TYPE CURVES FOR MULTIAQUIFER WELL

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LIST OF SYMBOLS

E ₁ (x)	-	exponential integral
Q _i (n)	-	contribution of the i th aquifer during time step n,i=1,2
Q _p (n)	-	pumping rate during time step n
r	-	radial distance to the observation well
rw	-	radius of the well screen
S _{iw} (n)	-	drawdown in the piezometric surface at the well point in
		the i th aquifer at the end of time step n, i=1,2
S _r (n)	-	drawdown at the observation well at time n after on set
		of pumping
S _{ri} (n)	-	drawdown in the i th aquifer at distance r at the end of
		time step n
t	-	time after pumping commenced
T	-	transpose of the maxtrix
T _i	-	transmissivity of the i th aquifer, i=1,2
φ,	-	storage coefficient of the i th aquifer, i=1,2
β _i	-	hydraulic diffusivity = T_i/ϕ_i of the i th aquifer, i=1,2
<pre> ³rwi⁽ⁿ⁾, ³rwi⁽ⁿ⁾ </pre>)-	discrete kernel coefficient at time step n of the i th
		aquifer, (i=1,2), at the well point
∂ _{ri} (n)	-	discrete kernel coefficient at time step n of the i th
		aquifer (i=1,2), at distance r
W(U ₁ (n))	-	well function for two aquifer system
U ₁ (n)	-	nondimensional time factor for two aquifer system
u	-	¢r ² /4Tt

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ABSTRACT

Type curves pertaining to aquifer test conducted in multiaquifer well, which is open to two aquifers, have been presented. Using these type curves the storage coefficient and transmissivity of each aquifer can be predicted. In a two aquifer system an observation well may tap a single aquifer or it may be open to both the aquifers. Some times the multiaquifer pumping well may also be used as an observation well. The type curves presented in the report include all these three cases. Use of the type curves for parameter estimation has been demonstrated using synthetic drawdown data.

INTRODUCTION

In a sedimentary ground water basin occurrence of multiple aquifers separated by confining layers of low and negligible permeability is quite common. A water well in such basin may have to be constructed tapping more than one aquifer in order to have requisite yield. If the aquifers are separated by confining layers of negligible permeability (aquiclude) interaction among the aquifers tapped by the well, is only through the well screens. A well tapping two or more water bearing strata, which have different hydraulic properties and which are not closely connected except by the well itself, is referred to as a multiaquifer well (Papadopulos, 1966). A solution for unsteady flow to a well, which taps two confined aquifers with different potentiometric surfaces prior to well construction, has been obtained by Papadopulos. Laplace transform technique has been used to obtain exact expression for head distribution but the solution is intractable for numerical calculation. Subsequently asymptotic solutions for both head and discharge distribution, amenable to computation which yield results accurate enough for practical application, have been derived by Papadopulos. However, no numerical results have been presented by him. Using integral transform technique unsteady flow to a multiaquifer well, open to two aquifers has also been analyzed by Khader and Verankutty (1975), who have presented numerical results for contribution of individual aquifer to the total discharge of a well. But the type curves required for identification of aquifer parameters have not been developed by them. The analysis of unsteady flow to a well tapping two aquifers separated by an aquiclude has been done by Mishra et al (1985) with discrete kernel approach. Their analysis does not include the inverse problem. In the present report analysis of unsteady flow in a two aquifer system having a multiaquifer pumping well and a multiaquifer observation well has been done. Type curves have been presented for observation well open to both the aquifers besides for observation wells open to individual aquifer separately.

1.0

2.0 REVIEW

The analysis for the direct problem of calculating drawdown in and around a well can be done using the known values of aquifer parameters and well discharge. The inverse problem of calculating aquifer parameters from field measurement of drawdown is also equally important. The aquifer parameters can be evaluated making use of the drawdown observed in observation well in response to a known pumping rate.

The transmissivity and storage coefficient of nonleaky isotropic artesian aquifers can be determined from pumping test conducted in a fully penetrating well with negligible diameter using Theis (1935) type curves. Variations of the well function, W(u), with the nondimensional time factor u and 1/u, where, $u=\phi r^2/4Tt$, r=the radial distance of the observation point, ϕ =the storage coefficient, T=the transmissivity of the aquifer and t=the time measured since pumping started have been presented by Theis(1935).

If the drawdowns which have been recorded at an observation point during a pump test are plotted on a double log paper whose scale is same as that of the type curves, and the time drawdown curve happens to match with the Theis type curve, the aquifer conforms to be a confined aquifer and the parameters can be evaluated making use of the relation

$$T = \frac{Q}{4\pi S^*(r,t^*)} W^*(u), \text{ and } \phi = \frac{u^* 4Tt^*}{r^2}$$

where S*(r,t), W*(u), u*,t* correspond to a match point.

Type curves are also available for predicting parameters of unconfined aquifer (Boulton, 1963) and leaky confined aquifer (Hantush, 1956, Walton, 1962) which have been well documented by Kruseman and De Ridder (1970).

Analysis of unsteady flow to a multiaquifer production well have been made by Khader and Verankutty(1975), Mishra et al (1986), Wikramaratna(1984)

and Mishra and Chachadi (1986). Using these analysis, the drawdown at an observation well which taps only one of the aquifers can be predicted and accordingly type curves can be prepared. However, the type curves for multiaquifer system are not yet available. Also solution to unsteady flow to a multiaquifer production well in the presence of multiaquifer observation well has not been given so far. In the present report using discrete kernel approach unsteady flow to a multiaquifer production well and a multiaquifer observation well has been analysed. Type curves for finding aquifer parameters from drawdown data observed at an observation well open to a single aquifer or both the aquifers during a pumping test in a multiaquifer well have been presented.

3.0 STATEMENT OF THE PROBLEM

A schematic cross-section of a production well tapping two aquifers which are separated by an aquiclude is shown in Figure 1. Each of the aquifers is homogeneous, isotropic, and infinite in areal extent. Prior to the pumping the aquifers are assumed to be at rest condition. The radius of the well screen is r_w . Drawdowns in the piezometric surfaces are caused by discharge from respective aquifers, consequent to a uniform rate of pumping from the well. If there exists an observation well which is open to both the aquifers drawdown to piezometric surface in each aquifer will be caused due to aquifer's own contribution to pumping. An observation well is located at a radial distance r from the pumping well. It is required to find the well function at various nondimensional time if the observation well is open to both aquifers or it is open to one of the aquifers.

4.0 ANALYSIS

The following assumptions have been made in the analysis:

- i) The radii of the pumping well and observation well are small and hence the well storages are neglected.
- ii) At any time the drawdowns in both the aquifers at well face are same but vary with time.
- iii) The time parameter is discrete.Within each time step, the abstraction rates of water derived from each of the aquifers are separated constants.
- 4.1 Well Function for Observation Well Open to one of the Aquifers:

When the two aquifers are tapped by a single well and the well is pumped, there is contribution from each aquifer to the pumping through the respective well screens. Let $Q_1(n)$ and $Q_2(n)$ be the contributions from aquifer 1 and 2 respectively at time step n. At any time the algebraic sum of the abstractions from both aquifers will be equal to the pumping rate. Hence,



FIG.1-SCHEMATIC SECTION OF A WELL TAPPING TWO CONFINED AQUIFERS SEPARATED BY AN AQUICLUDE

$$Q_1(n) + Q_2(r) = Q_p(n)$$
 (1)

in which $Q_p(n)$ is the pumping real auring non time step.

The drawdown at the well face at the end of time step n in aquifer 1 is given by (Morel Seytoux, 1975)

$$S_{1w}(n) = \sum_{\gamma=1}^{n} Q_1(\gamma) \partial_{rw1}(n-\gamma+1)$$
(2)

where,

$$\partial_{\mathbf{rw1}}(\mathbf{m}) = \frac{1}{4\pi T_1} \left[E_1 \left(\frac{r_w^2}{4\beta_1 \mathbf{m}} \right) - E_1 \left(\frac{r_w^2}{4\beta_1 (\mathbf{m}-1)} \right) \right]$$
 (3)

and $\beta_1 = T_1 / \phi_1$.

Similarly the drawdown at the well face at the end of time step n in aquifer 2 is given by

$$S_{2w}(n) = \sum_{\gamma=1}^{n} Q_2(\gamma) \partial_{rw2}(n-\gamma+1)$$
(4)

where

$$\Theta_{\mathbf{rw2}}(\mathbf{m}) = \frac{1}{4\pi T_2} \left[E_1 \left(\frac{\mathbf{r}_{\mathbf{w}}^2}{4\beta_2 \mathbf{m}} \right) - E_1 \left(\frac{\mathbf{r}_{\mathbf{w}}^2}{4\beta_2 (\mathbf{m}-1)} \right) \right]$$
(5)

and $\beta_2 = T_2 / \phi_2$.

 β_1 and β_2 are the hydraulic diffusivity of the 1st and 2nd aquifer respectively. Since $S_{1w}(n) = S_{2w}(n)$, therefore, from equations [2] and [4]

$$\sum_{\gamma=1}^{n} Q_1(\gamma) \partial_{\mathbf{rw1}}(\mathbf{n}-\gamma+1) = \sum_{\gamma=1}^{n} Q_2(\gamma) \partial_{\mathbf{rw2}}(\mathbf{n}-\gamma+1)$$
(6)

Rearranging,

$$Q_{1}(n)\partial_{rw1}(1)-Q_{2}(n)\partial_{rw2}(1)$$

$$= \sum_{\substack{\Sigma \\ \gamma=1}}^{n-1} Q_{2}(\gamma)\partial_{rw2}(n-\gamma+1) - \sum_{\substack{\Sigma \\ \gamma=1}}^{n-1} Q_{1}(\gamma)\partial_{rw1}(n-\gamma+1) \qquad (7)$$

Using equations (1) and (7) the contribution of first aquifer to pumping is found to be

(8)

$$\frac{Q_{1}(n)}{Q_{p}(n)} = \frac{1}{1 + (\frac{T}{T_{1}}) \frac{\partial'_{rw1}(1)}{\partial'_{rw2}(1)}} [1 - (\frac{T_{2}}{T_{1}}) \cdot \frac{Q_{p}(n)^{-1}}{\partial'_{rw2}(1)} \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial'_{rw1}(n-\gamma+1)$$

+
$$\frac{Q_{p}(n)^{-1}}{\partial r_{w2}(1)} \sum_{\gamma=1}^{n-1} (Q_{p}(\gamma) - Q_{1}(\gamma)) \partial r_{w2}(n-\gamma+1)$$

in which

and

$$\partial_{\mathbf{rw1}}'(\mathbf{n}) = E_{1}(\frac{\phi_{1}r_{w}^{2}}{4T_{1}n}) - E_{1}(\frac{\phi_{1}r_{w}^{2}}{4T_{1}(n-1)})$$

$$\partial_{\mathbf{rw2}}'(\mathbf{n}) = E_{1}(\frac{\phi_{2}r_{w}^{2}}{4T_{2}n}) - E_{1}(\frac{\phi_{2}r_{w}^{2}}{4T_{2}(n-1)}) = E_{1}(\frac{\phi_{2}}{\phi_{1}}, \frac{T_{1}}{T_{2}}, \frac{\phi_{1}r_{w}^{2}}{4T_{1}n})$$

$$-E_{1}(\frac{\phi_{2}}{\phi_{1}}, \frac{T_{1}}{T_{2}}, \frac{\phi_{1}r_{w}^{2}}{4T_{1}(n-1)})$$

$$\frac{Q_1(n)}{Q_1(n)}$$
 ca

 $\frac{1}{Q_p}$ can be found in succession starting from time step 1.

$$\frac{Q_1(n)}{Q_p} \text{ can be expressed as a function of } \partial'_{rw1}(n), \partial'_{rw2}(n),$$

$$\frac{T_1}{T_2}, \frac{\varphi_1}{\varphi_2}, \frac{\varphi_1 r_w^2}{4T_1 n} .$$

The contribution of the second aquifer is given by

$$\frac{Q_2(n)}{Q_p(n)} = 1 - \frac{Q_1(n)}{Q_p(n)}$$
(9)

Thus $Q_1(n)$ and $Q_2(n)$ can be solved in succession starting from time step one using the equations (8) and (9) for known values of $T_1, T_2, \phi_1, \phi_2, r_w$ and $Q_p(n)$. Knowing $Q_1(n)$ and $Q_2(n)$, the drawdown $S_{ri}(n)$, in the ith aquifer at any distance r from the centre of the well can be found using the relation

$$S_{ri}(n) = \sum_{\gamma=1}^{n} Q_{i}(\gamma) \partial_{ri}(n-\gamma+1), \quad i = 1,2$$
(10)

where

$$\Theta_{ri}(n) = \frac{1}{4\pi T_{i}} \left[E_{1} \left(\frac{r^{2}}{4\beta_{i}n} \right) - E_{1} \left(\frac{r^{2}}{4\beta_{i}(n-1)} \right) \right]$$
(11)

The drawdown at the end of time step n at an observation well located at a distance r from the pumping well in the first aquifer can be expressed as

$$S_{r1}(n) = \sum_{\gamma=1}^{n} Q_1(\gamma) \partial_{r1}(n-\gamma+1)$$
(12)

Splitting the summation into two parts

$$S_{r1}(n) = Q_{1}(n) \partial_{r1}(1) + \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1)$$
 (13)

Replacing $Q_1(n)$ by the expression given at equation (8) and simplifying

$$\frac{s_{r1}(n)}{q_{p}(n)} = \frac{\vartheta_{r1}'(1)}{1 + \frac{T_{2}}{T_{1}}} \frac{\vartheta_{rw1}'(1)}{\vartheta_{rw2}'(1)} \left[1 - \left(\frac{T_{2}}{T_{1}}\right) \frac{1}{\vartheta_{rw2}'(1)q_{p}(n)} \frac{n-1}{\gamma=1} Q_{1}(\gamma)\vartheta_{rw1}'(n-\gamma+1) \right] \\
+ \frac{1}{\vartheta_{rw2}'(1)q_{p}(n)} \frac{n-1}{\gamma=1} Q_{1}(\gamma)\vartheta_{rw2}'(n-\gamma+1) \left[\frac{1}{\vartheta_{rw2}'(1)q_{p}(n)} \frac{n-1}{\gamma=1} Q_{1}(\gamma)\vartheta_{rw2}'(n-\gamma+1) \right] \\
+ \frac{1}{q_{p}(n)} \frac{n-1}{\gamma=1} Q_{1}(\gamma)\vartheta_{r1}'(n-\gamma+1)$$
(14)

Thus for a given value of T_1/T_2 , ϕ_1/ϕ_2 , r_w , the right hand side of equation (14) is only a function of $\phi_1 r^2/4T_1 n$.

Let the pumping be carried out at a constant rate and let it equal to $\ensuremath{\mathbb{Q}_{\mathrm{P}}}$. Therefore,

$$\frac{S_{r1}(n)}{Q_{p}} = W(U_{1}(n))$$
(15)

where $W(U_1(n))$ is the right hand side of equation (14) and can be regarded as well function for a two aquifer system and

$$U_1(n) = \frac{\phi_1 r^2}{4T_1 n}$$

Taking the logarithm, of terms on either side of equation (15)

$$\log_{10} S_{r1}(n) = \log_{10} (\frac{Q_{p}}{4\pi T_{1}}) + \log_{10} W(U_{1}(n))$$

of $U_{1}(n)$
$$\log_{10}(n) = \log_{10} (\frac{\varphi_{1}r^{2}}{4T_{1}}) + \log_{10} (\frac{1}{U_{1}(n)})$$

Therefore the variation of $W(U_1(n))$ with $\frac{1}{U_1(n)}$ when plotted on double log paper will match with the log-log plot of $S_r(n)$ with n for an observation well.

4.2 Well Function When Production Well Serves as an Observation Well: When the observations are recorded at the pumping well the pertinent well function is given by (Mishra and Chachadi, 1986)

$$\frac{\frac{S_{w}(n)}{Q_{p}}}{\frac{4\pi T_{1}}{2}} = \frac{\frac{n-1}{\gamma_{=1}}}{-\left[\frac{T_{1}}{T_{2}} \frac{\partial \mathbf{r}_{w2}(1)}{\frac{1}{T_{2}}}\right] \left[\frac{T_{1}}{T_{2}} \partial \mathbf{r}_{w2}(n-\gamma+1) + \left(\frac{T_{1}}{T_{2}}\right) \partial \mathbf{r}_{w2}(1) - \left(\frac{T_{1}}{T_{2}} \frac{1}{\frac{1}{T_{2}}} \frac{\partial \mathbf{r}_{w2}(1)}{\frac{1}{T_{2}}}\right] \left[1 - \left(\frac{T_{1}}{T_{2}}\right) \frac{1}{\partial \mathbf{r}_{w2}(1)} \frac{n-1}{\gamma_{=1}} \frac{Q_{1}(\gamma)}{Q_{p}} \partial \mathbf{r}_{w1}(n-\gamma+1) + \left(\frac{T_{1}}{T_{2}}\right) \frac{1}{\partial \mathbf{r}_{w2}(1)} \frac{n-1}{\gamma_{=1}} \frac{Q_{1}(\gamma)}{Q_{p}} \partial \mathbf{r}_{w1}(n-\gamma+1) + \left(\frac{T_{1}}{T_{2}}\right) \frac{1}{\partial \mathbf{r}_{w2}(1)} \frac{n-1}{\gamma_{=1}} \frac{Q_{1}(\gamma)}{Q_{p}} \partial \mathbf{r}_{w2}(1) + \frac{1}{\partial \mathbf{r}_{w2}(1)} \frac{n-1}{\gamma_{=1}} \frac{Q_{1}(\gamma)}{Q_{p}} \partial \mathbf{r}_{w2}(n-\gamma+1)\right]$$
(16)

4.3 Well Function for an Observation Well Open to Both the Aquifers: If an observation well is open to both the aquifers there will be interaction between the two aquifers through the observation well. Let $Q_3(n)$ and $Q_4(n)$ be the quantities of flow coming out of the first and the second aquifer respectively during time step n. If the observation well storage is neglected, a mass balance over n^{th} time step leads to

$$Q_{2}(n) + Q_{4}(n) = 0$$
 (17)

A negative sign of $Q_3(n)$ means the first aquifer is receiving water from the second aquifer through the observation well. Similarly if $Q_4(n)$ is negative, the second aquifer is receiving water during time step n. Drawdown at the pumping well in the first aquifer is given by

$$S_{rw1}(n) = \sum_{\gamma=1}^{n} Q_1(\gamma) \partial_{rw1}(n-\gamma+1) + \sum_{\gamma=1}^{n} Q_3(\gamma) \partial_{r1}(n-\gamma+1)$$
(18)

Similarly drawdown at the pumping well in the second aquifer is given by

$$S_{rw2}(n) = \sum_{\gamma=1}^{n} Q_2(\gamma) \partial_{rw2}(n-\gamma+1) + \sum_{\gamma=1}^{n} Q_4(\gamma) \partial_{r2}(n-\gamma+1)$$
(19)

Since $S_{rw1}(n) = S_{rw2}(n)$, equating equation (18) with equation (19) and rearranging

$$\begin{aligned} & Q_{1}(n) \partial_{rw1}(1) + Q_{3}(n) \partial_{r1}(1) - Q_{2}(n) \partial_{rw2}(1) - Q_{4}(n) \partial_{r2}(1) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{rw1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r1}(n-\gamma+1) \\ &+ \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \partial_{rw2}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{4}(\gamma) \partial_{r2}(n-\gamma+1) \\ &= 0, \\ &\text{and} \quad \begin{bmatrix} Q_{p}(n), & & & & \\ 0, & & & \\ 0, & & & \\ [- \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{rw1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r1}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &\partial_{rw2}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{4}(\gamma) \partial_{r2}(n-\gamma+1)], \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r01}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &\partial_{r2}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{4}(\gamma) \partial_{r02}(n-\gamma+1)]_{\bullet} \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r01}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &\partial_{r2}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{4}(\gamma) \partial_{r02}(n-\gamma+1)]_{\bullet} \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r01}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &\partial_{r2}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{4}(\gamma) \partial_{r02}(n-\gamma+1)]_{\bullet} \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r01}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r01}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r0}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r0}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) - \sum_{\gamma=1}^{n-1} Q_{3}(\gamma) \partial_{r0}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{2}(\gamma) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) + \sum_{\gamma=1}^{n-1} Q_{1}(\gamma) \partial_{r1}(n-\gamma+1) \\ &= - \sum_{\gamma=1}^{n-1} Q_{1}(\gamma)$$

 $Q_1(n),\;Q_2(n),\;Q_3(n)$ and $Q_4(n)$ can be solved in succession starting from time step 1 using the relation

 $[B] = [A]^{-1}$. [C]

In particular for time step 1, matrix [C] is given by

 $[C] = [Q_{p}(1), 0, 0, 0]^{T}$

5.0 RESULTS

With assumed values of $T_1^{}, \phi_1^{}, T_2^{}, \phi_2^{}, r_w^{}$ and r the discrete wernels are generated. Depending on the type of observation well $Q_1(n)$, $Q_2(n)$, $Q_3(n)$, $\boldsymbol{Q}_4(n)$ are solved in succession starting from time step 1 for a known pumping rate. Then the drawdown S (n) at various times have been calculated. The variation of well function, $S(r,n)/(\frac{Q}{4\pi T_1})$ with nondimensional time factor $4T_1n/\phi_1r^2$ are plotted for adopted values of $T_1/T_2, \phi_1/\phi_2, r_w$ and r. The radius of the observation well, r₀, is assumed to be equation to radius of the pumping well. A set of type curves for $T_1/T_2=1,0.5,0.2,0.1;$ $\phi_1/\phi_2 = 10,000,1000,100,10,1,0.1,0.01,0.001,0.0001$ and $r/r_w = 1,1000,2000$ have been presented in Figures 2(a) through 2(d), 3(a) through 3(d), 4(a) through 4(d), 5(a) through 5(d), and 6(a) through 6(d) for the three types of observation wells. It could be seen that if an observation well is open to both the aquifers, the shape of the type curves differs significantly than that of the time drawdown curve recorded at an observation well open one of the aquifers. When an aquifer test is conducted in a two aquifer system which are separated by an aquiclude the drawdown at the abstraction well could be observed and a graph of drawdown vs time could be plotted as shown in Figure 2(e). The time drawdown curve should be plotted in a log-log scale same as that of the available type curve. The drawdown vs time curve could be matched with one member of the type curves. While searching for a match, the abscissa of the type curve and

time drawdown curve should be kept parallel. Once a matching has been identified (as shown in Fig.2(e) the T_1/T_2 and ϕ_1/ϕ_2 values, a set of $W(U_1(t))$, and $U_1(t)$ and corresponding S and t values are noted. The values of T_1 and ϕ_1 will be given by $T_1 = Q/4\pi S[W(U_1(t))]$ and $\phi_1 = 4T_1 t U_1(t)/r_w^2$. The values of T_2 and ϕ_2 can be predicted the ratios T_1/T_2 , and ϕ_1/ϕ_2 being known for the matched type curve.

6.0 CONCLUSIONS

A mathematical model has been developed for analysis of unsteady flow in a two aquifer system in which a pumping well and an observation well are open to both aquifers. A set of type curves for the two aquifer. system separated by an aquiclude has been presented. Using variation of drawdown at the abstraction well during an aquifer test conducted in a multiaquifer well the parameters of each layer can be estimated using these type curves.

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FIG.2(c)-VARIATION OF $S_w(n)/(Q/4\pi T_1)$ with $4T_1n/\phi_1r_w^2$ FOR AN ABSTRACTION WELL OPEN TO BOTH THE AQUIFERS





FIG.2(e)-EXAMPLE OF TYPE CURVE MATCHING AT THE ABSTRACTION WELL





































