TUTORIAL-4

ESTIMATION OF T-YEAR FLOOD AND IT'S STANDARD ERROR

Example :

For Narmada at Garudeshwar (1948—1979) estimate 500 and 1000 years floods and their 95% confidence limits assuming that peak discharge data follow (a) log normal distribution, (b) Log Pearson type III distribution and (c) Gumbel EV-1 distribution.

Solution :

The mean, standard deviation and coefficient of skewness of original and log transformed data are as follows :

	original series	log transformed series
Mean	29556.9	10.179
Standard deviation	14864.4	0.488
Coeff. of skewness	1.052	$0.104 \simeq 0.1$

LOG NORMAL DISTRIBUTION

T-Year Flood

The calculations for 500 and 1000 years flood are shown in table given below :

Return Period T (years)	Prob. of exceedance (P=1/T)	Frequency Factor, K_T	X_T in log domain	X_T in original domain
500	.002	2.87816	11.5835	107312.5
1000	.001	3.09023	11.687	119014.43

The frequency factors have been taken from Appendix-IV corresponding to P=.002 and .001 and coefficient of skewness equal to zero.

So 500 and 1000 years floods are 107312.5 and 119014.43 cumecs respectively.

Standard Error and Confidence Limits :
500 year flood :

$$S_E = \frac{X_T}{2} \left(e^{S_T} - e^{-S_T}\right)$$

$$S_T = \frac{\delta \cdot \sigma_y}{\sqrt{N}}$$

$\delta = (1 + t^2/2)^{1/2}$

For 500 year flood, $P = \frac{1}{500} = .002$

t for p = 0.002 for log normal distribution is 2.87816

so $\delta = \left(1 + \frac{2.87816^2}{2}\right)^{1/2} = 2.268$

$$S_T = \frac{2.268 \times 0.488}{\sqrt{32}}$$

$$= 0.1956$$

$$S_E = \frac{107312.5}{2}\left(e^{.1956} - e^{-.1956}\right)$$

$$= 21124.42$$

so 95% confidence limits will be given by

$X_T \pm t_{1-\alpha/2,\ N-1} \cdot S_E$

here N = 32, α = 5%, so $t_{1-\alpha/2,} = t_{0.975,31}$

$t_{0.975,31} = 2.04$ (From Appendix V)

95% confidence limits will be

= 107312.5 ± 2.04 X 21124.42

= 150406.32 and 64218.68 Cumecs

1000 year flood :

$$\delta = \left(1 + \frac{3.09023^2}{2}\right)^{1/2} = 2.403$$

$$S_T = \frac{2.403 \times 0.488}{\sqrt{32}} = 0.2073$$

$$\text{so } S_E = \frac{1190/4.43}{2}\left(e^{.2073} - e^{-.2073}\right)$$

$$= 24848.5$$

so 95% confidence limits will be 119014.43 ± 2.04 × 24848.8

= 169705.98 and 68322.878 cumecs

LOG PEARSON TYPE III DISTRIBUTION

T-Year Flood

The calculations for 500 and 1000 years flood are given below :

Return Period T (years)	Prob. of exceedance (P=I/T)	Frequency factor, K_T	X_T in log domain ($X_T = \overline{X} + K_T \cdot S$)	X_T in original domain
500	.002	2.99978	11.6428	113868.5
1000	.001	3.23322	11.7568	127618.4

The frequency factors have been taken from Appendix IV corresponding to P = 0.002 and 0.001 and coefficient of skewness equal to 0.1. So 500 and 1000 years floods are 113868.5 cumecs and 127618.4 cumecs respectively.

Standard Error and Confidence Limits

500 year flood

$C_s = \gamma_1 = 0.1$

$t = 2.87816$

$$\frac{\partial K}{\partial \gamma_1} = \frac{t^2-1}{6} + \frac{4\,(t^3-6t)}{6^3}\gamma_1 - \frac{3\,(t^2-1)}{6^3}\gamma_1^2 + \frac{4t}{6^4}\gamma_1^3 - \frac{10}{6^6}\gamma_1^4$$

$= 1.225$

$$S_{T,y}^2 = \frac{\sigma_y^2}{n}\left(1 + K\gamma_1 + \frac{K^2}{2}\,(3\gamma_1^2/4+1) + 3K\,\frac{\partial K}{\partial \gamma_1}\,(\gamma_1 + \gamma_1^3/4) + 3\left(\frac{\partial K}{\partial \gamma_1}\right)^2 \times\right.$$

$$\left.(2 + 3\gamma_1^2 + 5\,\gamma_1^4/8)\right)$$

where K = frequency factor = 2.99978

so $S_{T,y}^2 = 0.0074 \times 16.077$

$= 0.1196$

So $S_{T,y} = 0.345$

$$S_{T,X} = \frac{X_T\,(e^{S_{T,y}} - e^{-S_{T,y}})}{2} = 40068.4$$

95% confidence limits will be 113868.5 $\pm$ 2.04 $\times$ 40068.4

= 195608.04 and 32129.0 cumecs.

Similarly confidence limits for 1000 years flood can be calculated.

GUMBEL EV-1 DISTRIBUTION

T year Flood

The parameters (u and α) of EV1 distribution are estimated from ths following two equations using method of moments.

Mean $= u + 0.5772\,\alpha$

$$\sigma^2 = \frac{\pi^2 \alpha^2}{6}$$

so $29556.9 = u + 0.5772\,\alpha$

$$14864.4^2 = \frac{\pi^2 \alpha^2}{6}$$

From the above equations, the value of u and α come out to be 22867.3 and 11589.725 respectively.

The reduced variate $y_i = (x_i - u)/\alpha$ is given by

$$y_i = -\ln\left(-\ln\left(1 - \frac{1}{T}\right)\right)$$

$$= -\ln\left(-\ln\left(\frac{T-1}{T}\right)\right)$$

$$= -\ln.\ln\left(\frac{T}{T-1}\right)$$

So $y_{500} = -\ln.\ln\left(\frac{500}{499}\right)$

$= 6.2136$

$y_{1000} = 6.9072$

$X_{500} = u + \alpha\, y_{500}$

$= 22867.3 + 11589.725 \times 6.2136$

$= 94881.215$ Cumecs

$X_{1000} = 22867.3 + 11589.725 \times 6.9072$

$= 102919.85$ Cumecs

So 500 and 1000 years floods are 94881.215 Cumecs and 102919.85 Cumecs respectively.

Standard Error and Confidence Limits

500 year flood

$K = -(0.45 + 0.7797 \ln(-\ln(1-1/T)))$

$= 4.394$

$$S_T^2 = \frac{\sigma^2}{n}(1 + 1.1396\,K + 1.1\,K^2)$$

$$= \frac{14864.4^2}{32}(1 + 1.139 \times 4.394 + 1.1 \times 4.394^2)$$

$= 1.88132 \times 10^8$

or $S_T = 13716.14$ Cumecs

so $\quad$ 95% confidence limit will be $X_T \pm 2.04 \times S_T$

$= 94881.215 \pm 2.04 \times 13716.14$

$= 122862.14$ and 66900.3 Cumecs

1000 Year Flood

$K = 4.935$

$S_T = 15049.8$

So 95% confidence limits will be

$102919.8 \pm 2.04 \times 15049.8 = 13362.39$ and 72218.2 cumecs

Problem :

For Narmada at Mortakka (1981-82) estimate 500 and 1000 years floods and their 95% confidence limits assuming that it follow (a) log normal, (b) Log Pearson type III and (c) Gumbel EV-1 distribution.