

TR-52

GEOELECTRICAL TECHNIQUES FOR STUDY OF
SOIL MOISTURE VARIATIONS

SATISH CHANDRA
DIRECTOR

STUDY GROUP

V C GOYAL
B P PARIDA

NATIONAL INSTITUTE OF HYDROLOGY
JAL VIGYAN BHAWAN
ROORKEE-247667(UP)
INDIA

1987-88

CONTENTS

	PAGE NO.
LIST OF FIGURES	3
LIST OF TABLES	4
ABSTRACT	5
1.0 INTRODUCTION	7
1.1 Soil Moisture Variations in Zone of Aeration	7
1.2 Geoelectrical Techniques Applied in Hydrology	12
1.3 Resistivity and S.P. Dependence on Moisture Variations	14
2.0 REVIEW	18
2.1 Four-electrode Resistivity Sounding Method	18
2.2 S.P. Method	21
3.0 PROBLEM DEFINITION	22
4.0 METHODOLOGY	26
4.1 Potential and Conductivity Distribution in Layered Earth	26
4.2 Inversion of Resistivity Field Measurements	31
4.3 Relationship between Resistivity and Water Content	35
4.4 Field and Laboratory Procedures	38
5.0 APPLICATION	40
6.0 RESULTS	43
7.0 CONCLUSION	60
REFERENCES	62
APPENDICES	66

LIST OF FIGURES

Figure No.	Title	Page No.
1.1	Idealised cross-section showing vadose and saturated zones.	8
4.1	Conventional electrode arrangements	27
4.2	A two-layered medium	29
4.3	Switch controlled multi-electrode Wenner array	41
6.1	Electrode arrangement for S.P. measurements	44
6.2.6.7	Plots of variation of S.P., rainfall and evapotranspiration data	45- 50
6.8 - 6.10	Plots of variation of moisture content and resistivity profiles	58- 59
IV.1	A generalised n- layered earth model	74

LIST OF TABLES

Table No.	Title	Page No.
4.I	Ghosh's short-filter coefficients	34
6.I	Grain size Vs depth for soils at test site	151
6.II	Values of parameters measured on groundwater samples from various tubewells located in U.O.R. Campus and around	53
6.III	Values of parameters measured on soil samples from site 1	54
6.IV	Values of parameters measured on soil samples from site 2	55
6.V	Temporal variation of resistivity profiles	57
III.I	Clay mineral identification scheme	72
III.2	Quantisation scheme for major clay minerals	72

ABSTRACT

A systematic investigation of streaming potentials (S.P.) shows that they vary considerably with time. To study the influence of water movements on S.P. anomaly-patterns, repeated measurements were conducted during May-September, 1987, at site comprising of alluvium formation. An electrode array having six-electrodes at regular intervals of one foot, was permanently installed in a borehole and potential gradients as well as potential differences were measured on daily basis. An inverse correlation was observed between the S.P. anomalies and the rainfall events suggesting that S.P. measurements may become a valuable and standard technique in hydrology.

Studies of moisture migration in the vadose zone were conducted at two field sites using a Wenner resistivity array. Gravimetric moisture measurements on soil samples taken at each of the field sites were made to a maximum depth of 1.50 meters, and these results were compared with resistivity values. Resistivity was measured using a multi-electrode, switch-controlled Wenner-array, enabling repeated measurements at various electrode positions without disturbing physical location of the electrodes.

Effect of rainwater infiltration was studied by monitoring resistivity before and after rainfall events. Archie's empirical relationship was utilised to correlate moisture and resistivity measurements. Results indicate that (i) moisture is retained for long periods of time in clay/silt rich soils, (ii) moisture migration is slow below

a moist soil zone and is not readily detected by surface resistivity measurements, and (iii) near-surface moisture variation can be defined by surface resistivity measurements. Small anomalies encountered in monitoring resistivity for study of moisture-migration caution a high precision field measurements and interpretation of resistivity data.

1.0 INTRODUCTION

Earth's uppermost part is affected by most of the men's engineering and cultural activities. This zone is characterized by a significant spatial-temporal variation of the physical properties of rocks, including unconsolidated soils, and saturating waters. It is also characterized by a high mobility of groundwater, regular vertical and horizontal transfer of moisture and salts, and prolonged or seasonal changes in the aggregate state of water.

The porous media that contains groundwater is a three phase system. It consists of a solid phase (soils), a gaseous phase (air) and a liquid phase (water). The solid phase may consist of consolidated rocks such as limestones, granites, lava, etc. It may be semiconsolidated materials such as sandstones and shales or it may be unconsolidated alluvial deposits and soils formed in place by weathering processes. For the purposes of geoelectrical methods dealt with in this report, description has been limited to the unconsolidated formations, such as alluvium. The solid phase of soils and unconsolidated sediments consist of individual particles of various sizes. These particles are classified according to their sizes as cobbles, gravel, sand, silt and clay.

1. Soil Moisture Variations in Zone of Aeration

1.1.1 Zone of aeration defined

The zone of aeration, also called vadose zone, is the geological profile from the ground surface to the upper surface of the principal water bearing strata. Figure 1.1 gives an idealized cross section showing the vadose and

saturated zones. The vadose zone has been subdivided into three regions, designated as the soil zones, the intermediate vadose zone, and the capillary fringe. Weathering of native geological material together with the process of eluviation

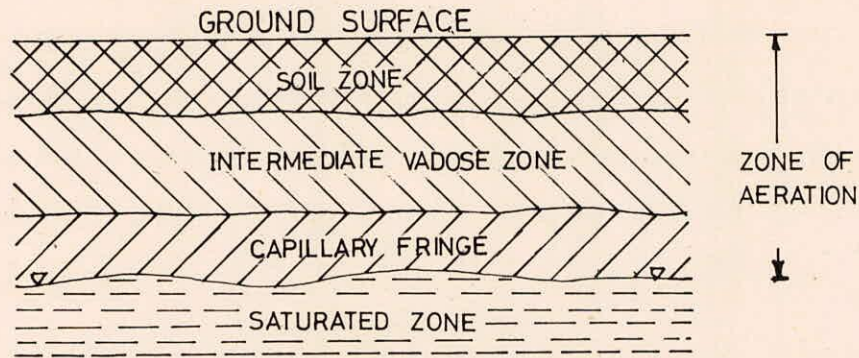


FIG. 1.1- A TYPICAL CROSS SECTION OF VADOSE ZONE
(EVERETT, et.al. 1984)

and illuviation of colloidal materials, to develop more or less well-developed soil profiles is generally recognised to take place and thus defines the limits of the soil zone. Water movement in the soil zone is generally in the unsaturated state i.e. that state in which the water exists under pressures that are less than atmospheric, the principle transport mechanisms associated with unsaturated flow are infiltration, percolation, redistribution and evaporation. Saturated regions may develop in the soil zone in response to surface flooding, especially in soils that contain horizons of low permeability.

The soil zone gradually merges with the underlying

intermediate vadose zone through a transition from weathered to generally unweathered native material. Thickness of intermediate vadose zone rarely remains uniform, as it varies with precipitation, evaporation, etc.

The base of the vadose zone, the capillary fringe, merges with underlying saturated material of the principal water bearing formation. This zone is not characterised as much by the nature of geological materials as by the presence of water under conditions of saturation or near saturation. In general, the thickness of the capillary fringe is greater in fine materials than in coarse deposits.

1.1.2 Monitoring in the zone of aeration

Monitoring in the saturated zone, for parameters like permeability, storage coefficient, specific yield, etc., utilizes well developed techniques familiar to hydrologists, geologists and engineers. Pumping tests provide adequate information to assess the storage and water movement through the formation. Chemical analysis of groundwater samples generally completes the required information needed to assess the conditions of the aquifer. In the vadose zone, where a large percentage of the flow is unsaturated, standard pump tests can not always be used although researchers are still interested in similar parameters. A wide variety of alternative methods are used to derive information on these vadose zone characteristics.

In general, vadose zone monitoring techniques can be arbitrarily divided into two categories, based on whether sample material is removed from the test location or the

test is conducted in-situ. Sampling methods can obtain either liquid or solid material. Extraction techniques for pore fluid from soil samples is dependent on whether inorganics, organics, or micro organics are under investigation. Liquid sampling methods are dependent on whether fluid is taken from the saturated or unsaturated regions of the vadose zone.

Non sampling methods do not remove material from the test location and therefore provide in-situ; indirect indication of the vadose zone characteristics. They include (Everett, et. al. 1984) surface and downhole geophysical methods, numerical and water budget type techniques.

1.1.2 Observation of soil moisture in the zone of aeration

Observations of soil moisture in the zone of aeration are made in-situ: (a) to calculate the moisture content and its variation with time; (b) to compute the soil moisture balance in this zone and to study its relationship with the groundwater balance; (c) to follow moisture movement in the zone of partial saturation under the influence of different factors; and (d) to study moisture dynamics and the transport of water-soluble salts.

In selecting soil-moisture observation stations care should be taken to locate optimum sized sites where the general geomorphological and groundwater conditions are typical of the regions selected for detailed water-balance study. Station locations reflect the objective of the particular experiment but they should also take into consideration

the manner in which soil moisture is affected by the character of vegetation, the nature of the soil, the micro-relief, the agronomic methods being used in the cultivated fields and the changes in groundwater factors with time and depth.

Two methods are commonly used for soil-moisture determination namely weighing and drying, and neutron-scattering techniques using one of several suitable radioactive isotopes. The first method, though more time consuming, permits study of the problems referred to above but its repetitive use destroys the site. The second method is convenient to apply on small plots of homogeneous soil structure and enables repeated measurements in the same borehole of the soil-moisture regime at different depths.

Over years, agriculture engineers have adopted four-electrode resistivity measurement technique, a well-established technique for groundwater exploration, for determination of soil salinity, leaching fraction, etc. on a regional basis. Indirectly, this technique also enables measurement of water content in soil profiles.

The primary problem in measuring soil moisture is determining how many measurements must be made to obtain a representative moisture value and where these measurements should be made. The problem that must be faced no matter which procedure is used in the horizontal and vertical variability of the soil characteristics. The selection of a procedure should be based on how the value for soil moisture content is to be used. If very detailed information is needed

for research modelling purposes, a sophisticated measurement-technique may be chosen. But in many engineering, design and operational programs a less detailed procedure can supply adequate information while costing less and taking less time. Geophysical techniques may prove their worth in such applications by providing measurement on a regional basis at low operational costs.

1.2 Geoelectrical Techniques Applied in Hydrology

Direct current resistivity measurements have a long history for use in ground water studies. The usual interest has been in detecting the water table and/or detecting and monitoring leachate from waste disposal sites. Resistivity studies are often effective at disposal sites because the resistivity of porous earth material is primarily a function of the amount of fluid which fills the pores, the conductivity of the fluid, and the clay content of the matrix. The relationship of resistivity to the first two parameters is fairly well understood for saturated conditions in clay-free formations. Archie's (1942) formula is one such relationship between formation resistivity, resistivity of the fluid and porosity. The presence of clay in the formation affects this relationship, both by altering the effective porosity and by altering the apparent pore water resistivity. Simple physical models describing ionic conduction in clayey sands and its use in interpreting lithology from apparent resistivity data have been described in literature (e.g. Elliott and Thomas, 1986).

Monitoring of variations in soil moisture by resistivity

for unsaturated conditions is not well documented. However, besides the conventional procedures, agricultural engineers have developed and successfully used four-electrode resistivity method for in-situ measurement of soil-salinity at different saturation conditions and to determine leaching fraction. Although with scanty available literature, the author took up the reported study after going through some earlier paper on the subject. Potential of the technique is also described by Kean et.al., (1987) as "the recent results of Krolow (1982) suggest that surface resistivity measurements could be useful in monitoring the passage of water through the unsaturated zone". Resistivity measurements can be applied as a useful supplemental technique to the existing conventional ones as a rapid method to determine the amount of water in a formation without disturbing it, and which would be sensitive to variation in water content at distances of several inches to several feet (Keller, 1962). Some of the work carried out at the National Institute of Hydrology, Roorkee has also been reported.

Regarding S.P. method, earlier its use had been restricted to mineral prospecting only. Subsequently, it was used in some engineering hydro-geologic studies such as seepage detection, etc., but with an assumption that S.P. measurements are reproducible within $\pm 5\text{mV}$ (Corwin and Hoover, 1979). This in effect meant that any time-variations in S.P. measurements are due to noise. Later, it was reported (Ernsteon and Scherer, 1986) that a component of these time-variations have definite relationship with hydrometeorological factors such as evapotranspiration,

precipitation and groundwater recharge. An analysis of repeated measurements of streaming potentials (S.P) may lead to decomposition into various components of different wavelengths, which in turn relate to the respective influencing parameters. However, at present only qualitative analysis of S.P. measurements, in terms of hydrometeorological parameters, have been attempted and relationship between them discussed in this report.

1.3 Resistivity and S.P. Dependence on Moisture Variations

1.3.1 Resistivity

All rocks at the earth's surface are porous. Highly porous rocks such as pumice or poorly compacted mudstones may have pore volumes as great as 50 percent or more of the total rock volume. Igneous rocks, evaporities and dense carbonate rocks have far lower porosities, but no surface rock is completely non-porous. Under any reasonable circumstances, these pores are partly or completely filled with water. This water usually carries some salt in solution, so that the water content of a rock has a far greater capacity for carrying current than does the solid matrix of the rock, unless highly conducting minerals are present.

The relative ability of materials to conduct electricity when a voltage is applied are expressed as conductivity; conversely, the resistance offered by a material to current flow is expressed in terms of resistivity. The resistivity of a medium is one of three physical properties which determine the behaviour of electromagnetic fields in the medium,

the other two properties being the dielectric constant and magnetic permeability. Resistivity is usually the most important of the properties in determining electrical current flow.

For most rocks near the earth's surface, conduction is of electrolytic nature, the conducting medium being an aqueous solution of common salts, distributed in a complicated manner through the pore structure of a rock. The resistivity of a water bearing formation will depend on the amount of water present, the salinity of this water and the way in which this water is distributed in the rock. The resistivity of water-bearing formation decreases with increasing water content. In fully saturated rocks, water content may be equated with porosity, but in partially saturated rocks, the effect of desaturation on resistivity must be considered. The texture of a rock also has some effect on the resistivity.

In formations whose pores are partly filled with water, circulation of water in the zone of aeration is an important factor in determining near surface resistivities. This circulation of water involves three major steps: (1) the infiltration of rain water from the surface into the soil immediately beneath the surface during rain periods, (2) the downward or lateral movement of this water through an aerated or partially saturated zone above the water table, and (3) return of the water to the atmosphere during dry periods or transpiration through plants. Influence of these factors on near-surface resistivity measurements has been discussed in this report.

1.3.2 Streaming potential (S.P)

In porous or fissured formation the relative movement between the solid phase and the water filling the pores and fractures is accompanied by certain electrical phenomena which are known as electrokinetic effects. These phenomena are ascribed to the presence of a potential difference and the zeta potential at the interface between the two phases. If the relative motion is brought about through a pressure drop, a difference of potential between any two points in the direction of motion will result. The electric potential difference E , known as streaming potential, is related to the pressure difference P across the rock by the relation:

$$E = C \Delta P, \text{ where} \quad \dots(1.1)$$

C = streaming potential coefficient, is dependent in a complex way on a great number of parameters like the resistivity, the dielectric constant and the viscosity of the fluid in the rock, the zeta potential, the grain size, the opening of fissures, the shape and tortuosity of the water path and others.

The quantitative evaluation of field anomalies in terms of size and magnitude of the source is generally difficult to make, mainly due to the lack of knowledge of in-situ streaming coefficients. Besides, real situations cannot always be consistent with simple models. However, a qualitative interpretation of field data, based on a comparison with the pattern of the anomalies caused by simple source geometries, have been attempted in hydrogeological studies.

From technical capability and economic feasibility point of view, electrical resistivity and S.P. methods have been chosen to study moisture variations in a soil profile and influence of precipitation and evapotranspiration on these variations.

The study area of investigations has comparatively simple lithology. It comprises of mainly alluvium with intermixed bands of clay resulting in fairly homogenous formation, which facilitates applications of resistivity and S.P.methods for soil-moisture measurement.

2.0 REVIEW

2.1 Four-Electrode Resistivity Sounding Method

The four-electrode resistivity technique for measuring the electrical conductivity of bulk soil has been known for many years. It has been regularly used in groundwater exploration and some engineering applications. Other important applications where resistivity technique has been effectively used include studies of groundwater contamination, soil salinity, soil moisture profile etc.

For groundwater exploration, resistivity technique is useful in delineating water-table and estimation of aquifer properties like transmissivity (Van Zijl, 1977), permeability (Keller and Frischknecht, 1977), specific capacity (Bardossy et.al., 1986). Resistivity method in conjunction with S.P. Technique are being routinely used in various engineering applications, such as assessment of foundation lithology (Khattari et.al., 1985), seepage studies in reservoir floors and earth dams (Bogoslovsky and Ogilvy, 1970), studies of engineering properties of rocks and soil (Arora and Sharma, 1986, Whiteley, 1983), detection of concentration of fractures or deep weathering and, therefore zones of weakness (Keller, 1974), detection of zones of vertical and lateral water movement in unstable masses such as landslides and general groundwater movement within spatially limited aquifers (Dobacki and Romig, 1985), etc.

With increasing awareness of groundwater contamination hazards, mapping of pollution from municipal, industrial agricultural and radioactive wastes has been attempted using

S.P. and resistivity techniques. Groundwater pollution is mostly governed by dispersion, except when its movement, due to the velocity of the water, is much more important than its mixing with the water of aquifer. In dispersion process, when two miscible fluids are brought into contact, there is a sharp interface at the beginning which vanishes into a transition zone, as the difference between physical properties (concentrations, for example) tend to be leveled with time. This effect results from the simultaneous action of several physio-chemical phenomena, such as molecular diffusion and permeability contrasts of the porous medium. Tracer experiments are routinely performed to study dispersion phenomenon. Resistivity technique could be used to tackle both types of problems in groundwater pollution, i.e. determination of dispersion coefficients as well as direction and intensity of aquifer velocity (Fried, 1975). A variety of electrical resistivity methods can be used (Sweeney, 1984) for investigation of groundwater pollution.

Field applications of resistivity technique in search for oil date back as early as 1930 s (Gish, 1932). Keller(1933) attempted to obtain empirical data on in-situ resistivities of different kinds of rocks at shallow depths. In historical perspective, it is noticed that not much attention has been paid to the use of resistivity technique for soil moisture measurements. Conventionally, soil moisture has been measured using laboratory methods, for example gravimetric method. Some field surveys indirectly touched upon this problem. For example, investigations carried out

for groundwater exploration on river-beds (Sarma, 1961) included effect of moisture on resistivity data. Kessels et.al. (1985) have reported results of investigations carried out in a salt mine in order to select sites for disposal of radioactive material. Water content was estimated in salt formations to identify areas with particularly high water content which should be avoided in radioactive waste disposal. Even occasional attempts of measuring soil moisture by resistivity method, for example in irrigation scheduling (Robertson 1949), could not lead to its widespread application.

Later, Rhoades and Ingvalson (1971) introduced concept of determining salinity in fields using soil resistance measurements. Since then, this technique has gained enough momentum so as to establish the concept. Further developments in theory (Bohn et.al. 1982: Frenkel et.al., 1983: Nadler et.al., 1984: Nadler, 1980), instrumentation (Nadler et.al., 1982: Rhoades, 1979: Austin and Rhoades, 1979) and field practicality (Nadler, 1982: Rhoades, et.al., 1981) have brought this technique to a level that now it is possible to use resistivity measurements for soil moisture and salinity estimation, at least as a potential supplement to other existing method. Rhoades (1980/81) has also reported study of soil electrical measurements for determining leaching fractions.

Attempts have also been made to utilize electrical methods for rapid, on-farm measurements of water content in order to be able to schedule irrigation optimally (Lauter et.al.). It has been reported that apart from larger areal

coverage capability, electrical methods have particular applicability in areas where the surface soil contains a high percentage of gravel (Robertson, 1949).

2.2 Streaming Potential (S.P) Method

Self-potential (S.P) method has long been used in prospecting for minerals. It has also been applied in some engineering (Corwin, 1948), for example seepage detection at reservoir and dam sites, and geothermal applications. Subsurface movement of water as manifested by the presence of springs has been mapped (Schiavone and Quarto, 1984) by interpreting S.P. anomalies of electrokinetic origin. A systematic study of time variation of S.P. measurements, incorporating potential hydrogeological factors like evapotranspiration, groundwater recharge, infiltration etc., has revealed that the water content in unsaturated zone, varying as a result of the above process, can be mapped with the help of S.P. method (Ernstson and Scherer, 1986).

For last two years, author has been engaged in developing use of resistivity technique in soil moisture measurements. Preliminary studies conducted at the National Institute of Hydrology, Roorkee suggest that there exist an inverse relationship between moisture-content and soil-resistivity (Goyal et.al., 1986). An empirical relationship has been developed between moisture-content and soil-resistivity for a site within the Campus. The relationship is being used for estimation of moisture content and its temporal variation in the soil profile at the study site.

3.0 PROBLEM DEFINITION

The main drawback in laboratory methods of soil moisture estimation, for example gravimetric method, is the non-representativeness of the in-situ properties of the sample. Once the sample is removed from its host soil mass, using whatever method of sampling, some physical properties, e.g. texture, change which ultimately affect moisture content of the sample. Besides, when sampling is required over long periods of time to monitor moisture movement or amount over time and space, this method can be very destructive to the site.

Tensiometric techniques provide direct measurements of soil water suction but only direct measurement of soil moisture characteristics for the soil. One possible advantage with such systems is their use to obtain data during freezing and thawing conditions, with the help of some liquids like ethylene glycol solution (Schmugge et.al., 1980).

The other commonly practised technique of determining soil moisture content is the neutron scattering method. A major limitation in use of this technique is the necessity of calibration for each soil type which alongwith installation of access-tubes for measurement makes this technique site specific and the data is available in form of point-measurements. The sphere of influence of the neutron probe measurement is the volume over which the average moisture content is calculated and depends on the amount of moisture in the soil. In many cases the diameter of the sphere of influence cannot be easily related to depth resolution because of the heterogeneity with soil depth (Schmugge et.al.,

1980). The vertical resolution is critical to many studies, especially those dealing with monitoring soil moisture in time and space.

With a view to popularize an alternative technique in order to supplement the existing techniques in terms of overcoming some of their respective limitations, an attempt was made to use geoelectrical techniques for measurement of soil moisture. Idea was to develop a relatively inexpensive method which can measure moisture content at various depths in a soil profile and its temporal variation.

Resistivity measurements have a long history of their use in ground-water studies. The usual interest has been in detecting the water table and/or detecting and monitoring leachate from disposal sites. The resistivity of porous earth material is primarily a function of the amount of fluid which fills the pores, the conductivity of the fluid, the formation porosity, and the clay content of the matrix in case formation is not clay-free. The relationship of resistivity to the first three parameters is fairly well understood for saturated conditions. However, monitoring moisture variations by resistivity for unsaturated conditions is not very well documented. The recent results of Goyal et. al. (1986) and Kean et.al. (1987) suggest that surface resistivity measurement could be useful in monitoring the passage of water through the unsaturated or vadose zone.

Keller (1962) has reported a relationship between formation resistivity and water content, in clay-free sedimentary rocks, as follows:

$$R = a R_w (S \emptyset)^{-n}, \text{ where} \quad \dots(3.1)$$

R is the bulk resistivity of the formation,

R_w is the resistivity of the water contained in the rock,

S is the fraction of pore space in the rock filled with water,

\emptyset is the volume fraction of pores in the rock, i.e. porosity

a and n are empirically determined constants

for particular rock types. The value of 'a' generally falls in the range $0.6 < a < 1.2$ and that for 'n' in the range $1.6 < n < 2.2$.

Having determined the value of R_w in a region, total water content (S \emptyset) in the formation at various depths can be estimated from surface resistivity measurements using equation (3.1). Under the reported project, the initial intent of the program was to monitor moisture migration caused by rainfall events and to establish a relationship between the changes in field resistivity and moisture content.

Owing to their link with water flow through the electrokinetic phenomenon, the natural electric potentials (S.P) have been used to study water movements. A systematic investigation of self-potentials shows that they vary considerably with time. In recent years both the use of the S.P. method and the effort to study the mechanisms of generation of potential anomalies have been on the increase since a strong correlation between S.P. anomalies and geothermal activity was discovered. Due to a low level of the signals

in hydrogeological studies a careful data acquisition procedure is necessary for obtaining meaningful anomalies. Although field procedure and operative methodology of the S.P. method are fairly developed, its quantitative interpretation in hydrogeological problems is still not well documented. The qualitative interpretation generally employs comparison of the observed anomalies, of electrokinetic origin, with theoretical and experimental anomalies generated by simple source geometries.

It has been established that the self-potentials at the near-surface must be regarded primarily as being of a streaming-potential character. Based on their findings, Ernston and Scherer (1986) have claimed "because of the close relation between self-potentials and hydrogeologic parameters, S.P. measurements may become a valuable and standard technique in hydrogeology".

In the reported project, S.P. measurements have been recorded on a routine basis with a view to establish a qualitative correlation between S.P. anomalies and water movements caused by rainfall events. No quantitative interpretation has been attempted in this study.

4.0 METHODOLOGY

4.1 Potential and Conductivity Distribution in Layered Earth

In the resistivity method a direct, commuted or low frequency alternating current is introduced into the ground by means of two electrodes (e.g. metal stakes) connected to the terminals of a portable source of e.m.f. The resulting potential distribution on the ground, mapped by means of two probes (metal stakes or non-polarizable electrodes), can be interpreted to yield the distribution of electric resistivity below the surface.

4.1.1 Potential distribution in a homogeneous earth

Considering a point electrode on the surface of a homogeneous isotropic earth extending to infinity in the downward direction and having a resistivity ρ , potential at a distance r from a point current source is given by:

$$V(r) = \frac{I\rho}{2\pi} \frac{1}{r}, \text{ where } \dots(4.1)$$

I is the strength of current passing into the earth.

The total potential at any point is $V = V(r) - V(r')$, where r' is the distance from the negative current electrode. In a homogeneous earth the fraction of the total current confined between the surface and the horizontal plane at a depth z is $(2/\pi) \tan^{-1}(z/L)$, where L is half the distance between the current electrodes. From this, it can be worked out that as much as 50 percent of the total current in a homogeneous earth never penetrates below the depth $Z = L$ and as much as 70.6 percent never below $Z = 2L$. The current will penetrate deeper, the greater the electrode separation.

4.1.2 Apparent (field) resistivity and its measurement

In case of a layered earth, let C_1, C_2 be the current electrodes, positive and negative (fig. 4.1), and P_1, P_2 the potential probes. If ΔV is the potential difference between P_1 and P_2 , it follows from equation 4.1.

$$\rho = 2\pi \frac{\Delta V}{IG} \quad , \quad \text{where} \quad \dots(4.2)$$

$$G = \frac{1}{C_1 P_1} - \frac{1}{C_2 P_1} - \frac{1}{C_1 P_2} + \frac{1}{C_2 P_2}$$

In an actual case, ρ will vary on altering the geometrical arrangement of the four electrodes or on moving them on the ground without altering their geometry, implying that R will not be directly proportional to G as on a homogeneous earth. The value of ρ , obtained on substituting the measured R and the appropriate G in (4.2), is called the apparent resistivity (ρ_a). It can be calculated for given electrode

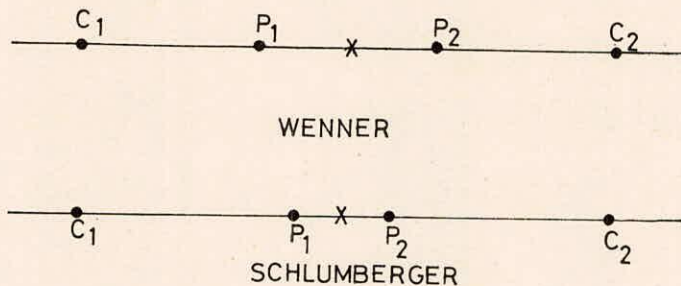


FIG. 4.1- CONVENTIONAL ELECTRODE CONFIGURATIONS

arrangements for a number of subsurface resistivity distributions. The apparent resistivity represents the combined effect of the number of layers and it should not be confused as an average

resistivity of the earth or any other similar magnitude.

Although a variety of electrode arrangements have been reported in literature, the two most commonly applied, especially in hydrogeological applications, configurations are shown in figure 4.1. In the Wenner configuration the separations between the adjacent electrodes are equal (a) so that (4.2) reduces to:

$$\rho_a = 2 \pi a R \quad \dots(4.3)$$

In the Schlumberger configuration $P_1 P_2 (=2\ell) \ll C_1 C_2 (=2L)$. At the centre of the system, the apparent resistivity can be computed as:

$$\rho_a = \frac{\pi L^2}{2\ell} \cdot \frac{\Delta V}{I} = \frac{\pi L^2}{I} \left(- \frac{dv}{dr} \right) \quad \dots(4.4)$$

where, $\frac{dv}{dr}$ is the surface gradient of V , that is, the electric field at the centre. If $\ell \ll L/5$, eq. (4.4) gives a ρ -value correct to within few percent.

Since the fraction of the current penetrating to deeper levels increases with the electrode separation, electric sounding primarily provides information about the variation of electric conductivity (or resistivity) with the depth.

4.1.3 Potential distribution in layered earth

Assumption of homogeneous earth is merely simplification of the actual geological conditions. In practical situations we often encounter much complicated geological structures. One type of conductivity (or resistivity) distribution that adequately describes, especially in groundwater prospecting, many geological situations is that represented by an earth composed of several horizontal strata. Out of

layered earth models, the simplest case where a horizontal layer of thickness h_1 and resistivity ρ_1 overlies a second homogeneous medium of resistivity ρ_2 (fig. 4.2). The potential of a point electrode C, through which a current I is passing

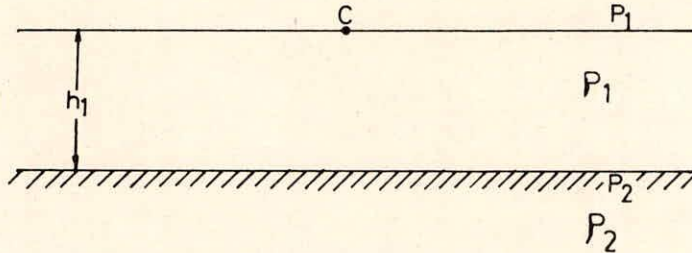


FIG.4.2- A TWO LAYERED MEDIUM

into such an earth, can be given by:

$$V(r) = \frac{I\rho_1}{2\pi} \cdot \frac{1}{r} \left[1 + 2r \int_0^\infty K(\lambda, k, h) J_0(\lambda r) d\lambda \right] \dots(4.5)$$

where $r = CP$, P is the point of measurement,

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}, \text{ known as resistivity-contrast coefficient}$$

$$K(\lambda) = k \exp(-2\lambda h) / (1 - k \exp(-2\lambda h)), \text{ and}$$

$J_0 =$ is the Bessel function of order zero

The potential distribution on an earth comprising of any arbitrary number of layers is again given by equation (4.5), but with $k(\lambda)$, known as kernel function, as a function of all strata thickness and resistivities. For a three-layer earth, for example,

$$K(\lambda) = \frac{k_1 u_1 + k_2 u_1 u_2}{1 - k_1 u_1 - k_2 u_1 u_2 + k_1 k_2 u_2} \dots(4.6)$$

where, $u_1 = \exp(-2\lambda h_1)$, $h_1 =$ thickness of first layer of resistivity ρ_1 .

$u_2 = \exp(-2\lambda h_2)$, h_2 = thickness of second layer resistivity ρ_2 , overlying an infinite sub-stratum of resistivity ρ_3 ,

$$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad \text{and}$$

$$k_2 = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2}$$

from (4.4) and (4.5), for the Schlumberger array:

$$\rho_a = \rho_1 \left(1 + 2r^2 \int_0^\infty K(\lambda) J_1(\lambda r) \lambda d\lambda \right) \quad \dots(4.7)$$

where,

$J_0'(\lambda r) = -J_1(\lambda r)$, J_1 is the Bessel function of order 1. Here it may be noted that in obtaining (4.7) from (4.5) the potential gradient must be doubled to include the effect of the negative current electrode of the array also. In (4.7) r is half the current electrode separation.

Using the following limit of a Lipschitz integral in Bessel function theory,

$$\lim_{C \rightarrow 0} r^2 \int_0^\infty e^{-\lambda C} J_1(\lambda r) \lambda d\lambda = 1$$

eq. (4.7) for the Schlumberger array becomes,

$$\rho_{as}(r) = r^2 \int_0^\infty T(\lambda) J_1(\lambda r) \lambda d\lambda \quad \dots(4.8)$$

and that for the Wenner array becomes,

$$\rho_{aw}(r) = a \int_0^\infty T(\lambda) [J_0(\lambda a) - J_0(\lambda 2a)] \lambda d\lambda$$

where, $T(\lambda) = \rho_1 (1+2k(\lambda))$, is known as the resistivity transform and can be easily written down for an arbitrary number of layers by means of recurrence formulae (Parasnis, 1979). Applying Hankel's transformation in Bessel function theory,

to (4.8), we have:

$$T(\lambda) = \int_0^{\infty} \frac{\rho_a}{r} J_1(\lambda r) dr \quad \dots(4.9)$$

This equation shows that the resistivity transform can be computed unambiguously from a measured ρ_a curve by numerical integration and implicitly contains all the information about the layered earth.

4.2 Inversion of Resistivity Field Measurements

In resistivity field measurements, basically surface potential is measured. Dividing surface potential by the applied current yields the resistance values, which when multiplied by a geometric factor pertaining to the electrode configuration used provides the apparent resistivity values corresponding to various electrode separations. This apparent resistivity (ρ_a) curve is interpreted, using suitable inversion procedures, to determine the thicknesses and resistivities of various individual layers. Different curve matching or computerised inversion methods are in use for interpretation of apparent resistivity data. Computerised inversion techniques based on direct inversion, trial-and-error, or optimization approach are numerically (and geologically) more efficient than the conventional curve matching procedure, especially for multi-layered problems .

4.2.1 Auxiliary point method

In auxiliary point method, apparent resistivity curve is matched by comparing with different curves from an album of pre-computed theoretical resistivity sounding curves. Master curves for two-layer and three layer earth models are easily available. The method can be extended

in principle to a sounding curve on any number of layers by the alternative use of the two layer master curve & three-layer auxiliary curves.

4.2.2 Pekeris' direct method

It is easy to measure the surface potential $v(r)$ of a single point electrode by removing the other current electrode to great distance. We compute the function

$$k(\lambda) = \lambda \int_0^\infty V(r) J_0(\lambda r) r dr \quad \dots(4.10)$$

by numerical integration and plot $\ln|f_1(\lambda)|$ against λ where

$$f_1(\lambda) = \frac{k(\lambda) + 1}{k(\lambda) - 1}$$

for large λ , the points will be on a straight line with a slope $2h_1$ and an intercept $\ln(1/k_1)$ with the y-axis where

$$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

If all the points lie on a straight line $h_2 = \alpha$ (two-layer case) ; otherwise closely similar function $f_2(\lambda)$, the plot of whose logarithm gives in turn h_2 and $k_2 = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2}$. The process is continued by calculating a third function f_3 if all the points do not lie on a straight line, that is, if $h_3 \neq \alpha$ and so on.

This method implies no restrictions on the relative magnitudes of h_1, h_2, h_3, \dots and is thus quite general. But it requires considerable computations. The main task in the direct method is the rapid and accurate calculation of the kernel function (or the resistivity transform) from measured data. Fast methods of calculating $T(\lambda)$ and constructing $k(\lambda)$ from the observed ρ_a curve have now developed the method into a rapid practical procedure.

4.2.3 Ghosh's method

In this trial-and-error method, the entire ρ_a curve of a model layered structure is calculated for comparison with the measured curve.

In essence Ghosh evaluates the right-hand side of equation (4.8), but with the help of a set of filter (called short filter) coefficients (Table 4.1), taking advantage of the fact that $T(\lambda)$ in (4.9) is an algebraic expression involving no more complicated functions than the exponential. A practical procedure of the method is as follows:

(a) In the first stage, sample values $T_m(m=0, 1, 2, \dots)$ of $T(\lambda)$ are calculated from the recurrence relations of Appendix IV, for successive values of λ in the ratio $10^{1/3}$, that is, $2.154 \times 1/\lambda$ is the distance along the x-axis and the sampling rate corresponds on a logarithmic plot to $3T_m$ values per decade. Any starting value of λ may be used but it is convenient to start with $\lambda = 1$ and calculate for $\lambda < 1$ as well as $\lambda > 1$.

(b) In the second stage, the sampled values of apparent resistivity are obtained as:

$$(\rho_a)_m = \sum_{j=-3}^5 b_j T_{m-j} \quad (m=0, 1, 2, \dots) \quad \dots(4.11)$$

where b_j are the 9 coefficients in Table 4.1. Due to the particular manner in which b_j 's have been calculated by Ghose, each $(\rho_a)_m$ value obtained from (4.11) refers not to the abscissa of the corresponding T_m but to an abscissa that is 5% to the left. Thus, for example, $(\rho_a)_m$ corresponding to T_m at $1/\lambda = 21.54$ must be plotted at $1/\lambda = 20.47$ etc.

TABLE 4.I : Ghosh's Short Filter Coefficients

a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5	a_6
-0.212	-0.1199	0.4226	0.3553	0.1664	0.0873	0.0345	0.0208	0.0118

From (4.11) it is seen that in order to obtain $(\rho_a)_m$ values within a particular range of current electrode separation it is necessary to have 5 extra T_m samples to the left of the range and 3 to the right.

4.2.4 Optimization methods

If a large computer is available it is not necessary to restrict the number of coefficient to 9 or the number of sampled $T(\lambda)$ values to 3 per decade. However, in any case, the trial-and-error in this method can be tedious and a considerable computer time can be saved by letting the computer seek the optimum model by an interactive procedure.

Considering a n -layer model, let P_j ($j=1,2,\dots,m$), be the n resistivities and $n-1$ layer thicknesses ($m=2n-1$).

Let y_i be the measured values of ρ_a for the separations between two current-electrodes x_i , $i=1,2,\dots,n$. The problem is to minimise the sum of squares.

$$S = \sum_{i=1}^n [y_i - \rho_a(x_i, P_j)]^2 \quad \dots(4.12)$$

with respect to P_j where $\rho_a(x_i, P_j)$ is the model values given by equation (4.8). ρ_a and P_j and non-linearly connected. Equa-

tion (4.8) is linearized by expanding ρ_a in a Taylor series around a starting model P_j^0 and discarding all terms of higher order than the first. This gives

$$S = \sum_{i=1}^n [y_i - \rho_a(x_i, P_j^0) - \sum_{j=1}^m \frac{\partial \rho_a}{\partial P_j} \Delta P_j]^2 \dots (4.13)$$

where the derivatives are evaluated at the point P_j^0 .

The minimization (4.13) determines the correction ΔP_j . The process is repeated by setting $P_j = P_j^0 + \Delta P_j$ until there is no further decrease in S . The P_j values in this case give the desired optimum parameters. In another approach, y_i and ρ_a are replaced by $\ln y_i$ and $\ln \rho_a$ respectively, thus linearising the problem to a great extent to start with, and ρ_a and the derivatives are calculated by Ghose-type coefficients. This approach leads to a considerable decrease in the time necessary to obtain the optimum solution.

4.3 Relationship Between Resistivity and Water Content

The only direct method for accurate estimation of soil moisture content is the gravimetric method. Although it involves a time-consuming process, but the method is still used as standard in calibration procedures for all other methods. Complete procedure of the gravimetric method is described in Appendix I.

It is because of the electrolytic conduction of current that the nature and amount of dissolved salts are important parameters in determining the soil resistivity. Actually bulk resistivity of the soil is affected by its physio-chemical properties, e.g. salt content, organic content, clay content, porosity, etc. and electric resistivity of

the solution (water) saturating the formation. To sum up, soil water content varie with the weather conditions, the nature of the sub-soil and depth to the water table. The resistivity of a solution is inversely proportional to concentrations, mobilities and activity coefficients of the anions and cations in solution. As a result, different salts have different effects on soil - resistivity. Generally, the common consituent in soil.

Any change in apparent (field) resistivity of soil which is observed at the surface is due to the change of bulk resistivity within the soil formation. Empirical relationship between bulk resistivity, porosity and saturation was originally presented by Archie (1942). General form of this formula for estimating bulk resistivity (ρ) of rock formation is given by:

$$\rho = F \rho_w \quad \dots(4.14)$$

where, ρ_w is the resistivity of electrolyte,

F is the formation-factor, and is given by

$$F = a \phi^{-m} S^{-n} \quad \dots(4.15)$$

where,

ϕ = Porosity

S = saturation, and

n, a, m are constants.

The saturation exponent of $n \approx 2$, whereas value of the empirical constants a and m may vary between limits (Telford et.al., 1976):

$$0.5 \leq a \leq 2.5$$

$$1.3 \leq m \leq 2.5$$

the low limits of the constants represent loose or poorly consolidated formations, and the upper limits are that for the consolidated formations.

In equation (4.15), ϕ and S are defined as:

$$\phi = \frac{V_P}{V_T} \quad , \text{ and} \quad \dots(4.16)$$

$$S = \frac{V_W}{V_P} \quad \dots(4.17)$$

where, V_T = total volume of formation
 V_P = pore volume inside the formation, and
 V_W = volume of electrolyte in the pores.

Combining equation (4.16) and (4.17), the volumetric soil-moisture content (θ) is defined as:

$$\theta = S \cdot \phi = \frac{V_W}{V_P} \cdot \frac{V_P}{V_T} = \frac{V_W}{V_T} \quad \dots(4.18)$$

From equations (4.14) to (4.18) it is seen that any change in physio-chemical or meteorological conditions may lead to changes in V_P , V_W or ρ_W , and as consequence the resistivity ρ of the formation will change.

Rewriting equation (4.15) in terms of soil-moisture content θ from equation (4.18)

$$F = a \phi^{-m} \left(\frac{\theta}{\phi}\right)^{-n} \quad \dots(4.19)$$

or $F = a \phi^{n-m} \theta^{-n}$

Substituting value of F from equation (4.19) equation (4.14) can be written as:

$$\rho = a \phi^{n-m} \theta^{-n} \rho_W \quad \dots(4.20)$$

As defined earlier, the saturation exponent $n \approx 2$. The value of 'a' has initially been assumed to be 0.5 as the formation in the study area consists of unconsolidated sandy soil. For known value of ρ , ϕ , ρ_w and θ and assumed values of 'a' the exponents m was evaluated for different sets of ρ and θ using the following equation:

$$m = \ln \frac{\rho \theta^2}{a \rho_w} / \ln \phi \quad \dots(4.21)$$

The average value of m for a set of field data (Goyal, et.al. 1986) was found to be 0.74. Then the final expression relating bulk resistivity with soil moisture, porosity and electrolyte resistivity for the study area is given by:

$$\rho = 0.5 \phi^{0.74} \theta^{-2} \rho_w \quad \dots(4.22)$$

In terms of saturation, the above expression can be written as :

$$\rho = 0.5 \theta^{-1.26} S^{-2} \rho_w \quad \dots(4.23)$$

Kelly (1985) used the following form of the Archie's formula for recharge studies in soil formations:

$$\rho = \phi^{-1.5} S^{-2} \rho_w \quad \dots(4.24)$$

From equation (4.23) and (4.24), it is seen that the formation in the study area is made up of unconsolidated sandy soils, as indicated by low value of a (=0.5) and m (=1.26).

4.4 Field and Laboratory Procedures

In view of the discussions in section 4.1, it was felt that resistivity technique for measurement of moisture variations in a soil profile needed characterisation of the particular site before any reliable interpretation is attempted.

For this, the mentioned properties of the study site were determined for a soil profile upto 1.5 m depth. A sampling auger of 1" dia. was used to collect soil samples at intervals of 15 cms, upto a depth of 1.5 m. The standard procedures for determination of these properties are described in Appendices II and III. Two possible source of water present in the soil profile are rainwater and groundwater. To ascertain resistivity of the saturating solution, electrical conductivity (EC) of water samples from various nearby wells and rainwater analysed in the laboratory for pH, EC and total dissolved solids (TDS) values.

Wenner sounding array (fig. 4.1) was used to measure apparent resistivity in the field. A multi-electrode switch controlled system has been developed to expedite field data collection. For a 10 point electrical sounding, it consisted of a total of forty electrodes, twenty each for potential and current purposes, controlled by a 4-pole-10 ways switch through multi-core cables. All the electrodes were installed simultaneously and successive measurements at expanding electrode-separations taken by moving switch positions at a central location. This system was hooked up with an available resistivity meter, which was used for measuring resistance of the sub-surface at various electrode positions. With the use of switch-controlled system, a 10 point sounding takes just 15 minutes time.

The apparent resistivity values obtained from the field were transformed into corresponding bulk resistivities using a computerised curve matching procedure (Schimschal, 1981).

5.0 APPLICATION

This study discussed the relationship of moisture content and resistivity in the unsaturated zone at two sites within the University of Roorkee campus, of similar lithologies but with different vegetation patterns. Both the sites have fairly homogenous lithologies, comprising of graded sands with occasional bands of clays/silt intermixed with sands. The water table at the NIH Campus (site 1) varies from 4.0 to 4.5 m, whereas at a nearby agricultural farm (site 2) it varies between 5-5.5 m. It was felt that geology at these sites was simple enough that the dependence of resistivity changes on water movement in the vadose zone could be determined. These results could have applications to a variety of problems in hydrology, water resources management and agronomy. The ability to monitor the progress of a wetting front would have additional usefulness in the monitoring of in-situ leaching operations and in monitoring the effect and progress of leachate plumes at waste disposal sites.

A switch controlled, multi-electrode Wenner array (fig. 4.3) was used to conduct electrical soundings at the two sites. This system was merely an extension of the conventional four-electrode Wenner array, in which electrode separations are controlled by switching at a central location between number of electrodes installed before hand and connected through a multicore cable. Apparent resistivity is calculated using equations for a Wenner array. The main advantage in using the multi-electrode array, fabricated

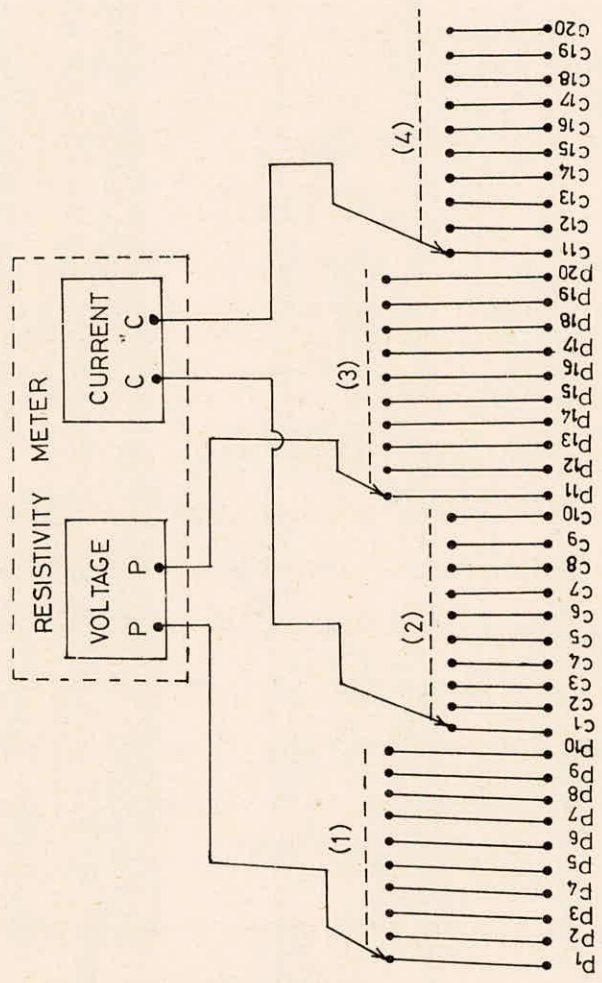


FIG. 4.3 - SWITCH CONTROLLED MULTI-ELECTRODE WENNER ARRAY.

at the National Institute of Hydrology, has been the ease in continuous monitoring of a Wenner sounding over time. The array used 20 potential and 20 current electrodes with 'A' spacings varying from .25 to 5.66 m, thereby giving a maximum depth of penetration upto 2m.

The electrodes consisted of .46m long circular bars of galvanised metal which were driven .1m into the ground. Separate wires ran from each electrode to a central board, where they were connected to the resistivity meter according to the respective switch positions. The electrodes remained emplaced in the ground at the site during the entire study. IGIS ACR-1 A.C. Resistivity Meter was used for field measurements.

Apparent resistivity data was collected at the site 1 during the months of June and July 1987, while that for the site 2 was collected during August 1987. The moisture contents at corresponding depths were determined in laboratory using standard gravimetric method, for each of the sites. Sieve analysis, followed by clay analysis was performed in order to ascertain the clay content and the type of clay minerals present in the soil samples from various depths. Geochemical analysis of the soil samples was also done to determine total dissolved solids (TDS) and organic content.

In order to assign a near-realistic value to the ρ_w (resistivity of saturating fluid) term in equation 4.22, EC values of groundwater samples from various tube wells in the U.O.R. Campus and around were measured. A sample of rainwater was also analysed to measure EC.

6.0 RESULTS

As already mentioned, S.P. measurements were included in the study to establish the variation of S.P. value with precipitation and evapotranspiration. During May, 1987 potentials were measured between the deepest electrode (6 feet below surface) and other successive electrodes at 5,4,3,2 and 1 feet respectively (please refer figure 6.1), effectively measuring the potential gradients. During June, July and September, 1987 potential gradients as well as potential differences (please refer figure 6.1) were measured. Variation of potential gradient and potential difference with rainfall and evapotranspiration during these four months is shown in figures 6.2-6.7. A general correlation between S.P. measurements and rainfall events is evident in all these figures. However, potential gradient shows a strong correlation with the rainfall events (figures 6.2, 6.3, 6.5, 6.7(2)). A time lag between the S.P. anomalies and the rainfall events corresponds to the time taken by a wetting front to reach the respective depth of measurement.

In order to apply equation (4.22) for estimation of soil moisture contents at various depths using resistivity data, the first step was to ascertain the values of the porosity (ϕ) and the electrolyte-resistivity (ρ_w) at the site of investigations. Soil samples were collected from various depths and analysed to determine the grain sizes. Grain-sizes in different soil-samples from varying depths are given in Table 6.I, which shows that the soil profile comprises of mainly coarse sand with occasional

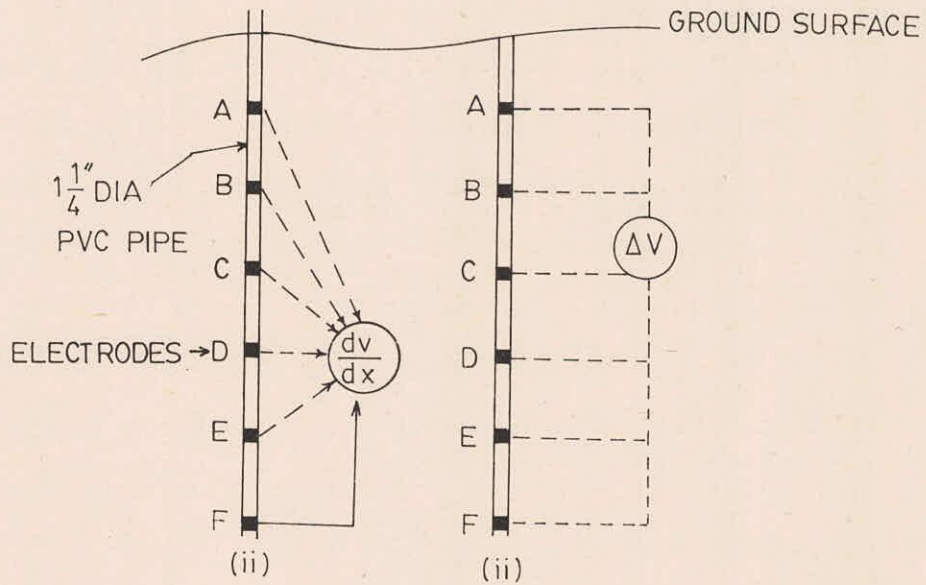


FIG. 6.1 ELECTRODE ARRANGEMENT FOR S.P. MEASUREMENTS
 (i) POTENTIAL GRADIENT (ii) POTENTIAL DIFFERENCE

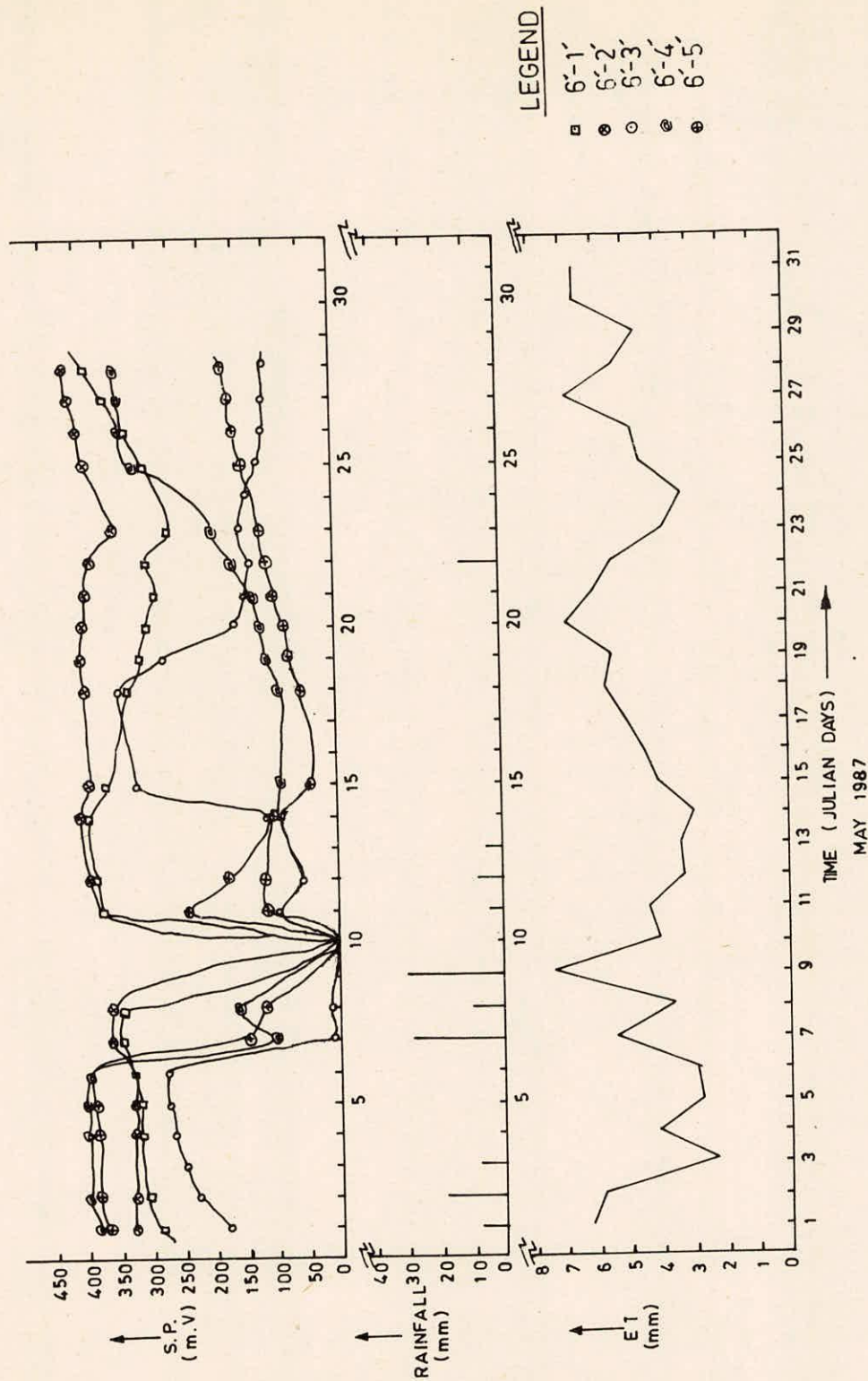


FIG. 6.2 - VARIATION OF S.P., RAINFALL AND EVAPOTRANSPIRATION DATA.

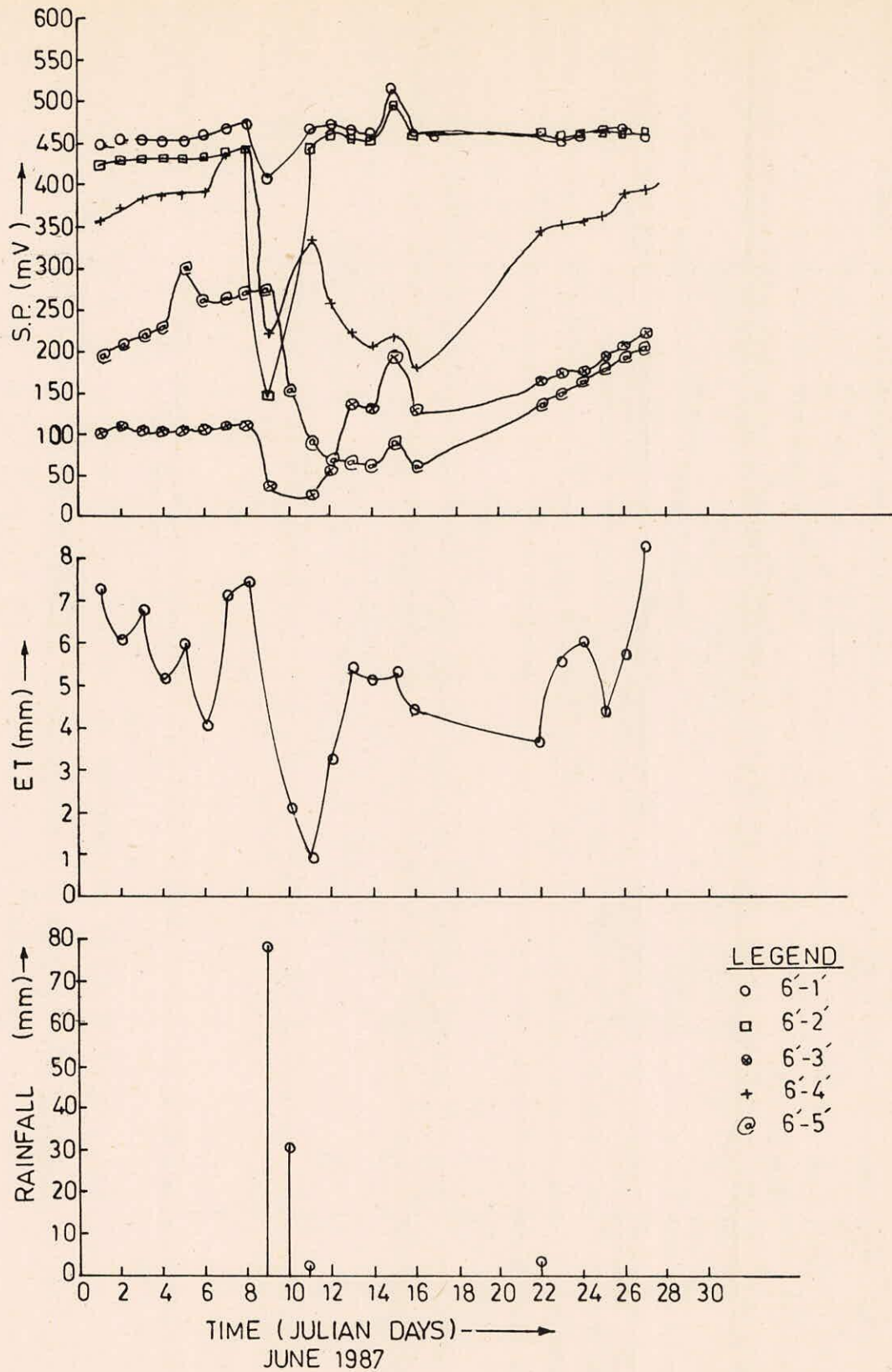


FIG. 6.3 - VARIATION OF S.P., RAINFALL AND EVAPOTRANSPIRATION DATA.

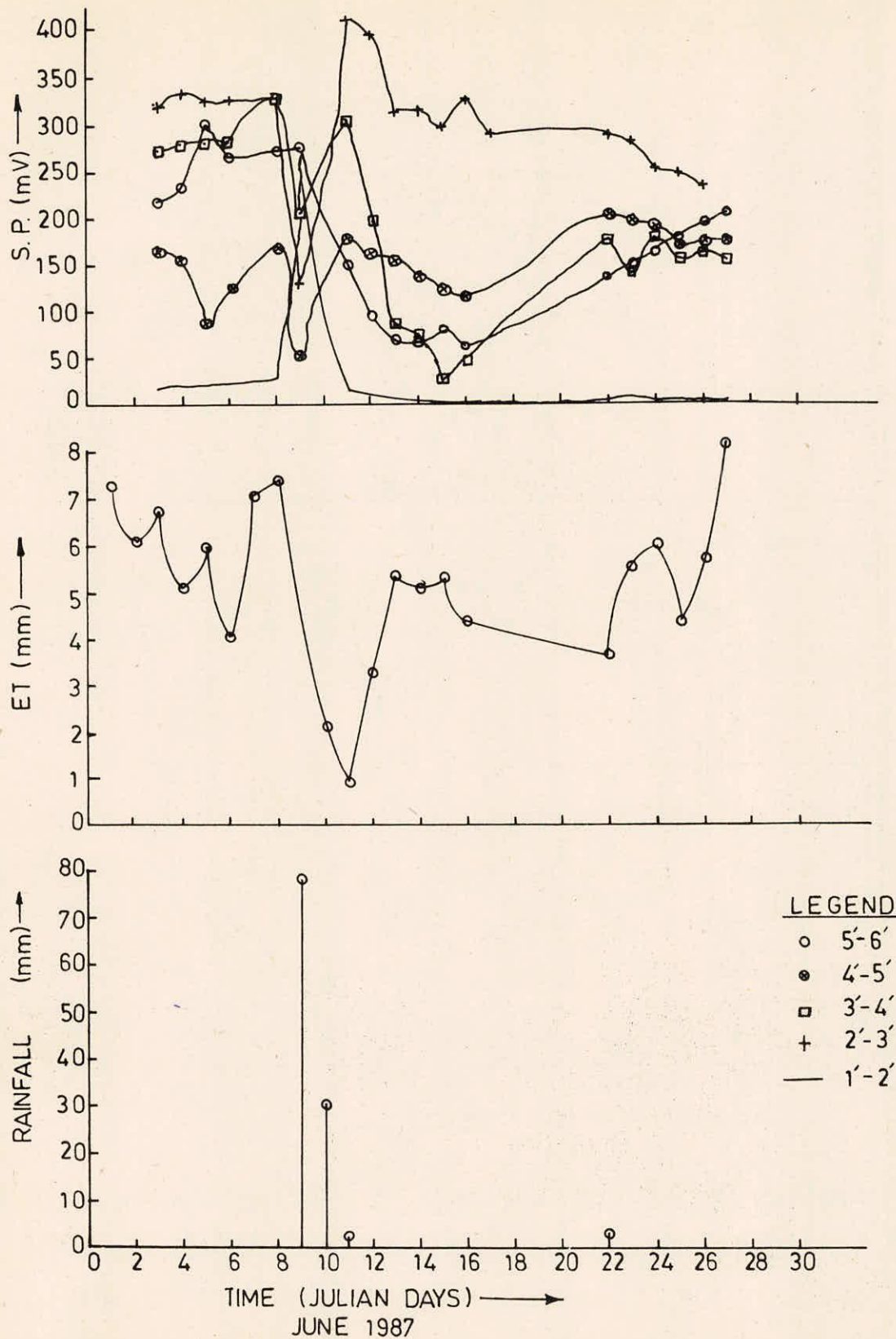


FIG. 6.4 - VARIATION OF S.P., RAINFALL AND EVAPOTRANSPIRATION DATA

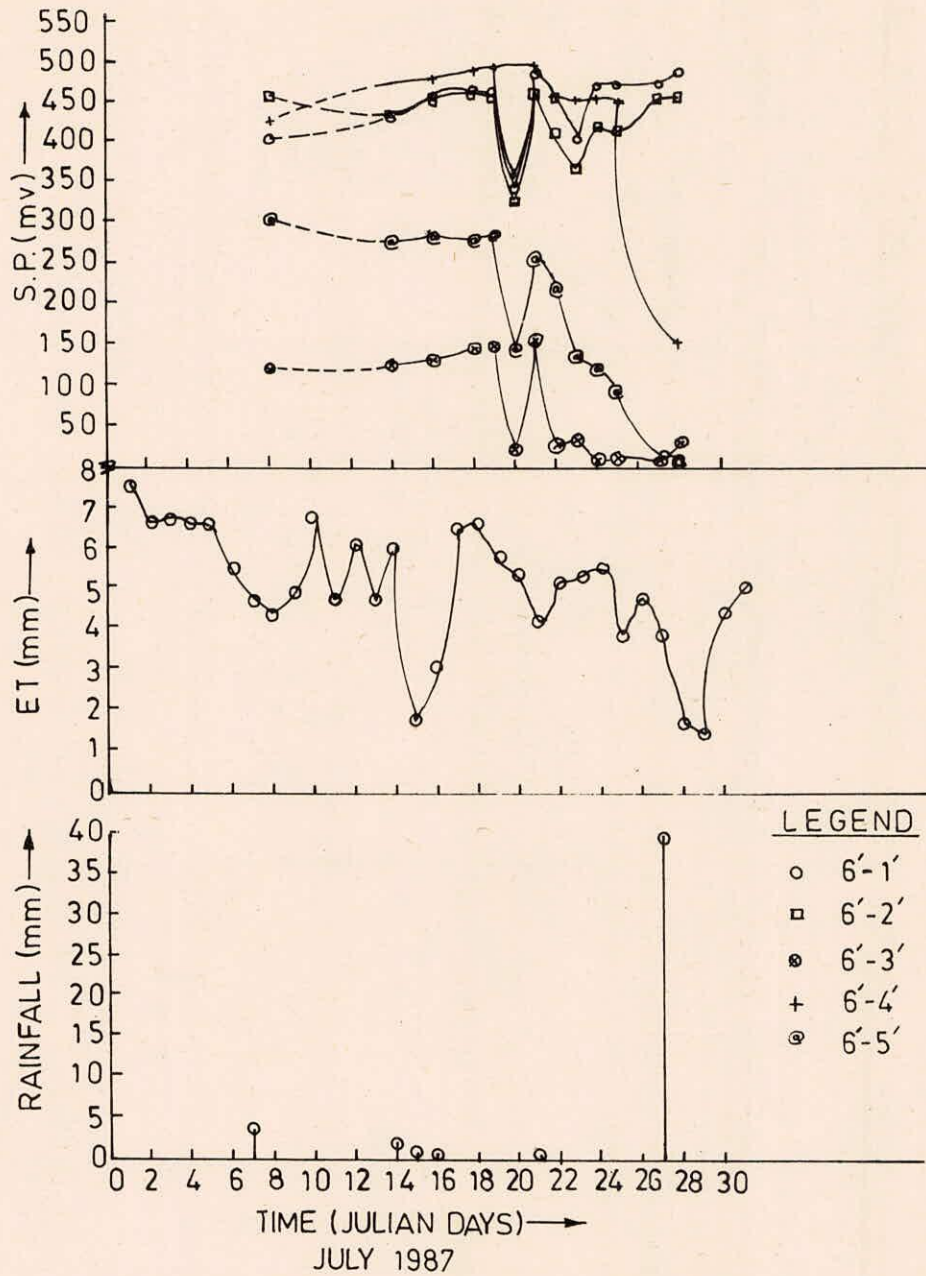


FIG.6.5-VARIATION OF S.P., RAINFALL AND EVAPOTRANSPIRATION DATA.

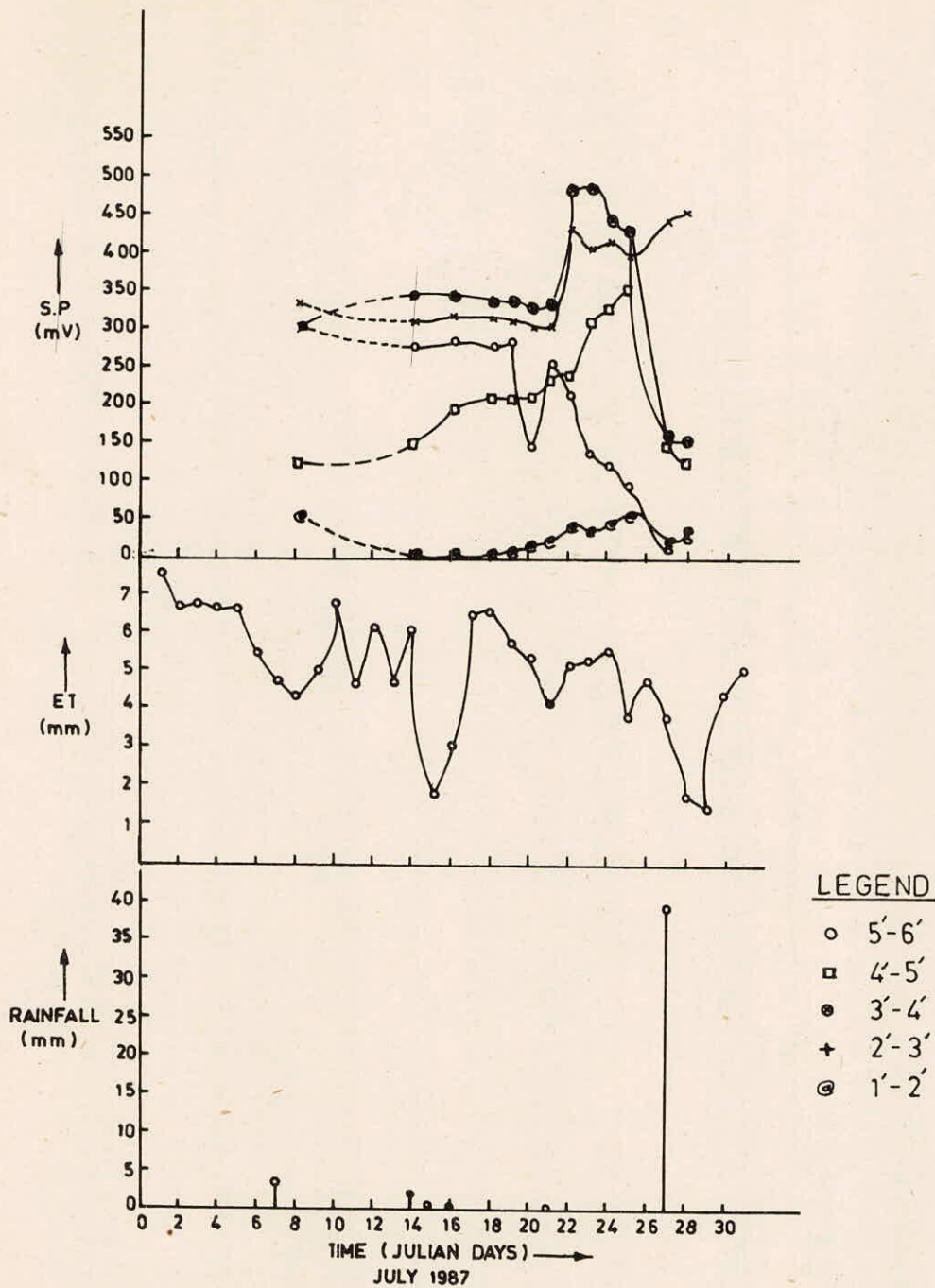
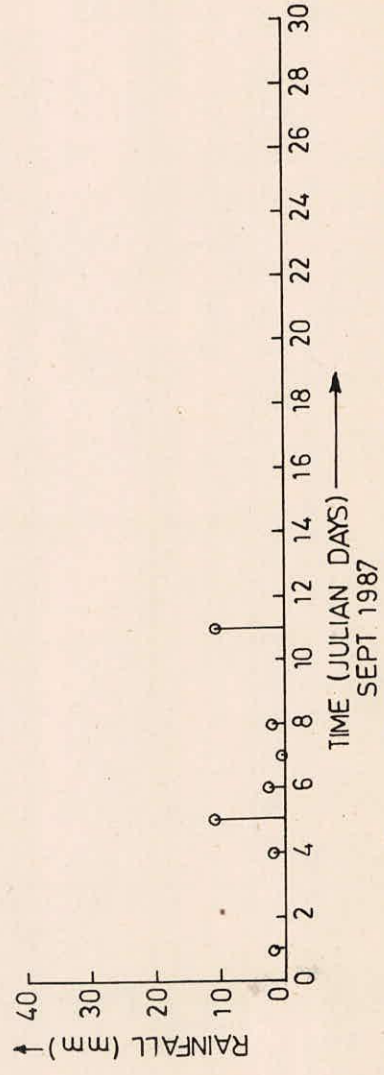
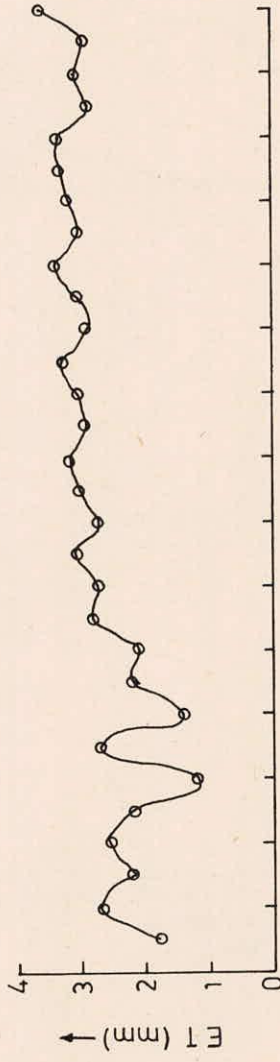
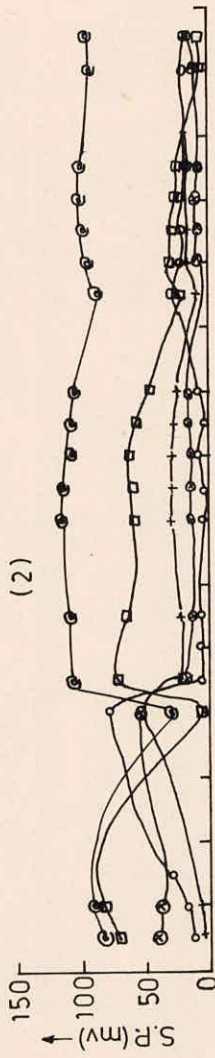
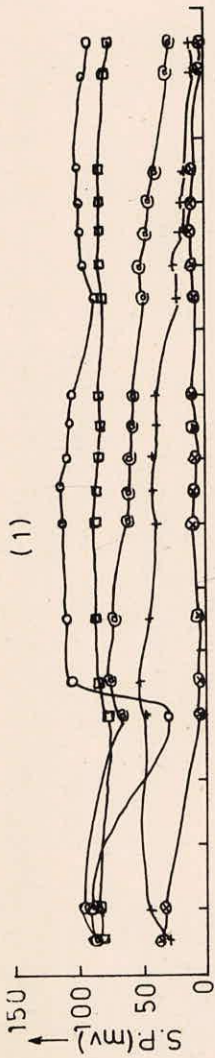


FIG.6.6 - VARIATION OF S.P., RAINFALL AND EVAPOTRANSPIRATION DATA.



LEGEND	
(1)	(2)
○	5'-6'
□	4'-5'
●	3'-4'
+	2'-3'
⊙	1'-2'
	6'-1'
	6'-2'
	6'-3'
	6'-4'
	6'-5'

FIG. 6.7- VARIATION OF S.P., RAINFALL AND EVAPOTRANSPIRATION DATA.

Table 6.I : Grain size VS depth for Soil at test site 1

Sl.No.	Depth (Cms)	Percent between .84-.50 mm (coarse sand)	Percent between .50-.30 mm (medium sand)	Percent less than .30 mm
1.	20-40	63.8	3.1	26.4
2.	40-50	51.7	5.3	42.7
3.	50-60	80.9	4.6	12.4
4.	60-70	82.9	2.4	9.4
5.	70-80	82.1	1.8	10.7
6.	80-90	87.8	1.2	7.3
7.	90-100	82.7	1.8	15.3
8.	100-110	80.9	1.8	8.6
9.	110-120	83.4	3.1	8.6
10.	120-130	81.6	2.5	9.8
11.	130-140	91.4	1.2	6.2
12.	140-150	90.0	0.6	5.0
13.	150-160	90.1	1.2	4.9
14.	160-170	88.3	1.2	4.9
15.	170-180	95.4	3.2	7.8
16.	180-190	85.2	1.9	7.0
17.	190-200	89.8	2.5	4.9
18.	200-210	85.0	2.5	6.9
19.	210-220	90.7	2.5	5.6
20.	220-230	89.4	3.0	11.3
21.	230-240	87.0	4.1	5.9
22.	240-250	94.2	1.9	9.6
23.	250-260	82.8	4.3	12.4
24.	260-270	81.5	2.8	11.2
25.	270-280	80.4	2.1	14.8
26.	280-290	82.1	3.1	12.6
27.	290-300	92.3	2.2	5.6

bands of fine sand. Further analysis to determine the clay content using the conventional pipette method indicated that clay (particle size < 0.0039 mm) content is relatively high at depths between 90-150 cms (Table 6.III). Samples from depth range 90-150 cms were then analysed on X-ray diffraction equipment to ascertain the relative abundance and the nature of clay minerals. Interpretation of X-ray diffractograms showed that Illite is the main clay mineral alongwith small amounts of Kaolinite in the soil profile at site 1.

Water samples were collected from various tube-wells located in and around the site of investigations, so that to have a general idea of the quality of groundwater, which was supposed to be the major controlling source of the moisture present in the soil formations. A sample of rain-water was also collected on August 13, 1987 to supplement the data from groundwater samples. Results from analysis of these samples in terms of pH, EC, and TDS are given in table 6.II. Value of TDS for rainwater and any of the groundwater samples is remarkably different, which suggest a corresponding difference in the value of electrolyte-resistivity (ρ_w). This fact must be taken into account when using equation (4.22) to invert resistivity data for estimation of soil moisture content.

Soil samples from the two sites were also analysed to determine organic content, salinity (TDS) and moisture content in the soil profiles. Values of these parameters for the sites 1 and 2 are given in Table 6.III and 6.IV

Table 6.II Values of Parameters Measured on Groundwater-Samples from Various Tube-wells located in U.O.R. Campus and Around. One sample of Rainwater Also Included.

Sl.No.	Location of tube-well	Date of sampling	EC m mho cm	TDS (ppm)	pH
1.	V C Lodge (U.O.R)	19.12.86	.345	220.8	6.48
2.	Azad Wing (U.O.R)	-do-	.329	210.5	6.23
3.	Sarojini Bhawan (U.O.R)	-do	.458	293.1	6.84
4.	Cautley Bhawan (U.O.R)	-do-	.450	288.0	6.68
5.	WRDTC Farm (U.O.R)	-do-	.399	255.4	6.85
6.	C.B.R.I	-do-	.402	257.3	6.72
7.	N.I.H	-do-	.585	374.4	-
8.	Solani Kunj (U.O.R)	3.08.87	.564	361.0	7.10
9.	N.I.H.	-do-	.419	268.2	7.46
10.	Gandhi Vatika (Civil Lines)	-do-	.356	227.8	7.70
11.	Rain Water	13.08.87	.076	48.6	7.54

Tabel 6.III: Values of Parameters Measured on Soil Sample from
Site 1. Samples Collected on 17.09.87

S.No.	Depth (cm)	Moisture (% Vol)	EC mmho/ cm	TDS (ppm)	Organic content (%)	Clay content (%)	
						8 ϕ (<.0039mm)	9 ϕ (<.0020 mm)
1.	0-15	15.05	.181	115.8	1.38	.73	NIL
2.	15-30	15.28	.173	110.7	1.52	2.16	1.44
3.	30-45	14.41	.199	127.4	1.09	.97	1.01
4.	45-60	14.30	.268	171.5	1.37	6.33	3.64
5.	60-75	22.44	.159	101.8	2.36	7.82	5.79
6.	75-90	15.40	.169	108.2	2.29	15.00	4.30
7.	90-105	27.01	.164	105.0	2.64	22.4	12.8
8.	105-120	28.36	.183	117.1	2.28	28.9	15.12
9.	120-135	24.92	.166	106.2	2.92	26.2	12.90
10.	135-150	27.19	.169	108.2	2.78	27.94	11.13

Table 6.IV: Values of Parameters Measured on Soil Samples from

Site - 2. Samples Collected on 26.08.87

Sl.No.	Depth (cm)	Moisture (% Vol.)	EC mmho/ cm	TDS (ppm)	Organic Content (%)	clay content (%)	
						8 ϕ (<.0039mm)	9 ϕ (<.0020mm)
1.	0-15	23.3	0.214	137.0	1.50		
2.	15-30	20.1	0.185	118.4	1.65		
3.	30-45	25.3	0.171	109.4	1.30		
4.	45-60	27.3	0.173	110.7	1.80	2.	.9
5.	60-75	25.7	0.179	114.5	1.65		
6.	75-90	26.4	0.167	106.9	1.85		
7.	90-105	26.0	0.173	110.7	1.75	6.7	4.9
8.	105-120	25.2	0.170	108.8	1.83	5.6	6.7
9.	120-135	25.8	0.171	109.4	1.80		
10.	135-150	23.1	0.195	124.8	1.70		
11.	150-165		0.190	121.6			
12.	165-180		0.189	121.0			

respectively. It is seen that organic content and salinity do not vary much in the soil profiles and hence their effect on resistivity of the formation can safely be neglected.

Apparent (field) resistivity data was inverted using a computerised curve matching procedure (Schimschal, 1981) and interpreted in terms of resistivities and layer thicknesses in the soil profile for the two sites. Resistivity and layer-thickness parameters for selected resistivity soundings are given in Table 6.V. Resistivity soundings marked with (W) are from site 2, while other four soundings are from site 1. Comparative variation of resistivity and soil moisture content corresponding to these soundings are shown in figures 6.8 - 6.10. A correlation between variation in the two parameters is observed in all these plots.

Table 6.V : Temporal Variation of Resistivity Profiles

LAYER	LAYER PARAMETERS	VES-1 30.09.85	VES-2 17.12.85	VES-3 23.06.87	VES-4 17.09.87	VES-5 14.08.87(W)	VES-6 26.8.87(W)
1.	RESISTIVITY (ohm-m)	44.4	85.8	63.1	34.2	30.5	76.5
	THICKNESS (m)	0.22	0.14	0.29	0.30	0.27	0.18
2.	RESISTIVITY (ohm-m)	69.7	148.4	79.3	66.4	66.2	134.0
	THICKNESS (m)	0.31	0.52	0.26	1.02	0.92	0.55
3.	RESISTIVITY (ohm-m)	30.5	35.7	40.4	43.7	52.8	50.5
	THICKNESS (m)	0.43	0.23	0.42	2.57	0.45	0.57
4.	RESISTIVITY (ohm-m)	9.0	8.0	9.7	30.0	60.4	13.0
	THICKNESS (m)	0.74	0.45	0.26	-	-	1.21
5.	RESISTIVITY	28.0	34.5	16.8	-	-	35.2

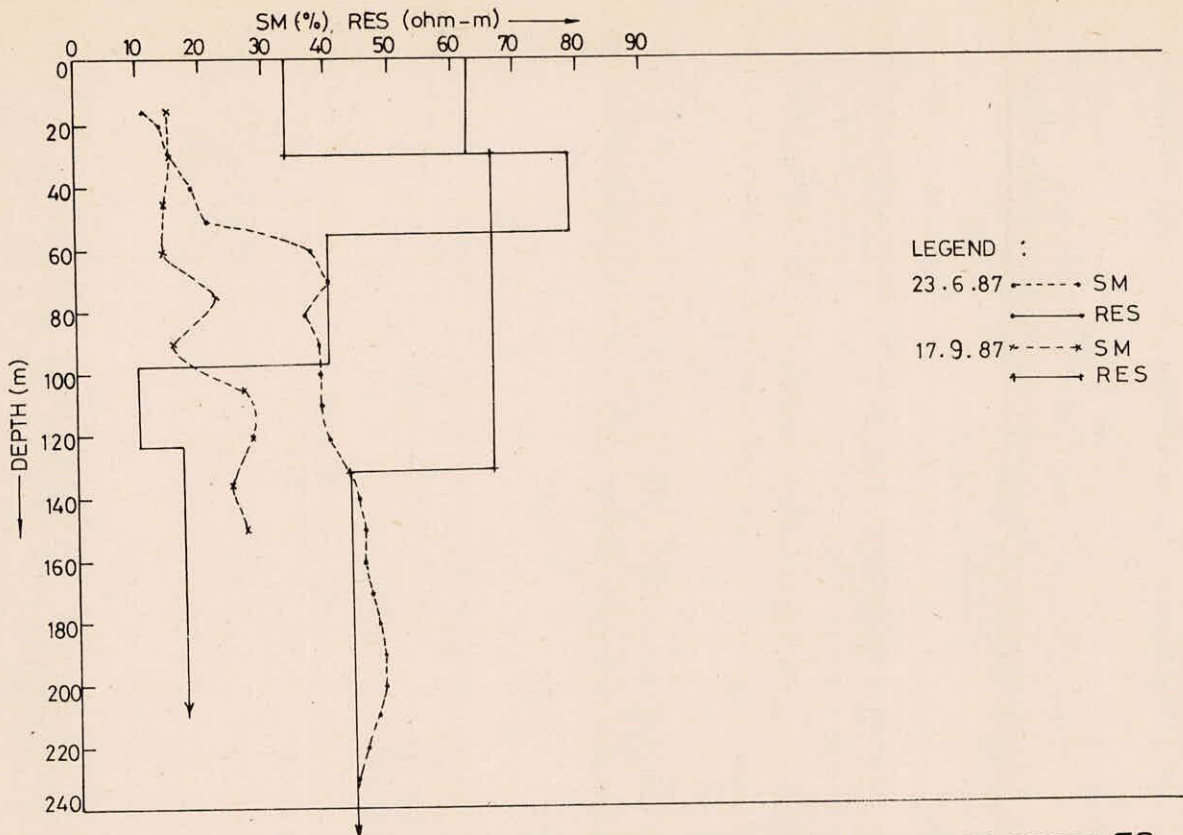


FIG. 6.8-VARIATION OF MOISTURE CONTENT AND RESISTIVITY PROFILES

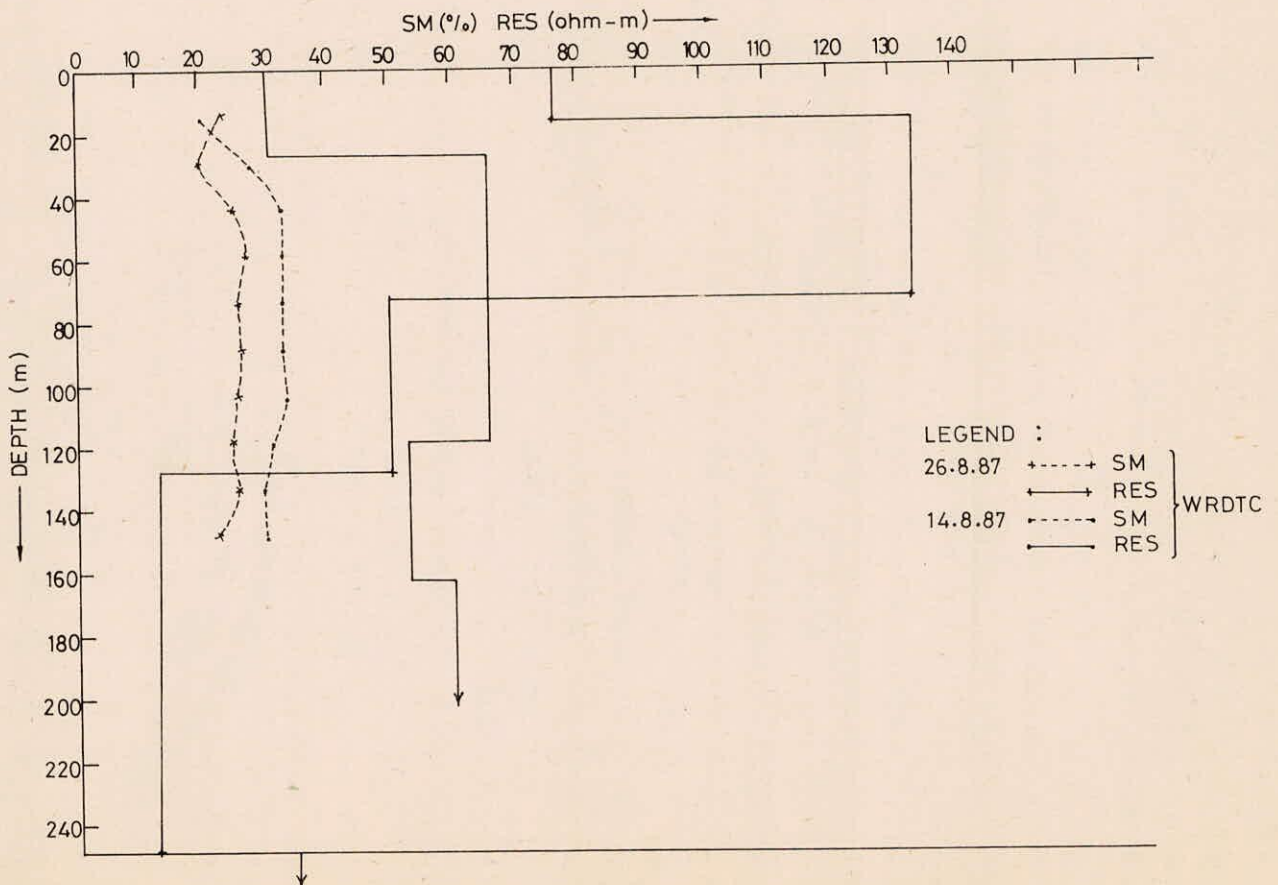


FIG. 6.9-VARIATION OF MOISTURE CONTENT AND RESISTIVITY PROFILES

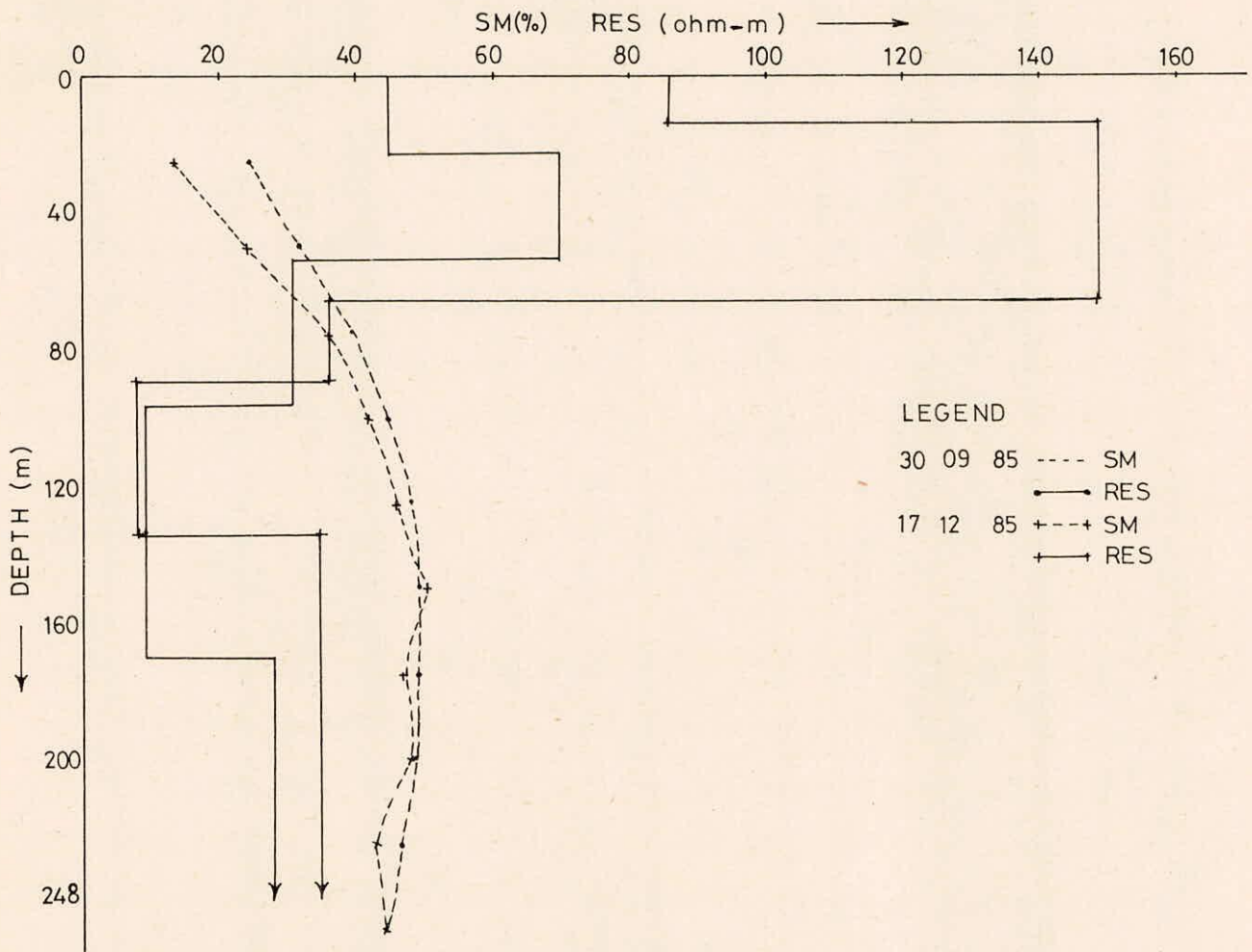


FIG. 6.10 - VARIATION OF MOISTURE CONTENT AND RESISTIVITY PROFILES.

7.0 CONCLUSION

Streaming potential (S.P) and electrical-resistivity techniques were utilised to monitor soil-moisture variations in surface formations. Effect of rainfall in altering the soil-moisture profile which ultimately affects the S.P. and resistivity measurements was also studied.

A qualitative correlation was observed between S.P. measurements and rainfall events. This technique can be further developed to establish a quantitative relationship between the soil-moisture movement and the S.P. anomalies. Since field measurement of S.P. anomaly involves a very simple instrumentation and minimal field-personnel, it can be used effectively as supplementary technique to the existing methods.

Monitoring of moisture-contents in a soil profile using resistivity measurements has shown promising results. However, as compared to groundwater exploration applications, resistivity measurements as well as interpretation in this particular application should be of high precision as the signals are comparatively weak and any inaccuracy would lead to erroneous results. This necessitates use of high-precision field instruments and automated interpretational procedures. In order to be able to monitor temporal variations of moisture content at a particular site, a multi electrode system would be required in which all the electrodes are installed before hand and readings are taken at a number of sounding positions, one after another in a sequence.

One such system was developed at the National Institute of Hydrology, Roorkee and was used for the field measurements in this study.

In the present study, the technique was applied at two nearby sites within the University of Roorkee campus and results are found to be encouraging. However, application of this technique on routine basis would require further establishment of empirical relationship between soil-moisture and resistivity for various types of formations. Studies to observe effect of electrolyte-resistivity on this relationship also need be done.

REFERENCES

1. Archie, G.E., 1942. Electrical Resistivity log as an aid in determining reservoir characteristics, Trans. A.I.M.E (Pet), 146 pp. 54-62.
2. Arora, J.K. and A.C. Sharma, 1986. Effect of Temperature and moisture on resistivity of concrete: in Proc. of 53rd A.R. and D. session of C.B.I.P., (Publication No. 184), pp. 111-120.
3. Austin, R.S. and J.D. Rhoades, 1979. A compact, low-cost circuit for reading four-electrode salinity sensors, Soil Sci. Soc. Am.J., Vol. 43, pp. 808-810.
4. Awasthi, A.K., 1979. Sedimentological studies of the Jodhpur group in the districts of Jodhpur and Nagaur, Rajasthan, India, Ph.D. Thesis, University of Roorkee 314 pp. (unpublished).
5. Bardessy, A., I. Bogardi and W.E. Kelly, 1986. Geostatistical analysis of geoelectrical estimates for specific capacity, J. of Hydrology, Vol. 84, No. 1/2, pp. 81-96.
6. Bogoslovsky, V.A. and A.A. Ogilvy, 1970. Application of geophysical method for studying the technical status of earth dams, Geophysical Prospecting, Vol. 18, No. 5, pp. 758-773.
7. Bohn, H.L., J.B. Asher, H.S. Tabbara and M. Marwan, 1982. Theories and tests of electrical conductivity in soils, Soil Sci. Soc. Am.J., Vol. 46, pp. 1143-1146.
8. Corwin, R.F., 1984. The self potential method and its engineering applications: An overview, Proc. Society of Exploration Geophysicists Conference, Ottawa, 11, 152-154.
9. Dobecki, T.L. and P.R. Romig, 1985. Geotechnical and groundwater geophysics, Geophysics, Vol. 50, No. 12, pp. 2621-2636.
10. Elliott, P.J. and L.T. Thomas, 1986. Interpretation of lithology from the formation resistivity of unconsolidated argillaceous/arenaceous sediments, Exploration Geophysics, 17, pp. 75-79.
11. Ernstson, K. and H.U. Scherer, 1986. Self potential variations with time and their relation to hydrogeologic and meteorological parameters, Geophysics, Vol. 51, No. 10, pp. 1967-1977.
12. Everett, L.G., E.W. Hoyleman, L.G. Wilson and L.G. McMillion, 1984. Constraints and categories of vadose zone monitoring devices, Ground-Water Monitoring Review, Winter, pp. 26-32.

13. Frenkel, H., A. Nadler and I. Shainberg, 1983. Effect of entrapped air on ECa Vs ECW calibration curves as measured in a soil column, Soil Sci. Soc. Am.J., Vol. 47, pp. 1036-1038.
14. Fried, J.J., 1975. Groundwater Pollution: Developments in Water Science 4, Elsevier Scientific Publishing Company, p. 83-110.
15. Gish, O.H., 1932. Use of geoelectric methods in search for oil, Geophysics, Vol. III, pp. 497-508.
16. Goyal, V.C., H. Sinvhal and A. Sinvhal, 1986. Use of resistivity technique in soil moisture studies - A preliminary evaluation, Hydrology Journal of IAH, Vol. IX, No. 4, pp. 48-55.
17. Kean, W.F., M.J. Waller and H.R. Layson, 1987. Monitoring moisture migration in the vadose zone with resistivity, Ground Water, Vol. 25, No. 5, pp. 562-571.
18. Keller, G.V., 1962. Electrical resistivity of rocks in the area 12 Tunnels, Nevada Test Site, Nye County, Nevada, Geophysics, Vol. XXVII, No. 2, pp. 242-252.
19. Keller, G.V., 1974. Engineering application of electrical geophysical methods: in Subsurface exploration for underground and heavy construction, A.S.C.E., N.York, pp. 128-143.
20. Keller, W.D., 1933. Earch resistivities at depths less than one hundred feet, Geophysics, Vol. IV, pp. 561-584.
21. Keller, G.V. and F.C. Frischknecht, 1977. Electrical methods in geophysical prospecting, Pergamon Press, pp. 90-196.
22. Kelly, W.E., 1985. Electrical resistivity for estimating groundwater recharge, J. Irrig. and Drain. Engg., 111(2), pp. 177-180.
23. Kessels, W., I. Flentge and H. Kolditz, 1985. D C geoelectric sounding to determine water content in the salt mine ASSE (FRG), Geophysical Prospecting 33, 436-446.
24. Khattri, K.N., Sri Niwas, B.B.S. Singhal and H.Sinvhal, 1985. Geoelectrical investigation for assessment of foundation geology at Mundiwalarao super passage of Khara power project, University of Roorkee.

25. Kirk, K.G. and H. Rouch, 1977. The application of tripotential method of resistivity prospecting for groundwater exploration and land use planning in karst terrains: in Tolson, J.S. and Doyle, F.L. (Eds.), Karst Hydrogeology, Int. Assoc. of Hydrogeologists Memoirs Vol. XII, pp. 285-299.
26. Lauter, C., L. Makoko and L.W. De Becker. A method of determining, on the farm, the amount of water and the frequency of irrigation.
27. Mckim, H.L., J.E. Walsh and D.N. Arion, 1980. Review of techniques for measuring soil moisture in-situ, special Report 80-31, U.S. Army corps of Engineers, Cold Regions Research and Engg. Lab. , U.S.A. pp. 17.
28. Nadler, A., 1980. Determining the volume of sampled soil when using the four-electrode technique, Soil Sci. Soc. Am.J., Vol. 44, pp. 1186-1190.
29. Nadler, A., 1982. Estimating the soil water dependence of the electrical conductivity soil solution electrical conductivity bulk soil ratio soil, Sci.Soc. Am.J., pp. 722-726.
30. Nadler, A., M. Magaritz, Y. Lapid and Y. Levy, 1982. A simple system for repeated soil resistance measurements at the same spot, Soil Sci. Soc. An.J., Vol.46, pp. 661-663.
31. Nadler, A., H. Frankel and A. Mantell, 1984. Applicability of the four-probe technique under extremely variable water contents and salinity distribution, Soil Sci. Soc. Am.J., Vol. 48, pp. 1258-1261.
32. Ogilvy, A.A., V.A. Bogoslovsky and E.N. Kuzmina, 1980. Goelectric models in engineering geophysics, Geophysical Prospecting, 28, pp. 945-955.
33. Parasnis, D.S., 1979. Principles of Applied Geophysics, Chapman and Hall, John Wiley & Sons, N.York, p. 256.
34. Rhoades, J.D., 1979. Inexpensive four-electrode probe for monitoring soil salinity, Soil Sci. Soc. Am.J. Vol. 43, pp. 817-818.
35. Rhoades, J.D.. 1980/81. Determining leaching fraction from field measurements of soil electrical conductivity, Agricultural Water management, 3, pp. 205-215.
36. Rhoades, J.D., D.L. Corwin, G.J. Hoffman, 1981. Scheduling and controlling irrigations from measurements of soil electrical conductivity, ASAE Publication No. 23-81, pp. J6-115.

37. Robertson, E.I., 1949. Measurement of Soil moisture at Winchmore by resistivity methods: in Report No. 94, Geophysics Division, Dept. of Scientific and Industrial Research, New Zealand, pp. 20-40.
38. Sarma, V.V.J., 1961. Resistivity investigations along Gosthani river, Proc. Sym. on Geophysics in Relation to Geological Problems, National Institute of Sciences of India, pp. 77-93.
39. Sarma, V.V.J. and V.B. Rao, 1962. Variation of electrical resistivity of river sands, calcite and quartz powders with water content, Geophysics, Vol. XXVII, No. 4, pp. 470-479.
40. Schimschal, U., 1981. Resistivity data analysis of a multilayered medium by computer curve matching techniques, Bureau of Reclamation, Colorado, USDI, p. 34.
41. Schmugge, T.J., T.J. Jackson and H.L. Mckim, 1980. Survey of methods for soil moisture determination, Water Resources Research, Vol. 16, No. 6, pp. 961-979.
42. Sweeney, J.J., 1984. Comparison of electrical resistivity methods for investigation of groundwater conditions at a landfill site, Ground Water Monitoring Review, Winter 1984, pp. 52-58.
43. Telford, W.M., Geldart, L.P., Sheriff, R.E. and Keys, D.A., 1976. Applied Geophysics, Cambridge University Press, London, p. 295.
44. Van Zijl, J.S.V., 1977. A practical manual on the resistivity method, Geophysics Division, NPRL, CSIR, p. 132.
45. Whiteley, R.J. 1983. Recent developments in the application of geophysics to geotechnical investigation: in Ervin, MC(Ed.), In-situ testing for geotechnical investigations, pp. 87-111.

APPENDIX I

GRAVIMETRIC MEASUREMENT OF WATER CONTENT IN SOIL SAMPLES

The water content is a measure of water per unit soil mass. The moisture content can be expressed as:

$$w = \frac{m_1}{m_2} ,$$

where m_1 = mass of water

m_2 = mass of dry soil

In this method, disturbed or undisturbed wet samples are weighed, dried to constant weight in an oven at 105° and reweighed. From the different weight measurements the water content on dry mass basis can be calculated.

Procedure:

Place the sample of soil in weighing metal cans with tight-fitting lids. Weigh the samples immediately or store them in such a way that evaporation is prevented. Place the sample in a drying oven (105°) with the lid off and dry it to constant weight. Remove the sample from the oven, replacing the cover, and place it in a desiccator containing active desiccant until cool. Weigh it again and also determine the tare weight of the sample container. Compute the water content.

Calculation:

The water content is calculated as follows:

$$w = \frac{m_{S+W} - m_S}{m_S} (\%) ,$$

where, m_{S+W} = mass of wet soil

m_S = mass of dry soil

It is a direct and reproducible measurement. The oven drying at 105°C is itself arbitrary. Some clays may still contain appreciable amounts of adsorbed water even at 105°C. On the other hand, some soil organic matter may oxidize and decompose at this temperature so that the weight loss may not be due entirely to the evaporation of water.

APPENDIX II

1. DETERMINATION OF ORGANIC CONTENT

Organic content is determined by oxidising it with hydrogen peroxide and computing the loss in weight of the soil sample.

Procedure:

A known amount of soil sample is treated with 5 ml of hydrogen peroxide (H_2O_2) and heated at approximately $90^\circ C$ until dry. The weight loss is measured. The process is repeated until no further loss in weight is observed.

Calculation:

Organic content is calculated as:

$$w = \frac{m_{s+oc} - m_s}{m_{s+oc}} (\%), \text{ where}$$

m_{s+oc} = mass of soil including organic matter

m_s = mass of dry soil after treatment

2. DETERMINATION OF ELECTRICAL CONDUCTIVITY (EC)

Electrical conductivity (EC), in mho/cm, is measured in extracts prepared from 1:5 solution of soil samples.

Procedure:

10 gm. of soil sample is dissolved in 50 ml. of water and kept for settling after thorough stirring. Process is repeated until suspended solution is obtained. Filtrate is used for measurement of electrical conductivity using conductivity meter. Measured value is expressed in mho/cm or converted in mmho/cm.

APPENDIX III

DETERMINATION OF CLAY CONTENT

Pipette method of sampling is based on sieving and sedimentation procedure. The latter consists of removing with a pipette a sample of known volume from a given depth in a water suspension of dispersed soil at a specific time after sedimentation has begun.

Procedure:

Dry and crumble the field sample. Separate the fine soil fraction by sieving through a 2 mm sieve. For effective fractionation the soil (<2mm) must be dispersed in water. This may require dissolution of free carbonate by acidification (HCl), of organic matter by oxidation (H_2O_2) and of sparingly soluble salts by washing.

For this, 20 gm. of sieved soil is dissolved in 50 ml. of water and stirred thoroughly. 5 ml. of hydrogen-peroxide (H_2O_2) is added and the solution is heated at $90^\circ C$ till the organic matter is removed. Any surplus hydrogen peroxide is removed through washing.

Free carbonates are dissolved by treating the solution with dilute HCl. Afterwards the particles greater than $50\mu m$ are removed by wet sieving. The fraction less than $2\mu m$ is determined by sedimenting the suspension of dispersed particles < $50\mu m$, made with 10 ml. of sodium Hexa Meterphosphate (2N) in a 500 ml. graduated cylinder. 25 ml. of suspended sample is taken from a depth of 5 cm. with the help of a pipette after 49 minutes of suspension. This gives clay content

with particles of size corresponding to 8 ϕ . Another sample may be taken after 3 hrs. 15 minutes of suspension to determine clay content with particles of size 9 ϕ . The 25 ml of solution in each case is dried and weighed.

Calculation:

The clay content is calculated as follows:

$$\% \text{ Clay} = (m_{25} \times 20 - M_{\text{SHMP}}) \times 5,$$

where

m_{25} = mass of clay after drying 25 ml. of pipette sample,

M_{SHMP} = mass of sodium hexa metaphosphate dissolved in 500 ml. of soil suspension,

a factor 5 is multiplied when 500 ml. solution was prepared from 20 gm. of soil sample.

X-ray Diffraction Analysis of Clay Samples:

1. Sample preparation:

First of all, samples are given pre-treatment to remove organic matter, free Fe oxides and carbonates that may be present in the samples. Further, cementing agents are removed using chemical as well as mechanical dispersion.

Slurry from the dispersion of original sample (<2 micron particle size) is deposited on a glass slide and allowed to dry at room temperature. The amount of clay deposited on the slide should be sufficient to prevent diffraction from the crystalline material in the slide, as well as to ensure that the relative intensities of low and high angle peaks, between 0° and 30° 2 θ , are not a function of specimen

thickness. Place the samples in dessicator at constant humidity when clay appears dry (for about 4-8 hours).

A second set is prepared by placing the samples in a dessicator containing about 1/2 pint Ethylene Glycol. Dessicator is placed in oven at 60°C for at least one hour; there after dessicator is removed and set aside to cool. This set of samples are marked as 'glycolated' samples. Heat treatment may be employed to a third set of the samples to distinguish further between some of the clay minerals. For example, the 060 reflection of the kaolinite is removed by heating the clay to 550°C while that of illite remains. Samples are allowed to cool in a dessicator.

2. Interpretation of Diffractograms:

X-ray diffraction apparatus gives the pattern of the principal crystallographic planes that cause the diffraction of X-rays. Since no two minerals have exactly the same interatomic distances in three dimensions, the angles at which diffraction occurs will be distinctive for a particular mineral. Hence, by interpreting the diffractograms minerals present in the mixture can be found out.

Identification of major clay minerals can be easily done using an identification scheme given in Table III.1. Quantitative estimates of major clay minerals is based on the principle that the intensity of X-ray diffraction by a mineral is related to the amount of that mineral. A quantisation scheme to obtain relative estimates of major clay minerals is given in Table III.2.

Table III.1 : Clay Mineral Identification Scheme (After Awasthi, 1979)

S.No.	Mineral	untreated slide	Peaks Glycol Treated slide	Heated slide (550°C)
1.	Montmorillonite	5.80° (15A°)	5.20° (17A°)	8.84° (10A°)
2.	Chlorite	6.30° (14.2A°) 12.40° (7.1A°) 18.40° (4.8A°)	No change No change No change	No change No change No change
3.	Vermiculite	6.30° (14.2A°) 18.00° (4.9A°)	No change No change	8.80° (10A°) 17.72° (5A°)
4.	Illite	8.80° (10A°) 17.70° (5A°) 26.60° (3.3A°)	No change No change No change	No change No change No change
5.	Kaolinite	12.30° (7.1A°) 24.90° (3.5A°)	No change No change	Complete destruction Complete destruction

Table III.2 : Quantisation scheme for major clay minerals (after Awasthi, 1979)

S.No.	Mineral Ratio	Peak used	Remarks
1.	Illite/Kaolinite	10 A°/3.58 A° or (8.8°/24.9°)	3.58A° peak of kaolinite is supposed to have 100% intensity and it is also free from chlorite, vermiculite, etc.
2.	Chlorite/Kaolinite	$\frac{4.78A^\circ}{3.58A^\circ} \times \frac{100}{80}$ or $\frac{(18.5^\circ)}{(24.9^\circ)} \times \frac{100}{80}$	The intensity of chlorite peak at 4.78A° is only 80%. Hence the correction.
3.	Illite/Montmorillonite	10A°/17A° or (8.8°)/(5.2°)	Peak at 10A° and 17A° from the diffractogram after glycolation are used.

APPENDIX IV

POTENTIAL OF A POINT CURRENT ELECTRODE ON THE SURFACE OF
A HORIZONTALLY-LAYERED EARTH

Let $n-1$ layers rest on an n th 'layer', the infinite sub-stratum (Fig. IV.1). Choosing a cylindrical coordinate system R, θ, z with the electrode C at the origin and z positive downwards, Laplace's equation for the electric potential V in each layer can be written as

$$\frac{\partial^2 V}{\partial R^2} + \frac{1}{R} \frac{\partial V}{\partial R} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \dots(1)$$

since, by symmetry, V is independent of θ .

Assuming $V(R, z) = F(R)G(z)$ where F is a function of R only and G of z only, the equation is separated into the two equations

$$\frac{d^2 G}{dz^2} - \lambda^2 G = 0 \quad \dots(2)$$

$$\frac{d^2 F}{dR^2} + \frac{1}{R} \frac{dF}{dR} + \lambda^2 F = 0 \quad \dots(3)$$

where λ is a constant independent of R and z .

Equation (3) is Bessel's equation of order zero with fundamental solutions $J_0(\lambda R), Y_0(\lambda R)$. $J_0(\lambda R)$ is finite for $R \rightarrow \infty$ only if λ is real, and since $Y_0(R)$ is always ∞ for $R \rightarrow \infty$ it must be rejected. The solutions of (2) are $\exp(-\lambda z)$ and $\exp(\lambda z)$.

The most general solution for the potential in any layer j ($j \neq 1$ or n) is then

$$V_j = \int_0^\infty [A_j(\lambda) e^{-\lambda z} + B_j(\lambda) e^{\lambda z}] J_0(\lambda R) d\lambda \quad \dots(4)$$

on account of the occurrence of $\exp(\lambda z)$ as well as $\exp(-\lambda z)$, there is no loss of generality in restricting to positive real values only.

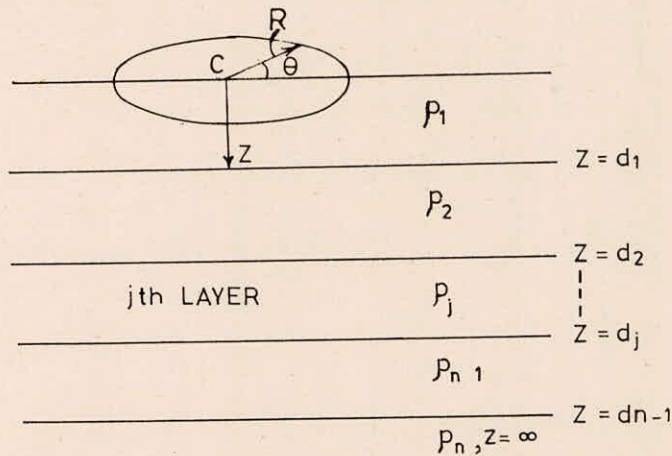


FIG. IV.1. AN n -LAYERED EARTH MODEL

For $J = n$, the term in $\exp(\lambda z)$ must be excluded since the potential in the sub-stream must be finite as $z \rightarrow \infty$

Thus:

$$V_n = \int_0^\infty A_n(\lambda) e^{-\lambda z} J_0(\lambda R) d\lambda \quad \dots (5)$$

In the topmost layer ($j=1$), V_1 is the sum of the normal potential

$$V_o = \frac{I \rho_1}{2 \pi} \frac{1}{(R^2 + z^2)^{1/2}}$$

and a disturbance potential

$$V_d = \int_0^\infty A_1(\lambda) (e^{-\lambda z} + e^{\lambda z}) J_0(\lambda R) d\lambda$$

The coefficients of $\exp(-\lambda z)$ & $\exp(\lambda z)$ in V_d are equal since $(1/\rho_1)(\partial V_d / \partial z) = 0$ at $z = 0$ (no current flow across the earth's surface except at C). Using Lipschitz's integral in Bessel function

theory for V_0 we have

$$V_1 = \frac{I \rho_1}{2\pi} \int_0^\infty e^{-\lambda z} J_0(\lambda R) d\lambda + \int_0^\infty A_1(\lambda) (e^{-\lambda z} + e^{\lambda z}) J_0(\lambda R) d\lambda \quad \dots(6)$$

From the continuity of V_j and the current density normal to the layer interfaces we have, for depth $z = d_j$ ($j=1,2,\dots,n-1$),

$$V_{j-1} = V_j \quad \dots(7)$$

$$\frac{1}{\rho_{j-1}} \frac{\partial V_{j-1}}{\partial z} = \frac{1}{\rho_j} \frac{\partial V_j}{\partial z} \quad \dots(8)$$

There are altogether $2n-2$ unknown functions $A(\lambda)$, $B(\lambda)$, to be determined ($2n-4$ in (4) and 1 each in (5) and (6)). They can be determined by solving the system of $2n-2$ linear equations obtained from the conditions (7). The solution is tedious but straightforward.

Identifying $A_1(\lambda)$ as $(I \rho_1 / \pi) K(\lambda)$ and R and r along the ground surface, it is easily seen that Equation (6) is the same as Equation (4.5)

$K(\lambda)$ is a function of the layer parameters, and recurrence formulae for building it for any number of layers on top of each other, starting from its expression for two layers, can be found by solving the system of Equation(7). Here the recurrence formulae for the transform $T(\lambda)$ of Equation (4.8) will be given instead, in terms of the layer thickness h_j ($j=1,\dots,n-1$), rather than the interface depths d_j .

For a layer (ρ_{n-1}, h_{n-1}) on top of a sub-stratum (ρ_n)

$$T_{n-1}(\lambda) = \rho_{n-1} \frac{1 - k_{n-1} \frac{u_{n-1}}{u_{n-1}}}{1 + k_{n-1} \frac{u_{n-1}}{u_{n-1}}}$$

where

$$u_{n-1} = \exp(-2h_{n-1}\lambda)$$

$$k_{n-1} = (\rho_{n-1} - \rho_n) / (\rho_{n-1} + \rho_n)$$

For the transform T_j for a layer (ρ_j, d_j) on top of the sequence $(\rho_{j+1}, \dots, \rho_n; h_{j+1}, \dots, h_n)$ with the transform T_{j+1} , we have

$$T_j(\lambda) = \frac{W_j(\lambda) + T_{j+1}(\lambda)}{1 + W_j(\lambda) T_{j+1}(\lambda) / \rho_j^2}$$

where

$$W_j(\lambda) = \rho_j \frac{1 - u_j}{1 + u_j}$$

Starting from $T_{n-1}(\lambda)$ the transform $T_1(\lambda) = T(\lambda)$ of Equation can be obtained by recursive application of the expression for $T_j(\lambda)$.