LECTURE—10

RISK AND RELIABILITY CONCEPTS IN FREQUENCY ANALYSIS

OBJECTIVES

The objectives of this lecture are to

(i) explain concepts of risk analysis including Binomial and Poison distributions, and
(ii) enlist different frequency levels used in hydrologic design.

10.1 INTRODUCTION

While designing any hydraulic structure some of the questions which may come in the mind of any designer are :

1. Why should we do risk analysis ?
2. What should be the return period for which the structure should be designed ?
3. What is the risk involved when we design a structure having a design life of n years for a T year return period flood ?
4. How much risk is permissible ?

This lecture answers some of the above questions.

Since design and planning are concerned with future events which are stochastic in nature, concepts of probability or frequency are used to define them. The risk acceptable, depends upon economic and policy considerations. The hydraulic structures can be designed for the worst possible event but the cost involved will be very high. So risk is calculated for a given frequency and based on risk acceptable, the structures are designed. Risk analysis concepts are explained in subsequent sections.

10 2 RISK ANALYSIS

The most important question facing the designer of any hydraulic structure is : what is the risk of failure ? In a recent study of over 300 dams disasters (Biswas and Chatterjee, 1971) it was found that about 35% of the failures were due to inadequate spillway design.

The simplest procedure in the frequency analysis estimation of design flood is to select a return period and use either graphical technique or a mathematical distribution to derive the corresponding event magnitude.

The return period for which a structure should be designed is calculated based on the risk acceptable. Risk is nothing but the probability of occurrence of a flood at least once during successive years of design life. Risk acceptable depends upon economic and policy considerations, The general formula for risk calculation is

$$R = 1 - \left(1 - \frac{1}{T}\right)^n \qquad (10.1)$$

where,

R is the risk,

T is the return period for which the structure should be designed, and

n is the design life of the structure.

Example 10.1 :

For a hydraulic structure having a design life of 100 years, what will be the risk involved if it is designed for (i) 50 years return period flood (ii) 1000 years return period flood.

Solution :

In the first case the risk involved will be

$$R = 1 - \left(1 - \frac{1}{50}\right)^{100}$$

$$= 0.867$$

$$= 86.7\%$$

and in the second case

$$R = 1 - \left(1 - \frac{1}{1000}\right)^{100}$$

$$= 0.095$$

$$= 9.5\%$$

Based on the risk acceptable the return period for which the structure should be designed can be ascertained.

10.2.1 Derivation of Equation 10.1

The probability of occurrence of T year flood in any year $= 1/T$

The probability of nonoccurrence $= 1 - 1/T$

The probability of nonoccurrence in successive n years $= \left(1 - \frac{1}{T}\right)^n$

The probability of occurrence of one, or more floods in successive n years $= 1 - \left(1 - \frac{1}{T}\right)^n$

or $R = 1 - \left(1 - \frac{1}{T}\right)^n$

Table 10.1 gives return periods associated with various degrees of risk and expected design life using equation 10.1.

TABLE 10.1

RETURN PERIODS ASSOCIATED WITH VARIOUS DEGREES OF RISK AND EXPECTED DESIGN LIFE

Risk %	Expected design life							
	2	5	10	15	20	25	50	100
75	2.00	4.02	6.69	11.0	14.9	18.0	35.6	72.7
50	3.43	7.74	11.9	22.1	29.4	36.6	72.6	144.8
40	4.44	10.3	20.1	29.9	39.7	49.5	98.4	196.3
30	6.12	14.5	28.5	42.6	56.5	70.6	140.7	281.0
25	7.46	17.9	35.3	52.6	70.0	87.4	174.3	348.0
20	9.47	22.9	45.3	67.7	90.1	112.5	224.6	449.0
15	12.8	31.3	62.0	90.8	123.6	154.3	308.0	616.0
10	19.5	48.1	95.4	142.9	190.3	238.0	475.0	950.0
5	39.5	98.0	195.5	292.9	390.0	488.0	976.0	1949.0
2	99.5	248.0	496.0	743.0	990.0	1238.0	2475.0	4950.0
1	198.4	498.0	996.0	1492.0	1992.0	2488.0	4975.0	9953.0

Example 10.2 :

What return period must a highway engineer use in his design of a critical underpass drain if he is willing to accept only 10% risk that flooding will occur in the next five years ?

Solution :

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$

or $$0.10 = 1 - \left(1 - \frac{1}{T}\right)^5$$

or $$T = 48.1 \text{ years.}$$

This means that there are 10% chances that a 48.1 years flood will occur once or more in next five years.

If one is interested in knowing exactly once, twice or thrice occurrences of a flood then concepts of Binomial distribution or Poisson distribution should be used.

10.3 BINOMIAL DISTRIBUTION

The Binomial distribution is a discrete distribution and is based on Binomial theorem which states that probability of exactly x successes in n trials is

$$P(x) = \binom{n}{x} p^x \quad q^{n-x} \tag{10.2}$$

where,

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

$q = 1-p$,

p = probability of exceedance/success,

q = probability of nonexceedance/failure,

x = number of exceedances/successes,

n = total number of events

The assumptions for Binomial distribution are same as for Bernoulli trials. Tossing a coin or drawing a card from a pack are examples of Bernoulli trials which operate under the following three conditions :

(i) Any trial can have either success or failure, true or false, rain or no rain.

(ii) Successive trials are independent

(iii) Probabilities are stable.

Binomial distribution is valid under above three conditions.

Example 10.3

If a dam is having project life of 50 years then what is the probability that flood with return period of 100 years will occur (i) once (ii) twice, during the life of the dam.

Solution :

Probability of exceedance = p

$$\text{here } p = \frac{1}{T} = \frac{1}{100} = 0.01$$

$$\begin{aligned} q &= 1 - p \\ &= 1 - 0.01 \\ &= 0.99 \end{aligned}$$

$$\begin{aligned} P(1) &= \binom{50}{1} \,.\, 0.01^{1} \,.\, 0.99^{49} \\ &= 0.306 \\ &= 30.6\% \end{aligned}$$

$$P(2) = \binom{50}{2} \cdot 0.01^2 \cdot 0.99^{48}$$

$$= 0.0755$$

$$= 7.55\%$$

This means that there are 30.6% chance that 100 years return period fiood will occur once during the project life and there are 7.56% chance of its two time occurrence.

10.4 POISSON DISTRIBUTION

The terms of a Binomial expansion are little inconvenient to compute in any large number. If n is large (> 30) and p is small (< 0.1) then Binomial distribution tends to Poisson distribution.

$$P(x) = \frac{\lambda^x . e^{-\lambda}}{x!} \qquad (10.3)$$

where $\lambda = np$

The conditions for this approximation are :

(i) The number of events is discrete.

(ii) Two events can not coincide.

(iii) The mean number of events in unit time is constant.

(iv) Events are independent.

The above example can be solved with Poision distribution also. The results are

$$P(1) = \frac{\left(50 \text{ X } \frac{1}{100}\right)^1 e^{-(50/100)}}{1\,!} = 0.303 = 30.3\%$$

$$P(2) = \frac{\left(50 \text{ X } \frac{1}{100}\right)^2 e^{-(50/100)}}{2\,!}$$

$$= 0.0758 = 7.58\%$$

The results obtained by Binomial distribution and Poisson distribution are almost same.

10.5 DESIGN FREQUENCIES

The designer is generally concerned with the return period for which the structure should be designed. The design frequencies shown in table 10.2 are typical of levels generally encountered in minor structure design (Viessman et al., 1977).

TABLE—10.2

MINOR STRUCTURE DESIGN FREQUENCIES

Type of minor structures	Return Period (T in years)	Frequency $\left(P=\frac{1}{T}\right)$
Highway cross road drainage		
0-400 ADT*	10	0.1
400-1700 ADT	10-25	0.1 — 0.04
1700-5000 ADT	25	0.04
5000 ADT	50	0.02
Airfields	5	0.2
Storm drainage	2-10	0.5 to 0.1
Levees	2-5	0.5 — 0.02
Drairnge Ditches	5-50	0.20 — 0.02

* ADT — Average Daily Traffic

Selection criteria given by Snyder (1964) for spillway design of dams are given in Table 10.3.

TABLE—10.3

SPILLWAY DESIGN FLOOD FOR DAMS

Category	Impoundment danger potential: Storage (acre-feet)	Impoundment danger potential: Height (ft)	Failure damage potential: Loss of life	Failure damage potential: Damage	Spillway design flood
Major	50,000	60	Considerable	Excessive or as a matter of policy	Probable Maximum Flood (a)
Intermediate	1000-50,000	40-100	Possible but small		Standard Project Flood (b)
Minor	1000	50	None	of same magnitude as cost of dam	50-100 years rccurrence Inserval

(a) Probable Maximum Flood:

Probable maximum flood is the flood caused by probable maximum precipitation. Probable maximum flood is generally obtained by using unit hydrograph and rainfall estimates of PMP. The probable maximum storm is defined as the most severe storm considered reasonably possible to occur. The ASCE Hydraulics Division Committee on Hydrometeorology (1973) has suggested that the probable maximum flood is perhaps equivalent to a design return period of 10,000 years.

(b) Standard Project Floods :

SPF is the flood caused by standard project storm which is generally obtained from a survey of severe storms in the general viscinity of the drainage basin or severe storms experienced in meteorologically similar areas.

In India the present practice (IS code) is to specify the design flood for various structures in terms of frequency of return period as indicated below :

Structure	Return period of design flood (years)
1. Major dams with storage more than 6000 ha.m. (50,000 Ac. ftl)	1000
2. Minor dams with storage less than 6000 ha.m.	100*
3. Barrages and pick up weirs	
(a) Free board	500+
(b) Items other than free board	50–100
4. River Training Works (Calculation of Scour)	50–100@
5. Water way of bridges	50

* IS 5477 — Method of Fixing capacities of Reservoirs–Part IV

\+ IS 6966 — Criteria for Hydraulic Design of Barrages and Weirs

@ IS 3408 — Criteria for River Training Works for Barrages and Weirs in Alluvium.

In case of spillways for major and medium reservoirs with storages more than 6000 ha.m. design is generally done for probable maximum flood which is the physical upper limit of the flood in the catchment.

CWC (1969) recommends the following design frequencies :

	Structure	return period
(i)	Major dam with storage more than 50,000 acre feet	probable maximum flood or frequency not less than once in 1000 years
(ii)	Permanent barrages and minor dams with storage less than 50,000 acre feet	standard project flood or 100 year flood whichever is higher
(iiii)	Pick up weirs	50–100 years
(iv)	Canal aqueducts	
	(a) Waterways	50–100 years
	(b) foudations and freeboard	50–100 years

References

1. ASCE (1973), 'Re-evaluating Spillway Adequacy of Existing Dams', Report of the Task Committee on on the Re-evaluation of the Adequacy of Spillways of Existing Dams of the Committee on Hydrometeorology of the Hydraulics Division, Proceedings, ASCE, Vol. 99, No. 1742, pp. 337–372.

2. Biswas, A.K. and Chatterjee, S' (1971), 'Dam Disasters : An Assessment / J. Engineering Institute of Canada, Vol. 54, No. 3, pp. 3–8.

3. CWC (1969), 'Estimation of Design Flood Recommended Procedures', Ministry of Irrigation & Power.

4. Snyder, A.F. (1964), 'Hydrology of Spillway Design; Large Structures—Adequate Data', ASCE, Journal of Hydraulics Division, Vol. 90, No. HY-3.

5. Viessman, W., Knapp, J.W., Lewis, G.L., Harbaugh, T.E. (1977), 'Introduction to Hydrology', T. Harper and Row, publishers, New York.