READ(8,*)(QOUTO(L),L=0,NP)
      ENDIF
      ELSE IF(LATOPT.EQ.2)THEN
      LLN=4
      DO LL=0,NP
      YLAT(LL)=0.0
      QOUTO(LL)=0.0
      ENDDO
      RETURN
      ENDIF
      IF(IPRINT.EQ.1)THEN
      WRITE(11,605)
      WRITE(11,22) (QOUTO(I),I=0,NP)
      ENDIF
605   FORMAT(/)
21    FORMAT(1X,I3/)
22    FORMAT(1X,6F8.2)
C**************************************************************
      CALL OPTL(QIN,QOUTO,DTH,NP,NZ,YLAT,INPUT,IPRINT)
C**************************************************************
```

```
C       OPTION: NOW WHERE TO JOIN THE LATERAL INFLOW
C       1 = LATERAL INFLOW JOINS WITH THE INFLOW
C       2 = LATERAL INFLOW JOINS AT THE MIDDLE OF THE REACH
C       3 = LATERAL IFLOW JOINS WITH THE OUTFLOW
C       4 OR ANY OTHER NO. FOR NO OR < THAN 5% LATERAL INFLOW
C*************************************************************
        IF(NUL.EQ.1)THEN
        IF(INPUT.EQ.1)THEN
746     WRITE(*,*)'ENTER THE OPTION NO. FOR LATERAL INFLOW:  '
        READ(*,*) LLN
        IF(LLN.LE.0) THEN
        WRITE(*,*)'NO. CANNOT BE ZERO OR NEGATIVE.',
     1  ' PLEASE TRY AGAIN.'
        GO TO 746
        ENDIF
        ELSEIF(INPUT.EQ.2)THEN
        READ(8,*)LLN
        ENDIF
        IF(IPRINT.EQ.1)THEN
        WRITE(11,749)LLN
749     FORMAT(/,I2/)
        ENDIF
        ELSEIF(NUL.EQ.0)THEN
        LLN=4
        ENDIF
        RETURN
        END
C****************************************************************************
C    SUBROUTINE FOR LATERAL FLOW COMPUTATION
C****************************************************************************
        SUBROUTINE OPTL(QIN,QOUTO,DTH,NP,NZ,YLAT,INPUT,IPRINT)
        DIMENSION QIN(0:NZ),QOUTO(0:NZ),XLAT(0:100),YLAT(0:NZ)
        DOUBLE PRECISION QIN,QOUTO
        COMMON NUL
        SUMI=0.0
        DO  I=0,NP
        SUMI=SUMI+QIN(I)
        ENDDO
        SUMI=SUMI-0.5*(QIN(0)+QIN(NP))
        SUMI=3600.0*DTH*SUMI
        WRITE(10,*)'VOLUMETRIC ANALYSIS OF FLOW'
        write(10,95) sumi
95      format(/1x,'vol. of inflow    ',f14.2)
        SUMO=0.0
        DO I=0,NP
        SUMO=SUMO+QOUTO(I)
        ENDDO
        SUMO=SUMO-0.5*(QOUTO(0)+QOUTO(NP))
        SUMO=3600.0*SUMO
        write(10,96) sumo
96      format(1x,'vol. of outflow   ',f14.2)
        DIFF=(SUMO-SUMI)/SUMI
        DIFF1=SUMO-SUMI
        PERC=100.0*DIFF
        IF(SQRT(PERC*PERC).LT.5.00)THEN
        WRITE(*,*)'%AGE LATERAL INFLOW IS LESS THAN 5%',
```

```
     1 ' AND HENCE NEGLECTED'
       NUL=0
       RETURN
       ENDIF
       NUL=1
       IF(INPUT.EQ.1)THEN
1      WRITE(*,*)'ENTER THE HYPOTHETICEL TOTAL NO. OF LATERAL',
     1 ' INFLOW ORDINATES:  '
       READ(*,*) NORDL
       IF(NORDL.LE.0.) THEN
       WRITE(*,*)'THE NO. CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
       GO TO 1
       ENDIF
113    WRITE(*,*)'ENTER THE TIME INTERVAL OF THE DISTRIBUTION',
     1 ' GRAPH ORDINATES (HR):'
       READ(*,*) DTHL
       IF(DTHL.LE.0.) THEN
       WRITE(*,*)'THE INTERVAL CANNOT BE ZERO OR NEGATIVE.',
     1 ' PLEASE TRY AGAIN.'
       GO TO 113
       ENDIF
       DO J=0,NORDL
10     WRITE(*,*)'SUPPLY THE HYPOTHETICAL DISTRIBUTION OF ',
     1 ' LATERAL INFLOW:  '
       READ(*,*) XLAT(J)
       IF(XLAT(J).LT.0.)THEN
       WRITE(*,*)'IT CAN NOT BE NEGATIVE: PLEASE TRY AGAIN.'
       GO TO 10
       ENDIF
       ENDDO
       ELSEIF(INPUT.EQ.2)THEN
       READ(8,*)NORDL
       READ(8,*)DTHL
       READ(8,*)(XLAT(JJ),JJ=0,NORDL)
       ENDIF
       IF(IPRINT.EQ.1)THEN
       WRITE(11,746)NORDL
       WRITE(11,747)DTHL
       WRITE(11,748)(XLAT(JJ),JJ=0,NORDL)
746    FORMAT(/,I2)
747    FORMAT(F4.1)
748    FORMAT(8F8.1)
       ENDIF
       SUML=0.0
       DO J=0,NORDL
       SUML=SUML+XLAT(J)
       ENDDO
       SUML=3600.0*DTHL*SUML
       WRITE(10,21)DIFF1
21     FORMAT(1X,'VOLUME DIFFERENCE=',F14.2//)
       WRITE(10,*)'LATERAL FLOW COMPUTATION'
       write(10,97) suml
97     format(/1x,'vol. of arbitrary lateral inflow  ',f14.2)
       AF=DIFF1/SUML
       suml1=0.0
```

```
      DO J=0,NORDL
      YLAT(J)=AF*XLAT(J)
      suml1=suml1+ylat(j)
      ENDDO
      suml1=3600.0*dthl*suml1
      write(10,98) suml1
98    format(1x,'vol. of computed Lateral inflow   ',f14.2/)
      WRITE(10,*)'-----------------------------------------------------'
      WRITE(10,30)
30    FORMAT(1X,'ARBITRARY LATERAL INFLOW',3X,'CORRECTED LAT. INFLOW')
      WRITE(10,*)'-----------------------------------------------------'
      DO I=0,NORDL
      WRITE(10,22) XLAT(I),YLAT(I)
      ENDDO
22    FORMAT(1X,9X,2F14.2)
      WRITE(10,*)'-----------------------------------------------------'
      AK=DTH/DTHL
      LL=0
      DO I=0,NZ,AK
      YLAT(LL)=YLAT(I)
      LL=LL+1
      IF(LL.GE.NP)RETURN
      ENDDO
      RETURN
      END
```

```
*********************************************************************************
TEST NO.: T1 (TEST 1)                                  FILE : F1=1    MCDF1.DAT
*********************************************************************************
1000.00                                                             PEAK DISCHARGE
  5.00                                                                TIME TO PEAK
   .000868                                                           CHANNEL SLOPE
2                                                          OPTION FOR READING DATA
 400.00                                                                  FLOW AREA
   1.60                                                                       BETA
 100.00                                                                  TOP WIDTH
  14.40                                                      LENGTH OF THE CHANNEL
1                                                                  OPTION FOR FLOW
   .00                                                                   BASE FLOW
  10.00                                              TIME BASE OF TRIANGULAR HYD.
1                                                                NUMBER OF REACHES
   1.00                                                              TIME INTERVAL
*********************************************************************************


*********************************************************************************
TEST NO.: T1 (TEST 1)                                  FILE :  MCDF.OUT
*********************************************************************************


****************************************************
      ROUTING BY *MUSKINGUM-CUNGE* METHOD
****************************************************


*******************
CHECKS APPLICATION
*******************
(1)RESOLUTION CRITERIA OF(C+D)>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS
```

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	18.183
2.00	400.000	201.653
3.00	600.000	400.150
4.00	800.000	600.014
5.00	1000.000	800.001
6.00	800.000	963.634
7.00	600.000	796.694
8.00	400.000	599.699
9.00	200.000	399.973
10.00	.000	199.998
11.00	.000	18.183
12.00	.000	1.653
13.00	.000	.150
14.00	.000	.014
15.00	.000	.001
16.00	.000	.000

```
*********************************************************************************
TEST NO.: T2 (TEST 2)                                FILE : F1=1     MCDF1.DAT
*********************************************************************************

 1000.00                                                          PEAK DISCHARGE
    5.00                                                            TIME TO PEAK
     .000868                                                      CHANNEL SLOPE
 2                                                   OPTION FOR READING DATA
  400.00                                                               FLOW AREA
    1.60                                                                    BETA
  100.00                                                               TOP WIDTH
   14.40                                                LENGTH OF THE CHANNEL
 2                                                               OPTION FOR FLOW
 1                                                           NUMBER OF REACHES
    1.00                                                           TIME INTERVAL
*********************************************************************************
```

```
TEST NO.: T2 (TEST 2)                                FILE : F2=1     MCDF2.DAT
*********************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.

*********************************************************************************
```

TEST NO.: T2 (TEST 2) FILE : MCDF.OUT

****

****

ROUTING BY *MUSKINGUM-CUNGE* METHOD

****

****

CHECKS APPLICATION

****

(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	18.183
2.00	400.000	201.653
3.00	600.000	400.150
4.00	800.000	600.014
5.00	1000.000	800.001
6.00	800.000	963.634
7.00	600.000	796.694
8.00	400.000	599.699
9.00	200.000	399.973
10.00	.000	199.998
11.00	.000	18.183
12.00	.000	1.653
13.00	.000	.150
14.00	.000	.014
15.00	.000	.001
16.00	.000	.000

```
**********************************************************************************
TEST NO.: T3 (TEST 3)                                  FILE : F1=1     MCDF1.DAT
**********************************************************************************

1000.00                                                            PEAK DISCHARGE
   5.00                                                              TIME TO PEAK
    .000868                                                         CHANNEL SLOPE
1                                                        OPTION FOR READING DATA
 300.00                                            inverse SLOPE OF RATING CURVE
 100.00                                                                TOP WIDTH
  14.40                                                    LENGTH OF THE CHANNEL
1                                                                OPTION FOR FLOW
    .00                                                                BASE FLOW
  10.00                                           TIME BASE OF TRIANGULAR HYD.
1                                                              NUMBER OF REACHES
   1.00                                                            TIME INTERVAL
**********************************************************************************

TEST NO.: T3 (TEST 3)                                  FILE :  MCDF.OUT
**********************************************************************************
```

```
*******************************************************
        ROUTING BY *MUSKINGUM-CUNGE* METHOD
*******************************************************
```

```
********************
CHECKS APPLICATION
********************
(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED
```

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	1.655
2.00	400.000	150.838
3.00	600.000	337.818
4.00	800.000	534.482
5.00	1000.000	733.628
6.00	800.000	930.100
7.00	600.000	831.677
8.00	400.000	657.702
9.00	200.000	464.370
10.00	.000	266.078
11.00	.000	68.170
12.00	.000	17.466
13.00	.000	4.475
14.00	.000	1.146
15.00	.000	.294
16.00	.000	.075
17.00	.000	.019
18.00	.000	.005
19.00	.000	.001
20.00	.000	.000

```
***************************************************************************************
TEST NO.: T4 (TEST 4)                              FILE : F1=1      MCDF1.DAT
***************************************************************************************

 1000.00                                                           PEAK DISCHARGE
    5.00                                                             TIME TO PEAK
     .000868                                                        CHANNEL SLOPE
1                                                        OPTION FOR READING DATA
  300.00                                              inverse SLOPE OF RATING CURVE
  100.00                                                                TOP WIDTH
   14.40                                                    LENGTH OF THE CHANNEL
2                                                                OPTION FOR FLOW
1                                                              NUMBER OF REACHES
    1.00                                                            TIME INTERVAL
***************************************************************************************
```

```
TEST NO.: T4 (TEST 4)                              FILE : F2=1      MCDF2.DAT
***************************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.
***************************************************************************************
```

****

****
ROUTING BY *MUSKINGUM-CUNGE* METHOD
****

****
CHECKS APPLICATION
****
(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	1.655
2.00	400.000	150.838
3.00	600.000	337.818
4.00	800.000	534.482
5.00	1000.000	733.628
6.00	800.000	930.100
7.00	600.000	831.677
8.00	400.000	657.702
9.00	200.000	464.370
10.00	.000	266.078
11.00	.000	68.170
12.00	.000	17.466
13.00	.000	4.475
14.00	.000	1.146
15.00	.000	.294
16.00	.000	.075
17.00	.000	.019
18.00	.000	.005
19.00	.000	.001
20.00	.000	.000

```
*********************************************************************************
TEST NO.: T5 (TEST 5)                                  FILE : F1=1     MCDF1.DAT
*********************************************************************************

 1000.00
    5.00                                                          PEAK DISCHARGE
    0.000868                                                        TIME TO PEAK
 2                                                                CHANNEL SLOPE
  400.00                                              OPTION FOR READING DATA
    1.60                                                              FLOW AREA
  100.00                                                                   BETA
   14.40                                                              TOP WIDTH
 2                                                         LENGTH OF THE CHANNEL
 1                                                              OPTION FOR FLOW
    1.00                                                      NUMBER OF REACHES
                                                                  TIME INTERVAL
*********************************************************************************
```

```
TEST NO.: T5 (TEST 5)                                  FILE : F2=1     MCDF2.DAT
*********************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.

0. 250. 500. 700. 900. 1200. 1000. 700. 520. 220.

8

1.0

0. 100. 300. 500. 400. 300. 200. 100. 0.

1
*********************************************************************************
```

****
TEST NO.: T5 (TEST 5)   FILE : MCDF .OUT
****

****
ROUTING BY *MUSKINGUM-CUNGE* METHOD
****

VOLUMETRIC ANALYSIS OF FLOW

vol. of inflow 17640000.00
vol. of outflow 21168000.00
VOLUME DIFFERENCE= 3528000.00

LATERAL FLOW COMPUTATION

vol. of arbitrary lateral inflow 6840000.00
vol. of computed Lateral inflow 3528000.00

ARBITRARY LATERAL INFLOW	CORRECTED LAT. INFLOW
.00	.00
100.00	51.58
300.00	154.74
500.00	257.89
400.00	206.32
300.00	154.74
200.00	103.16
100.00	51.58
.00	.00

****
CHECKS APPLICATION
****
(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	22.872
2.00	400.000	258.348
3.00	600.000	555.352
4.00	800.000	843.883
5.00	1000.000	1005.042
6.00	800.000	1118.255
7.00	600.000	899.841
8.00	400.000	651.277
9.00	200.000	404.662
10.00	.000	200.424
11.00	.000	18.221
12.00	.000	1.657
13.00	.000	.151
14.00	.000	.014
15.00	.000	.001
16.00	.000	.000

```
*******************************************************************************
TEST NO.: T6 (TEST 6)                              FILE : F1=1     MCDF1.DAT
*******************************************************************************

 1000.00                                                        PEAK DISCHARGE
    5.00                                                          TIME TO PEAK
     .000868                                                     CHANNEL SLOPE
 1                                                     OPTION FOR READING DATA
  300.00                                       inverse SLOPE OF RATING CURVE
  100.00                                                             TOP WIDTH
   14.40                                                 LENGTH OF THE CHANNEL
 2                                                             OPTION FOR FLOW
 1                                                         NUMBER OF REACHES
    1.00                                                         TIME INTERVAL

*******************************************************************************
```

```
TEST NO.: T6 (TEST 6)                              FILE : F2=1     MCDF2.DAT
*******************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.

0. 250. 500. 700. 900. 1200. 1000. 700. 520. 220.

8

1.0

0. 100. 300. 500. 400. 300. 200. 100. 0.

1
*******************************************************************************
```

TEST NO.: T6 (TEST 6)   FILE : MCDF.OUT

****

****

ROUTING BY *MUSKINGUM-CUNGE* METHOD

****

VOLUMETRIC ANALYSIS OF FLOW

vol. of inflow   17640000.00
vol. of outflow   21168000.00
VOLUME DIFFERENCE=   3528000.00

LATERAL FLOW COMPUTATION

vol. of arbitrary lateral inflow   6840000.00
vol. of computed Lateral inflow   3528000.00

ARBITRARY LATERAL INFLOW	CORRECTED LAT. INFLOW
.00	.00
100.00	51.58
300.00	154.74
500.00	257.89
400.00	206.32
300.00	154.74
200.00	103.16
100.00	51.58
.00	.00

****

CHECKS APPLICATION

****

(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	2.081
2.00	400.000	190.164
3.00	600.000	463.840
4.00	800.000	758.164
5.00	1000.000	943.966
6.00	800.000	1098.655
7.00	600.000	951.164
8.00	400.000	726.253
9.00	200.000	481.933
10.00	.000	270.578
11.00	.000	69.323
12.00	.000	17.761
13.00	.000	4.550
14.00	.000	1.166
15.00	.000	.299
16.00	.000	.077
17.00	.000	.020
18.00	.000	.005
19.00	.000	.001
20.00	.000	.000

```
********************************************************************************
TEST NO.: T7 (TEST 7)                                FILE : F1=1     MCDF1.DAT
********************************************************************************

 1000.00                                                          PEAK DISCHARGE
    5.00                                                            TIME TO PEAK
    0.000868                                                       CHANNEL SLOPE
 2                                                      OPTION FOR READING DATA
  400.00                                                               FLOW AREA
    1.60                                                                    BETA
  100.00                                                               TOP WIDTH
   14.40                                                   LENGTH OF THE CHANNEL
 2                                                               OPTION FOR FLOW
 1                                                            NUMBER OF REACHES
    1.00                                                           TIME INTERVAL
********************************************************************************
```

```
TEST NO.: T7 (TEST 7)                                FILE : F2=1     MCDF2.DAT
********************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.

0. 250. 500. 700. 900. 1200. 1000. 700. 520. 220.

8

1.0

0. 100. 300. 500. 400. 300. 200. 100. 0.

2
********************************************************************************
```

****

****
ROUTING BY *MUSKINGUM-CUNGE* METHOD
****

VOLUMETRIC ANALYSIS OF FLOW

vol. of inflow 17640000.00
vol. of outflow 21168000.00
VOLUME DIFFERENCE= 3528000.00

LATERAL FLOW COMPUTATION

vol. of arbitrary lateral inflow 6840000.00
vol. of computed Lateral inflow 3528000.00

ARBITRARY LATERAL INFLOW	CORRECTED LAT. INFLOW
.00	.00
100.00	51.58
300.00	154.74
500.00	257.89
400.00	206.32
300.00	154.74
200.00	103.16
100.00	51.58
.00	.00

****
CHECKS APPLICATION
****
(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	46.317
2.00	400.000	307.369
3.00	600.000	606.699
4.00	800.000	825.106
5.00	1000.000	979.890
6.00	800.000	1092.524
7.00	600.000	874.057
8.00	400.000	625.488
9.00	200.000	402.317
10.00	.000	200.211
11.00	.000	18.202
12.00	.000	1.655
13.00	.000	.150
14.00	.000	.014
15.00	.000	.001
16.00	.000	.000

```
*************************************************************************************
TEST NO.: T8 (TEST 8)                                   FILE : F1=1      MCDF1.DAT
*************************************************************************************

 1000.00                                                             PEAK DISCHARGE
    5.00                                                               TIME TO PEAK
    0.000868                                                          CHANNEL SLOPE
2                                                          OPTION FOR READING DATA
  400.00                                                                  FLOW AREA
    1.60                                                                       BETA
  100.00                                                                  TOP WIDTH
   14.40                                                      LENGTH OF THE CHANNEL
2                                                                   OPTION FOR FLOW
1                                                                NUMBER OF REACHES
    1.00                                                              TIME INTERVAL
*************************************************************************************
```

```
TEST NO.: T8 (TEST 8)                                   FILE : F2=1      MCDF2.DAT
*************************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.

0. 250. 500. 700. 900. 1200. 1000. 700. 520. 220.

8

1.0

0. 100. 300. 500. 400. 300. 200. 100. 0.

3
*************************************************************************************
```

****

****

ROUTING BY *MUSKINGUM-CUNGE* METHOD

****

VOLUMETRIC ANALYSIS OF FLOW

vol. of inflow	17640000.00
vol. of outflow	21168000.00
VOLUME DIFFERENCE=	3528000.00

LATERAL FLOW COMPUTATION

vol. of arbitrary lateral inflow	6840000.00
vol. of computed Lateral inflow	3528000.00

ARBITRARY LATERAL INFLOW	CORRECTED LAT. INFLOW
.00	.00
100.00	51.58
300.00	154.74
500.00	257.89
400.00	206.32
300.00	154.74
200.00	103.16
100.00	51.58
.00	.00

****

CHECKS APPLICATION

****

(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	69.762
2.00	400.000	356.390
3.00	600.000	658.045
4.00	800.000	806.329
5.00	1000.000	954.738
6.00	800.000	1066.792
7.00	600.000	848.273
8.00	400.000	599.699
9.00	200.000	399.973
10.00	.000	199.998
11.00	.000	18.183
12.00	.000	1.653
13.00	.000	.150
14.00	.000	.014
15.00	.000	.001
16.00	.000	.000

```
********************************************************************************
TEST NO.: T9 (TEST 9)                                 FILE : F1=1     MCDF1.DAT
********************************************************************************

 1000.00                                                            PEAK DISCHARGE
    5.00                                                              TIME TO PEAK
    0.000868                                                         CHANNEL SLOPE
 2                                                         OPTION FOR READING DATA
  400.00                                                                 FLOW AREA
    1.60                                                                      BETA
  100.00                                                                 TOP WIDTH
   28.80                                                     LENGTH OF THE CHANNEL
 2                                                                 OPTION FOR FLOW
 2                                                               NUMBER OF REACHES
    1.00                                                             TIME INTERVAL
********************************************************************************
```

```
TEST NO.: T9 (TEST 9)                                 FILE : F2=1     MCDF2.DAT
********************************************************************************

9

0. 200. 400. 600. 800. 1000. 800. 600. 400. 200.

0. 250. 500. 700. 900. 1200. 1000. 700. 520. 220.

8

1.0

0. 100. 300. 500. 400. 300. 200. 100. 0.

3
********************************************************************************
```

****
ROUTING BY *MUSKINGUM-CUNGE* METHOD
****

VOLUMETRIC ANALYSIS OF FLOW

vol. of inflow	17640000.00
vol. of outflow	21168000.00
VOLUME DIFFERENCE=	3528000.00

LATERAL FLOW COMPUTATION

vol. of arbitrary lateral inflow	6840000.00
vol. of computed Lateral inflow	3528000.00

ARBITRARY LATERAL INFLOW	CORRECTED LAT. INFLOW
.00	.00
100.00	51.58
300.00	154.74
500.00	257.89
400.00	206.32
300.00	154.74
200.00	103.16
100.00	51.58
.00	.00

****
CHECKS APPLICATION
****
(1)RESOLUTION CRITERIA OF C+D>1 CHECKED
(2)MIN. & MAX. REACH LENGTH CRITERIA SATISFIED
(3) DELTA t CRITERIA SATISFIED

ROUTING ANALYSIS

TIME (H)	INFLOW (M3/S)	OUTFLOW (M3/S)
.00	.000	.000
1.00	200.000	57.921
2.00	400.000	244.792
3.00	600.000	617.496
4.00	800.000	850.709
5.00	1000.000	959.836
6.00	800.000	1054.479
7.00	600.000	1088.007
8.00	400.000	842.780
9.00	200.000	603.641
10.00	.000	400.308
11.00	.000	201.679
12.00	.000	33.362
13.00	.000	4.399
14.00	.000	.524
15.00	.000	.059
16.00	.000	.006
17.00	.000	.001

APPENDIX A: DERIVATION OF THE NUMERICAL DIFFUSION COEFFICIENT OF THE MUSKINGUM-CUNGE METHOD

Expanding the grid function $Q(j\Delta x, n\Delta t)$ (Fig. 3) in Taylor series about point $(j\Delta x, n\Delta t)$ leads to:

$$Q_j^{n+1} = Q_j^n + \left[\frac{\partial Q}{\partial t}\right]_j \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_j \Delta t^2 + o(\Delta t^3) \qquad \text{(A1)}$$

$$Q_{j+1}^{n+1} = Q_{j+1}^n + \left[\frac{\partial Q}{\partial t}\right]_{j+1} \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_{j+1} \Delta t^2 + o(\Delta t^3) \qquad \text{(A2)}$$

$$Q_{j+1}^n = Q_j^n + \left[\frac{\partial Q}{\partial x}\right]_n \Delta x + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_n \Delta x^2 + o(\Delta x^3) \qquad \text{(A3)}$$

$$Q_{j+1}^{n+1} = Q_j^{n+1} + \left[\frac{\partial Q}{\partial x}\right]_{n+1} \Delta x + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_{n+1} \Delta x^2 + o(\Delta x^3) \qquad \text{(A4)}$$

Substituting Eqs. A1 to A4 into Eq. (1) and neglecting third-order terms yields:

$$\begin{aligned} & X\left\{\left[\frac{\partial Q}{\partial t}\right]_j \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_j \Delta t^2\right\} \\ & + (1 - X)\left\{\left[\frac{\partial Q}{\partial t}\right]_{j+1} \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_{j+1} \Delta t^2\right\} \\ & + \frac{C}{2}\left\{\left[\frac{\partial Q}{\partial x}\right]_n \Delta x + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_n \Delta x^2\right\} \\ & + \frac{C}{2}\left\{\left[\frac{\partial Q}{\partial x}\right]_{n+1} \Delta x + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_{n+1} \Delta x^2\right\} = 0 \end{aligned} \qquad \text{(A5)}$$

in which $C = c(\Delta t/\Delta x)$ is the Courant number.

Expressing the derivatives at grid point $[(j+1)\Delta x, (n+1)\Delta t]$ in terms of the derivatives at grid point $(j\Delta x, n\Delta t)$ by means of Taylor series:

$$\left[\frac{\partial Q}{\partial t}\right]_{j+1} = \left[\frac{\partial Q}{\partial t}\right]_j + \left[\frac{\partial^2 Q}{\partial x \partial t}\right]_{j,n} \Delta x + o(\Delta x^2) \qquad \text{(A6)}$$

$$\left[\frac{\partial Q}{\partial x}\right]_{n+1} = \left[\frac{\partial Q}{\partial x}\right]_n + \left[\frac{\partial^2 Q}{\partial x \partial t}\right]_{j,n} \Delta t + o(\Delta t^2) \qquad \text{(A7)}$$

$$\left[\frac{\partial^2 Q}{\partial t^2}\right]_{j+1} = \left[\frac{\partial^2 Q}{\partial t^2}\right]_j + \left[\frac{\partial^3 Q}{\partial t^3}\right]_j \Delta x + o(\Delta x^2) \qquad \text{(A8)}$$

$$\left[\frac{\partial^2 Q}{\partial x^2}\right]_{n+1} = \left[\frac{\partial^2 Q}{\partial x^2}\right]_n + \left[\frac{\partial^3 Q}{\partial x^3}\right]_n \Delta t + o(\Delta t^2) \qquad \text{(A9)}$$

Source: Ponce, V.M. (1989), "Engineering Hydrology", Prentice Hall, Eaglewood Chiffs, New Jersey 07632

Substituting Eqs. A6 to A9 into A5 and neglecting third-order terms:

$$X\left\{\left[\frac{\partial Q}{\partial t}\right]_j \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_j \Delta t^2\right\}$$

$$+ (1 - X)\left\{\left[\frac{\partial Q}{\partial t}\right]_j \Delta t + \left[\frac{\partial^2 Q}{\partial x \partial t}\right]_{j,n} \Delta x \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_j \Delta t^2\right\}$$

$$+ \frac{C}{2}\left\{\left[\frac{\partial Q}{\partial x}\right]_n \Delta x + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_n \Delta x^2\right\}$$

$$+ \frac{C}{2}\left\{\left[\frac{\partial Q}{\partial x}\right]_n \Delta x + \left[\frac{\partial^2 Q}{\partial x \partial t}\right]_{j,n} \Delta x \Delta t + \frac{1}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_n \Delta x^2\right\} = 0 \qquad \text{(A 10)}$$

In Eq. A10, dividing by Δt and simplifying,

$$\left[\frac{\partial Q}{\partial t}\right]_j + c\left[\frac{\partial Q}{\partial x}\right]_n$$

$$+ \frac{\Delta t}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_j + \frac{c\,\Delta x}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_n$$

$$+ \Delta x\left\{(1 - X) + \frac{C}{2}\right\}\left[\frac{\partial^2 Q}{\partial x \partial t}\right]_{j,n} = 0 \qquad \text{(A.11)}$$

The first two terms of Eq. A11 constitute the kinematic wave equation, Eq. (1XA). The remaining terms are the error R of the first-order-accurate numerical scheme:

$$R = \frac{\Delta t}{2}\left[\frac{\partial^2 Q}{\partial t^2}\right]_j + \frac{c\,\Delta x}{2}\left[\frac{\partial^2 Q}{\partial x^2}\right]_n$$

$$+ \Delta x\left\{(1 - X) + \frac{C}{2}\right\}\left[\frac{\partial^2 Q}{\partial x \partial t}\right]_{j,n} = 0 \qquad \text{(A12)}$$

From Eq. (ixa):

$$\frac{\partial Q}{\partial t} = -c\,\frac{\partial Q}{\partial x} \qquad \text{(A.13)}$$

Therefore

$$\frac{\partial^2 Q}{\partial x \partial t} = -c\,\frac{\partial^2 Q}{\partial x^2} \qquad \text{(A 14)}$$

$$\frac{\partial^2 Q}{\partial t^2} = c^2\,\frac{\partial^2 Q}{\partial x^2} \qquad \text{(A15)}$$

Substituting Eqs. A14 and A15 into A.12 and simplifying:

$$R = c\,\Delta x\left(X - \frac{1}{2}\right)\frac{\partial^2 Q}{\partial x^2} \qquad \text{(A 16)}$$

Comparing Eq. A16 with the right-hand side of the diffusion wave equation, repeated here:

$$\frac{\partial Q}{\partial t} + c\,\frac{\partial Q}{\partial x} = \nu_h\,\frac{\partial^2 Q}{\partial x^2} \qquad \text{(A.17)}$$

it follows that the numerical diffusion coefficient of the Muskingum-Cunge method is:

$$\nu_n = c\,\Delta x\left(\frac{1}{2} - X\right) \qquad \text{(A.18)}$$

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